UNIT 5 OLIGOPOLY

Structure

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5.0 OBJECTIVES

After reading and studying the unit, you should be able to:

- understand the nature of strategic interaction in an Oligopolistic market structure in terms of various models;
- find the equilibrium under various oligopoly models;
- compute Cournot equilibrium using the Residual demand curve and by way of Reaction curves;
- throw light on the possibility of Collusions and Cartels among the rivals in an Oligopoly;
- appreciate the Bertrand model of oligopoly as an alternative to the Cournot approach;
- find the Stackelberg equilibrium; and
- explain the Dominant firm model.

5.1 INTRODUCTION

We have so far considered three different market structures, viz. Perfect competition, Monopoly and Monopolistic competition. Perfect competition and monopoly are the two extreme forms of market structures. In a perfect competitive market, there is free entry and exit, many buyers and sellers, perfect information, etc. Such an idealised market does not exist in reality. Moreover, except for some natural monopolies, one hardly finds any example of a monopolistic market structure. A step closer to the reality is the monopolistic competition market structure which assumes no barriers to entry and non-price competition (in the form of product differentiation) between firms. But firms under monopolistic competition are naïve and therefore do not involve in strategic interactions. This makes the market structure under monopolistic competition rarely experienced in reality. The present unit is a move towards a relatively more realistic market structure. Most markets are better described as oligopolies. These are markets where there exist more than one market player, yet where each firm is large enough to enjoy some *monopoly power*. There are barriers to entry which result in a market with a small number of dominant players dependent strategically on each other's decisions about output or price. The prominent example of an oligopolistic market structure is that of soft drink suppliers, Pepsi and Coca-cola. For instance, if price of Pepsi is lowered to attract more sales, it will necessarily attract a reaction from Coca-cola suppliers whose customers will get lured away by the lower Pepsi price. Unless Coca-cola is willing to bear the revenue loss (which is unlikely to happen), it will find itself obligated to respond to Pepsi's price cut, which in turn would affect Pepsi sales. Considering rival's reaction thus becomes significantly important in such a market condition. How do firms behave in an oligopolistic market is what will be covered in this unit.

5.2 OLIGOPOLY

In oligopoly market structure a few firms account for most or all of the total production. Barriers to entry and exit result in prevalence of a small number of dominant players that remain strategically dependent on each other. By strategically dependence it is meant, a firm while taking its optimising decisions must consider the expected reaction of its rival. This is because, output and/or price decision by one impacts output sales and hence the revenue earnings of the other players in the market. Like in perfect competition firms in oligopoly cannot take market price as it comes, neither do they have the price-setting power like that with a monopolist who can set the price and worry about no prospective retaliation. In oligopoly, firms possess certain degree of price-setting power (or the monopoly power, i.e., the rate at which it can set the price above its marginal cost) depending on their market structure, as they have to worry about retaliation from competitors. Any action by a firm in oligopolistic market is followed by a counter reaction from other firms in that market. This compels each firm to consider its rivals' expected reactions as it decides about output and pricing. It follows that accurately portraying the interactions between firms across a wide range of possibilities is the critical task of any model of oligopoly.

There are two different ways in which firms might interact with each other in an oligopolistic market structure. They may cooperate or choose not to cooperate. If they collude, that is, choose to cooperate with each other so as to maximise their joint profits, they form a cartel. For example, the Organisation of Petroleum Exporting Countries (OPEC) is a cartel of oil producing countries that cooperate in how much oil to produce in an attempt to move the price of oil up or down in the market. On the other hand, if they behave non-cooperatively, acting in their own self-interest, they take into account the actions of other firms. Examples of a non-collusive oligopoly model include— the Cournot model, the Bertrand model, the Stackelberg model and the Dominant firm model. We will be covering these models in the present Unit.

5.2.1 Equilibrium in an Oligopolistic Market

A firm's equilibrium position refers to its profit optimising price and output decisions in different market situations. Under perfect competition and monopoly, outcomes are more or less certain with the former optimising at the point where market determined price equals the marginal cost of production; in the case of the latter, however, the firm possessing the market power optimises by setting marginal revenue equal to the marginal cost of production. However, under oligopoly no such certainty exists due to the presence of a small number of dominant players who control the major share of the market and who are strategically interdependent in terms of pricing and output decisions. There exist several ways in which individual oligopolists may respond to rivals' price and output decisions. Consequently, several different models of oligopoly, viz. the Cournot model, the Cartel, the Bertrand model, the Stackelberg model, the Dominant firm model, have been developed, underpinned by different analytical approaches and assumptions about the nature of oligopolistic market behaviour. The discussion of these models is provided in the subsequent sections. However before that, let us try to understand the nature of monopoly power of the oligopolistic firm. Let us assume there are only two symmetric firms in the market having the same constant marginal cost C (and zero fixed cost) and the firms are involved in quantity competition. The demand function faced by a firm $(i = 1, 2)$: $P = D_i(Q_1, Q_2)$, where P represents the market price and Q_1, Q_2 are the quantity produced by firm 1 and 2, respectively. Let the profit function of the firm i $(i = 1, 2)$ be:

$$\pi_i(Q_1, Q_2) = P \times Q_i - C(Q_i) = D_i(Q_1, Q_2)Q_i - C(Q_i)$$

Where $C(Q_i)$ is the cost function faced by firm i. First-order condition for profit maximisation for any of the firm i $(i = 1, 2)$ involves differentiating the profit function with respect to Q_i and put it equal to 0.

$$\Rightarrow \frac{\partial \pi_i\ (Q_1, Q_2)}{\partial Q_i} = 0 \Rightarrow \frac{\partial D_i(Q_1, Q_2)}{\partial Q}\frac{\partial Q}{\partial Q_i} Q_i + \ D_i(Q_1, Q_2) - C = 0$$

$$\Rightarrow \frac{\partial D_i(Q_1,Q_2)}{\partial Q}\frac{\partial Q}{\partial Q_i} Q_i + P - C\ = 0$$

$$[\because\ P = D_i(Q_1, Q_2)]$$

Multiplying and dividing the first term on the LHS with Q×P, we get

$$\Rightarrow \left[\frac{\partial D_i(Q_1,Q_2)}{\partial Q}\frac{Q}{P}\left(\frac{PQ_i}{Q}\right) + P - C\right] = 0$$

$$[\because \frac{\partial Q}{\partial Q_i} = 1\ ]$$

Taking first term of the LHS on the RHS and dividing both the sides by P

$$\Rightarrow \frac{P - C}{P} = \left(-\frac{\partial D_i(Q_1, Q_2)}{\partial Q}\frac{Q}{P}\right)\left(\frac{PQ_i}{Q}\right)\frac{1}{P}$$

$$\Rightarrow \frac{P-C}{P} = \ -\frac{1}{\varepsilon_d} s_i$$

⟹This is the equation for Lerner' s Index for Oligopoly.

Here, $s_i = \frac{PQ_i}{PQ}$ is the share of firm i in the total value of output; the reciprocal of market elasticity of demand is given by : $\frac{\partial D_i(Q_1,Q_2)}{\partial Q}\frac{Q}{P} = \frac{1}{\varepsilon_d}$. The term $\frac{1}{\varepsilon_d} s_i$ measures how far the i^{th} firm can raise the price above the marginal cost C. Recall we have derived a similar equation for Lerner's Index in the case of Monopoly (Unit 3) which is $\frac{P-C}{P} = -\frac{1}{\varepsilon_d}$. Given that s_i is the share of the i^{th} firm in the total value of market output

$\Rightarrow\ \ 0 \le s_i \le 1$. Thus the monopoly power (as measured by $\frac{P-C}{P}$) of an oligopolistic firm is less than that of the Monopoly.

5.3 THE COURNOT MODEL

A Cournot model is an Oligopoly model in which all the firms decide on their profit maximising output simultaneously with each firm assuming that its rivals will continue producing their current output levels. Given the rival's quantity, each firm attains equilibrium by producing the quantity where marginal revenue equals marginal cost, i.e. MR = MC. Introductory Microeconomics course (BECC 101) of Semester 1 introduced the Cournot model by considering two firms in the market. In such case both the firms decide simultaneously about their profit-maximising level of output. Each firm considers its rival's output as given while making its output decision. The relationship between a firm's profit-maximising output and the given output of its rival's is summarised by a reaction function. These functions are first obtained separately for each firm, and then solved simultaneously to obtain Nash equilibrium. The market price is then determined using the total output of both firms.

5.3.1 Equilibrium using Residual Demand Curve

A Residual demand curve for the first firm is that portion of the market demand curve that remains for this firm assuming that the second firm supplies a fixed amount of quantity Q_2 in the market. From the Fig. 5.1, residual demand curve facing the first firm is ascertained by shifting the vertical axis to the right by the amount of output assumed to be sold by the second firm (i.e. Q_2). Let the market demand curve (DD' in Fig. 5.1) be given by $P = A - BQ$, where $Q = Q_1 + Q_2$ with Q_1 and Q_2 be the respective quantities supplied by firm 1 and 2, P be the market price, and A and B any constants. Then residual demand curve faced by the first firm will be given by

$$P = A - BQ$$

$$P = A - B(Q_1 + Q_2)$$

$$P = [A - BQ_2] - BQ_1$$

where $[A - BQ_2]$ represent the price intercept of the residual demand curve facing the first firm. The MR curve will be given by first finding the total revenue (TR) function of the first firm, which equals quantity × price, that is,

$$TR_1: P \times Q_1 = [AQ_1 - BQ_2 Q_1] - BQ_1^2$$

and then taking the first derivative of this function with respect to Q_1 to get MR_1

$$MR_1 = A - BQ_2 - 2BQ_1 \quad (1)$$

In the Fig. 5.1, MR_1 originates from the point of intersection of the vertical line $Q_1 = 0$ with the market demand curve, $P = A - BQ$. At point A, MR_1 equals 0 which should be the case as at that point firm 1 produces no output. Now, on assuming marginal cost to be 0, we get the profit maximising quantity of firm 1 at point E where MR_1 = MC. Using (1) and that MC = 0, in equilibrium, we get

$$A - BQ_2 - 2BQ_1 = 0$$

Now, we solve for profit maximising level of Q_1 (let it be denoted by Q_1^*) from the above equation, which we get as a function of given quantity of Q_2 (also known as the reaction curve for firm 1):

$$Q_1^* = \frac{A}{2B} - \frac{1}{2}Q_2$$

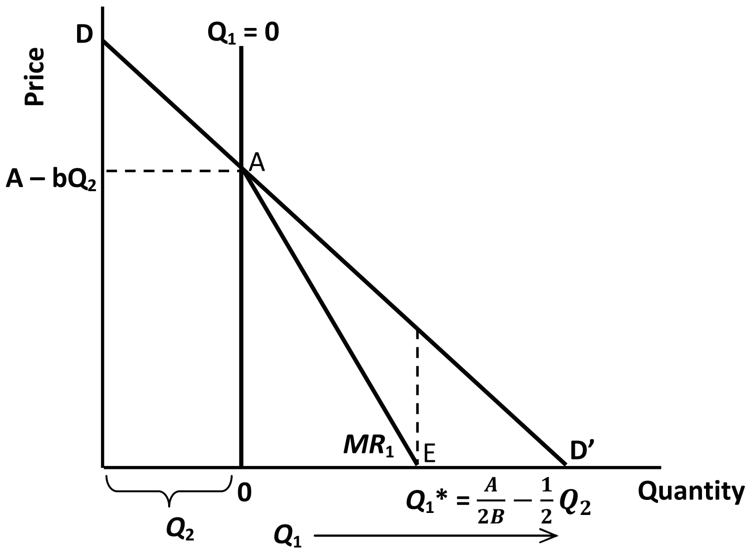

Fig. 5.1 : Equilibrium under Cournot Model using Residual Demand Curves

Similarly, profit maximising level of Q_2 can be obtained (assuming first firm supplies a fixed amount of quantity Q_1 in the market) as a function of Q_1. Both the functions can then be solved to get the optimising quantities supplied by each firm in the market.

5.3.2 Equilibrium using Reaction Curves

Reaction curves show relationship between an Oligopolist's profit maximising output and the amount it thinks its rivals will produce. If there are only two firms in the model, then firm 1st profit maximising output will depend upon what it thinks firm 2nd will produce, and this relationship will derive firm 1's reaction curve. Similarly firm 2nd reaction curve shows its output as a function of how much it thinks firm 1st will produce. The equation of the reaction curve is given by the profit maximising condition, MR = MC. In the above case, the reaction curve equation for firm 1, considering MC = 0, will be given by,

$$R_1(Q_2): MR_1 = MC \Rightarrow Q_1 = \frac{A}{2B} - \frac{1}{2}Q_2 \quad (2)$$

where, $R_1(Q_2)$ represents the reaction function of firm 1 which gives the optimal amount of output supplied by firm 1 as a function of given quantity supplied by firm 2 (Q_2). Similarly, the reaction curve equation for firm 2 will be given by,

$$R_2(Q_1): MR_2 = MC \Rightarrow Q_2 = \frac{A}{2B} - \frac{1}{2}Q_1 \quad (3)$$

To note here is that if both demand and cost functions are linear, reaction function will be linear as well. In Fig. 5.2, we plot these reaction functions by marking output supplied by firm 1 and firm 2 on the vertical and the horizontal axis, respectively. Reaction curve of firm 1, given by Equation (2) has the vertical intercept given by $Q_1 = \frac{A}{2B}$ when $Q_2 = 0$, and horizontal intercept given by $Q_2 = \frac{A}{B}$ when $Q_1 = 0$. Similarly, we plot reaction curve of firm 2.

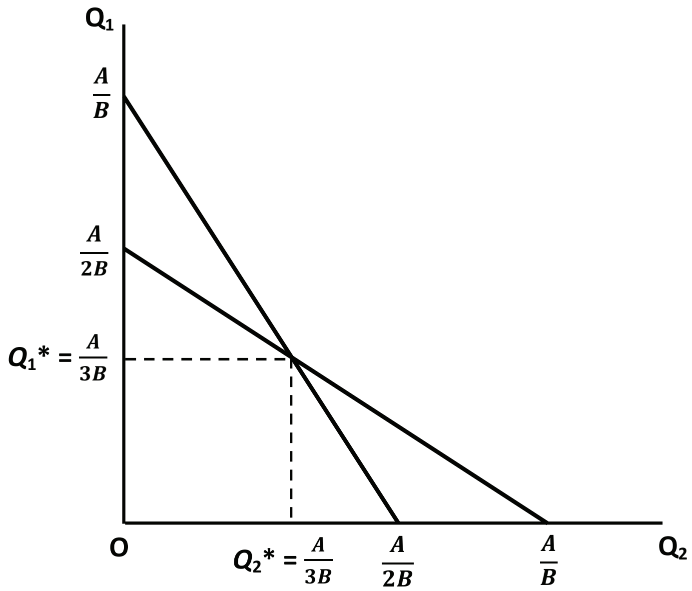

Fig. 5.2: Equilibrium under Cournot Model using Reaction Curves

Now, equilibrium output for firm 1 and firm 2 will be given by the intersection point of the two reaction curves. This is also called "Cournot-Nash Equilibrium", as each firm is doing the best it can given the behaviour of rival firms. This can be ascertained by substituting reaction curve for Firm 2 into the reaction curve for firm 1:

$$Q_1 = \frac{A}{2B} - \frac{1}{2}\left[\frac{A}{2B} - \frac{1}{2}Q_1\right]$$

$$Q_1 = \frac{A}{2B} - \frac{A}{4B} + \frac{1}{4}Q_1 \Rightarrow \; Q_1{*} = \frac{A}{3B}$$

Inserting the equilibrium value of $Q_1{*}$ in equation (3), we get

$$Q_2 = \frac{A}{2B} - \frac{1}{2}\left(\frac{A}{3B}\right) \Rightarrow Q_2{*} = \frac{A}{3B}$$

Total equilibrium quantity, $Q^* = Q_1{*} + Q_2{*} \Rightarrow Q^* = \frac{2A}{3B}$. Insert this in market demand curve to get Cournot price: $P = A - BQ$

$$P = A - B\left(\frac{2A}{3B}\right) \Rightarrow P = \frac{A}{3}$$

Cournot equilibrium profit of the firm 1 and 2 :

$$\pi_1(Q_1, Q_2) = PQ_1 = \pi_2(Q_1, Q_2) = PQ_2 = \left(\frac{A}{3} \times \frac{A}{3B}\right) = \frac{A^2}{9B}$$

Cournot equilibrium profit of the industry: $\pi_1(Q_1, Q_2) + \pi_2(Q_1, Q_2) = \frac{2A^2}{9B}$

If firm act as a monopoly, then the optimising output will be given by setting MR = MC:

Total revenue (TR): $P \times Q = (A - BQ) \times Q$;
Marginal Revenue (MR): $\frac{dTR}{dQ} = A - 2BQ$

For monopoly equilibrium, we set MR = MC $\Rightarrow A - 2BQ = 0 \Rightarrow Q_M = \frac{A}{2B}$, where Q_M gives the profit maximising monopoly output. Inserting this in the demand curve equation, we get monopoly price $P_M = \frac{A}{2}$. So the Monopoly profit : $P_M \times Q_M = \frac{A}{2B} \times \frac{A}{2} = \frac{A^2}{4B}$

Hence, we find that Cournot duopoly model has a higher total quantity and a lower price as compared to Monopoly quantity and price, respectively. Moreover firms under the Cournot competition earns less profit (even at the industry level) as compared to the monopoly $\Longrightarrow \frac{A^2}{4B} > \frac{A^2}{9B}$ and $\frac{A^2}{4B} > \frac{2A^2}{9B}$.

Example

Consider a duopoly of firm 1 and 2 producing a homogenous product, the demand of which is described by the following demand function:

$$Q = \frac{1}{2}(100 - P)$$

Where Q is total production of both firms (*i.e.*, $Q = Q_1 + Q_2$). Also let the marginal cost of production faced by both firms be Rs. 40, *i.e.*, $MC_1 = MC_2 = 40$. Calculate the residual demand function for both the firms. Using them ascertain their reaction curves and the Cournot-Nash equilibrium quantity produced by each firm?

Solution

Residual demand function of a firm is ascertained by fixing the quantity produced by the other firm.

We will first be finding the inverse demand function from the given demand function. Given, $Q = \frac{1}{2}(100 - P)$, the inverse demand function would be

$$P = 100 - 2Q \Rightarrow P = 100 - 2(Q_1 + Q_2)$$

The residual demand function faced by firm 1 will be given by assuming quantity Q_2 produced by firm 2 as fixed:

$$P = [100 - 2Q_2] - 2Q_1$$

where $[100 - 2Q_2]$ represent the price intercept. Now, MR_1 will be given by $\frac{\partial TR_1}{\partial Q_1}$

TR_1: $P \times Q_1 = 100Q_1 - 2Q_2Q_1 - 2Q_1^2$; MR_1: $\frac{\partial TR_1}{\partial Q_1} = 100 - 2Q_2 - 4Q_1$

For reaction function, we make use of the condition, $MR_1 = MC$

$$100 - 2Q_2 - 4Q_1 = 40 \quad \Rightarrow \quad Q_1 = 15 - \frac{1}{2}Q_2 \text{ [Reaction curve for firm 1]}$$

Similarly, residual demand function faced by firm 2 will be given by:

$$P = [100 - 2Q_1] - 2Q_2$$

And the corresponding reaction curve for firm 2 will be: $Q_2 = 15 - \frac{1}{2}Q_1$

We solve both the reaction curves to get the Cournot-Nash equilibrium,

$Q_1^* = Q_2^* = 10$, and considering the demand function, we can derive the equilibrium price, P* = 60

Profit by firm 1 = Profit by firm 2 = Total Revenue – Total Cost

$$= (60 \times 10) - (40 \times 10) = 200$$

Hence total industry profit = 400

Let us compare this result with the monopoly outcome. For the monopolist, the aggregate inverse demand curve will be $P = 100 - 2Q$.

Total revenue (TR): $P \times Q = 100Q - 2Q^2$;

Marginal Revenue (MR): $\frac{\partial TR}{\partial Q} = 100 - 4Q$

For monopoly equilibrium, we set MR = MC $\Rightarrow 100 - 4Q = 40 \Rightarrow Q_M = 15$, where Q_M gives the profit maximising monopoly output. Inserting this in the demand curve equation, we get monopoly price $P_M = 70$.

Monopoly profit = total revenue – total cost

$$= (15 \times 70) - (15 \times 40) = 450$$

Hence, we find that Cournot duopoly model has a higher total quantity (= 20) and a lower price (= 60) as compared to Monopoly quantity (= 15) and price (= 70), respectively. Also the total industry profit in case of Cournot model (= 400) is less than that of a monopolistic industry profit (= 450).

5.3.3 Cournot Equilibrium with Different Costs

In the above example we assumed that the two firms are symmetric i.e., having the same marginal cost of production as 40. However, cost conditions may vary from firm to firm. That is, a firm may be more cost efficient as compared to the other firm.

Considering the demand function $P = 50 - 2Q$. Let the marginal cost faced by firm 1 be Rs. 2, and that faced by firm 2 be Rs. 8. Find the Cournot-Nash equilibrium in such a case.

We make use of the optimising condition to find the reaction curves for each of the firms.

TR_1: $P \times Q_1 = 50Q_1 - 2Q_2Q_1 - 2Q_1^2$; MR_1: $\frac{\partial TR_1}{\partial Q_1} = 50 - 2Q_2 - 4Q_1$

For reaction function, we make use of the condition, $MR_1 = MC$

$$50 - 2Q_2 - 4Q_1 = 2 \quad \Rightarrow \quad Q_1 = 12 - \frac{1}{2}Q_2 \text{ [Reaction curve for Firm 1]}$$

Similarly, reaction function for firm 2 is given by: $Q_2 = \frac{21 - Q_1}{2}$

On solving the two reaction functions, we get the Cournot-Nash solution as $Q_1 = 9$, $Q_2 = 6$, Q (total quantity) = 15, P (Cournot price) = 20, with profit of Firm 1 = 152, profit of Firm 2= 60.

Hence, when firms have different costs, they choose different output levels, with the firm having low-cost (here firm 1) enjoying higher share of the market and making higher profits than the firm incurring high-cost (here firm 2).

Check Your Progress 1

1) Consider a Cournot duopoly of firm 1 and 2 producing output Q_1 and Q_2, respectively. Let market demand curve be P = 60 – Q, where $Q = Q_1 + Q_2$. Assume marginal cost of production is null for both the firms. Calculate Cournot-Nash equilibrium output of both the firms.

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2) Let market demand curve be P = a – bQ, where $Q = Q_1 + Q_2$. Let there be two firms 1 and 2 producing output Q_1 and Q_2, respectively at a constant marginal cost of 'c'. Calculate output of both the firms under the Cournot model.

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5.4 COLLUSION AND CARTELS

Collusion is said to take place when rival firms enter into an agreement in terms of price, market share, etc. in an attempt to realise higher profits than what they would have realised individually in an independent scenario. Firms may explicitly enter into an agreement resulting in an explicit collusion, or the collusion may take place without a formal agreement, which then is referred to as a Tacit collusion. Cartel results when few firms formally agree upon a certain level of output or a certain price of the goods in order to maximise joint profits of the industry, which they can share among themselves on a mutually binding agreement. Let us consider a case of two firms under Cournot market structure as well as under collusion. Coming back to the earlier example where demand curve is as follows:

$$P = A - BQ, \text{ with } Q = Q_1 + Q_2$$

along with the assumption that MC = 0.

If both the firms produce individually under Cournot market structure then (we know from our earlier section) the equilibrium outputs of each firm: $Q_1 = Q_2 = \frac{A}{3B}$

If the two firms collude and form a cartel, then total Revenue of the cartel will be given by:

TR = $(A - BQ)\, Q = AQ - BQ^2$; Marginal Revenue (MR): $\frac{\partial TR}{\partial Q} = A - 2BQ$

For cartel equilibrium, the two firms jointly behave like a monopolist and jointly control the entire market share. So from the equilibrium condition:

MR = MC $\Rightarrow A - 2BQ = 0 \;\Rightarrow Q^* = \frac{A}{2B}$ (same as the monopoly solution). Therefore total equilibrium quantity is Q = $\frac{A}{2B}$. Using the demand function, we get the equilibrium price as $P = \frac{A}{2}$.

The goal of the cartel is to set the industry output at a level that maximises industry profits. A rule governing the cartel behaviour specifies how the industry output and profits must be shared among the cartel members. In our example, we assume the two firms are producing homogeneous goods and are sharing similar cost structure which enabled the firms to equally share the industry output and thus profits, which means $Q_1 = Q_2 = \frac{A}{4B}$. This is less than the Cournot equilibrium output of $\frac{A}{3B}$ units produced by each firm. By independently maximising their own profits, firms produce more total output than they would if they collusively maximised industry profits. This pursuit of self interest does not typically maximise the well being of the industry as well as their individual profits. In terms of our numerical example, we saw that the total industry output of the duopoly was higher than that of the monopolistic industry, but total industry profit of monopolistic industry (Cartel) was higher as compared to the duopoly.

Conditions in oligopolistic industries tend to promote collusion since the number of firms is small and firms recognise their interdependence. The advantages can be more profits and decreased uncertainty. However it is hard to retain the collusion and firms tend to cheat each other and thus collusive arrangement often breaks down. Consequently, as long as a cartel is not maintained by legal provisions, there is a constant threat to its existence.

5.5 THE BERTRAND MODEL

In the Cournot model, each firm takes quantity as a strategic variable, and the resulting total output determines the market price. However there exists an alternative model, the Bertrand model where firms take price as the strategic variable, with each firm selecting a price and standing ready to meet all the demand it faces given the prices chosen by all the other firms. However unlike Cournot competition, each firm faces separate demand function based on the price charged by it and its rivals. Let a homogeneous good is produced by '*n*' firms in the industry competing on price, with each firm producing Q_i units of a good at a cost of $C_i (Q_i)$, where i = 1,2...*n*. The model further assumes that if firms set different prices, all demand shifts to the firm charging the lowest price, which in turn produces enough output to meet this demand. Further price rationing rule states that with more than one firm charging the lowest price, output demand is shared between those firms equally. Any firm charging above the lowest price charged in the market, receives no market share.

Let us assume a Bertrand model of duopoly where there are two firms, 1 and 2, producing homogenous product at a constant marginal cost "C". The firms choose prices P_1 and P_2, simultaneously. All sales go to the firm with the lowest price. Sales are split equally if $P_1 = P_2$. In contrast to the Cournot or Stackelberg models, the only Nash Equilibrium is the perfectly competitive outcome i.e., $P_1 = P_2 = C$. This results from the fact that each firm in the Bertrand model has an incentive to undercut price as long as production remains profitable. If one firm cut its price than the rival's price, then even for a slightest undercutting, it can appropriate the entire market share, thereby, inducing the rival out of the market. Similarly, the same firm may face zero market share if its rival outcompetes it by a slightest undercutting of their price. Given that each firm has an incentive to undercut its price in order to grab the market share, the firms would engage in reciprocal price undercutting until the price gets pushed down to the level of marginal cost "C". At this price, economic profits would be zero. Hence firms would have no incentive to undercut its price further. This would be Nash equilibrium as there would be no incentive for either firm to change its price once $P_1 = P_2 = C$. If either firm lowers their price below marginal cost, they would incur losses. If either raised their price, then it would be no better off, because it would lose the entire market share to the rival. Thus, quantity sold by each firm depends on both prices. If $P_1 < P_2$, then firm 1 serves the entire market demand $D(P_1)$; the opposite happens, that is, firm 2 serving the entire market demand $D(P_2)$ when $P_1 > P_2$; and *when* $P_1 = P_2$, each firm supplies half of the total market demand, that is, $\frac{D(P)}{2}$. If $P_1 = P_2 > C$, even though the firms share the market equally, it is not an equilibrium strategy. In this case either of the firm has an incentive to undercut the price and grab the entire market and still enjoy positive profit.

Assuming Π_i denote the profit earned by firm *i,* where *i* = 1, 2. Then, Nash equilibrium will be given by the pair of price (P_1*, P_2*) such that,

$$\left.\begin{array}{l} \Pi_1 (P_1^*, P_2^*) \geq \Pi_1 (P_1, P_2^*) \; ; \forall P_1 \\ \Pi_2 (P_1^*, P_2^*) \geq \Pi_2 (P_1^*, P_2) \; ; \forall P_2 \end{array}\right\} \qquad 5.1$$

where symbol ∀ stands for "*for all*".

5.5.1 Bertrand Paradox

The unique pair of prices satisfying condition 5.1 is given *by* $P_1 = P_2 = C$ (the given constant marginal cost) at which $\Pi_1 = \Pi_2 = 0$. This situation is referred to as the condition of *Bertrand Paradox*. The paradox results from the fact that just two firms are sufficient to dissipate the market power and yield an outcome similar to the perfectly competitive market (which usually assumes many sellers). In other words, with the number of firms rising from one to two, the price decreases from the monopoly price to the competitive price and stays at the same level even when the number of firms increases further. This is contrary to our perception, where we think that markets with a small number of firms possessing market power typically charge a price above the marginal cost.

5.5.1.1 Condition to be Satisfied for the Bertrand Paradox

For the Bertrand model to generate the Bertrand paradox, i.e., a situation when a perfect competitive outcome results with just two firms (engaged in reciprocal price undercutting above their marginal cost) are: 1. Firms involved in price competition must possess unlimited capacities. If initially the price condition is given by, $P_1 = P_2 = P$ (say) $> C$, with each firm sharing market equally, then either firm would be tempted to undercut its price slightly (say by $\varepsilon > 0$) and grab the entire market. In order to throw the rival out of the market and solely cater to the market demand, either firm needs a drastic expansion of capacity. The firm can satisfy this increased demand only when it faces no capacity constraints. If the firm faces a capacity constraint, price undercutting would not be profitable and it would not be able to drive its rival out of the market. Rather, that would leave some residual demand for the higher-priced rival firm and would decrease the incentive to undercut. Moreover Bertrand paradox prevails when firms are assumed to be in a homogeneous product industry i.e., the products of each firms are close substitutes, so that consumer cannot distinguish one firm's product from the other firms. If the firms are in the differentiated product industry (where consumer can exercise a brand preference as the products are similar but not exactly close substitutes) each firm enjoys some monopoly power and hence $P_1 = P_2 = C$ is not the Nash equilibrium. Rather the Nash equilibrium may be $P_1 \lesseqgtr P_2 > C$, depending upon their brand values.

5.5.2 Bertrand Equilibrium using Reaction Curves

In case of a Bertrand model, reaction function of a firm will be in price terms. In other words, a reaction function will give the optimum price at which firm chooses to supply its output, given the price of its rival's. Firm 1's reaction curve equation will be given by $R_1(P_2)$, giving the optimal price charged by firm 1 given that the price set by firm 2 is P_2. Similar description goes for the reaction curve equation of firm 2, that is, $R_2(P_1)$. Refer Fig. 5.3, where we represent Bertrand equilibrium using reaction curves assuming same marginal cost C is faced by both the firms.

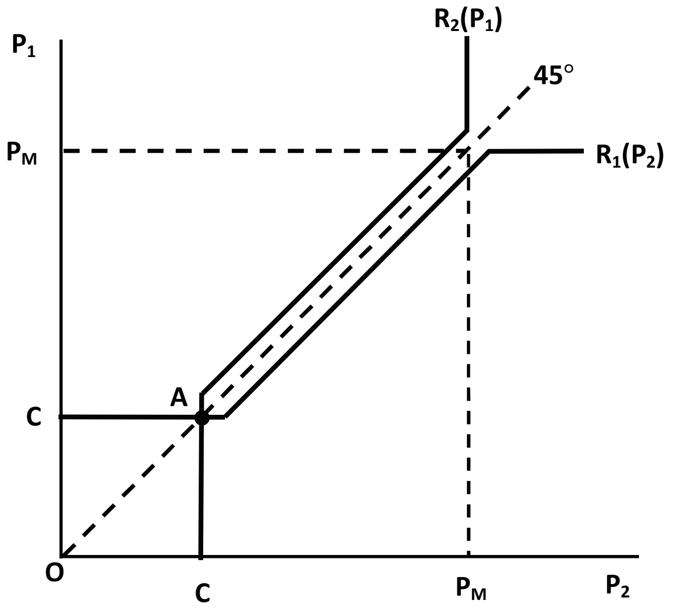

Fig. 5.3: Bertrand Equilibrium using Reaction Curves

In the above figure, firm 1's reaction function $R_1(P_2)$ shows when firm 2's price is below the marginal cost, i.e., when $P_2 < C$, it is optimal for firm 1 to set its price at the marginal cost, i.e., $P_1 = C$ as shown by the horizontal segment CA of the firm 1's reaction curve $R_1(P_2)$. The 45° line represents prices where $P_1 = P_2$. P_M represent the monopoly prices charged for the good when both the firms collude and act as a monopolist. Now, again look at the reaction curve of firm 1. When firm 2's price is above the marginal cost (C) but below the monopoly price (P_M), then it is optimal for firm 1 to set price just below firm 2, i.e. $P_1 < P_2$. Whereas, when firm 2's price is above the monopoly price, it is optimal for firm 1 to charge the monopoly price, i.e. $P_1 = P_M$. With both facing the same marginal cost (C), reaction function of firm 2, $R_2(P_1)$ will be symmetrical with respect to the 45° line. Symbolically, we can represent a reaction function as follows:

$$R_1(P_2) = \begin{cases} P_M & \text{if } P_2 > P_M \\ P_2 - \varepsilon & \text{if } C < P_2 \le P_M \\ C & \text{if } P_2 \le C \end{cases} \quad \text{and}$$

$$R_2(P_1) = \begin{cases} P_M & \text{if } P_1 > P_M \\ P_1 - \varepsilon & \text{if } C < P_1 \le P_M \\ C & \text{if } P_1 \le C \end{cases}$$

where, $\mathcal{E}$ represents a small positive number. Bertrand equilibrium is given by the intersection of both the reaction curves at Point A, where $P_1 = P_2 = C$, which is mutually best response for both the firms. At any other price above the marginal cost, either firm would always find it is in their best interest to undercut its rival's price a little (say by $\mathcal{E} > 0$) and serve the entire market itself. It is only at the price equals to the marginal cost that firms have no incentives to deviate from the equilibrium prices.

5.5.3 Comparison between the Cournot and the Bertrand Model

In the Cournot model, firms engage in quantity competition, so quantity is the strategic variable of the firm whereas in the Bertrand model firms engage in price competition, so price is the strategic variable. In the Cournot model the equilibrium price is generally above the marginal cost and the quantity approaches the perfectly competitive market situation only when the number of firms becomes large. In contrast to this, in the Bertrand model even with two firms, price competition dissipates the monopoly power and ensures competitive price which is equal to the marginal cost. Another difference is that in the Cournot model, the firms take their rival's output as given and cannot "steal" any consumers away from their rivals by lowering their prices. Whereas, a Bertrand rival believes that it can lure customers from its rivals by small cuts in price.

5.6 THE STACKELBERG MODEL

Similar to firms in Cournot model, in the Stackelberg model of oligopoly, firms produce homogeneous product and engage in quantity competition. The principal difference between the two models is that instead of simultaneous output or quantity choice of the rival firms (as in the Cournot model) the firms in Stackelberg model is based on sequential output or quantity choice. So instead of a static-move game like that of Cournot model, the firms in the Stackelberg model are engaged in the dynamic-move game. In case of a Stackelberg model of duopoly, we assume one of the firm moves first, followed by it the other firm. Thus, the model basically becomes a model of two periods, where first mover (say firm 1) decides about its quantity choice in period 1, then the second mover (say firm 2) after observing the firm 1's move, decides about its optimal quantity choice in period 2, and there the game ends with the appropriation of profits by both the firms. How does this affect the equilibrium of this game? We analyse the result using the example considered above in case of the Cournot model of duopoly. Now, let the firm 1, also referred to as the Stackelberg leader, moves first to produce quantity Q_1. Firm 2, the Stackelberg follower, observes firm 1's quantity choice Q_1, and then chooses to produce quantity Q_2. The quantity chosen by the follower must therefore be along its reaction function. Since, firm 1 knows that firm 2 will take firm 1's output as given and optimally decides its output along its reaction function, firm 1 (the Stackelberg leader) being the first mover enjoys a strategic advantage over firm 2 (the Stackelberg follower) by knowing the reaction function of the firm 2. Firm 1 can influence the behaviour of firm 2 by altering its own output and it takes into account the effects of its own output on firm 2's behaviour.

In sequential games, the optimal solution is obtained by the backward induction techniques, where, we first solve the firm 2's optimal quantity choice problem in the second period and then proceed backward to solve the firm 1's optimal quantity choice problem in the first period. Such approach is called backward induction as it is a process of reasoning backwards in time. In period 2, firm 2 chooses Q_2 given firm 1 has chosen Q_1 in period 1. This gives us the reaction curve of firm 2 as a function of Q_1, $R_2(Q_1)$. This reaction function of firm 2 is then considered by firm 1 in period 1 as given to decide its own profit maximising quantity Q_1. We illustrate this below:

Consider the same demand function that we considered in the earlier example of Cournot model. Here we assume firm 1 to be the leader and firm 2, to be the follower. The solution is worked out as follows:

$$P = A - BQ \qquad [\text{where, } Q = Q_1 + Q_2]$$

Given marginal cost is $MC_1 = MC_2 = 0$.

Backward Induction: Consider period 2 first. Here, firm 2 will choose its optimal output according to its own Reaction function i.e.

$$R_2(Q_1): Q_2 = \frac{A}{2B} - \frac{1}{2}Q_1 \qquad (4)$$

(as derived in the earlier example)

Now consider period 1: Firm 1 will choose its output by maximising its own profit function and considering the reaction function of firm 2, $R_2(Q_1)$. The profit function of firm 1 (Π_1) is :

$$\Pi_1(Q_1, Q_2) = TR_1 - TC_1$$

where TR_1 is total revenue of firm 1 and TC_1 is total cost of firm 1

$$\Pi_1(Q_1, Q_2) = TR_1 - TC_1 = [A - B(Q_1 + Q_2)]\, Q_1 - 0 \times Q_1$$

$$\Rightarrow \Pi_1(Q_1, Q_2) = AQ_1 - BQ_1^2 - BQ_2 Q_1$$

$$= AQ_1 - BQ_1^2 - BQ_1 \times R_2(Q_1)$$

Firm 1 being the first mover knows the reaction function of firm 2 (as shown in Eq. 4) and therefore incorporate it in its own profit function in order to solve for its own optimal output choice:

$$\Pi_1(Q_1, Q_2) = AQ_1 - BQ_1^2 - BQ_1\left(\frac{A}{2B} - \frac{1}{2}Q_1\right)$$

$$= AQ_1 - BQ_1^2 - \frac{AQ_1}{2} + \frac{BQ_1^2}{2}$$

So the first order condition for the optimisation of firm 1's profit is obtained by setting the first - order partial derivative with respect to Q_1 equal to zero.

$$\frac{\partial \Pi_1(Q_1, Q_2)}{\partial Q_1} = 0 \;\Rightarrow A - 2BQ_1 - \frac{A}{2} + \frac{2BQ_1}{2} = 0$$

Solving for optimal Q_1, gives $Q_1 = \frac{A}{2B}$.

Now, substituting Q_1 back in Eq. (4), we get the optimal output of firm 2: $Q_2 = \frac{A}{4B}$. So the equilibrium output choice of firm 1 and 2 are:

$[Q_1 = \frac{A}{2B}, Q_2 = \frac{A}{4B}]$.

Equilibrium price under Stackelberg model : P = $[A - B(Q_1 + Q_2)] = [A - B\frac{3A}{4B}] = \frac{A}{4}$

Now profit of the leader (firm 1) : $\Pi_1(Q_1, Q_2) = [A - B(Q_1 + Q_2)]\, Q_1 =$
$[A - B\frac{3A}{4B}]\ \frac{A}{2B} \Rightarrow \Pi_1(Q_1, Q_2) = \frac{A}{4}\frac{A}{2B} = \frac{A^2}{8B}$

Profit of the follower (firm 2): $\Pi_2(Q_1, Q_2) = [A - B(Q_1 + Q_2)]Q_2 = [A - B\frac{3A}{4B}]\ \frac{A}{4B} \Rightarrow \Pi_2(Q_1, Q_2) = \frac{A}{4}\frac{A}{4B} = \frac{A^2}{16B} \Rightarrow \Pi_1(Q_1, Q_2) > \Pi_2(Q_1, Q_2) \Rightarrow$ Thus the leader enjoys higher market share and higher profit than the follower in equilibrium.

Unlike the Cournot outcome which was symmetric, that is both the firms produced the same level of output, in the Stackelberg model, leader firm i.e., firm 1 enjoys greater market share (produces more output) as compared to the follower i.e., $Q_1 > Q_2$ in the equilibrium. One may compare the Stackelberg equilibrium outcome with that of the Cournot outcome and may check that the leader (firm 1) produces more than produced by a firm in Cournot equilibrium whereas the follower produces less than Cournot equilibrium quantity. Thus, there is an advantage of being the first mover.

Now, the total industry output will be $Q_1 + Q_2 = \frac{3A}{4B}$, which is more than the total industry output under Cournot equilibrium of $\frac{2A}{3B}$ for a given value of A and B. Profit of the Stackelberg leader is higher than the firm under Cournot competition $\Rightarrow \Pi_1(Q_1, Q_2)^S > \Pi_1(Q_1, Q_2)^C \Rightarrow \frac{A^2}{8B} > \frac{A^2}{9B}$. Moreover profit of the Stackelberg follower is lower than the firm under Cournot competition $\Rightarrow \Pi_2(Q_1, Q_2)^S < \Pi_2(Q_1, Q_2)^C \Rightarrow \frac{A^2}{16B} < \frac{A^2}{9B}$. Industry profit under Stackelberg model is greater than that of the Cournot competition

$$\Rightarrow \Pi_1(Q_1, Q_2)^S + \Pi_2(Q_1, Q_2)^S > \Pi_1(Q_1, Q_2)^C + \Pi_2(Q_1, Q_2)^C$$

$$\Rightarrow \frac{3A^2}{16B}\left(= \frac{A^2}{8B} + \frac{A^2}{16B}\right) > \frac{A^2}{9B}$$

5.7 THE DOMINANT FIRM MODEL

In some oligopolistic models, one large firm dominates the market share and many small fringe firms are the followers catering the residual demand and acting competitively. A Dominant firm (also known as the leader), typically having a larger share in market, behaves as a price-setter that faces smaller price-taking firms (also known as Fringe firms) each having a very small share in the market. The leader or the dominant firm sets the price for the commodity that maximises its own profits and assumes that its rivals will behave as competitive firms or in other words as price-takers that will take the price set by the dominant firms as given in determining their output in that price. Let us explain how this market structure operates with the help of a diagram below.

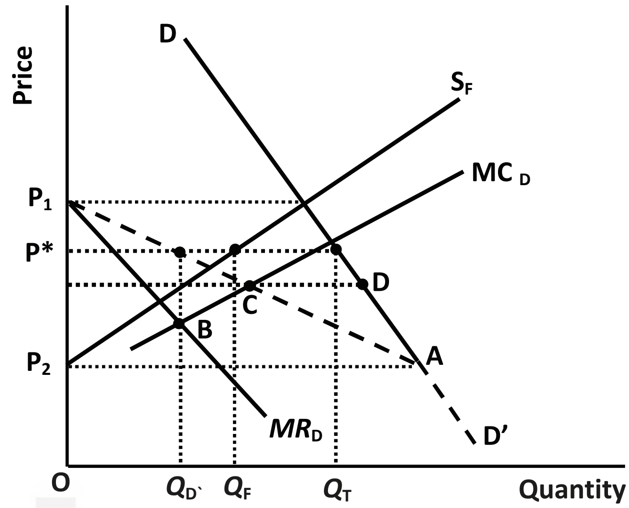

Fig. 5.4: Dominant Firm Model

In Fig. 5.4 DD' curve represents the market demand which is served by the dominant firm and the fringe firms collectively. Fringe firms are assumed to behave as competitive firms supplying where their marginal cost equals the price set by the dominant firm. This way fringe firms collectively behave as a competitive industry whose output can be determined by the supply curve S_F. The dominant firm's problem is to find a price which maximises its own profits. To solve this problem we need demand curve of the dominant firm. The extent of demand that the dominant firm gets to serve is equal to the total quantity demanded at that price (given by DD') minus the quantity the fringe firms supply at that price (given by S_F).This is represented by the residual demand curve P_1AD'. In other words, P_1AD' represents the horizontal difference between the fringe firm's supply curve S_F and the market demand curve DD'. At price P_1, dominant firm produces nothing with fringe firms serving the entire market demand at that price. On the other hand, at a price as low as P_2 dominant firm gets to serve the entire market demand with supply by fringe firms falling to zero. The dominant firm's residual demand curve between these two extreme prices is given by P_1A. Corresponding to the residual demand curve of the dominant firm, we have a marginal revenue curve represented by MR_D. MC_D likewise represents the marginal cost curve of the dominant firm.

The dominant firm optimise at point B where marginal revenue equals marginal cost resulting in profit maximising output Q_D sold at the market price P*. At this price fringe firms will supply Q_F amount of output as will be given by their supply curve. Total output at P* equals Q_T which is the sum of Q_D and Q_F. Total output generated in this model will be less than competitive output. This is clear by the figure above. Dominant firm optimise at a point where its price is above the marginal cost it faces. An efficient optimum in this case will be given by the point C where MC_D intersects the residual demand curve resulting in higher total output as is given by the point D. In this case, not only fringe firms but also the dominant firm is producing where MC equals price.

Check Your Progress 2

1) Explain the difference between the Bertrand model and the Stackelberg model of Oligopoly.

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2) For the following demand curve P = a – 2b and constant marginal cost curve 'c', find equilibrium price, quantity and profit according to:

i) Stackelberg Model

ii) Bertrand Model

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5.8 LET US SUM UP

The present unit is an attempt to move relatively closer to the real market conditions. The extreme market structures like perfect competition and monopoly have already been taken into account in order to discuss a relatively real market situation of that of an oligopolistic market. Oligopoly is a form of market structure where only a few firms account for most or all of production. The market also assumes barriers to entry which allows a few firms to act as dominant players. These few firms are characterised by strategic interdependence on output and pricing decisions. The unit discussed several oligopolistic models with varying assumptions, decision parameters and consequently the equilibrium outcome. In oligopolistic market, the concept of Nash equilibrium is more appropriate as it gives due consideration to the strategic interdependence among the firms in their decision-making. A Nash-equilibrium is a situation in which each firm adopts the best response strategy, given the strategy of its rival firm. In the Cournot model, each firm set its profit maximising quantity assuming its rival's output as given. On the other hand, in Bertrand model, each firm set the profit maximising price assuming its rival's price as given. Then after these two simultaneous-move models, we discussed a sequential-move model, i.e. the Stackelberg models, where we have a leader and a follower, with leader deciding first about the optimum output giving due consideration to the reaction function of the follower. Subsequently, the Dominant firm model is described, where a Dominant firm (the leader), behaves as a price-setter, with the smaller price-taking firms (Fringe firms) taking the price set by the dominant firms as given in determining their output in that price. Each model has its own significance with respect to the assumptions and the respective equilibrium outcome.

5.9 SOME USEFUL REFERENCES

- Hal R. Varian, *Intermediate Microeconomics,* a Modern Approach, W.W. Norton and company/Affiliated East- West Press (India), 8th Edition, 2010.
- Pindyck R. S and Rubinfeld D. L, *Microeconomics,* Pearson India, 2009
- C. Snyder and W. Nicholson, *Fundamentals of Microeconomics,* Cengage Learning (India), 2010.

5.10 ANSWERS OR HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) Firm 1

$TR_1 = P\,Q_1 = (60 - Q)\,Q_1$

$= 60\,Q_1 - (Q_1 + Q_2)\,Q_1$

$= 60\,Q_1 - Q_1^2 - Q_2\,Q_1$

$\therefore MR_1 = 60 - 2Q_1 - Q_2$

Setting $MR_1 = MC_1$

Firm 1's Reaction curve is: $Q_1 = 30 - \frac{1}{2}Q_2$

Similarly we can get Firm 2's Reaction curve as $Q_2 = 30 - \frac{1}{2}Q_1$

The equilibrium levels of Q_1 and Q_2 are given by solving the two reaction curves.

$\therefore$ Cournot-Nash equilibrium $Q_1 = Q_2 = 20$, and Cournot price, $P = 20$

2) Solve the question like the above question.

$Q_1 = Q_2 = \frac{a-c}{3b}$, total industry output, $Q = \frac{2(a-c)}{3b}$, Cournot price, $P = \frac{a+2c}{3}$

Check Your Progress 2

1) Bertrand model is based on price competition, where the only Nash equilibrium is when $P_1 = P_2 = C$. In the Stackelberg model, the first mover, firm 1 has an advantage, it has a higher market share, hence higher profits than firm 2, price is also above marginal cost.

2)

Market form	Firms output	Price
Stackelberg	$Q_1 = \frac{a-c}{2b}$ $Q_2 = \frac{a-c}{4b}$	$P = \frac{a+3c}{4}$
Bertrand	$Q_1 = Q_2 = \frac{a-c}{2b}$	$P = C$