UNIT 4 NUMBER SYSTEM, NUMERATION AND FOUR FUNDAMENTAL OPERATIONS

Structure

4.1 INTRODUCTION

The development of Natural Numbers and Counting is one of the most fascinating discoveries of mankind. The idea of number is abstracted from the idea of 'collection' by putting the question 'How many?' For example, a collection {x, x, x} represents the number 'three'. Since most of the earlier collections were taken from Nature or the surroundings of ancient man such as a herd of sheep, a bunch of arrows, a cluster of tress or huts, the name given to these number ideas was 'Natural' numbers. Further, since these numbers answered the question 'How many' in a collection, the phrase 'counting' numbers was also used for these.

The development of the method of recording these ideas in symbols was a more time-consuming exercise. It took more than two thousand years to finally accept a numeration system. Earlier recordings of number idea have been found as notches cut on sticks, knots put on ropes, tally marks and fingers, Each civilization developed its own numerals and methods of representing numbers using the numerals. The present Numeration System is based upon the ancient Indian (Hindu) system which used the idea of base 10 and a symbol '0' called zero ('kwU;) meaning 'naught'.

The set of 'Natural' numbers is represented as

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ...

When we include '0' in this set as a number, we get the set of whole numbers. In this unit we will study the important features and properties of whole numbers and the numeration systems.

4.2 OBJECTIVES

After going through this unit, you will be able to make your students:

- understand the concept of Natural and Whole Numbers and the order property of Whole Numbers;
- recognize numbers written in numerals from different State of India/the Roman system;
- understand the concept of place value and expanded notation;

- develop a skill in the four fundamental operations on whole numbers;
- classify whole numbers as even numbers, odd numbers, prime numbers and composite numbers; and
- understand the concept of multiples and factors, and calculate H.C.F. and L.C.M. of numbers.

4.3 NUMBERS AND NUMERALS: DIGIT, PLACE VALUE AND ORDERING OF NUMBERS

Activity 1: To give the notion of numbers

The idea of number is abstracted from 'collections' which can be identified with the help of a property common to all its members.

Fig. 4.1: Collection of fruits, collection of children, collection of keys.

Students should be given experience in identifying collections. The idea of comparing collections should be intuitively given by drawing arrow diagrams. The vocabulary more, less 'as many as' should be used to show 'manyness' property of a collection.

Stage 1

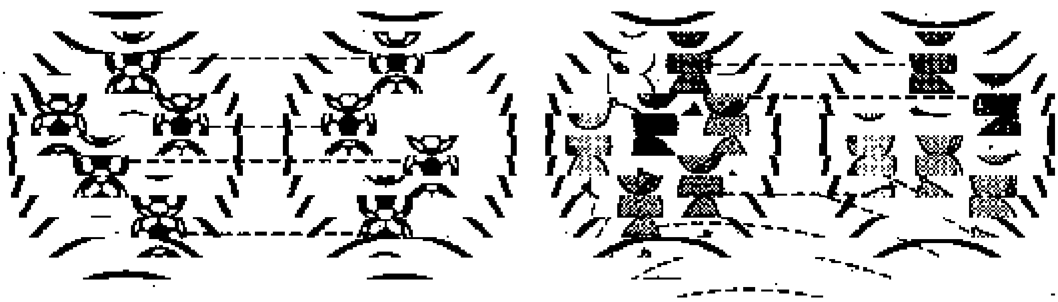
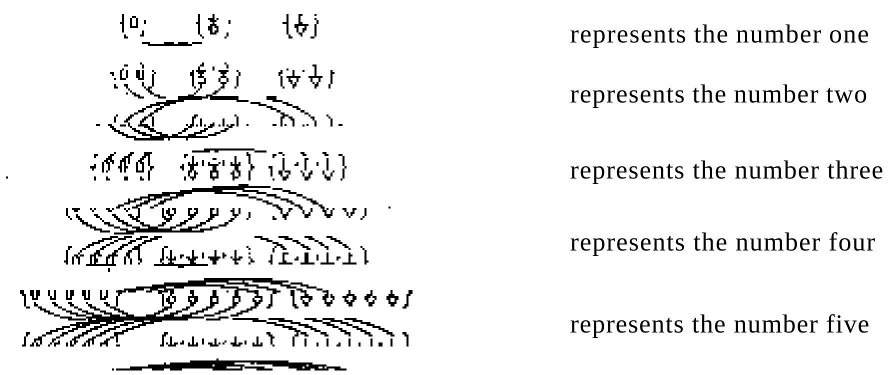


Fig. 4.2

Stage 2 : Equivalent sets should be identified for 'manyness' through matching.



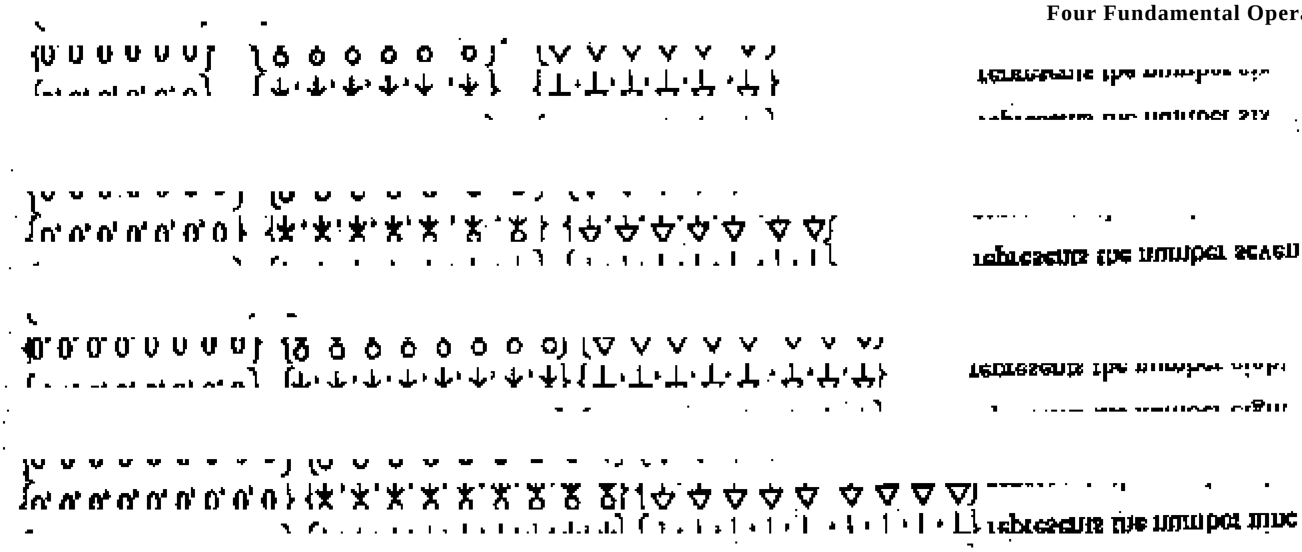


Fig. 4.3

The students should be encouraged to find the property which helps in creating natural numbers.

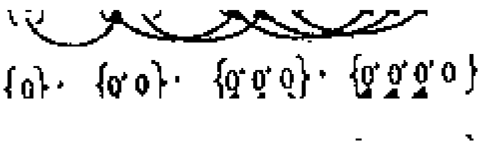


Fig. 4.4

One is the first natural number

Each number is 'one more' than the number that comes 'before'.

i.e. Two is one more than one

Five is one more than four

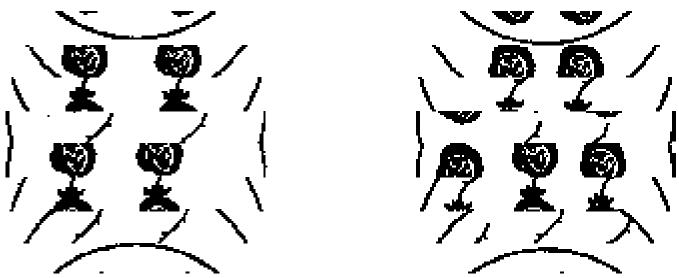
The notion that there is no largest number can be given.

Check Your Progress

Notes: a) Write your answers in the space given below.

b) Compare your answers with the one given at the end of this unit.

1.



Group I Group II

i) How will you illustrate ?

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ii) Which group has more flowers?

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2. Elicit:

i) Seven comes before ....

ii) Four comes after ....

3. i) Write the first number.

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ii) Can you write the largest natural number?

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Activity 2 : To give the idea of numeration

A numeral is a symbol for the idea of number. Numbers are innumerable and we would like to have only a few symbols to express them. These symbols will have rules to write the numerals.

PICTURE	NUMBER NAME	NUMERAL
□	one	1
□ □	two	2
□ □ □	three	3
□ □ □ □	four	4
□ □ □ □ □	five	5
□ □ □ □ □ □	six	6
□ □ □ □ □ □ □	seven	7
□ □ □ □ □ □ □ □	eight	8
□ □ □ □ □ □ □ □ □	nine	9

In this international system, the symbols 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 are used. However, in practice numerals are used as numbers.

The first nine symbols are symbols for the numbers one to nine and a tenth symbol '0' is called zero. Thus we use ten symbols to represent numbers. These are called 'digits'.

To represent numbers greater than nine in symbols the teacher should demonstrate on an abacus or use counting trays.

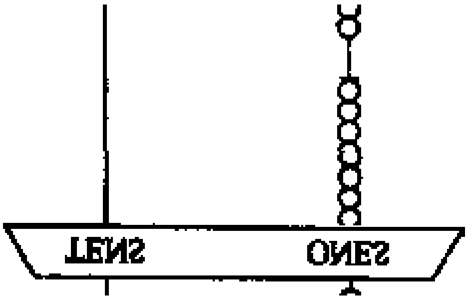

Fig. 4.5

We use ‘grouping by tens’ to count. Hence when one more is put with nine, the whole lot is shifted to the next column, as a single unit called tens column. We thus get 10 for ten.

The system of numeration uses ‘grouping by tens’. Hence we say that the base of our system is ‘ten’. The numerals use an addition principle. Thus:

Eleven is one more than ten and has the numeral 11.

Twelve is one more than eleven and has the numeral 12 or we can say ten and two more is 12. Proceeding in the same way.

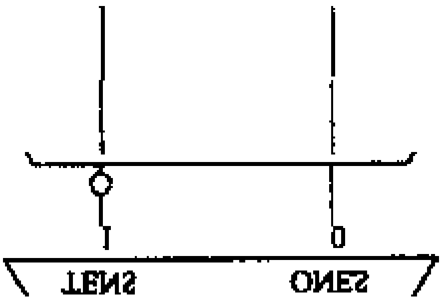

Fig. 4.6: 0 stands for empty place

Fifteen is one more than fourteen and has the numeral 15 or we can say ten and five more is 15.

Symbols up to ninety nine can be developed in the same way using the units/one column and the tens column.

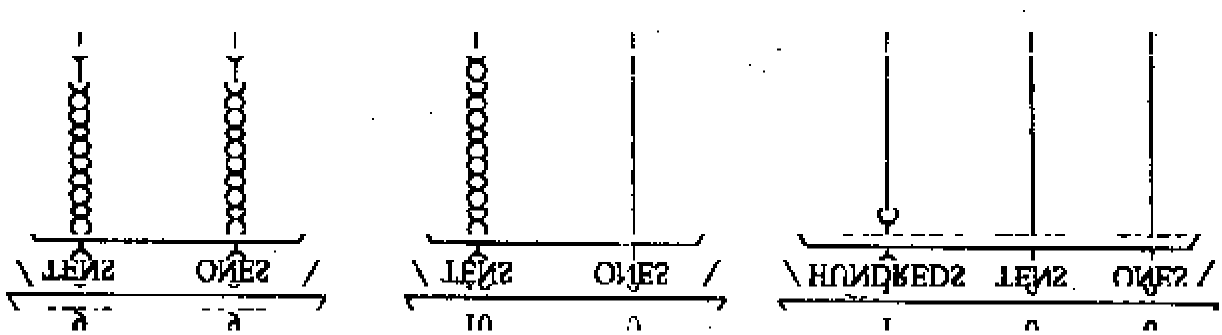

Fig. 4.7

To write one hundred which is one more than ninety nine in symbols demonstrate that ten ones make one ten and we get tens in tens column. These are again shifted to hundreds column and make 1 hundred. The larger numbers can thus be represented by opening new places on the left of hundreds and following the principle of addition. The place value table thus has the following places:

PLACE-VALUE CHART

CRORES		LAKHS		THOUSANDS		ONES		
TEN CRORES 100000000	ONE CRORE 10000000	TEN LAKHS 1000000	ONE LAKH 100000	TEN THOUSANDS 10000	ONE THOUSAND 1000	HUNDREDS 100	TEN 10	ONE 1
					2	3	4	5

The numeral 2345 written below the place value chart represents the number two thousand three hundred forty five. This is a four digit numeral. Each digit in a numeral has a place value due to the place it occupies. Thus 2 has a place value 2000. 3 has a place value 300. 4 has a place value 40 and the place value of 5 is five ones or 5.

We can write 2345 as $2 \times 1000 + 3 \times 100 + 4 \times 10 + 5 \times 1$

or

$$2000 + 300 + 40 + 5$$

Using the addition principle. This is also called expanded notation.

Check Your Progress

Notes: a) Write your answers in the space given below.

b) Compare your answers with the one given at the end of this unit.

How will you explain to:

4. Find the place value of (i) 4 and 3 in 43 (ii) 2 and 7 in 237 (iii) 4, 7 and 5 in 4375.

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5. Write the following in expanded notation

(i) 579 (ii) 9275 (iii) 43921 (iv) 20047.

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6. Read the numbers and mark the periods:

   (i) 921075 (ii) 4358078

   ....

   ....

   ....

   ....

Ordering of Numbers

The meaning of 'Order' is developed with the help of activities which involve the use of vocabulary 'comes *after*' and 'comes *before*'. This is then extended intuitively to numbers using matching of objects in sets.

Make pupils infer from such activities that 2 comes after 1, 3 comes after 2, etc; change it to 2 is 1 more than 1, 3 is one more than 2 etc.; next use 2 is greater than 1,3 is greater than 2 etc. The inverse should be similarly introduced to elicit '1 is less than 2', 2 is less than 3, etc.

The use of symbols should then be encouraged to order the one digit numbers.

$$1 < 2 < 3 < 4 < 5 < 6 < 7 < 8 < 9$$

$$9 > 8 > 7 > 6 > 5 > 4 > 3 > 2 > 1$$

This is the basic 'order' property which is then extended to compare two-digit, three-digit and greater numbers. Use place value idea to develop the generalizations:

i) 2 digit numbers are greater than 1 digit numbers.

ii) 3 digit numbers are greater than 2 digit and 1 digit numbers.

In general: A number with more digits is greater than a number with less digits.

Thus, to compare numbers we consider the number of digits in the numerals. When the numerals have same number of digits, we compare the left most digits.

729 is greater than 598 because 7 in the hundred's place is greater than 5 in the hundred's place.

999 is greater than 699 because 9 is greater than 6 in the hundred's place.

When the left-most digits are the same, then we compare the next digit on the right of the left-most.

Using the above property more than two numbers can be arranged in ascending or descending order.

Example: Arrange the following numbers in ascending order 11, 729, 425, 97, 4321, 8.

Solution

i) 8 is smallest number. It is the only 1 digit number.

ii) Out of 11 and 97 which are two digit numbers 97 is greater than 11 because 9 is greater than 1 in left most place.

iii) Out of 425 and 729 (which are 3 digit numbers) 729 is greater than 425 because 7 is greater than 4 in the left-most place.

iv) 4321 is greater than all other numbers because it has maximum number of digits.

We, therefore, write

8 < 11 < 97 < 425 < 729 < 4321

Check Your Progress

Notes: a) Write your answers in the space given below.

b) Compare your answers with the one given at the end of this unit.

7. Give explanations to:

Arrange the following in (a) ascending and (b) descending order.

3521, 490, 95, 2497, 827, 54

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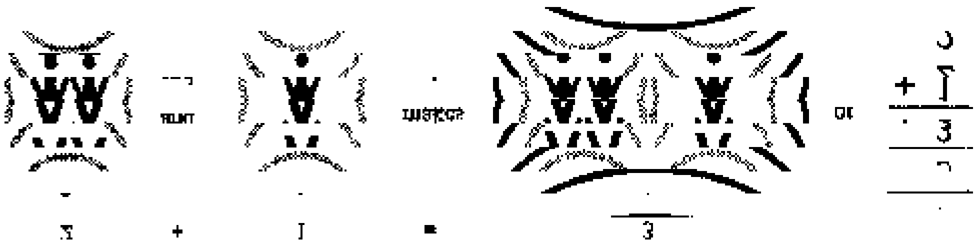
4.4 FOUR FUNDAMENTAL OPERATIONS ON NUMBERS

Addition

Counting very naturally leads to the addition of numbers. Addition is counting forward. It is closely related to the idea of putting together.



The teacher should organize activities with concrete objects and demonstrate 'putting together' to introduce properties of addition of natural numbers,

1. If we add two numbers (called addends) we get one number as the answer which is called the *sum*, The addition of any two natural numbers yields a number answer which belongs to natural numbers.

2. A pair of numbers can be added in any order:

   2 + 3 = 5 also 3 + 2 = 5

3. When it is necessary to add three or more numbers, we add any two numbers first and then add the third number to the sum. This property also illustrates why we can check any addition sum by adding in the opposite direction.

4. The property of zero is that its addition to any other number preserves the identity of the number

   a + 0 = 0 + a = a and 0 + 0 = 0

   The above properties should be illustrated with examples. No formal treatment is recommended at the primary level.

Examples: i) Add 6, 5 ii) Add 3 and 0

6	111111	5	11111	3	0
+ 5	11111	+ 6	111111	+ 0	+ 3
11	11111111111	11	11111111111	3	3

iii) Add 9, 3 and 4

9	12	
+ 3		16
+ 4		
16		

9	16	
+ 3	7	
+ 4		↑ check
16		

Addition of larger numbers

Consider the addition problem below:

What we are actually doing is regrouping in groups of

23	2	tens and 3 units		2 × 10 + 3 × 1
35	3	tens and 5 units or		3 × 10 + 5 × 1
41	4	tens and I unit		4 × 10 + 1 × 1
86	8	tens and 6 units		8 × 10 + 6 × 1
185	17	tens and 15 units		17 × 10 + 15 × 1
=	18	tens and 5 units	or	18 × 10 + 5 × 1

We illustrate that 15 units = 1 ten and 5 units; we take 1 ten to tens and make 18 tens. Then 18 tens = 1 hundred and 8 tens which gives the answer 1 hundred, 8 tens and 5 units.

When the child is introduced to column addition (using place-value) this grouping is demonstrated at first with concrete experiences using bundles of 1's, 10's, 100's etc. The child actually visualizes the meaning of carrying as a method of rearranging the groups.

Example: Add 6743, 2469, 538

Carry ⇒	1	1	2	
	Ten Th	Th	H	Units
	6	7	4	3
	2	4	6	9
		5	3	8
	9	7	5	0

Think: Write the numbers in columns:

i) Add units 3 + 9 + 8 = 20. Write 0 in units column and carry 2 to tens column

ii) Add tens [2] + 4 + 6 + 3 = 15. Write 5 in tens column and carry 2 to tens column

iii) Add hundreds [1] + 7 + 4 + 5 = 17. Write 7 in hundreds column and carry 1 to thousands column and carry 1 to thousands column

iv) Add thousands [1] + 6 + 2 = 9. Write 9 in thousands column

Everyday life problem on addition should also be analyzed and discussed with children.

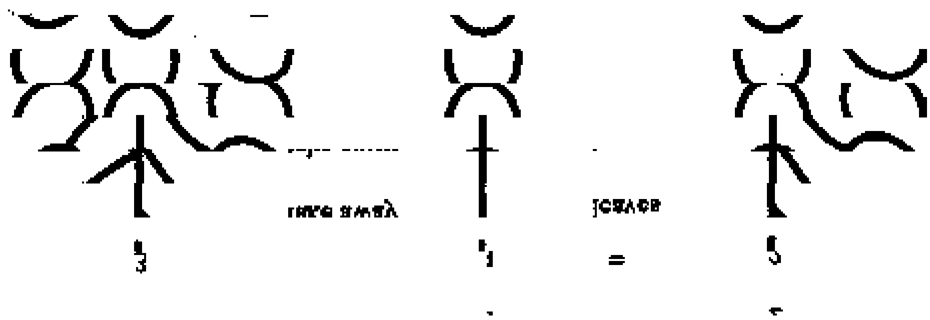
Example: This sales of a shopkeeper for Monday, Tuesday and Wednesday are respectively Rs. 3539, Rs. 2807 and Rs. 2982. Find the total sale for three days.

Solution	Sale for Monday	=	Rs.3539
	Sale for Tuesday	=	+ Rs. 2807
	Sale for Wednesday	=	+ Rs. 2982
	Total Sale	=	Rs. 9328

Subtraction

The operation of subtraction is the inverse operation of addition. It should be illustrated with activities involving 'taking away' or 'partitioning'. Pictures may be used to represent life situations.

Example



Three balloons. Take away one balloon. We are left with two balloons.

Next, the relationship with addition should be discussed. For any addition sum, say 5 + 8 = 13 we associate two subtraction sums, 15 – 5 = 8 and 13 – 8 = 5. The problem posed for children is 5 + ☐ = 13. What should be added to 5 to get 13; or ☐ + 8 = 13 what should be added to 8 to get 13. Thus by finding the missing addend the children discover the difference and are led to get the concept of subtraction as inverse of addition.

The property of zero should, also be illustrated with concrete experiences. I have 5 apples and I have eaten all. Then I am left with 'none' or zero apples. I have 5 apples and I eat 'none' or zero. Then I am left with 5 apples.

i) a – a = 0   ii) a – 0 = a   iii) 0 – 0 = 0

Just as in addition, in subtraction we subtract units from units, tens from tens, hundreds from hundreds, etc.

The idea of 'borrowing' should be illustrated by regrouping using place value.

Example

i)	5	111~~11~~	ii)	35	5 units – 2 units = 3 units
	– 2			– 12	3 tens – 1 ten = 2 ten
	3			23	

			Tens	Units	
ii)	42	⇒	3	12	We cannot take away 8 units
	– 18		– 1	8	from 2 units so we regroup
	24		2	4	42 as 3 tens and 12 units.

Example: Subtract 167 from 312.

3	1	2
– 1	6	7

H	T	U
3	0	12
– 1	6	7

⇒

H	T	U
2	10	12
– 1	6	7
1	4	5

Short method

	10	
2	~~0~~	12
~~3~~	~~1~~	2
–1	6	7
1	4	5

Think : 7 units cannot be taken away from 2 units, so, we regroup and make 12 units. We are left with 0 tens. Now 6 tens cannot be taken away from 0 tens. We change 1 hundred to 10 tens. Thus we are left 2 hundreds and we make 10 tens.

The idea of borrowing should be illustrated by concrete experiences using bundles of 1's, 10's and 100's etc. Suitable examples from everyday life should also be taken.

Example : There are 5000 bags of wheat in a godown. If 3529 bags are taken out, how many bags will remain in the godown?

Solution : Number of bags in godown = 5000

Number of bags taken out = 3529

Remaining number of bags = 1471

5000
– 3529

⇒

Th	H	T	U
4	9	9	10
3	5	2	9
1	4	7	1

Check Your Progress

Notes: a) Write your answers in the space given below.

b) Compare your answers with the one given at the end of this unit.

Give illustrations to:

8. Find the sum

(i)	(ii)	(iii)	(iv)
9	21	348	3625
+ 2	+ 19	+ 434	+ 2173
....	....	....	....

9. A company supplies cement to two villages. One village gets 3870 bags of cement. The other gets 5295 bags of cement. How many bags of cement are supplied to both the villages? How will you elicit.

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10. Subtract (i) 0 from 9 (ii) 12 from 30 (iii) 284 from 567. Give illustrations.

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11. There are 2365 students in a school. 1434 of them are boys. How many are girls? Elicit.

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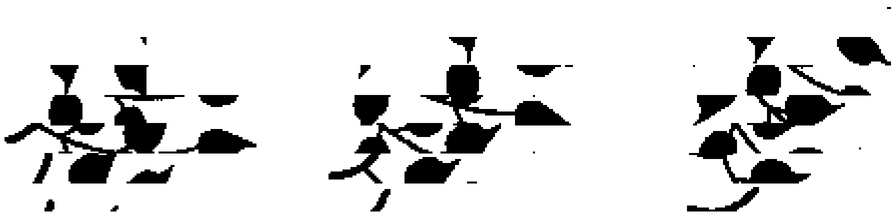
Multiplication

Multiplication is repeated addition. It should be illustrated with concrete activities and examples. If there are 3 branches and each has 5 leaves, how many leaves are there in all?

Naturally 15 is the answer, but how does it come?



1 branch has 5 leaves

3 branches have

5 + 5 + 5 = 15 leaves

or

5 + 5 + 5 = 3 × 5 = 15

(three fives)

We say ‘three fives are fifteen’ so, multiplication is the repeated addition of the same number. 15 is called the product. 3 is called the ‘multiplier’ and 5 is the ‘multiplieand’.

Multiplication table from 2 onwards upto twenty may be introduced by repeated addition. The properties of order and grouping may also be illustrated.

Example: Multiply 48 by 3

The column method may also illustrated by the distributive property.

H T O		
4 8	or 3×48	$= 3 \times (40+8)$
× 3		$= 3 \times 40 + 3 \times 8$
2 4		
1 2		$= 120 + 24$
1 4 4		$= 144$

3 times 8 ones = 24 ones

2 tens and 4 ones

Write 4 under 'ones' and carry 2 tens to 'Ten'

3 times 4 tens = 12 tens

1 hundred and 2 tens

Add 2 tens to the 2 tens carried over.

Write 4 under 'Tens'

Write 1 under 'Hundreds'

Product of 48 and 3 is 144.

Example: Multiply 2341 by 2

2 3 4 1	or 2×2341	$= 2 \times (2000 + 300 + 40 + 1)$
× 2		$= 2 \times 2000 + 2 \times 300 + 2 \times 40 + 2 \times 1$
4 6 8 2		$= 4000 + 600 + 80 + 2$
		$= 4682$

Explanation:

2 × 1 ones	= 2 ones
2 × 4 tens	= 8 tens
2 × 4 hundreds	= 6 hundreds
2 × 2 thousands	= 4 thousands

Example: Multiply 465 by 3

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   4  6  5
      ×  3
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1  3  9  5
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Explanation

3 × 5 ones	=	15 ones	
	=	1 ten + 5 oncs	carried to tens
3 × 6 tens	=	18 tens	
		+ 1	
		19 tens = 1 hundred + 9 tens	
3 × 4 hundreds	=	12 hundreds	carried to hundred
	+	1 hundred	
		13 hundred	
		465 × 3 = 1395	

Can you guess the multiplication by 10, 100, and 1000 ? Let us take some examples.

Example: Multiply 342 by 10

$$342 \times 10 = 342 \times 1 \text{ ten}$$
$$= (342 \times 1) \text{ tens}$$
$$= 342 \text{ tens}$$
$$= 3420$$

Example: Multiply 31 by 100

$$31 \times 100 = 31 \times 1 \text{ hundred}$$
$$= (31 \times 1) \text{ hundreds}$$
$$= 3100$$

Example: Multiply 7 by 1000

$$7 \times 1000 = 7 \times 1 \text{ thousand}$$
$$= (7 \times 1) \text{ thousands}$$
$$= 7 \text{ thousands}$$
$$= 7000$$

How will you multiply a two-digit number by a two-digit number?

Example: Multiply 76 by 23

$$23 = 20 + 3 \text{ (Two tens + 3 ones)}$$

So, $76 \times 23 = 76 \times (20 + 3)$

$$76 \times 20 + 76 \times 3$$

But $76 \times 20 = 1520$ and $76 \times 3 = 228$

$$76 \times 23 = 1520 + 228 = 1748$$

We can also do this multiplication as follows:

$$\begin{array}{rrrr} & & 7 & 6 \\ & \times & 2 & 3 \\ \hline & 2 & 2 & 8 \\ 1 & 5 & 2 & 0 \\ \hline 1 & 7 & 4 & 8 \\ \hline \end{array} \quad \begin{array}{l} \Leftarrow \; 3 \times 76 \\ \Leftarrow \; 20 \times 76 \\ \Leftarrow \; 23 \times 76 \end{array}$$

The answer is 1748.

Let us consider a problem based on multiplication of numbers.

Example: A wooden chair costs 175 rupees. What is the cost of 24 such chairs?

We will calculate thus:

Cost of 1 chair = Rs. 175

16 Cost of 24 chairs = Rs. 175 × 24

$$\begin{array}{rrrr} & 1 & 7 & 5 \\ & \times & 2 & 4 \\ \hline & 7 & 0 & 0 \\ 3 & 5 & 0 & 0 \\ \hline 4 & 2 & 0 & 0 \\ \hline \end{array}$$

Cost of 24 chairs = Rs. 4200

Can you guess the multiplication of a number by *zero?*

You know that 5 × 4 means 'five times four'

or

$$4 + 4 + 4 + 4 + 4 = 20$$

If we have to multiply '4' with '0', it will be written as 0 × 4, which means zero times four. When we take a number zero times, it means we are not taking that number at all, so the product will be zero. Therefore, 0 × 4 = 0. Thus, if we multiply a number by zero, the product is always zero.

Division

Division is inverse of multiplication. It should be introduced through concrete experience of 'equal sharing' and 'equal grouping'. These activities lead to the idea of repeated subtraction.

Example: There are 15 sweets to be distributed equally among 5 friends. How many sweets will each get?

To do this, find out how many times 5 can be subtracted from 15.

$15 - 5 = 10 \rightarrow 1$

$10 - 5 = 5 \rightarrow 2$

$5 - 5 = 0 \rightarrow 3$

Hence each friend will get 3 sweets.

Next introduce division as inverse of multiplication.

Each multiplication fact yields two division facts

3 × 4 = 12 yields 12/3 = 4 and 12/4 = 3

or 12 ÷ 3 = 4 and 12 ÷ 4 = 3

Example: Divide 12 by 3

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        4    →    Quotient
  3    12
     – 12
        0         Remainder
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Ask 3 × ☐ = 12

We recall the table of 3 to get 12, that is, 4 times 3 is 12. The answer is 4, which is also known as the quotient.

Let us make some more examples to clarify the division method.

Example: Divide 46 by 2

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                    T V
                 2 + 3 ⇒ 23
    23
 2) 46           2) 40+6
   −4              −40
    6                6
   −6               −6
    0                0
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Ask 'How many 2's in 4 tens'? 2 tens. So write 2 in tens column in quotient. Multiply 2 tens by 2 and take away 40 from dividend. Now we are left with 6 units. How many 2's in 6 units? 3. So, write 3 in Units column in quotient. Multiply 2 by 3 units and take away 6 from dividend. The quotient is 20 + 3 = 23

Check Your Progress

Notes: a) .... Write your answers in the space given below.

b) Compare your answers with the one given at the end of this unit.

Give illustrations for the following:

12. Multiply (i) 8 by 7 (ii) 24 × 2 (iii) 112 by 3.

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13. Ram purchased 3 packets of toffees. Each packet contained 125 toffees. How many toffees were there altogether in the 3 packets?

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14. Divide (i) 8 by 2 (ii) 15 by 3 (iii) 639 by 3 (iv) 785 by 5.

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15. If 4 ladoos can be packed in 1 packet. How many packets are needed to pack 48 ladoos?

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4.5 VARIOUS TYPES OF NUMBERS

Even and Odd Numbers

If we observe the numbers, then we find that some numbers are multiples of 2, while others are not multiples of 2.

The numbers which are multiples of 2 are called the *even numbers* and the numbers which are not multiples of 2 arc called *odd numbers.*

Example of even numbers

2, 4, 6, 8, 10. …………………..

Example of odd numbers

1, 3, 5, 7, 9………………

Prime Numbers

Let us find all the possible factors of 2, 3, 5

2 = 1 × 2

3 = l × 3

5 = l × 5

The possible factors of 2 are 1 and 2. The possible factors of 3 are 1 and 3. The possible factors of 5 are 1 and 5. These numbers have only two factors.

Can you suggest a name for such numbers which have only two factors?

Yes, such numbers are called **Prime Numbers.**

We conclude that 'The numbers which have only two factors, 1 and the number itself, are called **Prime Numbers'.**

For example 2, 3, 5, 7, 11, 13, etc. are **Prime Numbers.** Since 1 has only one factor, we do not call 1 as **Prime Number.**

Composite Number

Let us find all the factors of 4, 6, 12

4 = l × 4

4 = 2 × 2

The factors of 4 are 1, 2, 4

6 = l × 6

6 = 2 × 3

The factors of 6 are 1, 2, 3, 6

12 = 1 × 12, 12 = 2 × 6, 12 = 3 × 4

The factors of 12 are 1, 2, 3, 4, 6, 12. From the above, we see that each of the numbers 4, 6 and 12 has more than two factors. Such numbers are called composite numbers.

We conclude that the numbers which have more than two factors are called composite numbers.

For example 4, 6, 8, 9 are composite numbers.

Co-prime

Consider two numbers 16 and 35.

16 = 1 × 16, 16 = 2 × 8, 16 = 4 × 4

The factors of 16 are 1,2,4,8 and 16.

35 = 1 × 35; 35 = 5 × 7

The factors of 35 are 1, 5, 7, and 35

The common factor of 16 and 35 is 1.

Two numbers which have only 1 as the common factor are called **Co-prime.**

For example

2 and 3 are Co-prime Numbers.

Twin Primes

The pairs of Prime Numbers which have only one composite number between them are called **Twin Primes.**

For example 11 and 13 are twin primes because they have only one composite number 12 between them. Similarly 3, 5 and 5, 7 are also Twin Primes.

Check Your Progress

Notes: a) Write your answers in the space given below.

b) Compare your answers with the one given at the end of this unit.

How will you explain the following:

16. Encircle the even numbers.

    7, 10, 12, 15, 24, 36

17. Encircle the odd numbers.

    5, 8, 12, 17, 21, 38, 36

18. Write separately all the even numbers and odd numbers between 1 to 25.

    ....

    ....

19. Is 11 a Prime Number?

    ....

20. Write all the prime numbers between 11 to 41.

    ....

21. Write all the composite numbers upto 20.

    ....

22. Which of the following is a pair of Twin Prime numbers.

    (a) 3, 5 (b) 15, 17

    ....

    ....

4.6 MULTIPLES AND FACTORS

Multiples

$2 \times 1 = 2$

$2 \times 2 = 4$

$2 \times 3 = 6$

$2 \times 4 = 8$

The numbers 2, 4, 6, 8……..are obtained by multiplying 2 by 1, 2, 3, 4, 5…………..These are called multiples of 2.

If a given number is successively multiplied by 1, 2, 3,4, …………………..we get the multiples of the given number.

The multiples of 5 are 5, 10, 15, 20, 25................

The multiples of 8 are 8, 16, 24, 32, 40.............

When we multiply any two or more numbers we get a product.

The product is called a multiple of each of the numbers multiplied. This is a common multiple of the two given numbers.

Let us consider 5×4

We know that $5 \times 4 = 20$

20 is a multiple of both 5 and 4 and is a ‘common multiple’.

Example: Find tenth multiple of 12 and of 18.

$12 \times 10 = 120$

$18 \times 10 = 180$

We can find the multiple of any number we like. Let us take the multiples of 1, such as

$1 \times 24 = 24$

$1 \times 32 = 32$

$1 \times 41 = 41$

Here we find that

1. Every number is a multiple of itself.
2. Every number is a multiple of 1.

Factors: A factor is a divisor. We take a number, say 21. What are the divisors of 21? Clearly 1, 3, 7 and 21 are divisors because

$1 \times 21 = 21$

$3 \times 7 = 21$

Thus 1, 3, 7, 21 are called factors of 21.

Let us take another number, say 36

$36 = 1 \times 36$

$36 = 2 \times 18$

$36 = 3 \times 12$

36 = 4 × 9

36 = 6 × 6

Does any number other than above numbers exist which makes 36? No.

If we divide 36 by the numbers 1, 2, 3, 4, 6, 9, 18, 36, the remainder is zero. Thus we conclude that these numbers are the factors of 36.

We know that

21 = 1 × 21

36 = 1 × 36

43 = 1 × 43

159 = 1 × 159

So, we observe that one is the factor of all the numbers. Every number (other than zero) is a factor of itself.

Example: Let us write all the factors of 24.

24 = 1 × 24

24 = 2 × 12

24 = 3 × 8

24 = 4 × 6

1, 2, 3, 4, 6, 8, 12, 24 are the factors of 24.

Can you think of the smallest factor of a number?

Yes, it is one.

What is the greatest factor of a number?

The number itself.

LCM and HCF of Numbers

Highest Common Divisor or Factor (HCF)

Let us consider two numbers say 24 and 16. We find out the factors of 24 and 16.

24 = l × 24

24 = 2 × 12

24 = 3 × 8

24 = 4 × 6

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

Now 16. 16 = 1 × 16, 16 = 2 × 8, 16 = 4 × 4

The factors of 16 are 1, 2, 4, 8 and 16.

Can you find some common factors of 24 and 16?

Yes, 1, 2, 4 and 8 are the common factors of 24 and 16.

Which of the common factors is the highest?

It is 8.

What can we say about 8? We will say that 8 is the highest common factor of 24 and 16.

Example: Find the H.C.F. of 36 and 45.

36 = 1 × 36; 36 = 2 × 18; 36 = 3 × 12; 36 = 4 × 9; 36 = 6 × 6.

45 = l × 45; 45 = 3 × 15; 45 = 5 × 9.

The factors of 36 are 1,2,3,4,6,9,12,18 and 36.

The factors of 45 are 1,3,5,9, 15 and 45.

The common factors of 36 and 45 are 1,3,9. The H.C.F. of 36 and 45 is 9.

Let us find out H.C.F by *Prime Factorization Method* consider the number 36.

2	36	→	2 × 18
2	18	→	2 × 9
3	9	→	3 × 3
3	3	→	3 × 1
	1		

36 = 2 × 2 × 3 × 3

You have noticed above that all the factors of 36 are prime numbers. This kind of factorization is also known as Prime Factorization of a Number.

Example: Find the H.C.F. of 36 and 60 by Prime Factorization Method.

2	36		2	60
2	18		2	30
3	9		3	15
3	3		5	5
	1			1

36 = 2 × 2 × 3 × 3

60 = 2 × 2 × 3 × 5

H.C.F. of 36 and 60 = Product of all Common Prime Factors of 36 an 60 i.e. 2 × 2 × 3 which is 12.

Least Common Multiple (LCM)

Let us consider the multiples of 5 and 2

The multiples of 5 are 5, 10, 15, 20, ...

The multiples of 2 are 2, 4, 6, 8, 10, ...

Common multiples of 5 and 2 are 10, 20, 30, ...

The Least Common Multiple of 5 and 2 is 10

It is called L.C. M. of 5 and 2.

Example: Find the L.C.M. of 3 and 4

The multiples of 3 are 3, 6, 9, 12, 15, ...

The multiples of 4 are 4, 8, 12, 16, 20, ...

The least common multiple of 3 and 4 is 12.

Example: Find the L.C.M. of 6 and 18

The multiples of 6 are 6, 12, 18, 24, 30, ...

The multiples of 18 are 18, 36, 54, 72, ...

The common multiples of 6 and 18 are 18, 36, 54

The L.C.M. of 6 and 18 is 18.

Example: Find the L.C.M. of 12, 15 and 20

The multiples of 12 are 12, 24, 36, 48, 60,

The multiples of 15 are 15, 30, 45, 60, 75, 90, 105, ...

The common multiples of 12, 15 and 20 are 60, 120, 180, ...

The L.C.M. of 12, 15, and 20 is 60.

L.C.M. by Prime Factorization Method

Example: Find the L.C.M. of 60 and 75.

2	60
2	30
3	15
5	5
	1

3	75
5	25
5	5
	1

60 = 2 × 2 × 3 × 5

75 = 3 × 5 × 5

Common factors between the numbers 60 and 75 are 3 and 5.

For finding the L.C.M., we consider the common factors only once and the numbers left in the factors of both the given numbers which is 2 × 2 × 3 × 5 × 5 = 300.

Check Your Progress

Notes: a) Write your answers in the space given below.

b) Compare your answers with the one given at the end of this unit.

Give illustrations for the following:

23. Find (i) first five multiples of 5 (ii) first four multiples of 8.

....

....

....

24. (a) Is 24 a multiple of 5? (b) Find the 10th multiple of 9.

....

....

25. Write the factors of (i) 24 (ii) 35 (iii) 64.

....

....

....

....

....

26. Find the HCF of (i) 32 and 48 (ii) 35 and 63.

....

....

....

....

27. Find the LCM of (i) 12, 15, 10 (ii) 9, 27, 6.

....

....

....

....

....

28. Find the prime factors of (i) 90 (ii) 75 (iii) 48.

....

....

....

....

....

4.7 LET US SUM UP

- The set of natural numbers is 1, 2, 3, 4, ... 1 is the smallest natural number. There is no largest natural number.
- Our system of numeration is base ten since we use grouping by tens in counting. We use ten symbols 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0 to represent numbers. These are called digits.
- The numeration system follows a place-value principle. Place-value of each digit in a numeral is ten times the place-value of the digit on its right. The places and periods are as follows.

Periods ⇒	Crore		Lakh		Thousand		Units		
Places	Ten Crore	Crore	Ten Lakh	Lakh	Ten Thousand	Thousand	Hundreds	Tens	Unit
		3	8	5	2	4	6	0	1

- The number represented by a numeral is the sum of the place-value of the digits. The digits are read from left with the periods. For example the given numeral 38524601 is read as 3 crore 85 lakh 25 thousand 6 hundred one and it represents the number 30000000 + 8000000 + 500000 + 20000 + 4000 + 600 + 0 + 1
- The numbers can be arranged in ascending and descending order. The order relation for numbers 1 to 9 is

$$1 < 2 < 3 < 4 < 5 < 6 < 7 < 8 < 9$$

- To compare any two given numbers we compare total number of digits. A numeral with more digits is greater than a numeral with less number of digits. When the number of digits is same in two numerals, we compare their left most digits using the order relation of digits 1– 9.
- We can add any two given numbers. Addition has the property of order and grouping. The sum of a number and 0 is the number itself.
- Multiplication is a process of repeated addition. Multiplication also has the property of order and grouping. When a number is multiplied by zero, the product is zero.
- Subtraction is the inverse of addition.
- Division is a process of repeated subtraction. It is also the inverse of multiplication.
- Every number is a multiple of 1. A number is a multiple of itself.
- 1 is a factor of all the numbers. Every number (other than zero) is a factor of itself.
- The numbers which have only two factors, 1 and the number itself, are called prime numbers.
- The numbers which have more than two factors are called composite numbers. 1 is neither prime nor composite.
- Two pairs of prime numbers which have only one composite number between them are twin primes.

4.8 UNIT-END EXERCISES

1. What do you understand by base of a numeration system? How is the number of digits related to the base?
2. Find the place value of 4 in each of the numeral.

   40, 354, 4397, 943742
3. Find the difference between the largest 3 digit number and the smallest two-digit number.
4. How many numbers are there which have (i) 2 digits (ii) 3 digits in the numeral?
5. Arrange the following in ascending order.

   5729, 345, 4712, 59, 2890, 362, 39
6. How many primes are there between 1 – 100 ?
7. Find the L.C.M. of (i) 12, 18, 27, 35 (ii) 20, 30, 50, 60.
8. Find the H.C.F. of (i) 75, 20 (ii) 312, 936.

4.9 ANSWERS TO CHECK YOUR PROGRESS

1. Group II

2. i) 8 ii) 3

3. i) 1 ii) No

4. i) Place-value of 4 in 43 is 40

   Place-value of 3 in 43 is 3

   ii) Place-value of 2 in 237 is 200

   Place-value of 7 in 237 is 7

   iii) Place-value of 4 in 4375 is 4000

   Place-value of 7 in 4375 is 70

   Place-value of 5 in 4375 is 5

5. i) 579 = $5 \times 100 + 7 \times 10 + 9 \times 1$

   $= 500 + 70 + 9$

   ii) 9275 = $9 \times 1000 + 2 \times 100 + 7 \times 10 + 5 \times 1$

   $= 9000 + 200 + 70 + 5$

   iii) 43921 = $4 \times 10000 + 3 \times 1000 + 9 \times 100 + 2 \times 10 + 1 \times 1$

   $= 40000 + 3000 + 900 + 20 + 1$

   iv) 20047 = $2 \times 10000 + 0 \times 1000 + 0 \times 100 + 4 \times 10 + 7 \times 1$

   $= 20000 + 40 + 7$

6. i) 921075 → Nine lakhs twenty one thousands seventy five.

   ii) 4358078 → Forty three lakhs fifty eight thousands seventy eight.

7. a) 54 < 95 < 490 < 827 < 2497 < 3521

   b) 3521 > 2497 > 827 > 490> 95 > 54

8. i)

9	111111111
+ 2	11
11	11111111111

   ii)

21	$2 \times 10 + 1 \times 1$
+ 19	$1 \times 10 + 9 \times 1$
40	$3 \times 10 + 10 \times 1$
	or
	$4 \times 10 + 0 \times 1$

   iii)

   ①
   348
   + 434
   782

iv)
```
   3625
 + 2173
 ------
   5798
```

9. Number of bags of cement supplied to one village = 3870 (carries ① ①)

+

Number of bags of cement supplied to other village = 5295

Total number of bags of cement supplied to both villages = 9165

10. i) 9 – 0 = 9

ii)
```
  2 10
   30
 – 12
 ----
   18
```

iii)
```
 4 16
  567
– 284
-----
  283
```

11. Total number of students = 2365 (borrowing: ① ⑬)

– 1434

Number of boys = 0931

Number of girls = 931

12. i) 8 × 7 = 8 + 8 + 8 + 8 + 8 + 8 + 8 = 56

ii)
```
  2 4
  × 2
 ----
  4 8
```

iii)
```
  112
   ×3
 ----
  336
```

13. Toffees in 1 packet = 125

Toffees in 3 packet = 125 × 3

```
=   125
    × 3
   ----
    375
```

14. i) 8 ÷ 2

```
     4
 2) 8
   –8
    0
```

So, 8 ÷ 2 = 4

ii) 15 ÷ 3

```
   5
3) 15
  −15
    0
```

15 ÷ 3 = 5

iii) 639 ÷ 3

```
  213
3) 639
  − 6
   03
  − 3
    09
  −  9
     0
```

iv) 785 ÷ 5

```
  157
5) 785
  −5
   28
  −25
    35
  − 35
     0
```

785 ÷ 5 = 157

15. No. of packets needed to pack 4 ladoos = 1

No. of packets needed to pack 48 ladoos = 48 ÷ 4 = 12

```
   12
4) 48
  − 4
   08
  − 8
    0
```

16. (10), (12), (24), (36)

17. (5), (7), (21)

18. **Even numbers**

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24

Odd numbers

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 23, 25

19. Yes

20. 11, 13, 17, 19, 23, 29, 31, 37, 41

21. 4, 6, 8, 9, 10, 12, 14, 15, 16, 18

22. a) 3, 5 are twin primes as they have only one composite number (4) between them.

    b) 15, 17 are twin primes as they have only one composite numbers (16) between them.

23. i) 5, 10, 15, 20, 25

    ii) 8, 16, 24, 32

24. a) No

    b) 90

25. i) **Factors of 24**

    1 × 24 = 24

    2 × 12 = 24

    3 × 8 = 24

    4 × 6 = 24

    Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24

    ii) **Factors of 35**

    1 × 35 = 35

    5 × 7 = 35

    Factors of 35 are 1, 5, 7, 35

    iii) **Factors of 64**

    1 × 64 = 64

    2 × 32 = 64

    4 × 16 = 64

    8 × 8 = 64

    Factors of 64 are 1, 2, 4, 8, 16, 32, 64

26. i) **H.C.F. of 32 and 48**

    Factors of 32 = 1, 2, 4, 8, 16, 32

    Factors of 48 = 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

    Common factors of 32 and 48 = 1, 2, 4, 8, 16

    H.C.F. = 16

    ii) **H.C.F. of 35 and 63**

    Factors of 35 = 1, 5, 7, 35

    Factors of 63 = 1, 3, 7, 9, 21, 63

    Common factors of 35 and 63 = 1, 7

    H.C.F. = 7

27. i) **L.C.M. of 12, 15, 10**

Multiples of 12 are 12, 24, 36, 48, 60, 72 ....

Multiples of 15 are 15, 30, 45, 60, 75, ...

Multiples of 10 are 10, 20, 30, 40, 50, 60, ...

L.C.M. = Least common multiple of 12, 15 and 10 = 60

ii) 9, 27, 6

Multiples of 9 are 9, 18, 27, 36, 45, 54, 63, ...

Multiples of 27 are 27, 54, 81, ...

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, ...

L.C.M. of 9, 27, and 6 = 54

28. i)

$$\begin{array}{r|r} 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

90 = 2×3×3×5

ii)

$$\begin{array}{r|r} 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

75 = 3×5×5

iii)

$$\begin{array}{r|r} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

48 = 2×2×2×2×3