UNIT 5 THE NEOCLASSICAL GROWTH MODEL*

Structure

5.0 OBJECTIVES

After going through the unit, you will be able to:

- apply some essential techniques to study growth models in general;
- discuss the neo-classical growth model ;
- point out the difference between the Harrod-Domar model and neo-classical model(s);
- apply the neo-classical growth models in the analysis of certain economic topics like savings, fiscal policy, poverty, and so on;
- explain the comparative growth experiences of nations using the neo-classical model; and
- examine how the concepts of money and monetary processes have been brought into the neo-classical model.

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5.1 INTRODUCTION

The previous two units of this block have introduced you to the concepts of economic growth and development, as well as the Harrod-Domar growth model. You saw how important economic growth is, how it differs from development, why we should study growth models and what the limitations of economic growth can be. The Harrod-Domar model was presented to you both as a unified model, as well as separately the models of Harrod and Domar. You became familiar with the model's assumptions, its structure, limitations as well as some applications, including application in Indian economic planning.

This unit presents for discussion a model which was put forward by Robert Solow and Trevor Swan, independently of each other, in 1956, although brief anticipations of the basic idea was carried out by James Tobin. However it is commonly known as the Solow-Swan model, or even as the Solow model. Solow has done a lot to popularise the model through subsequent papers and books. The Solow model has proved to be one of the most used most robust, and standard models in all of economic theory. Several economists of more than one generation have built upon, extended and refined the Solow model. Even when some have put forward new models, they did so referring to the Solow model as the model that they critiqued and found limitations with. Empirical economists spent lots of time, effort, energy to examine data sets and use statistical tools to see if predictions which can be generated out of the Solow model, have actually been matched by the performances of group of countries. Solow justly received a Nobel Prize for his contribution to growth theory, and as he remarked with a touch of pride during the address he gave at the time of receiving the prize, his model started a "cottage industry [of model- building in growth theory]".

In this unit we study the neoclassical growth model. We begin by acquainting you with some tools and techniques you are going to need to study growth models, not only in this unit, but in subsequent units as well. Once you have familiarised yourself with these concepts, you will be introduced to the basic neoclassical model, with the assumptions stated and the structure spelled out and elaborated. Some applications and extensions of the model are presented , following which we bring in monetary factors into the basic neoclassical model. We close the discussion with a study of the very important and relevant topics of convergence and poverty traps.

5.2 SOME CONCEPTS AND TOOLS TO STUDY GROWTH MODELS

If we assume a constant rate of growth, the formula used for calculating the growth rate for more than one year is as follows:

$Y_t = Y_o (1+r)^n$

=) $\log Y_t = \log Y_o + n \log (1+r)$

$$\Rightarrow \log(1+r) = \frac{\log y_t - \log y_o}{n}$$

$$\text{or } \log(1+r) = \frac{\log(y_t/y_o)}{n}$$

$$\Rightarrow 1+r = \text{antilog}\left[\log\frac{(y_t/y_o)}{n}\right]$$

$$\therefore\ r = \left\{anti\log\left[\frac{\log(y_t/y_o)}{n}\right]-1\right\}$$

This assumes constant rate of growth. r (as in a constant rate of interest in compound interest formulations)

When the r.o.g is not constant, it is say

$x_1, x_2, \ldots\ldots x_i \ldots\ldots\ldots x_t$ in t=1,2,.......i,......t

then r = $(x_1, x_2, \ldots\ldots x_t)^{1/t}$

It is geometric mean.

Calculation of Growth Rates

Consider the following data on rice production:

Year	Rice Production (in lakhs of tonnes)
1990	8
1991	9
1992	13
1993	12
1994	13
1995	20
1996	19
1997	18
1998	18
1999	18
2000	23
2001	23
2002	26
2003	28
2004	27
2005	24

(i) A crude interpretation of the growth rate is given as an average percentage change from first year to the last year, in this example, average growth rate would be

$$\frac{\left(\frac{24-8}{8}\right)}{16} \times 100$$

$$= 13.3\%$$

This method overemphasises original and terminal values of data. Also, it does not use all observations.

(ii) Linear growth rate : The estimated regression using the given data turn out to be:

$$y_t = 8.1 + 1.2.45\ t$$

Three different growth rates may be completed by making following manipulations:

(a) growth rate = $\frac{\hat{\beta}}{y_0} \times 100 = \frac{1.245}{8} \times 100$

$= 15.57\%$

(b) growth rate = $\frac{\hat{\beta}}{\bar{y}} \times 100 = \frac{1.245}{18.6875} \times 100$

$= 6.66\%$

(c) growth rate = $\frac{\hat{\beta}}{HM(y)} \times 100 = \frac{1.245}{16.286} \times 100$

$= 7.64\%$

(iii) Compound growth rate: Value of $\hat{\beta}$ in the log-linear relation:

$$\ln\ y_t = \ln \alpha + \beta t$$

Growth

As an example, let's model population growth

$$\frac{dp}{dt} . \frac{1}{p} = \frac{dp/dt}{p}$$

$\frac{dp}{dt}$ is called the absolute growth rate,

$\frac{dp}{dt}/p$ is called relative growth.

US population in millions 1790-1990

Example: U.S. Population in million 1790-1990

Year	**Population**
1790	3.9
1800	5.3
1810	7.2
1820	9.6
1830	12.9
1840	17.1
1850	23.1
1860	31.4
1870	38.6
1880	50.2
1900	76.0
1910	92.0
1920	105.7
1930	122.8
1940	131.7

Suppose we went to estimate relative growth ($\frac{dp}{dt}/p$) from 1790 to 1860 let us take one particular year, say 1830. If we want to estimate $\frac{dp}{dt}/p$ in 1830 we take the rate of change in the population, and divide it by the population itself:

$$\frac{1}{p}\frac{dp}{dt} = \frac{1}{pop.in1830}\frac{popin1840 - popin1830}{10 years}$$

$$= \frac{1}{12.9}.\frac{17.1-12.9}{10} = 0.0326 = 3.26\%$$

$$P_{10} = P_0(1+3.47)$$

$$P_{10} = P_0(1+0.347\times 10)$$

Similar calculations for 1790, 1800,………, 1850 give percentages as shown below

Year	Relative growth rate
1790	3.59%
1800	3.58%
1810	3.33%
1820	3.44%

1830	3.26%
1840	3.57%
1850	3.53%

How do we calculate average relative growth rate for the period?

The simplest model is to answer that the relative growth rate is constant, i.le.

$$\frac{1}{p}\frac{dp}{dt} = k$$

1. Arithmetic growth (simple growth): growth by a constant amount in each time period.
2. Geometric growth (exponential growth): growth by a constant proportion in each time period.
3. Nominal rate of interest & effective rate of interest; compounding period.

   If we have a photo & enlarge it 3 times, new are is $9\times$ (old area).

   Correct are the following

   New area is 9 times the area of the original

   New area is 9 times as large as the original

   New area is 9 times as great as the original

   Incorrect are

   New area is 9 times greater than the original

   New area is 9 times more than the original

   New area is 9 times larger than the original

   "9 times greater than" means "10 times as great as".

B. An increase from 1 to 9 is an increase of 800%, Not 900%. If support for a political party decreases from 60% to 30% it has dropped 30 percentage points but decreased 50%.

C. When speaking of reductions also we must be careful. A reduction from 9 to 1 means the new amount is $1/9^{th}$ as much, $8/9^{th}$ less than, 11% as much as in89% less than the original.

<u>Growth and change</u>

Let x be an economic variable.

Let x_0 be the initial value.

x_1 be the subsequent value.

The proportional change in going from x_0 to x_1 is simply

$$\frac{x_1 - x_0}{x_0} = \frac{\Delta x}{x_0}$$

The percentage change in going from x_0 to x_1 is simply 100 times the proportionate change:

$\%\Delta x = 100\,(\Delta x/x_0)$

If x is itself in percentage it becomes tricky.

If unemployment decreases from 15% to 12% it is a reduction of 3 percentage points, but a $\frac{15-12}{15}\times 100 = \frac{3}{15}\times 100 = 20\%$ fall in unemployment.

Time as regressor

A linear trend relationships would be depicted as:

$$y = \alpha + \beta T + \mu \quad (1)$$

where T indicates time. One has to appropriately define the origin

Constant growth covers

$$y_t = \alpha + \beta T + \mu_t$$
$$y_{t-1} = \alpha + \beta T - 1 + \mu_{t-1}$$
$$\Rightarrow y_t - y_{t-1} = \beta + (\mu - \mu_{t-1})$$
$$\Delta y_1 = \beta + \mu_1 - \mu_{t.......1}$$

Ignoring the disturbances, the series increases (decreases) by a constant amount each period. For an increasing series ($\beta > 0$), this implies a decreasing growth rate, and for a decreasing series ($\beta < 0$), the specification gives an increasing decline rate.

If we wish to study a series with a constant growth rate, the formulation (1) is inappropriate. The appropriate specifications expresses the logarithm of the series as a linear function of time.

This can be seen as follows without disturbances, a constant growth series is given by the equation.

$$y_t = y_0(1+g)^t \quad (2)$$

where g = $\frac{(y_t - y_{t-1})}{y_{t-1}}$ is the constant proportionate rate of growth per period

Taking logs of both sides of (2) given

$$\ln y = \alpha + \beta t \quad (3)$$

where $\alpha = \ln y_0$ and $\beta = \ln(1+g)$

If one suspects that a series has a constant growth rate, plotting the log of the series against time provides a quick check. If the series is approximately linear, (3) can be fitted by least squares, regressing the log of y against time. The resultant slope coefficient then provides as estimates $\hat{g}$ of the growth rate, namely, b= $\ln(1+\hat{g})$ giving $\hat{g}$=e^b-1

The β coefficient of (1) represents the continuous rate of change $\partial \ln y_t / \partial t$, whereas g represents the discrete case. Formulating a constant growth series in continuous time gives

$$y_t = y_0 e^{\beta t}$$
$$\ln y_t = \alpha + \beta t$$

Note that taking first difference of equation (1) given

$\Delta m y_t = \beta = \ln(1+g)$; g

Thus, taking first difference of logs given the continuous growth rate, which in turn is an approximation to the discrete growth rate. This approximation is only reasonably accurate for small values of g.

Example

Bituminous coal O^t in the U8 1841-1910

Decade	**Av.annual O^t**	**lny**	**t**	**t(my)**
1841-1850	1837	7.5159	-3	-22.5457
1851-1860	4868	8.4904	-2	-16.9809
1861-1870	12411	9.4904	-1	-9.4263
1871-1880	32617	10.3926	0	0
1881-1890	82770	11.3238	1	11.3238
1891-1900	148457	11.9081	2	23.8161
1901-1910	322958	12.6853	3	38.0558
Sum		**71.7424**	**0**	**24.2408**

Plotting the log of output against time, we find a linear relationship. So we will fit a constant growth and estimate the annual growth rate. Setting the origin for time at the centre of the 1870s and taking a unit of time to be 10 years, we obtain the t series shown on the table. From the data in the table

$$a = \frac{\sum \ln y}{n} = \frac{71.7424}{7} = 10.2489$$

$$b = \frac{\sum t \ln y}{\sum t^2} = \frac{24.2408}{28} = 0.8657$$

The r^2 for this regression is 0.9945, confirming the linearity of the scatter. The estimated growth rate per decade is obtained from

$$\hat{g} = e^b - 1 = 1.3768$$

Thus the constant growth rate is almost 140 per cent per decade. The annual growth rate (agr) is then found from

$(1+\text{agr})^{10} = 2.3768$

which gives agr = 0.00904, or just over 9 per cent per annum. The equivalent continuous rate is 0.0866 or time as regressor, see Russell Davidsos and James G. Mackinon. Estimation and _______________ in Econometrics, OUP, 1993 pp 115-118

Growth Rates

Compound rates - geometric growth $\rightarrow$ continuous compound growth $\rightarrow$ exponential growth

$$v_t = v_o(1+g)^t$$

$$\Rightarrow \left(\frac{v_t}{v_o}\right) = (1+g)^t$$

$$\Rightarrow \left(\frac{v_t}{v_o}\right)^{1/t} = 1+g$$

$$\therefore g = \left(\frac{v_t}{v_o}\right)^{1/t} - 1$$

Let v_o equal the value of the variable in year 0 (the fast year) & v_t equal the value of the variable t years later. Further, let g equal the average compound annual growth rate. Then

$$v_t = v_o(1+g)^t$$

$$\Rightarrow g = \left(\frac{v_t}{v_o}\right)^{1/t} - 1$$

If the growth rate is continuously compounded, then

$$v_o e^{gt} = v_t$$

$$e^{gt} = \frac{v_t}{v_o}$$

$$g = \frac{1}{t}\ln\left(\frac{v_t}{v_o}\right)$$

Geometric mean growth rate is the same thing as compound growth rate.

Illustrations

Take company X's sales for 6 years

Year	**(Net sales)**	**Annual growth**
1979	216,283	
1980	260,404	20.4%
1981	294,145	13.0%
1982	285,954	-2.8%
1983	303,498	6.2%
1984	318,842	5.1%

Annual growth rate in year t

$$= \frac{S_t - S_{t-1}}{S_{t-1}} = \frac{S_t}{S_{t-1}} - 1$$

where S is sales. (t=1,2,3,4,5,)

= 1980

The average compound (geometric mean) annual growth rate over the 5 years from end of '79 to end of 84' for Co. X's net rates is

$$g = \left(\frac{S_t}{S_o}\right)^{1/t} - 1$$

$$\Rightarrow \left(\frac{S_5}{S_o}\right)^{1/5} - 1$$

$$= \left(\frac{318,842}{216,283}\right)^{1/5} - 1$$

$$= (1.474)^{1/5} - 1$$

$$= 1.081 - 1 = .081 = 81\%$$

The geometric mean growth rate is calculated as

$$\left[\prod_{t=1}^{n}(1+g_t)\right]^{1/n} - 1$$

In the above example,

$$g = \left[(1.204)(1.130)(0.972)(1.062)(1.051)\right]^{1/5} - 1$$

$$= (1.474)^{1/5} - 1$$

$$= 0.81$$

The arithmetic mean of the five annual growth rates (arithmetic mean growth rates)

$$\bar{g} = \frac{\sum_{t=1}^{n} gt}{n}$$

$$= \frac{1}{5}(20.4 + 13.0 - 2.8 + 6.2 + 5.1)$$

$$= 8.4\%$$

The arithmetic mean growth rate 8.4% is > the geometrics mean growth rate (8.1%). In fact values all give the same, A.M.G.K. will be > G.M.G.R.

Only if gi-n are constant, will amgv=gmgr.

5.3 THE SOLOW MODEL

5.3.1 Assumptions of the Solow model

1. The economy produces one composite good which can either be consumed or accumulated as a stock of capital. This is a simplified picture of reality. while we do not deny that lots of goods are produced in the economy, we consider, for purposes of building a model – a model, after all, is a parable or a fable-- only one 'composite' or 'aggregated' good.
2. Labour supply is homogeneous. In other words, we do not distinguish between workers with different skills or between say, blue- and white-collared workers.
3. There is a stock of capital which has been accumulated from the past. This capital and the labour are the factors of production, inputs to the production process.
4. The production function exhibits constant returns to scale. This means that if labour and capital are increased by a certain proportion, say λ, output increases by the same proportion λ. If labour and capital are doubled, output is doubled. Thus there is an aggregate production function which is continuous and which displays constant returns to scale.
5. The labour force grows at an exogenously given growth rate $g_L = n$. Thus labour force at time t is equal to $L_t = L_0e^{nt}$.
6. People save a constant proportion of Income. If S denotes saving then

   S = sY. This assumption is the same as in the Harrod-Domar model. Some people feel that Solow made this deliberately to make a comparison with Harrod-Domar model.
7. There is no foreign trade
8. The government does not intervene in the economy; there are no taxes or government purchase

5.3.2 Structure of The Model

Since we have assumed there is no depreciation, we consider the model here in the absence of depreciation. Later on, we shall discuss the case when there is depreciation of capital. To begin with consider the aggregate production function.

$Y=F(K,L)$

We have assumed that there are constant returns to scale. This means that if K & L are increases by a proportion λ, Y increases by the same proportion. The function F is homogenous of degree one.

$Y=F(\lambda K,\lambda L)$ for all $\lambda>1$

For simplicity, let $\lambda=1/L$

Then $\frac{Y}{L}=F\left(\frac{K}{L},\frac{L}{L}\right)$

Or $\frac{Y}{L}=F\left(\frac{K}{L},1\right)$

Let in denote quantities divided by L, by lower-case letters

So $y=F(k,1)$

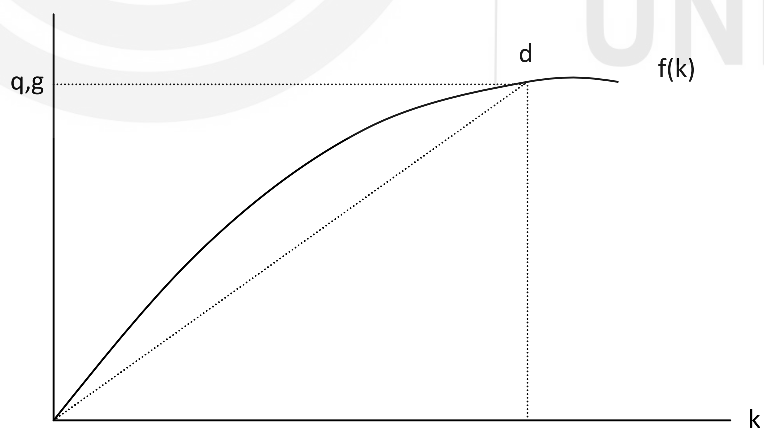
Or $y=f(k)$

This gives output per person as a function of capital labour ratio.

Sometimes income is used synonymously with output. Denoting output by Q and output-labour ratio by q, we have

$q = f(k)$

If we draw a picture of above the relationship we can show it as follows:



A ray od to any point d on the curve has a slobe that gives the ratio of output to capital. This is because the of this ray is $=\frac{Q/L}{K/L}=\frac{Q}{K}$

This is the is verse of capital output ratio v. Each point on the production function is the slow model is that ………….Harrad-Domar model, l is not fixed or exogenous different points on the production function show different, capital output ratios.

Equilibrium Growth

We have $k = K/L$

Taking natural logarithms (denoted by Ln), we get

$$\ln(k)=\ln\left(\frac{K}{L}\right)$$

$$\ln k=\ln\frac{K}{L}$$

or, $\ln k=\ln K-\ln L$

Differentiating with respect to figure, gives the proportional growth ratio:

$$\frac{d}{dt}(\ln k)=\frac{d}{dt}(\ln k)-\frac{d}{dt}(\ln L)$$

$$\frac{dk}{dt}.\frac{1}{k}=\frac{dK}{dt}.\frac{1}{K}-\frac{dL}{dt}.\frac{1}{L}$$

$$or\ \hat{k}=\hat{K}-\hat{L}$$

Now $\frac{dK}{dt}$ on the right hand side is equal to investment I

Investment = facing in equilibrium. So

$$\frac{dK}{dt}=\dot{K}=S$$

$S = SQ_t$ (as we have assumed)

$So\frac{dK}{dt}=sQ_t$ (t is the subscript denotes Q at a point of time)

The Second term on the right hand side is $\hat{L}=\frac{dL}{dt}.\frac{1}{L}$ which shows the proportional growth rate of labour. Which we have denoted n. So our equation

$$\hat{k}=\hat{K}-\hat{L}$$

can be written

$$\hat{k}=\frac{sQ}{K}-n$$

Dividing Q and K by L we get

$$\hat{k} = \frac{sq}{k} - n = \frac{sf(k)}{k} - n \text{ .... (A)}$$

This gives $\hat{k}$, the rate of growth of k, is term of k itself.

Equation (A) is the fundamental equation of the Solow model.

The equilibrium value of k is the one for which $\frac{dk}{dt}.\frac{1}{k} is - 0$, *i.e.* $\frac{dk}{dt} = 0$ or where k, once it reaches that value, does not change. Setting $\hat{k} = 0$ in the equation above, we get

$$\hat{k} = 0 = \frac{sf(k^*)}{k^*} - n$$

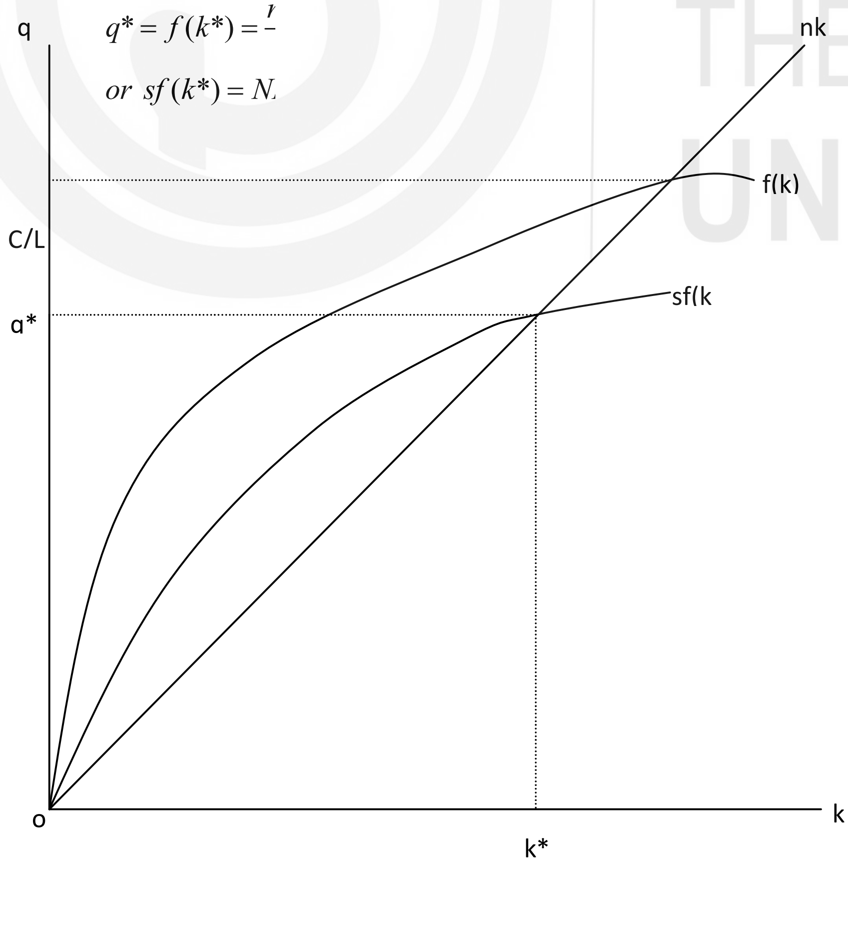
$$or \ \frac{sf(k^*)}{k^* = n}$$

where an * above k denotes its equilibrium value.

The equilibrium q value is obtained as

$q^* = f(k^*) = $

$or \ sf(k^*) = N.$



At any point to the left of k*, where k<k*,

$$f(k)\rangle\left[(n/s)k\right]$$

This implies

$$\frac{sf(k)}{k}\rangle n$$

And from equation (A), we can sea that in this case $\hat{k}\rangle 0,$ which mean that when $k\langle k^*, \hat{k}\, increase$

Similarly we can show that whenever k>k* (to the right of k*), $\hat{k}$ is rising and where k>k*, $\hat{k}$ is falling. Thus k* is a stable equilibrium point.

In equilibrium, when k equals k*, q reaches an equilibrium q*. As q* is a constant,

$$\frac{d\theta}{dt}.\frac{1}{t}=\frac{dL}{dt}.\frac{1}{t}=n$$

The economy thus converges to a steady state growth where $\hat{\theta}=\hat{K}$ and capital output ratio v is constant. However, $\hat{\theta}\, and\, \hat{K}$ are not greater than $\hat{L}$ but equal to it.

The equilibrium condition in the Solow model

$$\frac{sf(k^*)}{k^*}=n$$

can be written as

$$\frac{n}{s}=\frac{f(k)}{k}=\frac{q}{k}=\frac{Q/L}{K/L}=\frac{Q}{K}=\frac{1}{V^*}$$

Where V is the capital output ratio.

So we have $n=\frac{S}{V^*}$, the Harrod-Domar condition for balanced full employment growth. However the Solow model allows V to vary and explains how the economy will turn toward a growth bath along with the Harrod-Domar condition is and there in the Solow model, the capital output ratio V* emerges as an equilibrium value, and not as a necessary technology assumption.

CONSUMPTION IN THE SOLOW MODEL

We know that in a closed economy with no government intervention in equilibrium

$$Y=C+I$$

Where Y is aggregate output , C is aggregate consumption and I is investment.

Writing in per-worker ………., we have

$$\frac{Y}{L} = \frac{C}{L} + \frac{I}{L}$$

We know $\frac{Y}{L} = y = f(k)$

So $f(k) = \frac{C}{L} + \frac{I}{L}$ …………………………………..(B)

Now consider capital labour ratio

$$k = K/L$$

We have seen $\frac{dk}{dt}.\frac{1}{k} = \frac{dK}{dt}.\frac{1}{k} - \frac{dL}{dt}.\frac{1}{L}$

Or $\hat{k} = \hat{K} - \hat{L}$

Where $\wedge$ denotes proportional growth rate. We had already denoted $\hat{L}$ *by n*

So we have

$$\hat{k} = \hat{K} - n$$

or $\frac{\dot{k}}{k} = \frac{\dot{K}}{k} - n$ where $\dot{X}$ *denotes* $\frac{dX}{dt}$ Multiplying both sides by K/L we get

$$\frac{\dot{k}}{k}.\frac{K}{L} = \frac{\dot{K}}{K}.\frac{K}{L} - \frac{nK}{L}$$

or $\dot{k} = \frac{\dot{K}}{L} - nk$

Alternatively putting it;

$$\frac{\dot{K}}{L} = \dot{k} + nk \text{ ……………………………………………..(C)}$$

Since one of the assumptions we had made was that there is no depreciation, hence

$$\dot{k} = \frac{dK}{dt} = I$$

So we may write equation (C) as

$$\frac{I}{L} = \dot{k} + nK$$

In equation (B) we can replace $\frac{I}{L}$ by the right hand side of the above equation.

Equation (B) then becomes

$$f(k) = \frac{C}{L} + \dot{k} + nK \text{ ....(D)}$$

This equation states the following: Output per worker (since we are taking θ as equal to Y and hence q=y) is put to three uses which are shown on the right hand side. First, consumption per worker $\frac{C}{L}$; a portion of investment nk, that maintains the capital labour ratio constant in the face of growing labour force; and a portion of investment, $\dot{k}$ which increases the capital labour ratio. When capital goods increase faster then the increase in labour, so that the capital-labour ratio rises, it is called capital deepening, while when capital goods rise merely to keep pace with the rise in labour force so that the capital labour ratio remains constant, it is called capital widening. Thus output per worker, in equation D, is divided among consumption per worker, capital deepening and capital widening.

We can arrive at equation D by a different route, from our fundamental equation of the Solow Model.

Recall that the fundamental equation, equation (A) is

$$\hat{k} = \frac{sf(k)}{k} - n = \frac{sq}{k} - n$$

Since $\hat{k} = \frac{\dot{k}}{k}$, hence multiplying the above equation throughout by k, we get

$$\dot{k} = sq = nk$$

Recall our assumption that q = y. We have

$$\dot{k} = sy - nk$$

Since $y = \frac{Y}{L}$, we have $\dot{k} = \frac{sY}{L} - nk$

We had made the assumption that S = sY

Now in equilibrium S =Y-C = I

Hence we have $\dot{k} = \frac{Y}{L} - \frac{C}{L} - nk$

Now switching back in rotation f(k) for $\frac{Y}{L = y}$, we have

$$\dot{k} = f(k) - \frac{C}{L} - nk$$

or $f(k) = \frac{C}{L} + \dot{k} + nk$

which is equation (D).

What are the basic proposition and conclusion that we get from the Solow model? Does it provide some guidelines for studying the growth trajectories of actual economies? We give below some theoretical conclusions that emerge from the Solow model.

First, given the assumptions as stated earlier, there exists a steady state (balanced growth) solution for the model. The balanced growth solution is stable. Stability is there in that whenever the initial values of all the variables, the economy eventually mark to the study state equilibrium value of y and k. We have already seen this from the diagram.

The second conclusion we get that the balanced rate of growth (all ……….. grow at the same rate) is the constant exogenous grow the of labour force, which is n.

THE SOLOW MODEL WITH DEPRECIATION

We know that in absence of depreciation

$\dot{K} = I$ Here I denote grow investment.

Let us now answer that a certain proportion δK of capital depreciates (through wear and ………). We can now write

$$I = \dot{K} + \delta K$$

Where I now is net investment and δ is the constant rate of depreciation of capital stock.

Dividing by L, we obtain

$$\frac{I}{L} = \frac{\dot{K}}{L} + \frac{\delta K}{L} \text{ ………………………………………….(E)}$$

We know from equation (C), which we …………….earlier, that

$$\frac{\dot{K}}{L} = \dot{k} + nk$$

Substituting this expression for $\frac{\dot{K}}{L}$ into equation (E), we obtain

$$\frac{I}{L} = \dot{k} + nk + \delta k$$

$$or \frac{I}{L} = \dot{k} + (n+\delta)k$$

Writing $\frac{I}{L}$ as $\frac{S}{L}$ (in equation) and they as sy or sf(k)

We have $sf(k) = \dot{k} + (n+\delta)k \;\; or \;\; \dot{k} = sf(k) - (n+\delta)k$

Or $\dot{k} = \frac{sf(k)}{k} - (n+\delta)$

This is a modified form of the fundamental equation of the Solow model. The fasic analysis that we studied, carrier over for the case of depreciating capital; we merely need to replace n by (n+δ).

Poverty Traps

Empirically, the convergence by pother is has not held up very well. The neo-clinical model has not been very successful an showing why rates of growth differ across nations. One fact was staring everyone in the face: many rations of the world were poor. In a sense, what was going on was the exact opposite of what was suggested by the convergence hypothesis: Some countries were shifting stagnant growth, while other were progressing very fact. This idea that the poorer nations were actually caught is a traphas come to be called poverty trap. It is a trap because inspite of efforts. These nations stay a low-level 'equilibrium'.

There are two types of poverty traps: technological-Induced and population induced. They can both be demonstrated in the Solow system.

(a) Technological trap:

If we consider the Solow production function y = f (k), or $Y/L = F(K/L)$, and suppose there is a certain range where for certain values of L, K, the production function exhibits increasing returns to scale, then there will be multiple equilibrium. For certain values of k, say k* if the economy sports at a level lower than k*, the economy will slump back towards a very low income and outp0ut level because the level k* may show a unstable equilibrium.

The idea behind technological trap is that has such a hypothetical country received an initial injection of capital so as to give a value of k larger k*, it would have been pushed over the level. This idea is sometimes called the 'Big push Theory'of which you will read later in block 4. The idea of poverty trap being caused by low savings, low technology is sometimes called the vicious circle of poverty, about which you will read in block 4.

The second way in which poverty traps can arise is induce by population growth. In the neo-classical model, the rate of population growth was given exogenously. However for classical writers population growth was given endogenously. Robertr Methus in his 1798 work, of which you will study in block 4, suggested that the rate of growth of population depends on per capita income. As per capita income rises, the rate of population growth rises ever faster. This has come to be knows as the theory of demographic transition.

We can bring in the idea of demographic transition into the neo classical model. Recall that is the neo classical model population grows exogenously at a rate n. …………,now suppose population growth rate is dependent on y, as suggested by the theory of demographic transition. We know that y is a function of capital per person. $y = f(k/l)$. Thus n, the population growth rate indirectly becomes a function of the capital labour ratio:

$$n = g\left(k/l\right)$$

We may think of some interval $k_2 - k_1$ where for values of k below k_1, n is <0, but for values of k in the range [k_2, k_1] n is >0; again for values of k > k_2, n may become <0. Historically, in olden time in societies where k was below k_1, population was last theory wars, disease etc.

In our analysis, consider the range $k_2 - K_1$. Even here, n, although >0, can itself increase or fall. In other words, although the population is increasing. The rate of increase may itself vary. The idea is that, this leads to a situation where the saving (or investment) curve changes shape and may turn out to be S shaped. Then, we may have multiple equilibrium points of k most of which are unstable: any movement from these points pushes the system far then away rather than bringing it back into equilibrium. We have considered the interval (or range of values between) [k_2, K_1] . Supposing we can two equilibrium points k_a and k_b. Let k_a lie between k1 and k_2, i.e. $k_2 > k_a > k_1$. Let k_b be greater than k_2. Here k_a is the stable equilibrium while k_b is the unstable one. n is the stable equilibrium which creates the problem here. For any value of k leather k_2, the economy is pulled back to equilibrium level k_a. Only if an injection of capital is gives which pushed the k level above k_b (where the equilibrium is unstable) will the k level be given a "Big Push" and sent to higher and higher values and thus raising the level of y via the f-function.

We mentioned the demographic transition which roughly says that n depends only. But in the last century, due to advance is health care and ………., low y did not necessarily lead to low n. As death rates dropped, population increased. On the other hand sub-Saharan African nation like Ethiopia did see n very low (other nations are under populated) due to very low levels of y.

5.4 LET US SUM UP

The neoclassical growth model is the central, 'base-line' model which has saved on the spring board for almost all most researches into growth theory. The no-classical model, associated with the name of Robert Solow, presented the first major extension to the Harrod-Domar model, by endogenusing the capital output ratio.

In this unit, we learnt some concepts and tools useful for studying growth theory, and then we studied the structure of the neo-classical model, along with the assumptions. We extended the Solow model to consider depreciation and variable savings. We their process led to apply the Solow model to look at some applications, like convergence and poverty tragss. We also looked at some implications of the Solow model namely technical progress is more important than capital accumulation, and raising the savings ratio in the short run is not going to help. In the Solow model, technological progress and consequently, output growth is exogenous. When we get to unit 9 on endogenous growth, we will look at models that endogenous these.