UNIT 11 HETEROSCDASTICTY*

Structure

11.0 OBJECTIVES

After going through this unit, you should be able to

- explain the concept of heteroscedasticity in a regression model;
- identify the consequences of heteroscedasticity in the regression model;
- explain the methods of detection of heteroscedasticity;
- describe the remedial measures for resolving heteroscedasticity;
- show how the use of deflators can help in overcoming the consequences of heteroscedasticity; and
- identify the correct functional form of regression model so that heteroscedasticity is avoided.

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11.1 INTRODUCTION

A crucial assumption of the Classical Linear Regression Model (CLRM) is that the error term u_i in population regression function (PRF) is homoscedastic. It means that u_i has the same variance σ^2 throughout the population. An alternative scenario arises where the variance of u_i is σ_i^2. In other words, the error variance varies from one observation to another. Such cases are referred to as cases of heteroscedasticity.

11.2 HETEROSCEDASTICITY: DEFINITION

Let us first make a distinction between homoscedasticity and heteroscedasticity. This will help us in understanding the concept of heteroscedasticity better.

11.2.1 Homoscedasticity

Consider a 2-variable regression model, where the dependent variable Y is personal savings and the explanatory variable X is personal disposable income (or after-tax income).

As personal disposal income (PDI) increases, the mean or average level of savings also increases but the variances of savings around its mean value remains the same at all the levels of PDI. Such a case depicts the case of homoscedasticity or equal variance as shown in Fig. 11.1. In such cases, we have:

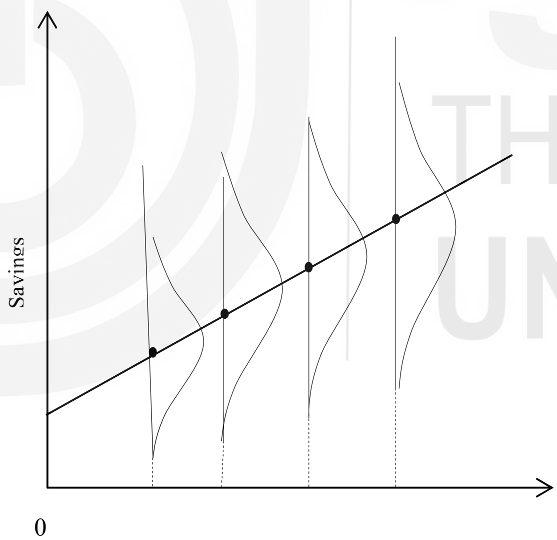

Fig.11.1: Case of Homoscedasticity

$E(u_i^2) = \sigma^2$ … (11.1)

We can alternatively express equation (11.1) as a case where:

$V(u_i) = \sigma^2$ … (11.2)

In Fig. 11.1, we see a case of homoscedasticity where the variance of the error term is a constant value, σ^2. This is expressed in the form of an equation as in

(11.2). Since the expected value of the error term is zero, the expression $V(u_i) = \sigma^2$ can also be written as $E(u_i^2) = \sigma^2$ as in equation (11.1).

11.2.2 Heteroscedasticity

As PDI increases, the average level of savings increases. However, the variance of savings does not remain the same at all the levels of PDI. This is the case of heteroscedasticity or unequal variance. In other words, high-income people, on average, save more than low-income people, but at the same time, there is more variability in their savings. This can be graphically represented as in Fig. 11.2. We now therefore have:

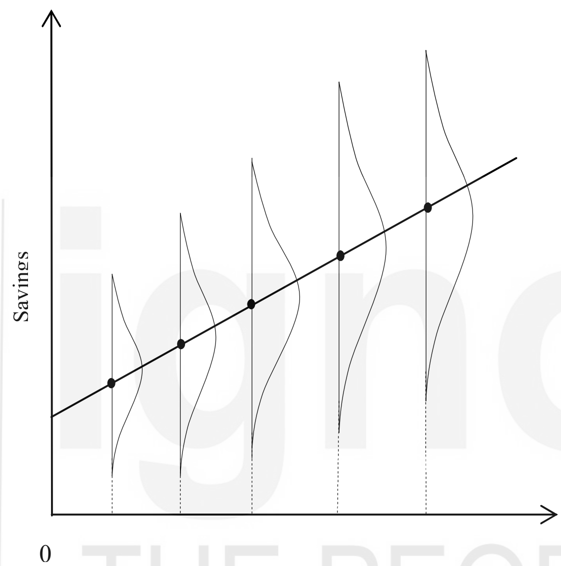

Fig. 11.2: Case of Heteroscedasticity

$E(u_i^2) = \sigma_i^2 \;\; or \;\; V(u_i) = \sigma_i^2$ … (11.3)

The case of heteroscedasticity reflected in Fig.11.2 indicates that the error variance is not constant. It rather changes with every observation, like

$$V(u_i) = \sigma_i^2.$$

It is observed that heteroscedasticity is usually found in cross-sectional data and not so much in time series data. The reason for its occurrence more in cross-sectional data is mainly because, in the case of cross-sectional data, the members of population are like individuals, firms, industries, geographical division, state or countries. The data in such cases is collected at a point in time. Hence, the members of the population may be of different sizes: small, medium or large. This is referred to as the scale effect. In other words, due to what is called in economics as the 'scale effect', in cross sectional data we find cases of heteroscedasticity more commonly.

In the case of time series, on the other hand, the data of similar variables vary over a period of time. For instance, GDP (gross domestic product) or savings or unemployment varies over a period (like 1960 to 2008).

Check Your Progress 1

1) What is meant by heteroscedasticity?

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2) Is the problem of heteroscedasticity related to data? Comment.

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11.3 CONSEQUENCES OF HETEROSCEDASTICITY

To avoid the problem of heteroscedasticity, we have made one of the assumptions in the classical linear regression model that the error term is homoscedastic. However, in many regression models and actual data, the disturbance variance varies across observations. Consequently, the model suffers from specific impacts due to heteroscedastic error term.

The following are the characteristics of the OLS model in the presence of heteroscedasticity.

(i) The OLS estimators are linear function of the variables. The regression equation is also linear in its parameters.

(ii) The ordinary least squares (OLS) estimators are unbiased. This means the expected value of estimated parameters is equal to the true population parameters.

(iii) The OLS estimators though unbiased, are no longer with minimum variance, i.e., they are no longer efficient. In fact, even in large samples, the OLS estimators are not efficient. Therefore, the OLS estimators are not BLUE both in small as well as asymptotically large samples.

(iv) In light of the above, the usual formula for estimating variances of OLS estimator is biased, i.e., they are either upward biased (positive bias) or downward biased (negative bias). Note that when the OLS overestimates the true variances of estimators, a positive bias is said to occur, and when it underestimates the true variances of estimators, we say that a negative bias occurs.

(v) The estimator of true population variance as given by $\hat{\sigma}^2 = \frac{\sum e_i^2}{df} = \frac{RSS}{df}$ is biased. That is

$$E(\hat{\sigma}^2) \neq \sigma^2 \quad \dots (11.4)$$

We know that the degrees of freedom for testing an estimated parameter is (n – k), where k is the number of parameters (or explanatory variables) in the regression model. For example, if there are three explanatory variables, d.f. = (n – 3). In the two variables case, df = (n – 2). Note that we are counting the intercept estimate for this purpose of determining the d.f.

(vi) Equation (11.4) implies that in the presence of heteroscedasticity, the estimated value of error variance is not equal to the true population error variance. In view of this, the usual confidence interval and hypothesis testing based on t and F distributions are unreliable (since, the estimator of the error variance is biased). Therefore, the possibility of making wrong inferences (Type–II error) is very high. As a result, in the presence of heteroscedasticity, the results of the usual hypothesis-testing are not reliable raising the possibility of drawing misleading conclusions.

Check Your Progress 2

1) State any two important consequences of heteroscedasticity.

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2) In the presence of heteroscedasticity, the OLS estimator will either overestimate or underestimate the error variance. Justify the statement.

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11.4 DETECTION OF HETEROSCEDASTICITY

So far, we have discussed the consequences of heteroscedasticity. Now let us discuss how heteroscedasticity can be detected. There are quite a few methods of detecting heteroscedasticity. Some of these methods are described below.

11.4.1 Graphical Examination of the Residuals

We can begin with examining the residuals obtained from the fitted regression line. The residual plot of squared residuals is an indicator of the existence of heteroscedasticity. Since the error terms u_i are not observable, we examine the residuals, e_i.

A plot of the residuals can give us various types of diagrams as in Fig. 11.3.

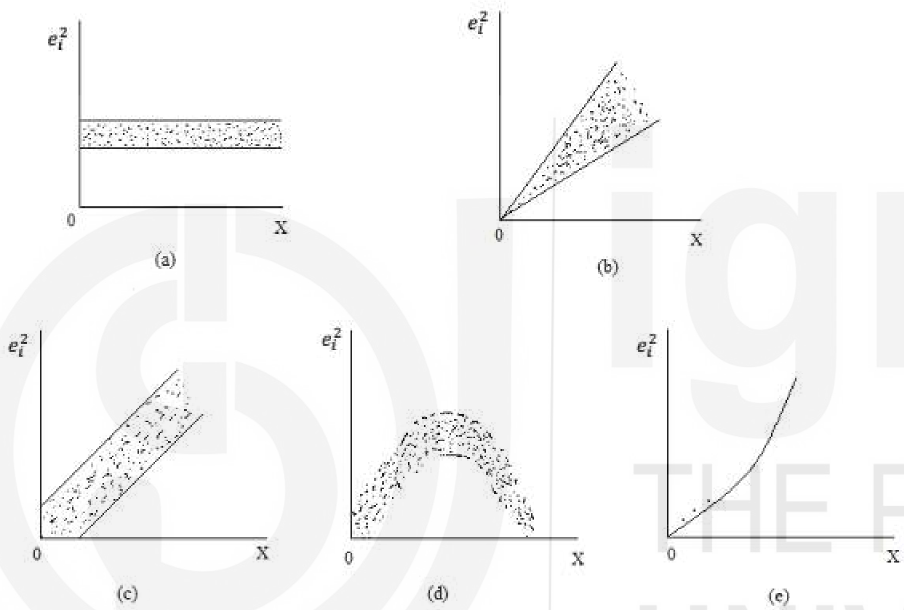

Fig. 11.3: Cases of Homoscedasticity and Heteroscedasticity

In the five situations depicted in Fig. 11.3, we see that Case (a) represents homoscedasticity, i.e., $V(u_i) = \sigma^2$ whereas in the remaining four cases viz., (b), (c), (d) and (e) represent heteroscedasticity, i.e., $V(u_i) = \sigma_i^2$.

11.4.2 Park-Test

If there is heteroscedasticity in a data set, the heteroscedastic variance σ_i^2may be systematically related to one or more explanatory variables. Therefore, we can regress σ_i^2on one or more explanatory variables such as

$$\sigma_i^2 = f(X_i)$$

$$\ln\sigma_i^2 = \beta_1 + \beta_2 \ln X_i + v_i \qquad \ldots (11.5)$$

In equation (11.5), a non-linear (double-log) regression is run to establish a relationship between the error variance and the explanatory variable with v_i

taken as the residual term. When σ_i^2 are not known, we take the residual term e_i as proxies for u_i. Therefore, we have

$$\ln e_i^2 = \beta_1 + \beta_2 \ln X_i + v_i \quad \ldots (11.6)$$

Now, Park test for detecting heteroscedasticity involves the following steps:

a) Run the original regression in equation (11.5) despite the heteroscedasticity problem.

b) From the regression obtain e_i and square them. Then take the logs of e_i^2.

c) Run the double-log form regression as indicated in equation (11.6) using an explanatory variable in the original model (in the case of more than one explanatory variable). Then run the regression against each *X* variable. In other words, we run the regression against $\hat{Y}_i$, the estimated value of Y_i.

d) Test the null hypothesis $\beta_2 = 0$, i.e., there is no heteroscedasticity.

e) A statistically significant relationship implies that the null hypothesis of no heteroscedasticity is rejected. It suggests the presence of heteroscedasticity which requires remedial measures.

f) If the null hypothesis is not rejected, then it means we accept $\beta_2 = 0$ and the value of β_1, that is, the value of the intercept can be accepted as the common, homoscedastic variance σ^2.

11.4.3 Glejser Test

The Glejser Test is similar to the Park Test. The steps to carry out the Glejser test are as follows:

a) Obtain the residual e_i from the original model.

b) Take absolute value $|e_i|$ of the residuals

c) Regress the absolute values of $|e_i|$ on the *X* variable that is expected to be closely associated with heteroscedastic variance σ_i^2.

d) You can take various functional forms of X_i. Some of the functional forms suggested by Glejser are

$$|e_i| = \beta_1 + \beta_2 X_i + v_i \quad \ldots (11.7)$$

$$|e_i| = \beta_1 + \beta_2 \sqrt{X_i} + v_i \quad \ldots (11.8)$$

$$|e_i| = \beta_1 + \beta_2 \left(\frac{1}{X_i}\right) + v_i \quad \ldots (11.9)$$

The above means that the Glejser test suggests various plausible (linear as well as non-linear) relationships between the residual term and the explanatory variable to investigate the presence of heteroscedasticity.

e) For each of the cases given, test the null hypothesis that there is no heteroscedasticity, i.e., $H_0: \beta_2 = 0$ (no heteroscedasticity).

f) If H_0 is rejected we conclude that there is evidence of heteroscedasticity.

You should note that the error term v_i can itself be heteroscedastic as well as serially correlated. Thus, in the case of Glesjer test also, we follow the same steps as in the Park Test. The difference between the two tests is in the functional forms to be considered.

11.4.4 White's General Test

Let us consider the following PRF:

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \quad \ldots (11.10)$$

The steps to carry out White's general test for heteroscedasticity are as follows:

a) Estimate the population regression equation (11.10) by OLS and obtain the residuals e_i.

b) Find the square of the residuals e_i^2.

c) Run the following auxiliary regression:

$$e_i^2 = A_1 + A_2 X_{2i} + A_3 X_{3i} + A_4 X_{2i}^2 + A_5 X_{3i}^2 + A_6 X_{2i} X_{3i} + v_i \ldots (11.11)$$

d) Obtain the coefficient of determination R^2 from the auxiliary regression under the null hypothesis that there is no heteroscedasticity (i.e., all the slope coefficient are zero). That is,

$$H_0: A_2 = A_3 \ldots A_6 = 0 \quad \ldots (11.12)$$

The null hypothesis given at equation (11.12) implies that all the partial slope coefficients are simultaneously zero. Note that we do not include the intercept term A_1 in equation (11.12).

e) Test the null hypothesis in equation (11.12) by using the chi-square distribution as follows:

$$nR^2 \sim \chi_{k-1}^2 \quad \ldots (11.13)$$

Equation (11.13) tells us that the product of sample size (n) and the coefficient of determination (R^2) follows χ^2 distribution with degrees of freedom (k–1). Here k is the number of regressors in the auxiliary regression (equation 11.11).

f) If $\chi_{calculated}^2 > \chi_{critical}^2$ we reject the H_0, and conclude that the null hypothesis of homoscedasticity is to be rejected, i.e., there is heteroscedasticity. Alternatively, we can also decide on the basis of the p value (readily given by econometric softwares). If the p value is < 0.05, we reject H_0. If $\chi_{calculated}^2 < \chi_{critical}^2$. On the other hand, if $p > 0.05$ we do not reject the null hypothesis of no heteroscedasticity. This implies the existence of homoscedasticity.

11.4.5 Goldfeld-Quandt Test

The Goldfeld-Quandt (G-Q) test is applicable if heteroscedasticity is related to only one of the explanatory variables. Let us assume that the error variance σ_i^2 is related to one of the explanatory variables (say, X_i) in the regression model.

Suppose σ_i^2 is positively related to X_i as given below.

$$\sigma_i^2 = \sigma^2 X_i^2 \qquad \dots (11.14)$$

In order to carry out the G-Q test we proceed as follows:

a) Arrange the observations in increasing order of X_i

b) Omit some of the observations (say, C out of the no observations in the sample) in the middle of the series. There is no hard and fast rule for the exact value of C and the choice is quite arbitrary. In practice about one fourth observations are omitted.

c) Run a regression on the first $n_1 = (n - C)/2$ observations. Find out the error sum of squares for this regression, i.e., ESS_1.

d) Run a regression on the last $n_2 = (n - C)/2$ observations. Find out the error sum of squares for this regression, i.e., ESS_2.

e) Take the following null hypothesis:

$$H_0: \sigma_i^2 = \sigma^2 \qquad \dots (11.15)$$

f) Find out the ratio:

$$\Lambda = \frac{RSS_1 / \frac{n_1 - C - 2k}{2}}{RSS_2 / \frac{n_2 - C - 2k}{2}}$$

In case $n_1 = n_2$, the above ratio becomes

$$\Lambda = \frac{RSS_1}{RSS_2} \qquad \dots (11.6)$$

The above ratio (Λ) follows F-distribution with degrees of freedom

$$\left(\frac{n_1 - C - 2k}{2}, \quad \frac{n_2 - C - 2k}{2} \right) \qquad \dots (11.17)$$

g) We compare the value of Λ obtained above with the tabulated value of F given at the end of the book. If $\lambda > F_{critical}$ we reject $H_0: \sigma_i^2 = \sigma^2$ and conclude that there is heteroscedasticity in error variance. It implies $\sigma_i^2 \neq \sigma^2$. If $\lambda < F_{critical}$ we do not reject H_0. We conclude that there is homoscedasticity in error variance, i.e., $\sigma_i^2 = \sigma^2$.

Check Your Progress 3

1) State the steps in conducting the Park test for detection of heteroscedasticity.

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11.5 REMEDIAL MEASURES OF HETEROSCEDASTICITY

Heteroscedasticity means that the OLS estimators are unbiased but no longer efficient; not even in large samples. Therefore, if heteroscedasticity is present, it is important to seek remedial measures. For proceeding with remedial measures, it is important to know if the true error variance σ_i^2 is known or not. In such cases, use of a 'deflator' may help rectify the problem of heteroscedasticity. We will learn about the use of deflators in this section.

11.5.1 Case I: σ_i^2 is Known

If we know σ_i^2, we can use the method of Weighted Least Squares (WLS). We explain the procedure of carrying out WLS below.

Let us consider the two-variable Population Regression Function (PRF).

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad \text{...(11.18)}$$

Let us assume that u_i has heteroscedastic error variance. Here, since the true variance is known, we can use it to divide the equation (11.18) by σ_i. By dividing both sides of (11.18) by σ_i, we obtain:

$$\frac{Y_i}{\sigma_i} = \beta_1\left(\frac{1}{\sigma_i}\right) + \beta_2\left(\frac{X_i}{\sigma_i}\right) + \frac{u_i}{\sigma_i} \qquad \text{... (11.19)}$$

Note that the error term gets transformed due to the division by σ_i. Let the new error term be v_i. Squaring the new error term we get:

$$v_i^2 = \frac{u_i^2}{\sigma_i^2} \qquad \text{...(11.20)}$$

Since the variance of error term is given by $var(v_i) = E(v_i^2)$, taking the expectation of both sides of the equation (11.20) we get:

$$E(v_i^2) = E\left(\frac{u_i^2}{\sigma_i^2}\right)$$

$$= \left(\frac{1}{\sigma_i^2}\right).E(u_i^2)$$

$$= \frac{\sigma_i^2}{\sigma_i^2} = 1$$

Thus, the transformed error-term v_i is homoscedastic. Therefore, equation (11.19) can be estimated by the usual OLS method. The OLS estimators of β_1 and β_2 thus obtained are called the Weighted Least Squares (WLS) estimators.

11.5.2 Case II: σ_i^2 is Unknown

When the error variance σ_i^2 is not known, we need to make further assumptions to use the WLS method. Here, we consider the following two cases.

(i) **Error variance σ_i^2 is Proportional to X_i**

In this case, we follow what is called as the square root transformation. The proportionality assumption means that:

$E(u_i^2) = \sigma^2 X_i$

Or, $V(u_i) = \sigma^2 X_i$ … (11.21)

Now, the square root transformation requires that we divide both sides of equation (11.18) by $\frac{1}{\sqrt{X_i}}$ to get:

$$\frac{Y_i}{\sqrt{X_i}} = \beta_1 \frac{1}{\sqrt{X_i}} + \beta_2 \frac{X_i}{\sqrt{X_i}} + \frac{u_i}{\sqrt{X_i}}$$

$$= \beta_1 \frac{1}{\sqrt{X_i}} + \beta_2 \sqrt{X_i} + v_i \quad \ldots (11.22)$$

where $v_i = \frac{u_i}{\sqrt{X_i}}$ … (11.23)

The error term in equation (11.23) is a transformed error term. In order to see whether v_i is devoid of heteroscedasticity, we square both the sides of equation (11.23) to get:

$$v_i^2 = \frac{u_i^2}{X_i} \quad \ldots (11.24)$$

Now, the variance of the transformed error term, i.e., equation (11.24) is:

$$E(v_i^2) = \frac{E(u_i^2)}{X_i} = \frac{\sigma^2 X_i}{X_i} \quad \ldots (11.25)$$

$= \sigma^2 \Rightarrow$ homoscedasticity

Thus, when we apply the square root transformation ($v_i = \frac{u_i}{\sqrt{X_i}}$), we could make the error variance to become homoscedastic.

(ii) Error Variance is Proportional to X_i^2

Here, we have:

$E(u_i^2) = \sigma X_i^2$ … (11.27)

$V(u_i) = \sigma X_i^2$

Dividing both sides of equation (11.18) by X_i,

$\frac{Y_i}{X_i} = \beta_1\left(\frac{1}{X_i}\right) + \beta_2 + \left(\frac{u_i}{X_i}\right)$

$= \beta_1\left(\frac{1}{X_i}\right) + \beta_2 + v_i$ … (11.28)

Equation (11.28) is the transformed PRF in which the error term is:

$v_i = \frac{u_i}{X_i}$, … (11.29)

Squaring both the sides of equation (11.29), we get:

$v_i^2 = \frac{u_i^2}{X_i^2}$ … (11.30)

The variance of the error term of the transformed equation in (11.30) is homoscedastic because:

$E(v_i^2) = \frac{E(u_i^2)}{X_i^2} = \frac{\sigma X_i^2}{X_i^2} = \sigma$ … (11.31)

11.5.3 Re-Specification of the Model

Instead of speculating about σ_i^2, sometimes choosing a different functional form can reduce heteroscedasticity. For instance, instead of running the usual regression model, we can estimate the model in its log form.

$\ln Y_i = \beta_1 + \beta_2 \ln X_i + u_i$ … (11.32)

In many cases transforming original model as above will take care of the problem of heteroscedasticity.

We used the word 'deflator' in the beginning of this section. The cases we have considered above basically involve dividing both sides of the original regression model by a known value to transform the variables. Such transformation of variables by division amounts to deflating the original values. The known values used to perform the division act are known as the 'deflators'.

Check Your Progress 4

1) How does the use of deflators work as a solution for the problem of heteroscedasticity?

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2) Explain how the usage of deflators serve to tackle the problem of heteroscedasticity when the error variance is proportional to X_i^2.

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11.6 LINEAR VERSUS LOG – LINEAR FORMS

The regression model can be run in various functional forms depending upon: (i) the relationship of dependent and independent variable, and (ii) the data. Suppose there is a choice of running two types of regression models: (i) a linear regression model, and (ii) a log-linear model. To help decide in such cases, a test for the selection of the appropriate functional form for regression is proposed by Mackinnon, White and Davidson (MWD). The MWD test is applied as follows:

Let there be two distinct functional forms of a regression like:

Model 1: $Y_i = \beta_1 + \beta_2 X_i + u_i$ …….(11.33)

Model 2: $\ln Y_i = \beta_1 + \beta_2 ln X_i + u_i$ …….(11.34)

In Model 1, the dependent variable is linearly related to one (or more than one) of the *X*s. In Model 2, the relationship between the dependent and independent variable is non-linear. The MWD test involves considering a null and an alternate hypothesis as follows:

H_0: Linear Model, i.e., *Y* is a linear function of regressors (equation (11.33))

H_1: Log- Linear Model, i.e., ln *Y* is a linear function of lnX_i (equation (11.34))

Following are the steps for carrying out the MWD test:

(i) Estimate the linear model and obtain the estimated *Y* values. Let the estimated *Y* values be denoted as Y_f.

(ii) Estimate the log-linear model and obtain the estimated ln*Y* values. Let the estimated values of the log-linear *Y* be denoted as $\ln Y_f$.

(iii) Obtain $Z_1 = (\ln Y_f - Y_f)$

(iv) Regress *Y* on X_s and Z_1 obtained in Step (iii) Reject H_0 if the coefficient of Z_1 is statistically significant by the usual *t*-test.

(v) Obtain $Z_2 = (\text{antilog} \ln Y_f - Y_f)$

(vi) Regress log of Y on the logs of X_s and Z_2. Reject H_1 if the coefficient of Z_2 is statistically significant by the usual t-test.

Suppose the linear model I in equation (11.33) is in fact the correct model. In that case, the constructed variable Z_1 should not be statistically significant in Step (iv). For, in that case the estimated Y values from the linear model and those estimated from the log-linear model (after taking their antilog values for comparative purposes) in equation (11.34) should not be different. The same logic applies to the alternative hypothesis H_1.

Check Your Progress 5

1) Outline the MWD test for choosing the appropriate functional form of the regression model between its linear and log-linear forms.

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11.7 LET US SUM UP

In this Unit, we have discussed the concept of heteroscedasticity in regression models. The unit outlines the consequences of the presence of heteroscedasticity and the methods of its detection. Various techniques to provide remedial measures are explained in the unit. The remedial measures involve understanding of the use of deflators. The unit has also explained a method for the choice of selecting the functional form by way of the MWD test.

11.8 ANSWERS/ HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) A crucial assumption of the Classical Linear Regression Model CLRM is that the error term u_i is population regression function (PRF) is homoscedastic, i.e., they have the same variance σ^2. However, if the variance of u_i is σ_i^2 (in other words, it varies from one observation to another), then the situation is referred to as heteroscedasticity.

2) Heteroscedasticity is usually found is cross-sectional data and not in time series data. This is because, in the case of cross-sectional data, the members of population are in the form of individual firms, industries, geographical division, state or countries. The data collected for such units at a point of time from the members of population may be of different sizes: small, medium or large firms. This is referred to as scale effect.

Due to the scale effect, in cross-sectional data, there is a greater chance of coming across heteroscedasticity in the error terms.

Check Your Progress 2

1) The OLS estimators are unbiased but they no longer have minimum variance, i.e., they are no longer efficient. Even in large samples the OLS estimators are not efficient. Therefore, the OLS estimators are not BLUE in small as well as large samples (asymptotically).

   The usual formula for estimating the variances of OLS estimator are biased i.e. there is either upward bias (positive bias) or downward bias (negative bias).

2) The OLS estimator of error variance is a biased estimator. Thus it will either overestimate or underestimate. In fact, the OLS estimator of error variance is inefficient, thereby meaning that it is very high; thus it is always an overestimate.

Check Your Progress 3

1) In the presence of heteroscedasticity, the heteroscedastic variance σ_i^2 may be systematically related to one or more explanatory variables. Therefore, we can regress σ_i^2 on one or more of *X*- variables as:

   $\sigma_i^2 = f(X_i)$ or $\ln\sigma_i^2 = \beta_1 + \beta_2 \ln X_i + v_i$

   where $v_i = new$ residual term. If σ_i^2 are not known, estimated e_i can be used as proxies for u_i. A statistically significant relationship implies that the null hypothesis of no heteroscedasticity is rejected suggesting the presence of heteroscedasticity which requires remedial measures. If null hypothesis is not rejected then it means we accept $\beta_2 = 0$ and value of β_1 can be taken as the common, homoscedastic variance σ^2.

2) Heteroscedasticity means that the OLS estimators are unbiased but estimators are no longer efficient, not even in large samples. This lack of efficiency makes the conventional hypothesis testing of OLS estimators unreliable. For remedial measures, it is important to know whether the true error variance σ_i^2 is known or not. In such cases, use of deflators will help rectify the problem of heteroscedasticity. Various deflators can be used to convert the error variance ti make them homoscedastic.

   When σ_i^2 is known, the method of Weighted Least Squares (WLS) can be considered. In this, the error variance σ_i^2 is used to divide both sides of the equation by σ_i. See Section 11.5 for details.

3) The estimated residuals show a pattern similar to earlier case I, but error variance is not linearly related to *X* but increases proportional to square of X. Hence, $E(u_i^2) = \sigma X_i^2$ and $V(u_i) = \sigma X_i^2$. Dividing both sides by X_i, we get:

$$\frac{Y_i}{X_i} = \beta_1\left(\frac{1}{X_i}\right) + \beta_2 + \left(\frac{u_i}{X_i}\right)$$

$$= \beta_1\left(\frac{1}{X_i}\right) + \beta_2 + v_i$$

$v_i = \frac{u_i}{X_i}, v_i^2 = \frac{u_i^2}{X_i^2}$

$E(v_i^2) = \frac{E(u_i^2)}{X_i^2} = \frac{\sigma X_i^2}{X_i^2} = \sigma$

Thus, the transformed equation is homoscedastic.

Check Your Progress 5

1) The test for selection of the appropriate functional form for regression as proposed by Mackinnon, White and Davidson is known as MWD Test. The MWD test is used to choose between the two models. See Section 11.6 for details.