UNIT 2 RELATIONS AND FUNCTIONS*

Structure

2.0 OBJECTIVES

You studied about sets in the previous unit. In this unit we shall talk about some ways that sets are combined and how new configurations emerge from the combination of sets, and ways that sets are combined. After reading the unit, you will be able to:

- find Cartesian Product of two Sets;
- describe the concept for Relations and Functions;
- explain the difference between Functions and Correspondences;
- define Real Space and Point-Sets; and
- explain set-functions.

2.1 INTRODUCTION

In the previous unit – the first of the course – the foundation was laid by introducing you to the concept of sets. We also looked at subsets, supersets and power sets. Please keep those concepts in mind since the present unit builds upon the concepts used in the previous unit. The first unit also familiarised you with some set operations like union, intersection and difference. This unit begins by introducing you to another operation involving sets, namely, the product of sets. Building upon this the unit proceeds to discuss subsets of product of sets. These subsets are called relations.

*Contributed by Shri Anup Chatterjee

Subsequently, the unit explains the concept of functions, which are particular type of relations with certain properties. Now functions are based upon certain components called domain and co-domain. These domain and co-domain are sets. Depending on whether the co-domain or domain have elements which are themselves sets, we get the concepts of correspondence and set-functions. The unit during the discussion about functions describes the various types of functions. You know that sets are collection of objects. Suppose these elements are real numbers. Then we have the set of real numbers. The unit discusses the properties of this set and the product of sets of real numbers. The unit also discusses some properties of the sets formed by the product of sets of real numbers.

2.2 ORDERED PAIRS AND CARTESIAN PRODUCTS

2.2.1 Ordered Pairs

When we write a set, say {a, b}, we do not worry about the order in which the elements of the set are written. The set {a, b} is the same set as set {b, a}. In this case, we say the pair of elements (a, b) is an unordered pair. If the ordering does matter, we can write two different ordered pairs (a, b) and (b, a). They have the property that (a, b) ≠(b, a). Here we have talked of ordered pairs, but we could easily have mentioned ordered triple, or quadruples or quintuples. In general, if there are *n* elements, we talk of an *n*-tuple. If the elements are ordered, we talk of an ordered *n*-tuple. Generally, ordered pairs, triples or *n*-tuples are called ordered sets. Ordered sets are denoted by parentheses (..) rather than by curly braces {..}.We can give an example of the use of ordered pairs.

Let us consider an ordered pair as (x, y), where $x \in X$ and $y \in Y$. Further assume that $X = \{x_1, x_2, x_3 \ldots x_n\}$ the age of students in a class, and $Y = \{y_1, y_2, y_3 \ldots y_n\}$ the weight of students in the class. Thus, (x, y) will give the pair of attributes, age and weight of a student in the class. To understand this formulation as an ordered pair, you have to assign numerical values to x and y.

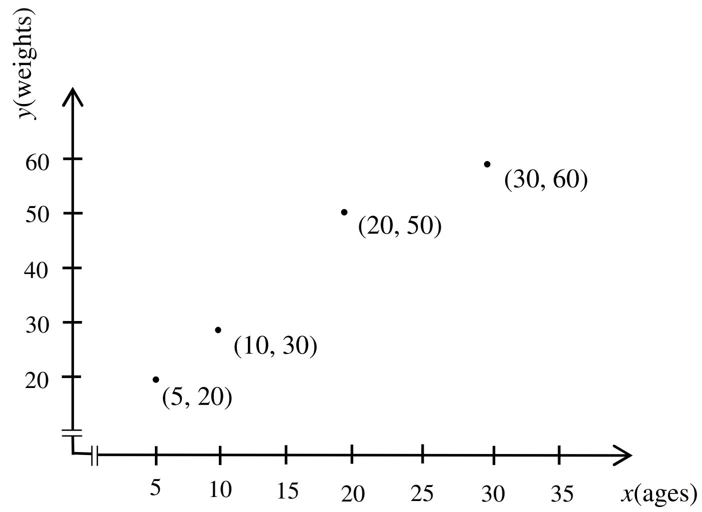

Figure 2.1

Suppose to take a specific case, $x = 12$ years and $y = 50$ kg. Then $\{x, y\}$ represents the ordering pair of age and weight. The first entry being age and the second being weight of a student. If you change the ordering and write $\{50,12\}$ instead of $\{12,50\}$, then you will get the student's age to be 50 years and weight to be 12 kg. To strengthen our understanding of ordered pair as the elements of a set, look at figure 2.1. The *x*-axis of the figure records the age of the students and *y*-axis the weights. The broken lines of the axis indicates that we are considering the variables after a gap from the initial values (*i.e.*, we do not consider the values from 0 to 5 on *x*-axis and 0-19 on *y*-axis). The points in Figure 2.1 show the ordered pairs. Note that (5, 20) will be different from (20, 5) as the elements of the sets *X* and *Y* will differ according to a changed ordering.

Definition: Let *X* and *Y* be any two non-empty sets, such that $x \in X$ and $y \in Y$. An ordered pair (x, y) is a pair such that $(x, y) \neq (y, x)$, if $x \neq y$. This implies that $(x, y) = (y, x)$, only if $x = y$. Also, $(p, q) = (r, s)$, if and only if, $p = r$ and $q = s$.

Note: In the ordered pair (x, y), *x* is known as the first element and *y*, the second element.

2.2.2 Cartesian Product of Sets

Consider two sets *X* and *Y* with their elements. The set of all ordered pairs (x, y), where $x \in X$ and $y \in Y$ is called the Cartesian product of *X* and *Y*. We write the cartesian product as

$$X \times Y = \{(x, y) \mid x \in X \text{ and } y \in Y\}$$

and read as cartesian product *X* cross *Y* equals to (x, y) such that (or given that) x belongs to *X* and y belongs to *Y*.

Example

Let $A = \{a, b, c\}$ and $X = \{2, 5\}$. Then

$A \times X = \{(a, 2), (a, 5), (b, 2), (b, 5), (c, 2), (c, 5)\}$.

From the above results, we can get the following:

1) More generally, we can define the ordered *n-tuples* $(x_1, x_2, ..., x_n)$ and the Cartesian product $A_1 \times A_2 \times A_3 \ldots \times A_n (n \in N, n > 2)$ in a similar way.
2) For $A \times A$ we often write A^2, and similarly, A^n stands for $\underbrace{A \times A \times ... \times A}_{n}$.

2.3 RELATIONS

Any subset of a Cartesian product of sets is called a relation. That is, a relation is a set of ordered pairs. If *X* and *Y* are two non-empty sets and there is a set ρ such that $\rho \subseteq X \times Y$, we say that ρ is a relation from *X* to *Y* (we can also say that ρ is a relation between the elements of *X* and *Y*). If $\rho \subseteq X \times X$, we say that ρ is a relation in *X*.

ExampleLet $X = \{1, 3, 5\}$ and $Y = \{2, 4, 6\}$, and *S* be the Cartesian product of set X and set Y.

Then, $S = \{(1,2); (1,4); (1,6); (3,2); (3,4); (3,6); (5,2); (5,4); (5,6)\}$.

Our objective is to get the relation (ρ) from *S*. For this, by imposing one of the following restrictions, we pick up a subset of *S*.

i) $(x+y)$ is an exact multiple of 3

ii) $(x+y)\leq 7$

iii) $x > y$

For these three cases we define the relations as

i) $(1,2);(3,6);(5,4)$

ii) $(1,2);(1,4);(1,6)$; (3, 2); (3, 4); (5, 2)

iii) $(3,2);(5,2);(5,4)$

Examples of some Relations

1) The relation of *equality* in a nonempty set *X*.

$\rho_1 = \{(x,x): x\in X\}$. Thus $(x, y) \in \rho_1 \subseteq X\times X$ if and only if $x = y$.

2) The relation of *divisibility* in N.

$\rho_2 = \{(m, n) \in N\times N : \exists k\in N\, n = k \cdot m\}$

Thus, $(m, n) \in \rho_2 \subseteq N\times N$ iff $m \mid n$, that is *n* can be divided by *m* without remainder (*n* is divisible by *m*).

Note: ∃ symbol stands for "there exists (at least one)"

3) The relation of '*is less than*' in $\mathbb{R}$.

$\rho_3 = \{(x, y) \in R\times R : x<y\}$. Thus $(x, y) \in \rho_3 \subseteq R\times R$ iff $y-x$ is a positive number.

4) The relation of "*greater or equal*" in $\mathbb{R}$.

$\rho_4 = \{(x, y) \in R\times R : x \geq y\}$, thus $(x, y) \in \rho_4 \subseteq R\times R$ iff $x-y$ is a nonnegative number.

5) Relation between the elements of the set *T* of all triangles of the plane and the elements of the set $\mathbb{R}_0^+$ of all nonnegative numbers,

$\rho_5 = \{(t, a) \in T\times \mathbb{R}_0^+$: the area of the triangle t is $a\}$.

6) Relation between the elements of the set $\mathbb{R}_0^+$ of all nonnegative numbers and the elements of the set *T* of all triangles of the plane, $\rho_6 = \{(a, t) \in \mathbb{R}_0^+ \times T$: the area of the triangle t is $a\}$.

7) Relation between the elements of the set *C* of all circles of the plane and the elements of the set *L* of all lines of the plane, $\rho_7 = \{(c, l) \in C\times L$: l is a tangent of $c\}$.

2.3.1 Domain and Range of Relations

Let *X* and *Y* be sets and ρ be a relation from *X* to *Y* (*i.e.*, $\rho \subseteq X\times Y$).

1) **Domain of the relation ρ:**

Domain of a relation ρ is the set of all first elements of the ordered pairs in a relation ρ from set X to Y. It is defined by $D(\rho) := \{x \in X : \exists y \in Y \ (x, y) \in \rho\}$.

2) **Range of the relation ρ:**

Range of a relation ρ is the set of all second elements of the ordered pairs in a relation ρ from set X to Y. It is defined by $R(\rho) := \{y \in Y : \exists x \in X \ (x, y) \in \rho\}$.

2.3.2 Properties of Relations

Let X be a set and ρ be a relation in X (*i.e.*, $\rho \subseteq X^2$).

1) **Reflexivity:**
ρ is called *reflexive*, if $\forall x \in X \ (x, x) \in \rho$.

Note: symbol ∀ stands for "for all."

2) **Irreflexivity:**
ρ is called *irreflexive*, if $\forall x \in X \ (x, x) \notin \rho$.

3) **Symmetry:**
ρ is called *symmetric*, if $\forall (x, y) \in \rho \ (y, x) \in \rho$, that is $(x, y) \in \rho$ implies $(y, x) \in \rho$.

4) **Antisymmetry:**
ρ is called *antisymmetric*, if $(x, y) \in \rho$ and $(y, x) \in \rho$, implies $x = y$.

5) **Transitivity:**
ρ is called *transitive*, if $(x, y) \in \rho$ and $(y, z) \in \rho$, implies $(x, z) \in \rho$.

2.3.3 Special Relations

Let X be a nonempty set and ρ be a relation in X (*i.e.*, $\rho \subseteq X^2$).

1) **Equivalence Relation:**
We say that ρ is an *equivalence* relation if it is
(a) reflexive, (b) symmetric, and (c) transitive.

2) **Order Relation:**
We say that ρ is an *order* relation if it is
(a) reflexive, (b) antisymmetric, and (c) transitive.

We say that the order relation ρ is a *total (linear) order* relation if for each $(x, y) \in X^2$ $(x, y) \in \rho$ or $(y, x) \in \rho$ is satisfied; otherwise ρ is said to be a *partial order* relation.

3) **Inverse Relation**

Let X and Y be two nonempty sets and ρ be a relation from X to Y (*i.e.*, $\rho \subseteq X \times Y$). The inverse of ρ (denoted by ρ^{-1}) is defined by $\rho^{-1} = \{(y, x) \in Y \times X : \ (x, y) \in \rho\}$

Classification of Sets

Let X be a nonempty set. A set of subsets of X, denoted by say Z is called a classification of X, if the following properties are satisfied:

i) $\forall A \in Z, A$ is nonempty subset of X,

ii) (A, $B \in Z$ and $A \neq B$) implies $A \cap B = \phi$,

iii) $\bigcup Z$ (*i.e.*, union of a set Z of subsets) = X.

The elements of Z are called the *classes* of the *classification*.

Equivalence Classes

Let X be a nonempty set and ρ be an equivalence relation in X.

For each $x \in X$ we define $A_x := \{y \in X : (x, y) \in \rho\} \in \mathcal{P}(X)$ (*i.e.*, the power set of X). A_x is called the *equivalence class* of x.

Check Your Progress 1

1) What is a Relation?

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2) What is Equivalence Relation?

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3) Define the concepts of Domain and Range of a relation.

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2.4 FUNCTIONS

We have seen above that an ordered pair associates y value with that of x. Therefore, we can postulate a relationship between y and x. For a given value of x one or more values of y can be specified by such a relation. Consider the following examples to understand this statement.

Example: Suppose that we have a set $X = \{2, -2\}$ and relation postulated between y and x is $y = (x^2)$, where $x \in X$. Then for the value of $x = 2$, $y = 4$, and for the other element of x, *i.e.*, $x = -2$, the value of y is also 4. Thus, two values of x have been related to one value of y.

Example: We have considered $X = \{1,3,5\}$ and $Y = \{2,4,6\}$ as two sets earlier as an example. For a relation $x > y$, we found that there were 3 ordered pairs, *viz*., $\{(3,2);(5,2);(5,4)\}$ to satisfy the specified relation. See that a single value of x, *i.e.* $x = 5$, has been associated with 2 values of y, *viz*., 2 and 4.

Observe from the above examples that when the value of x is given, it may not always be possible to determine a unique y from a relation. However, if a relation exists such that for each value of x there exists a unique corresponding value of y, then y is said to be a function of x, denoted as $y = f(x)$. A function is also referred to as a transformation. It is usually denoted by $f : X \to Y$ (reads as f is a function from X to Y). Symbolically, let X and Y be two nonempty sets, then, a relation $f \subseteq X \times Y$ is called a function from X to Y if

(i) $D(f) = X$ (ii) $\forall x \in X$ the set $\{y \in Y : (x, y) \in f\}$ has exactly one element, *i.e.*, $\forall x \in X$ there exists exactly one element $y \in Y$ such that $(x, y) \in f$.

Note: The notations $y = f(x)$ implies "y is a function of x", and $X \to Y$ mean that $(x, y) \in f$. $f(x)$ is the element associated with x. It is also known as the *image* of x under f or the value of f at x. When we write a function as $y = f(x)$, x is referred to as the argument of the function, and y is called the value of the function. In Economics, we often use x as the independent variable while y as dependent variable.

2.4.1 Domain, Range, Target and Codomain of a Function

Let X and Y be two nonempty sets and $y = f(x)$ be a function.

According to the definitions for relations, $D(f)$ is the *domain* of the function f and $R(f) = \{f(x) : x \in D(f)\}$ is the *range* of the function f.

If $X \subseteq W$, we can also write $f : W \supset \to Y$, which means that $D(f) \subseteq W$. Y is then called the *target* of the function f.

The *codomain* of a function $f : X \to Y$ is the set Y. Since the range of f is the set $f(X)$ defined as $\{f(x) : x \in X\}$, it follows that the range of f is always a subset of the codomain of f.

Restrictions of Functions

Let X, Y, A be nonempty sets such that, $A \subseteq X$,and $f : X \to Y$ be a function. The function $g : A \to Y, g(x) = f(x)$ is called the *restriction* of f to A, and we use the notation $f|_A = g$.

2.4.2 Injective, Surjective, Bijective Functions

Let X, Y be two nonempty sets and $f : X \to Y$ be a function.

1) **Injective function:**

   f is *injective* if ∀for all $x, z \in X$, $f(x) = f(z)$ implies $x = z$, *i.e.*, $x \neq z$ implies $f(x) \neq f(z)$. We also say that f is an injection, or a one-to-one correspondence. Thus, a function is one-to-one, if image of each distinct element in its domain under f are distinct.

2) **Surjective function:**

   f is *surjective* if $\forall y \in Y$, there exists $x \in X$ such that $f(x) = y$, that is $R(f) = Y$.

   We also say that f is a surjection, or f is a map onto Y. Thus, a function is surjective (onto) if every element in Y under f is the image of some element in X. Equivalently, the range of the function is equal to its codomain.

3) **Bijective function:**

   f is *bijective* if it is both *injective* and *surjective*. We also say that f is a *bijection* or *one-to-one and onto*. Thus, a function is bijective if for every y in Y there is exactly one x in X such that $f(x) = y$.

Equality of Functions

Let f and g be two functions. f and g are said to be *equal* (*i.e.* $f = g$), iff

(i) $D(f) = D(g)$ and (ii) $\forall x \in X$, $f(x) = g(x)$.

Inverse Function

Consider a bijective function $f : X \to Y$, so that $R(f) = Y$. The relation $f^{-1} \subseteq R(f) \times X$ is a function that we call the *inverse function* of f. That is the *inverse function* of f is the function f^{-1} defined by $f^{-1}: R(f) \to X, f^{-1}(y) = x$, where x is the unique element of X such that $f(x) = y$.

Composition of Functions

Let $f : X \to Y$ and $g : Y \to Z$ be functions.

We define the composition of f and g, denoted by $g \ o \ f$, as a function $g \ o \ f : X \to Z$, given as

$$g \ o \ f(x) = g(f(x)), \forall x \in X.$$

Moreover, we can also have formulations with $f(x) = (x_1, x_2 ...)$ *i.e.*, more than one independent variables related to the dependent variable.

2.4.3 Image and Inverse Image of Sets under Functions

Let $f : X \to Y$ be a function and *A*, *B* be any sets.

1) **Image of set A under *f* :**

   We define the set $f(A) := \{f(x) : x \in A\}$ and call $f(A)$ the image of set *A* under the function f.

   Note that $f(A) = f(A \cap X) \subset f(X) = R(f) \subset Y$.

2) **Inverse image of set *B* under *f* :**

   We define the set $f^{-1}(B) := \{x \in X : f(x) \in B\}$ and call $f^{-1}(B)$ the inverse image of set *B* under the function f.

   Note that $f^{-1}(B) = f^{-1}(B \cap Y) \subset f^{-1}(Y) = f^{-1}(R(f)) = X$.

It is important to see that the set $f^{-1}(B)$ can be defined for any set *B*, even if f is not injective (thus, the inverse function of f does not exist). The notation f^{-1} in "$f^{-1}(B)$" does not mean the inverse function of f. However, we can easily prove that if f is injective, then for any set *B*, $f^{-1}(B)$ is the image of *B* under the *inverse function* of *f* .

Check Your Progress 2

1) What is the definition of a function?

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2) Give an example of Surjective function.

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3) Define an Inverse function.

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2.5 REAL SPACE AND POINT-SETS

We have seen what are sets, relations and functions. Think of the number-line, with zero in the middle and the line extending on either side to infinity. This line depicts the set of real numbers, denoted by R. Now think of an interval [*a*, *b*] which contains all real numbers between *a* and*b*, including *a* and *b*. This interval can be thought to be a subset of R.

So now you understand the concept of a set of real numbers. Measurements in Economics are usually done using subsets of real numbers, so you must realize this set of real numbers is very important. We can think of these sets as set of points, because these are depicted as points on the number line. We can call these sets as point-sets. The number line depicting the real numbers is called the real line.

Let us now make some use of the concept of the Cartesian product of sets. We have this set of real numbers, R. Can we have a Cartesian product of this set with some other set? Of course, we can. What about if the other set is also R, that is R is multiplied by itself? Then we get a set R×R that can be denoted by R^2.

What are the elements of this set? This set consists of all ordered pairs. How do we depict this set graphically? This is usually depicted by the usual *x*-*y* axes that you are familiar with. The *x*-axis depicts real numbers from minus infinity to plus infinity. That is one R. But so does the *y*-axis. The y-axis also depicts numbers from minus infinity to plus infinity, just that these are depicted in a vertical manner. So the four quadrants that emerge consist of ordered pairs of numbers, the first number represents the number on the *x*-axis and the second number, the *y*-axis. The ordered pairs are written (*x*, *y*), that is within parentheses, separated by a comma. However, take care not to confuse an ordered pair with an open interval which is also depicted in the same way. Thus every ordered pair of real numbers is a member of R^2. R^2 is also a point-set.

Think of a consumer who consumes only apples and oranges. Various combinations of apples and oranges are pairs of numbers. Each of this pair is a member of R^2. But for this, we have to write one of the commodities first, and the other commodity later. For example, let the *x*-axis measure units of apples, and the *y*-axis measure units of oranges. Then (*x*, *y*)= (3,4) would denote a bundle consisting of three apples and four oranges. Of course, we are measuring non-negative quantities of apples and oranges. So it is the north-east quadrant from the four quadrants formed by the intersection of the *x* and *y*-axes, which is of relevance here. The north-west quadrant is a collection of ordered pairs of which x is the negative number, and y the positive number; in the south-west quadrant, both numbers are negative and in the south-east quadrant x is positive and y is negative. In any of the quadrants, the points are ordered pairs. Thus the set R^2 is also a point-set, that is, a set of points, with each point being an ordered pair. Keep in mind therefore, that although each ordered pair is a collection of two numbers, the ordered pair itself is a single point and thus a single element in the set R^2. So R^2 is a set with ordered pairs as elements; each of these ordered pairs is a point, and thus R^2 is a *point-set* (a set of points).

We have talked of two special point-sets, where the points are real numbers (the set is R) and where the points are ordered pairs of real numbers (the set is R^2). Can we extend the idea to sets whose elements are such that each element has more than two numbers? Certainly we can. We have already multiplied R with itself and got the set R^2. Let us multiply R with R^2. Then we will get R^3. This is $R \times R \times R$. Each element of this set is an ordered triple (x, y, z). So suppose we have three axes x-axis, y-axis and z-axis, then one element of the set R^3 will have a component of x and y and z . The entire set R^3 is a three dimensional structure unlike R^2, which is a plane and R, which is a line. But elements of each of these sets are points. You can conceptualise R^3 by thinking that our consumer consumes bananas along with apples and oranges. So each good (fruit) is now measured along an axis, and because there are three fruits, we need three axes to depict these.

Now think that there are n commodities ($n > 3$). Since we measure quantities of each commodity along an axis, we will need n-axes! How will we obtain n-axes? By having a Cartesian product of R with itself n times. So we have $R \times R \times R \times \ldots \times R$ (n times). We will get a new set R^n. What is a typical element of this set? Each element of this set is an ordered profile of n numbers. We can denote this by $(x_1, x_2, x_3 \ldots x_n)$. This is called an-tuple.

It would definitely be difficult (actually impossible) to draw a diagram depicting n-axes! But you can think of it in an abstract manner. Thus whether we consider R or R^2 or R^3 or R^n, all of these sets have points as elements and are called point-sets. The elements of R^n, the n-tuples are also called vectors (even ordered pairs and ordered triples are vectors). We will make great use of vectors in the second course on mathematical methods in economics, which you will study in the next semester. Vectors play a very important role in Economics, as we shall see.

2.6 CORRESPONDENCE AND SET FUNCTIONS

In an earlier section of the unit you were familiarised with the concept of a function. Let us review what the ingredients of a function are: you need two sets — a domain and a range, and a rule. This rule takes an element of the *domain* and 'transforms' it or 'sends' it or 'maps' it to an element in the *range*. This element in the range is called its *image*. We can say that a *function* is a rule according to which each element in the domain is associated with an element in the range.

2.6.1 Correspondence

Consider a set A. Let it be the domain. Now consider a set B as the codomain. This set can have several subsets. Think of a new set D which has as its members or elements various subsets of B. Let D be the range. So the range has as its elements some subsets. A mapping from elements of *A* to elements of *D* is a *correspondence*. An ordinary function maps an element of the domain to a single element in the range. Therefore, an ordinary function is called a *single valued* function. A correspondence, on the other hand, since it maps one element of the domain to an element, which is itself a subset and hence has several items, is called a *multi-valued* function. Let us try to explain the concept with an example.

Let $A = \{2,7,9,11,14\}$ and $B = \{a, b, c, d, e, f, g, h, i\}$. Now comes the important part: consider a set *D* which has as its elements some subset of *B*. In other words, *D* is actually a family of sets.

Let $D = \{\{a, c, f\}, \{b, a, d\}, \{i, c, h, g, d\}\}$. Now think of a mapping from the elements of *A* to elements of *D*. Suppose the element 9 of set *A* maps into the element $\{b, a, d\}$ of set *D* and the element 14 of set *A* maps into the element $\{a, c, f\}$ of the set *D*. Now understand the concept of correspondence, and why it is called multi-valued. In this example the single element 9 maps into the single element $\{b, a, d\}$, but this single element itself has three values. Thus, a correspondence maps an element into a subset of a set. The elements of set *A* map into elements of set *D*, which are subsets of set *B*.

2.6.2 Set-Functions

For a correspondence, we asked you to think of a set and its subsets, and we constructed a set whose elements were some (or all) of the subsets of this original set. This new set, the elements of which are the subsets of the original set, we considered as the range. Now we want you to consider a set, and think of a set whose elements are some or all of the subsets of this set. But now we want to take it as the domain. So suppose there is a set $H = \{12, 17, 3, 9, 8, 6\}$. Think of a set *J* with some of the subsets of *H* as its elements. Let $J=\{\{12, 17, 8\}, \{17, 9, 3\}, \{17, 12, 6, 9\}\}$. Now think of a mapping from *J* into some set *M* = $\{a, b, c, d\}$. This function or mapping would be an example of what is called a *set-function*. Let the element $\{17, 12, 6, 9\}$ map into *b* of the set *M*. This function would be single-valued no doubt because each element of the domain would map into one element of the range. But each element of the domain itself is a subset of some set (here set *H*) in our example. So each element of the domain of the set *J* is a set (remember we are considering a mapping from set *J* to set *M*, not set *H* to set *M*). This kind of mapping where elements that are themselves sets are mapped into single-valued elements in the range is called a *set-function*.

Check Your Progress 3

1) What do you understand by a set of points? What does 'point' mean in this context?

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2) Explain the concept of:

   a) Correspondence

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   b) Set-function

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2.7 LET US SUM UP

Building on the set operations, we extended the discussion to the idea of relations as subsets of ordered pairs, and have prepared the background of formulating functions specific relation. The concept of function, its presentation and commonly encountered forms have been discussed. The unit began by giving a detailed explanation of a Cartesian product of sets, and how a Cartesian product of two sets has ordered pairs as its elements. The unit subsequently went on to discuss the very important concept of relations as subsets of Cartesian products.

Following this, you were familiarised with functions. You are perhaps accustomed to thinking that a function depicts how a variable depends on another variable. Here we explain functions as mappings or transformations or association between elements of a set called the domain with the elements of another set called the range. You came to learn how a function is again a subset of a relation, that is, how certain conditions imposed on a relation can make it a function. So we see that a relation is a subset of a Cartesian product, and a function is a subset of a relation. The unit went on to discuss various types of mappings like injective, bijective and surjective.

After this the unit discussed the set of real numbers and real space, and their importance in Economics. In doing so, the unit also explained to you the important idea of a set of points, or point-sets. Finally, the unit discussed about special mappings, where the range or the domain has as their elements, some or all subsets of some other set. These are correspondence, or multi-valued functions, which are called correspondence; and set-functions, where the domain has as its elements subsets of another set.

2.8 ANSWERS/HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) Any subset of a Cartesian product of sets is called a relation. In other words, a relation is a set of ordered pairs. (Refer section 2.3 to explain further)

2) A relation ρ is an equivalence relation if it is reflexive, symmetric and transitive.

3) Domain is the set of all first elements in a relation ρ. Whereas, range is the set of all second elements called images of a relation ρ.

Check Your Progress 2

1) A relation ρ from set *X* to *Y* where every element of *X* has a unique image in *Y* is defined as a function from *X* to *Y*.

2) A function $f : X \rightarrow Y$ is surjective, or onto function if the range of *f* equals the codomain of *f*. For example, $f = \{(1,0),(2,0),(3,5)\}$, where $X = \{1,2,3\}$ and $Y = \{0,5\}$ is a surjective function. Similarly, other examples can be framed.

3) Refer section 2.4 to answer.

Check Your Progress 3

1) See section 2.5 to answer.

2) See section 2.6 to answer.