UNIT 6 PRESENTATION OF DATA

Objectives

After studying this unit, you should be able to:

- understand the need and significance of presentation of data
- know the necessity of classifying data and various types of classification
- construct a frequency distribution of discrete and continuous data
- present a frequency distribution in the form of bar diagram, histogram, frequency polygon, and ogives.

Structure

6.1 INTRODUCTION

In the previous unit, we discussed the various ways of collecting data. The successful use of the data collected depends to a great extent upon the manner in which it is arranged, displayed and summarised. In this unit, we shall be mainly interested in the presentation of data. Presentation of data can be displayed either in tabular form or through charts. In the tabular form, it is necessary to classify the data before the data is tabulated. Therefore, this unit is divided into two section, viz., (a) classification of data and (b) charting of data.

6.2 CLASSIFICATION OF DATA

After the data has been systematically collected and edited, the first step in presentation of data is classification. Classification is the process of arranging the data according to the points of similarities and dissimilarities. It is like the process of sorting the mail in a post office where the mail for different destinations is placed in different compartments after it has been carefully sorted cut from the huge heap.

6.3 OBJECTIVES OF CLASSIFICATION

The principal objectives of classifying data are:

i) to condense the mass of data in such a way that salient features can be readily noticed

ii) to facilitate comparisons between attributes of variables

iii) to prepare data which can be presented in tabular form

iv) to highlight the significant features of the data at a glance

6.4 TYPES OF CLASSIFICATION

Some common types of classification are:

1 Geographical i.e., according to area or region.

2 Chronological, i.e., according to occurrence of an event in time.

3 Qualitative, i.e., according to attributes.

4 Quantitative, i.e., according to magnitudes.

Geographical Classification. In this type of classification, data is classified according to area or region. For example, when we consider production of wheat statewise, this would be called geographical classification. The listing of individual entries are generally done in an alphabetical order or according to size to emphasise the importance of a particular .area or region.

Chronological Classification. When the data is classified according to the time of its occurrence, it is known as chronological classification. For example, sales figure of a company for last six years are given below:

Year	Sales (Rs. lakhs)	Year	Sales (Rs. lakhs)
1982-83	175	1985-86	485
1983-84	220	1986-87	565
1984-85	350	1987-88	620

Qualitative Classification. When the data is classified according to some attributes (distinct categories) which are not capable of measurement is known as qualitative classification. In a simple (or dichotomous) classification, an attribute is divided into two classes, one possessing the attribute and the other not possessing it. For example, we may classify population on the basis of employment, i.e., the employed and the unemployed. Similarly we can have manifold classification when an attribute is divided so as to form several classes. For example, the attribute education can have different classes such as primary, middle, higher secondary, university, etc.

Quantitative Classification. When the data is classified according to some characteristics that can be measured, it is called quantitative classification. For example, the employees of a company may be classified according to their monthly salaries. Since quantitative data is characterised by different numerical values, the data represents the values of a variable. Quantitative data may be further classified into one or two types: discrete or continuous. The term discrete data refers to quantitative data that is limited to certain numerical values of a variable. For example, the number of employees in an organisation or the number of machines in a factory are examples of discrete data.

Continuous data can take all values of the variable. For example, the data relating to weight, distance, and volume are examples of continuous data. The quantitative classification becomes the basis for frequency distribution.

When the data is arranged into groups or categories according to conveniently established divisions of the range of the observations, such an arrangement in tabular form is called a frequency distribution. In a frequency distribution, raw data is represented by distinct groups which are known as **classes.** The number of observations that fall into each of the classes is known as **frequency.** Thus, a frequency distribution has two parts, on its left there are classes and on its right there are frequencies.

When data is described by a continuous variable it is called continuous data and when it is described by a discrete variable, it is called **discrete data.** The following are the two examples of discrete and continuous frequency distributions.

No. of employees	No. of companies	Age (Years)	No. of workers
110	25	20-25	15
120	35	25-30	22
130	70	30-35	38
140	100	35-40	47
150	18	40-45	18
160	12	45-50	10

Discrete frequency distribution Continuous frequency distribution

Activity A

What do you understand by classification of data?

Why classification is necessary?

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Activity B

With the help of a suitable example, illustrate the difference between qualitative and quantitative data.

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6.5 CONSTRUCTION OF A DISCRETE FREQUENCY DISTRIBUTION

The process of preparing a frequency distribution_ is very simple. In the case of discrete data, place all possible values of the variable in ascending order in one column, and then prepare another column of `Tally' mark to count the number of times a particular value of the variable is repeated. To facilitate counting, block of five `Tally' marks are prepared and some space is left in between the blocks. The frequency column refers to. the number of `Tally' marks, a particular class will contain. To illustrate the construction of a discrete frequency distribution, consider a sample study in which 50 families were surveyed to find the number of children per family. The data obtained are:

3 2 2 1 3 4 2 1 3 4 5 0 2
1 2 3 3 2 1 1 2 3 0 3 2 1
4 3 5 5 4 3 6 5 4 3 1 0 6
5 4 3 1 2 0 1. 2 3 4 5

To condense this data into a discrete frequency distribution, we shall take the help of `Tally' marks as shown below:

No. of Children	No. of families	Frequency
0	𝍸	4
1	𝍸 𝍸	9
2	𝍸 𝍸	10
3	𝍸 𝍸 ||	12
4	𝍸 ||	7
5	𝍸 |	6
6	||	2
		Total 50

6.6 CONSTRUCTION OF A CONTINUOUS FREQUENCY DISTRIBUTION

In constructing the frequency distribution for continuous data, it is necessary to clarify some of the important terms that are frequently used.

Class Limits. Class limits denote the lowest and highest value that can be included in the class. The two boundaries (i.e., lowest and highest) of a class are known as the lower limit and the upper limit of the class. For example, in the class 60-69, 60 is the lower limit and 69 is the upper limit or we can say that there can be no value if! that class which is less than 60 and more than 69.

Class Intervals. The class interval represents the width (span or size) of a class. The width may be determined by subtracting the lower limit of one class from the lower limit of the following class (alternatively successive upper limits may be used). For example, if the two classes are 10-20 and 20-30, the width of the class interval would be the difference between the two successive lower limits, i.e., 20-10 = 10 or the difference between the upper limit and lower limit of the same class, i.e., 20-10 = 10.

Class Frequency. The number of observations falling within a particular class is called its class frequency or simply frequency. Total frequency (sum of all the frequencies) indicate the total number of observations considered in a given frequency distribution.

Class Mid-point. Mid-point of a class is defined as the sum of two successive lower limits divided by two. Therefore, it is the value lying halfway between the lower and upper class limits. In the example taken above the mid-point would be (10+20)/2 = 15 corresponding to the class 10-20 and 25 corresponding to the class 20-30.

Type of Class Interval. There are different ways in which limits of class intervals can be shown such as:

i) Exclusive and Inclusive method, and

ii) Open-end

Exclusive Method. The class intervals are so arranged that the upper limit of one class is the lower limit of the next class. The following example illustrates this point.

Sales (Rs. thousands)	No. of firms	Sales (Rs. thousands)	No. of firms
20-25	20	35-40	27
25-30	28	40-45	12
30-35	35	45-50	8

In the above example there are 20 firms whose sales are between Rs.20,000 and Rs. 24,999. A firm with sales of exactly Rs. 25 thousand would be included in the next class viz. 25-30. Therefore in the exclusive method, it is always presumed that upper limit is excluded.

Inclusive Method. In this method, the upper limit of one class is included in that class itself. The following example illustrates this point.

Sales (Rs. thousands)	No. of firms	Sales (Rs. thousands)	No. of firms
20-24.999	20	35-39.999	27
25-29.999	28	40-44.999	12
30-34.999	35	45-49.999	8

In this example, there are 20 firms whose sales are between Rs. 20,000 and Rs. 24,999. A firm whose sales are exactly Rs. 25,000 would be included in the next class. Therefore in the inclusive method, it is presumed that upper limit is included.

It may be observed that both the methods give the same class frequencies, although the class intervals look different. Whenever inclusive method is used for equal class intervals, the width of class intervals can be obtained by taking the difference between the two lower limits (or upper limits).

Open-End. In an open-end distribution, the lower limit of the very first class and upper limit of the last class is not given. In distribution where there is a big gap between minimum and maximum values, the open-end distribution can be used such as in income distributions. The income disparities, of residents of a region may vary between Rs. 800 to Rs. 50,000 per month. In such a case, we can form classes like:
Less than Rs. 1,000

1,000-2,000

2,000-5,000

5,000-10,000

10,000-25,000

25,000 and above

Remark. To ensure continuity and to get correct class intervals, we shall adopt exclusive method. However, if inclusive method is suggested then it is necessary to make an adjustment to determine the class interval. This can be done by taking the average value of the difference between the lower limit of the succeeding class and the upper limit of the class. In terms of formula:

$$\text{Correction factor} = \frac{\text{Lower Limit of second class - Upper Limit of the first class}}{2}$$

This value so obtained is deducted from all lower limits and added to all upper limits. For instance, the example discussed for inclusive method can easily be converted into exclusive case. Take the difference between 25 and 24,999 and divide it by 2. Thus correction factor becomes (25-24,999)/2 = 0.0005. Deduct this value from lower limits and add it to upper limits. The new frequency distribution will take the following form:

Sales (Rs. thousand)	No. of firms	Sales (Rs. thousand)	No. of firms
19.9995-24.9995	20	34.9995-39.9995	27
24.9995-29.9995	28	39.9995-44.9995	12
29.9995-34.9995	35	44.9995-49.9995	8

6.7 GUIDELINES FOR CHOOSING THE CLASSES

The following guidelines are useful in choosing the class intervals.

1 The number of classes should not be too small or too large. Preferably, the number of classes should be between 5 and 15. However, there is no hard and fast rule about it. If the number of observations is smaller, the number of classes formed should be towards the lower side of this limit and when the number of observations increase, the number of classes formed should be towards the upper side of the limit.

2 If possible, the widths of the intervals should be numerically simple like 5, 10, 25 etc. Values like 3, 7, 19 etc. should be avoided.

3 It is desirable to have classes of equal width. However, in case of distributions having wide gap between the minimum and maximum values, classes with unequal class interval can be formed like income distribution.

4 The starting point of a class should begin with 0, 5, 10 or multiples thereof. For example, if the minimum value is 3 and we are taking a class interval of 10, the first class should be 0-10 and not 3-13.

5 The class interval should be determined after taking into consideration the minimum and maximum values and the number of classes to be formed. For example, if the income of 20 employees in a company varies between Rs. 1100 and Rs. 5900 and we want to form 5 classes, the class interval should be 1000

$$\frac{(5900 - 1100)}{1000} = 4.8 \text{ or } 5$$

All the above points can be explained with the help of the following example wherein the ages of 50 employees are given:

22	21	37	33	28	42	56	33	32	59
40	47	29	65	45	48	55	43	42	40
37	39	56	54	38	49	60	37	28	27
32	33	47	36	35	42	43	55	53	48
29	30	32	37	43	54	55	47	38	62

In order to form the frequency distribution of this data, we take the difference between 60 and 21 and divide it by 10 to form 5 classes as follows:

Activity C

Distinguish between the following:

i) Discrete and continuous frequency distributions.

ii) Class limits and class intervals.

iii) Inclusive and Exclusive method.

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6.8 CUMULATIVE AND RELATIVE FREQUENCIES

It is often useful to express class frequencies in different ways. Rather than listing the actual frequency opposite each class, it may be appropriate to list either cumulative, frequencies or relative frequencies or both.

Cumulative Frequencies. As its name indicates, it cumulates the, frequencies, starting at either the lowest or highest value. The cumulative frequency of a given class interval thus represents the total of all the previous class frequencies including the class against which it is written. To illustrate the concept of cumulative frequencies, consider the following example

Monthly salary (Rs.)	No. of employees	Monthly salary (Rs.)	No. of employees
1000-1200	5	2000-2200	25
1200-1400	14	2200-2400	22
1400-1600	23	2400-2600	7
1600-1800	50	2600-2800	2
1800-2000	52		

If we keep on adding the successive frequency of each class starting from the frequency of the very first class, we shall get cumulative frequencies as shown below:

Monthly salary (Rs.)	No. of employees	Cumulative
1000-1200	5	5
1200-1400	14	19
1400-1600	23	42
1600-1800	50	92
1800-2000	52	144
2000-2200	25	169
2200-2400	22	191
2400-2600	7	198
2600-2800	2	200
Total	200	

Relative Frequencies. Very often, the frequencies in a frequency distribution are converted to relative frequencies to show the percentage for each class. If the frequency of each class is divided by the total number of observations (total frequency), then this proportion is referred to as relative frequency. To get the percentage for each class, multiply the relative frequency by 100; For the above example, the values computed for relative frequency and percentage are shown below:

Monthly salary (Rs.)	No. of employees	Relative frequency	Percentage
1000-1200	5	0.025	2.5
1200-1400	14	0.070	7.0
1400-1600	23	0.115	11.5
1600-1800	50	0.250	25.0
1800-2000	52	0.260	26.0
2000-2200	25	0.125	12.5
2200-2400	22	0.110	11.0
2400-2600	7	0.035	3.5
2600-2800	2	0.010	1.0
	200	1.000	100%

There are two important advantages in looking at relative frequencies (percentages) instead of absolute frequencies in a frequency distribution.

1 Relative frequencies facilitate the comparisons of two or more than two sets of data.

2 Relative frequencies constitute the basis of understanding the concept of probability.

Activity D

With the help of an example, explain the concept of relative frequency.

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6.9 CHARTING OF DATA

Charts of frequency distributions which cover both diagrams and graphs are useful because they enable a quick interpretation of the data. A frequency distribution can be presented by a variety of methods. In this section, the following four popular methods of charting frequency distribution are discussed in detail.

i) Bar Diagram

ii) Histogram

iii) Frequency Polygon

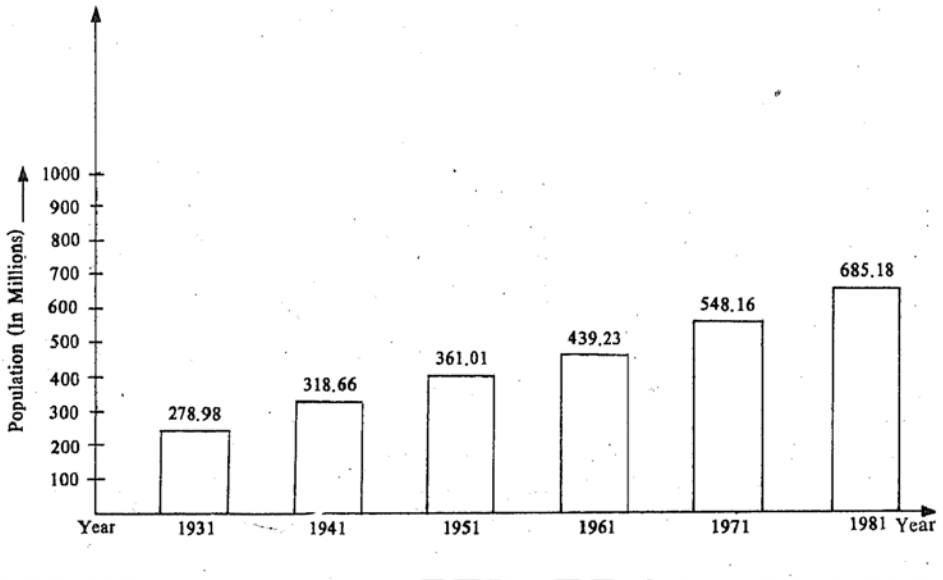
iv) Ogive or Cumulative Frequency Curve

Bar Diagram. Bar diagrams are most popular. One can see numerous such diagrams in newspapers, journals, exhibitions, and even on television to depict different characteristics of data. For example, population, per capita income, sales and profits of a company can be shown easily through bar diagrams. It may be noted that a bar is a thick line whose width is shown to attract the viewer. A bar diagram may be either vertical or horizontal.

In order to draw a bar diagram, we take the characteristic (or attribute) under consideration on the X-axis and the corresponding value on the Y-axis. It is desirable to mention the value depicted by the bar on the top of the bar.

To explain the procedure of drawing a bar diagram, we have taken the population figures (in millions) of India which are given below:

Bar Diagram



Take the years on the X-axis and the population figure on the Y-axis and draw a bar to show the population figure for the particular year. This is shown below: As can be seen from the diagram, the gap between one bar and the other bar is kept equal. Also the width of different bars is same. The only difference is in the length of the bars and that is why this type of diagram is also known as one dimensional.

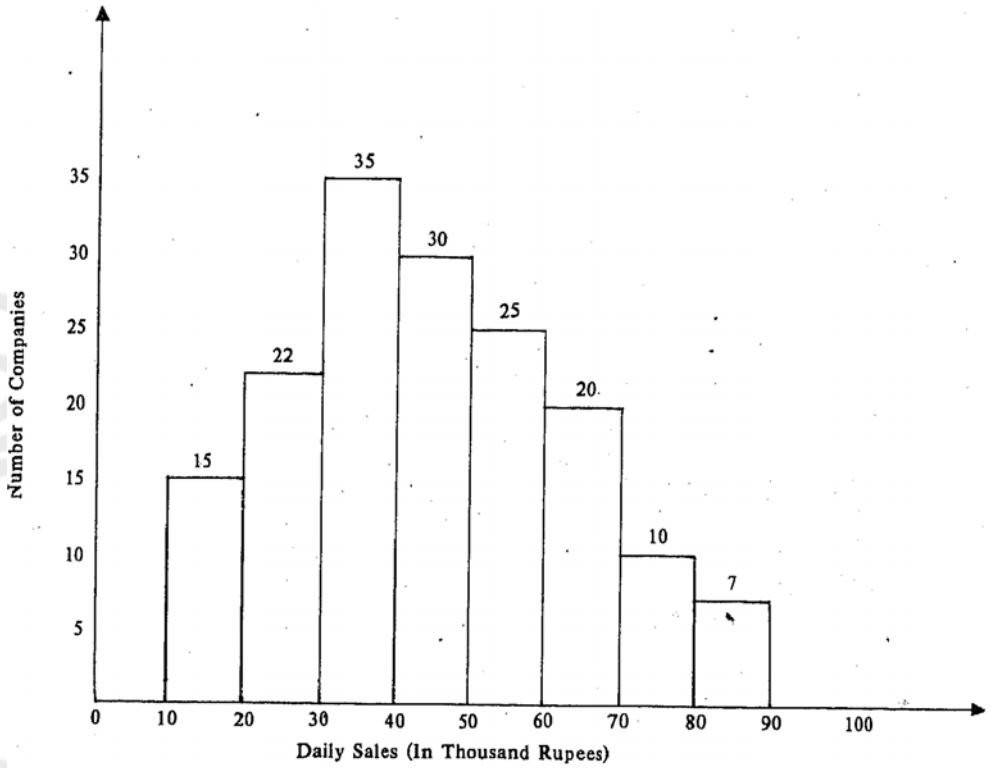
Histogram. One of the most commonly used and easily understood methods for graphic presentation of frequency distribution is histogram. A histogram is a series of rectangles having areas that are in the same proportion as the frequencies of a frequency distribution.

To construct a histogram, on the horizontal axis or X-axis, we take the class limits of the variable and on the vertical axis or Y-axis, we take the frequencies of the class intervals shown on the horizontal axis. If the class intervals are of equal width, then the vertical bars in the histogram are also of equal width. On the other hand, if the class intervals are unequal, then the frequencies have to be adjusted according to the width of the class interval. To illustrate a histogram when class intervals are equal, let us consider the following example.

Daily sales (Rs. thousand)	No. of companies	Daily sales (Rs. thousand)	No. of companies
10-20	15	50-60	25
20-30	22	60-70	20
30-40	35	70-80	16
40-50	30	80-90	7

In this example, we may observe that class intervals are of equal width. Let us take class intervals on the X-axis and their corresponding frequencies on the Y-axis. On each class interval (as base), erect a rectangle with height equal to the frequency of that class. In this manner we get a series of rectangles each having a class interval as its width and the frequency as its height as shown below:

Histogram with Equal Class Intervals



It should be noted that the area of the histogram represents the total frequency as distributed throughout the different classes.

When the width of the class intervals are not equal, then the frequencies must be adjusted before constructing the histogram.

The following example will illustrate the procedure:

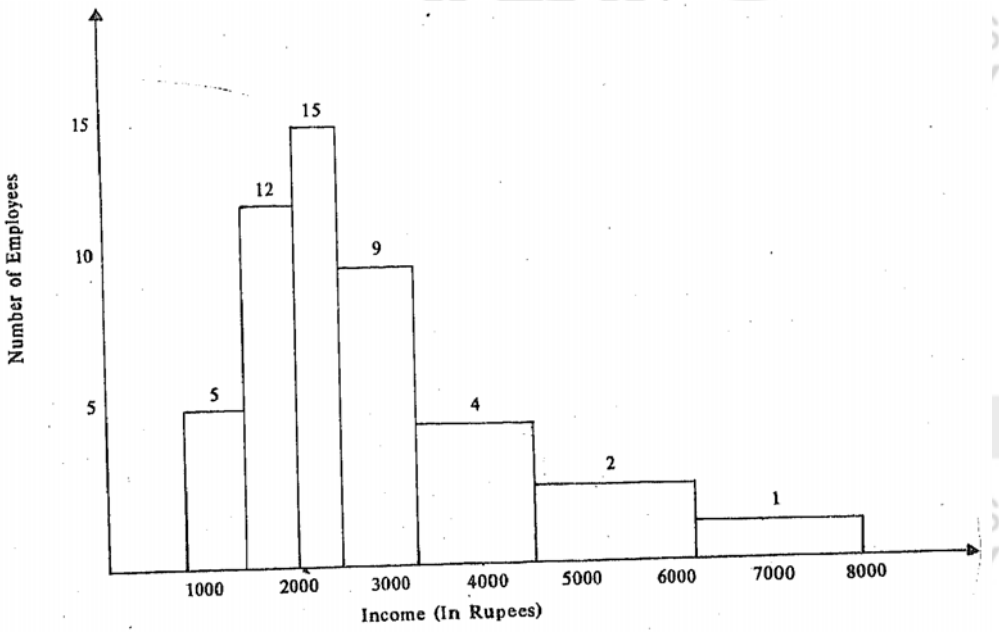
Income (Rs.)	No. of employees	Income (Rs.)	No. of
1000-1500	5	3500-5000	12
1500-2000	12	5000-7000	8
2000-2500	15	7000-8000	2
2500-3500	18		

As can be seen, in the above example, the class intervals are of unequal width and hence we have to find out the adjusted frequency of each class by taking the class with the lowest class interval as the basis of adjustment. For example, in the class 2500-3500, the class interval is 1000 which is twice the size of the lowest class interval, i.e., 500 and therefore the frequency of this class would be divided by two, i.e., it would be 18/2 = 9. In a similar manner, the other frequencies would be obtained. The adjusted frequencies for various classes are given below:

Income (Rs.)	No. of employees	Income (Rs.)	No. of employees
1000-1500	5	3500-5000	4
1500-2000	12	5000-7000	2
2000-2500	15	7000-8000	1
2500-3500	9		

The histogram of the above distribution is shown below:

Histogram with Unequal Class Intervals



It may be noted that a histogram and a bar diagram look very much alike but have distinct features. For example, in a histogram, the rectangles are adjoining and can be of different width whereas in bar diagram it is not possible.

Activity E

Draw a sketch of a histogram and a bar diagram and explain the difference between the two.

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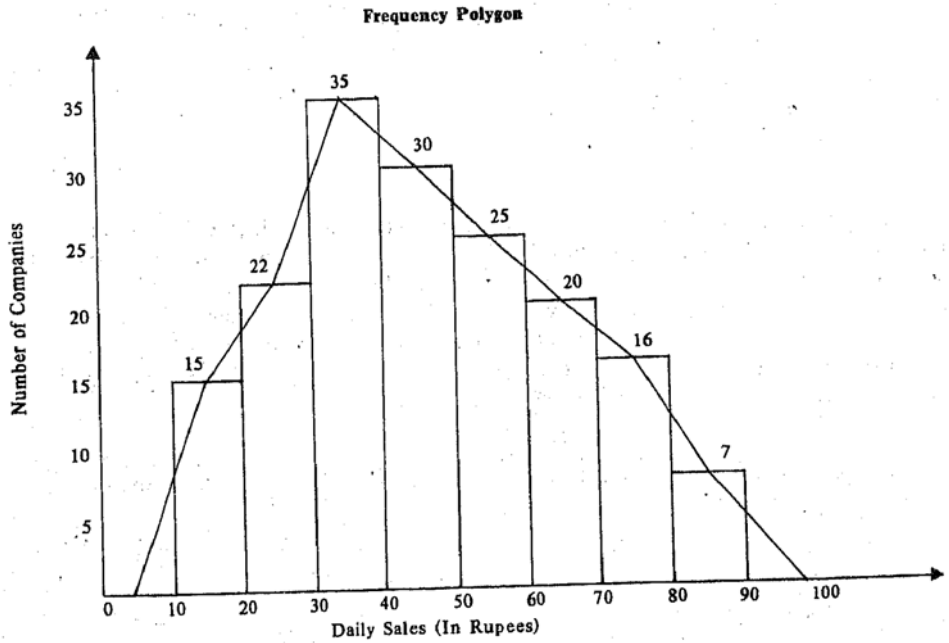
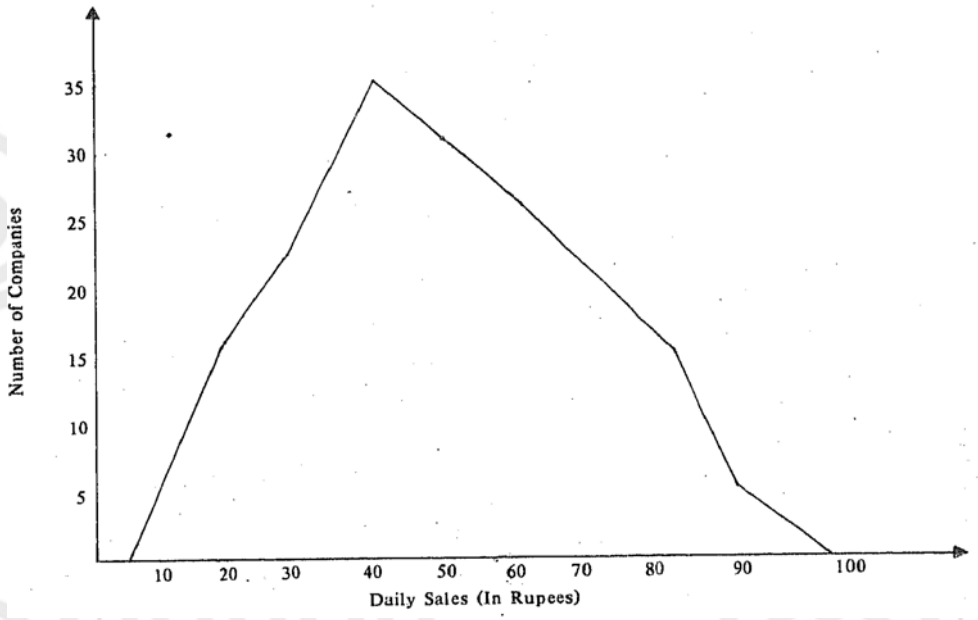
Frequency Polygon. The frequency polygon is a graphical presentation of frequency distribution. A polygon is a many sided closed figure. A frequency polygon is



constructed by taking the mid-points of the upper horizontal side of each rectangle on the histogram and connecting these mid-points by straight lines. In order to close the polygon, an additional class is assumed at each end, having a zero frequency. To illustrate the frequency polygon of this distribution is shown on page 22.

If we draw a smooth curve over these points in such a way that the area included under the curve is approximately the same as that of the polygon, then such a curve is m known as frequency curve. The following figure shows the same data smoothed out to form a frequency curve, which is another form of presenting the same data.

Frequency Curve



Remark. The histogram is usually. associated with discrete data and a frequency polygon is appropriate for continuous data. But this distinction is not always followed in practice and many factors may influence the choice of graph.

The frequency polygon and frequency curve have a special advantage over the histogram particularly when we want to compare two or more frequency distributions.

Activity F

What is the procedure of making a frequency polygon?

Illustrate with the help of suitable data.

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Ogives or Cumulative Frequency Curve. An ogive is the graphical presentation of a cumulative frequency distribution and therefore when the graph of such a distribution is drawn, it is called **cumulative frequency curve or ogive.** There are two methods of constructing ogive; viz.,

i) Less than ogive

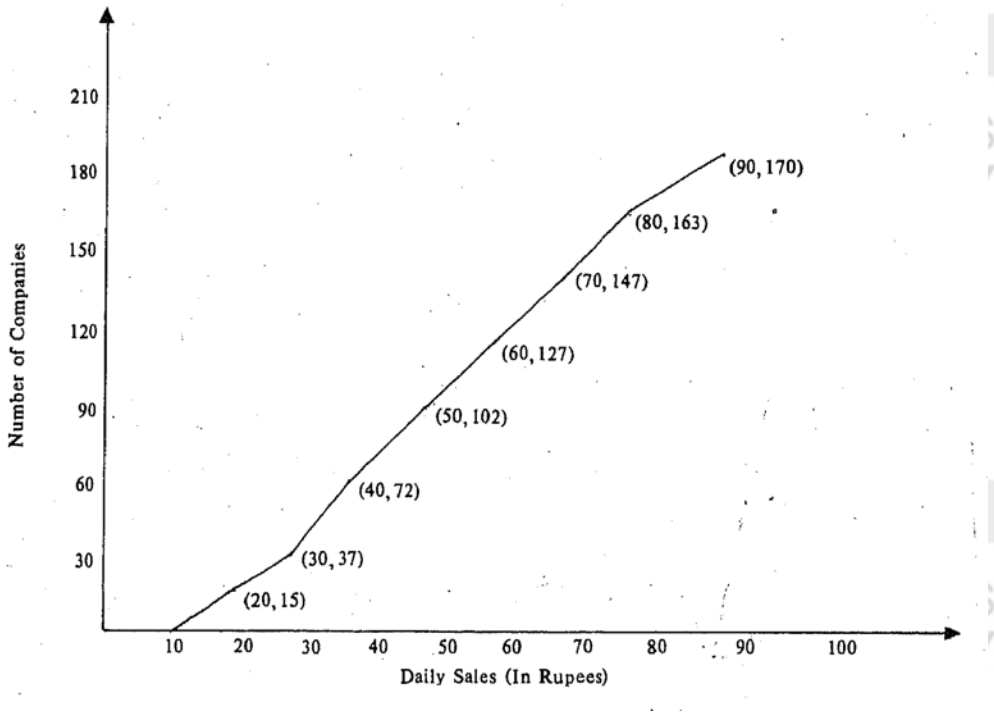
ii) More than ogive

Less than Ogive. In this method, the upper limit of the various classes are taken on the X-axis and the frequencies obtained by the process of cumulating the preceding frequencies on the Y-axis. By joining these points we get less than ogive. Consider the example relating to daily sales discussed earlier.

Daily sales (Rs. thousand)	No. of companies	Daily sales (Rs. thousand)	No. of companies
10-20	15	Less than 20	15
20-30	22	Less than 30	37
30-40	35	Less than 40	72
40-50	30	Less than 50	102
50-60	25	Less than 60	127
60-70	20	Less than 70	147
70-80	16	Less than 80	163
80-90	7	Less than 90	170

The less than Ogive Curve is shown below:

Less than Ogive

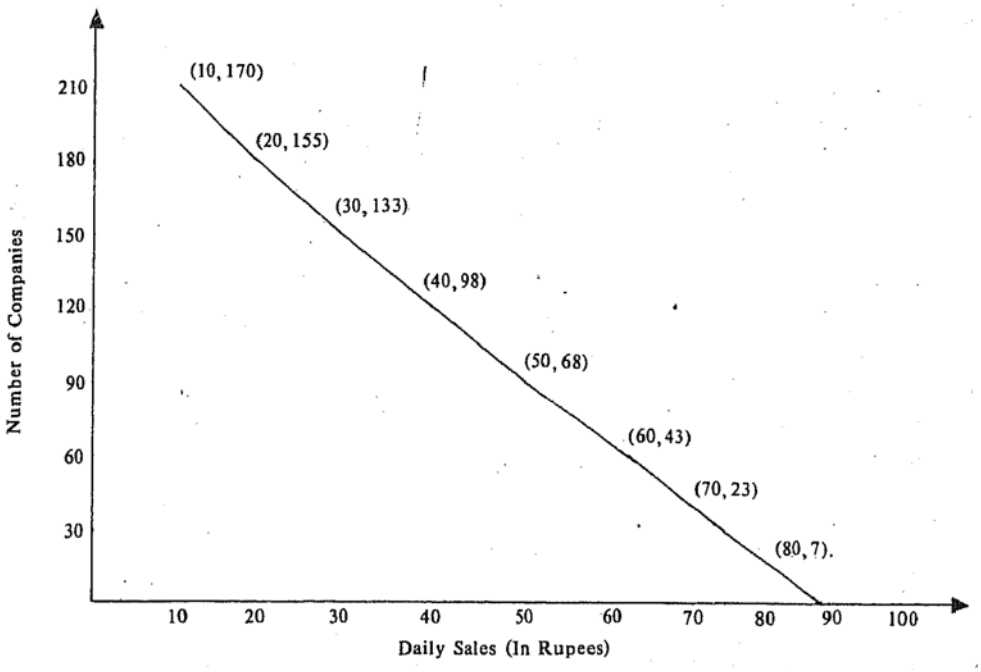


More than Ogive. Similarly more than ogive or cumulative frequency curve can be drawn by taking the lower limits on X-axis and cumulative frequencies on the Y-axis. By joining these points, we get more than ogive. The table and the curve for this case is shown below:

Daily sales (Rs, thousand)	No. of companies	Daily sales (Rs. thousand)	Cumulative frequency
10-20	15	More than 10	170
20-30	22	More than 20	155
30-40	35	More than 30	133
40-50	30	More than 40	98
50-60	25	More than 50	68
60-70	20	More than 60	43
70-80	16	More than 70	23
80-90	7	More than 80	7

The more than ogive curve is shown below:

More than Ogive



The shape of less than ogive curve would be a rising one whereas the shape of more than ogive curve should be falling one.

The concept of ogive is useful in answering questions such as: How many companies are having sales less than Rs. 52,000 per day or more than Rs. 24,000 per day or between Rs. 24,000 and Rs. 52,000 ?

Activity G

With the help of an example, explain the concept of less than ogive and more than ogive.

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6.10 SUMMARY

Presentation of data is provided through tables and charts. A frequency distribution is the principal tabular summary of either discrete or continuous data. The frequency distribution may show actual, relative or cumulative frequencies. Actual and relative frequencies may be charted as either histogram (a bar chart) or a frequency polygon. Two graphs of cumulative frequencies are: less than ogive or more than ogive.

6.11 KEY WORDS

Bar Chart is a thick line where the length of the bars should be proportional to the magnitude of the variable they present.

Class Interval represents the width of a class.

Class Limits denote the lowest and highest value that-can be included in the class.

Continuous Data can take all values of the variable.

Discrete Data refers to quantitative data that are limited to certain numerical values of a variable.

Frequency Distribution is a tabular presentation where a number of observations with similar or closely related values are put in groups.

Qualitative Data is characterised by exhaustive and distinct categories that do not possess magnitude.

Quantitative Data possess the characteristic of numerical magnitude.

6.12 SELF-ASSESSMENT EXERCISES

1 Explain the purpose and methods of classification of data giving suitable examples.

2 What are the general guidelines of forming a frequency distribution with particular reference to the choice of class intervals and number of classes?

3 Explain the various diagrams and graphs that can be used for charting a frequency distribution.

4 What are ogives? Point out the role. Discuss the method of constructing ogives with the help of an example.

5 The following data relate to the number of family members in 30 families of a village.

4	3	2	3	4	5	5	7	3	2
3	4	2	1	1	6	3	4	5	4
2	7	3	4	5	6	2	1	5	3

Classify the above data in the form of a discrete frequency distribution.

6 The profits (Rs. lakhs) of 50 companies are given below:

20	12	15	27	28	40	42	35	37	43
55	65	53	62	29	64	69	36	25	18
56	55	43	35	26	21	48	43	50	67
14	23	34	59	68	22.	41	42	43	52
60	26	26	37	49	53	40	20	18	17

Classify the above data taking first class as 10-20 and form a frequency distribution.

7 The income (Rs.) of 24 employees of a company are given below:

1800	1250	1760	3500	6000	2500
2700	3600	3850	6600	3000	1500
4500	4400	3700	1900	1850	3750
6500	6800.	5300	2700	4370	3300

Form a continuous frequency distribution after selecting a suitable class interval.

8 Draw a histogram and a frequency polygon from the following data:

Marks	No. of students	Marks	No. of students
0-20	8	60- 80	12
20-40	12	80-100	3
40-60	15		

9 Go through the following data carefully and then construct a histogram.

Income (Rs.)	No. of persons	Income (Rs.)	No. of persons.
500 1000	18	3000-4500	
1000-1500	20	4500-5000	12
1500-2500	30	5000-7000	5
2500-3000	25		

10 The following data relating to sales of 100 companies is given below:

Sales (Rs. lakhs)	No. of companies	Sales (Rs. lakhs)	No. of companies
5–10	5	25–30	18
10–15	12	30–35	15
15–20	13	35–40	10
20–25	20	40–45	7

Draw less than and more than ogives. Determine the number of companies whose sales are (i) less than Rs.13 lakhs (ii) more than 36 lakhs and (iii) between Rs. 13 lakhs and Rs. 36 lakhs.

4.13 FURTHER READINGS

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