UNIT 10 ESTIMATION OF PRODUCTION AND COST FUNCTIONS

Objectives

After going through this unit, you should be able to:

- **explain** various functional forms of production and costs;
- **understand** empirical determination of these theoretical functions;
- **identify** managerial uses of such empirical estimates.

Structure

10.1 INTRODUCTION

In the process of decision-making, a manager should understand clearly the relationship between the inputs and output on one hand and output and costs on the other. The short run production estimates are helpful to production managers in arriving at the optimal mix of inputs to achieve a particular output target of a firm. This is referred to as the 'least cost combination of inputs' in production analysis. Also, for a given cost, optimum level of output can be found if the production function of a firm is known. Estimation of the long run production function may help a manager in understanding and taking decisions of long term nature such as capital expenditure.

Estimation of cost curves will help production manager in understanding the nature and shape of cost curves and taking useful decisions. Both short run cost function and the long run cost function must be estimated, since both sets of information will be required for some vital decisions. Knowledge of the short run cost functions allows the decision makers to judge the optimality of present output levels and to solve decision problems of production manager. Knowledge of long run cost functions is important when considering the expansion or contraction of plant size, and for confirming that the present plant size is optimal for the output level that is being produced.

In the present Unit, we will discuss different approaches to examination of production and cost functions, analysis of some empirical estimates of these functions, and managerial uses of the estimated functions.

10.2 ESTIMATION OF PRODUCTION FUNCTION

The principles of production theory discussed in Unit 7 are fundamental in understanding economics and provide an important conceptual framework for analysing managerial problems. However, short run output decisions and long run planning often require more than just this conceptual framework. That is, quantitative estimates of the parameters of the production functions are required for some decisions.

Functional Forms of Production Function

The production function can be estimated by regression techniques (refer to MS-8, course on "Quantitative Analysis for Managerial Applications" to know about regression techniques) using historical data (either time-series data, or cross-section data, or engineering data). For this, one of the first tasks is to select a functional form, that is, the specific relationship among the relevant economic variables. We know that the general form of production function is,

$$Q = f\ (K,L)$$

Where, Q = output, K = capital and L = labour.

Although, a variety of functional forms have been used to describe production relationships, only the Cobb-Douglas production function is discussed here. The general form of Cobb-Douglas function is expressed as:

$$Q = AK^{a}\ L^{b}$$

where A, a, and b are the constants that, when estimated, describe the quantitative relationship between the inputs (K and L) and output (Q).

The marginal products of capital and labour and the rates of the capital and labour inputs are functions of the constants A, a, and b and. That is,

$$MP_K = \frac{dQ}{dK} = aAK^{a-1} L^{b}$$

$$MP_L = \frac{dQ}{dK} = bAK^{a}L^{b-1}$$

The sum of the constants (a+b) can be used to determine returns to scale. That is,

(a+b) > 1 Þ increasing returns to scale,
(a+b) = 1 Þ constant returns to scale, and
(a+b) < 1 Þ decreasing returns to scale.

Having numerical estimates for the constants of the production function provides significant information about the production system under study. The marginal products for each input and returns to scale can all be determined from the estimated function.

The Cobb-Douglas function does not lend itself directly to estimation by the regression methods because it is a nonlinear relationship. Technically, an equation must be a linear function of the parameters in order to use the ordinary least-squares regression method of estimation. However, a linear equation can be derived by taking the logarithm of each term. That is,

$$\log Q = \log A + a \log K + b \log L$$

A linear relationship can be seen by setting,

$$Y = \log Q, \quad A^* = \log A, \quad X_1 = \log K, \quad X_2 = \log L$$

and rewriting the function as

$$Y = A^* + aX_1 + bX_2$$

This function can be estimated directly by the least-squares regression technique and the estimated parameters used to determine all the important production relationships. Then the antilogarithm of both sides can be taken, which transforms the estimated function back to its conventional multiplicative form. We will not be studying here the details of computing production function since there are a number of computer programs available for this purpose. Instead, we will provide in the following section some empirical estimates of Cobb-Douglas production function and their interpretation in the process of decision making.

Types of Statistical Analyses

Once a functional form of a production function is chosen the next step is to select the type of statistical analysis to be used in its estimation. Generally, there are three types of statistical analyses used for estimation of a production function. These are: (a) time series analysis, (b) cross-section analysis and (c) engineering analysis.

a) **Time series analysis:** The amount of various inputs used in various periods in the past and the amount of output produced in each period is called time series data. For example, we may obtain data concerning the amount of labour, the amount of capital, and the amount of various raw materials used in the steel industry during each year from 1970 to 2000. On the basis of such data and information concerning the annual output of steel during 1970 to 2000, we may estimate the relationship between the amounts of the inputs and the resulting output, using regression techniques.

   Analysis of time series data is appropriate for a single firm that has not undergone significant changes in technology during the time span analysed. That is, we cannot use time series data for estimating the production function of a firm that has gone through significant technological changes. There are even more problems associated with the estimation a production function for an industry using time series data. For example, even if all firms have operated over the same time span, changes in capacity, inputs and outputs may have proceeded at a different pace for each firm. Thus, cross section data may be more appropriate.

b) **Cross-section analysis:** The amount of inputs used and output produced in various firms or sectors of the industry at a given time is called cross-section data. For example, we may obtain data concerning the amount of labour, the amount of capital, and the amount of various raw materials used in various firms in the steel industry in the year 2000. On the basis of such data and information concerning the year 2000, output of each firm, we may use regression techniques to estimate the relationship between the amounts of the inputs and the resulting output.

c) **Engineering analysis:** In this analysis we use technical information supplied by the engineer or the agricultural scientist. This analysis is undertaken when the above two types do not suffice. The data in this analysis is collected by experiment or from experience with day-to-day working of the technical process. There are advantages to be gained from approaching the measurement of the production function from this angle because the range of applicability of the data is known, and, unlike time-series and cross-section studies, we are not restricted to the narrow range of actual observations.

Limitations of Different Types of Statistical Analysis

Each of the methods discussed above has certain limitations.

1. Both time-series and cross-section analysis are restricted to a relatively narrow range of observed values. Extrapolation of the production function outside that range may be seriously misleading. For example, in a given case, marginal productivity might decrease rapidly above 85% capacity utilization; the production function derived for values in the 70%-85% capacity utilization range would not show this.
2. Another limitation of time series analysis is the assumption that all observed values of the variables pertains to one and the same production function. In other words, a constant technology is assumed. In reality, most firms or industries, however, find better, faster, and/or cheaper ways of producing their output. As their technology changes, they are actually creating new production functions. One way of coping with such technological changes is to make it one of the independent variables.
3. Theoretically, the production function includes only efficient (least-cost) combinations of inputs. If measurements were to conform to this concept, any year in which the production was less than nominal would have to be excluded from the data. It is very difficult to find a time-series data, which satisfy technical efficiency criteria as a normal case.
4. Engineering data may overcome the limitations of time series data but mostly they concentrate on manufacturing activities. Engineering data do not tell us anything about the firm's marketing or financial activities, even though these activities may directly affect production.
5. In addition, there are both conceptual and statistical problems in measuring data on inputs and outputs.

It may be possible to measure output directly in physical units such as tons of coal, steel etc. In case more than one product is being produced, one may compute the weighted average of output, the weights being given by the cost of manufacturing these products. In a highly diversified manufacturing unit, there may be no alternative but to use the series of output values, corrected for changes in the price of products. One has also to choose between 'gross value' and 'net value'. It seems better to use "net value added" concept instead of output concept in estimating production function, particularly where raw-material intensity is high.

The data on labour is mostly available in the form of "number of workers employed" or "hours of labour employed". The 'number of workers' data should not be used because, it may not reflect underemployment of labour, and they may be occupied, but not productively employed. Even if we use 'man hours' data, it should be adjusted for efficiency factor. It is also not advisable that labour should be measured in monetary terms as given by expenditure on wages, bonus, etc.

The data on capital input has always posed serious problems. Net investment i.e. a change in the value of capital stock, is considered most appropriate. Nevertheless, there are problems of measuring depreciation in fixed capital, changes in quality of fixed capital, changes in inventory valuation, changes in composition and productivity of working capital, etc.

Finally, when one attempts an econometric estimate of a production function, one has to overcome the standard problem of multi-collinearity among inputs, autocorrelation, homoscadasticity, etc.

10.3 EMPIRICAL ESTIMATES OF PRODUCTION FUNCTION

Consider the following Cobb-Douglas production function with parameters A=1.01, a = 0.25 and b=0.75,

$$Q = 1.01K^{0.25} \quad L^{0.75}$$

The above production function can be used to estimate the required capital and labour for various levels of output. For example, the capital and labour required for an output level of 100 units will be given by

$$100 = 1.01K^{0.25} \quad L^{0.75}$$

Þ $$99 = K^{0.25} \, L^{0.75}$$

By substituting any value of L (or K) in this equation, we can obtain the associated value of K (or L). For example, if L=50, the value of K will be given by

$$99 = K^{0.25} \, (50)^{0.75}$$

Þ $$\log 99 = 0.75 \log 50 + 0.25 \log K$$

Þ $$1.9956 = 0.75\,(1.6990) + 0.25 \log K$$

Þ $$\log K = \frac{1}{0.25} \; (1.9956 - 1.2743) = 2.8852$$

Þ $$K = \text{antilog } 2.8852 = 768$$

Similarly, for any given value of K we can find out the corresponding value of L.

As explained in Unit 7, an isoquant for any given output level or an isoquant map for a given set of output levels can be derived from an estimated production function.

Consider the following Cobb-Douglas production function with parameters A=200, a = 0.50 and b = 0.50,

$$Q = 200K^{0.50} \quad L^{0.50}$$

For different combinations of inputs (L and K), we can construct an associated maximum rate of output as given in Table 10.1 For example, if two units of labour and 9 units of capital are used, maximum production is 600 units of output. If K=10 and L=10 the output rate will be 2000. The following three important relationships are shown by the data in this production Table.

1. Table 10.1 indicates that there are a variety of ways to produce a particular rate of output. For example, 490 units of output can be produced with any one of the following combinations of inputs.

Table 10.1: Output Table for different Combinations of Inputs for the Production Function $Q = 200K^{0.50}\ L^{0.50}$

Rate of labour input (L)

Rate of capital input (K)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	200	283	346	400	447	490	529	566	600	632
2	283	400	490	566	632	693	748	800	849	894
3	346	490	600	693	775	849	917	980	1039	1095
4	400	566	693	800	894	980	1058	1131	1200	1265
5	447	632	775	894	1000	1095	1183	1265	1342	1414
6	490	693	849	980	1095	1200	1296	1386	1470	1549
7	529	748	917	1058	1183	1296	1400	1497	1587	1673
8	566	800	980	1131	1265	1386	1497	1600	1697	1789
9	600	849	1039	1200	1342	1470	1587	1697	1800	1897
10	632	894	1095	1265	1414	1549	1673	1789	1897	2000

Combination of inputs		**Output**
K	L	
6	1	490
3	2	490
2	3	490
1	6	490

This shows that there is substitutability between the factors of production. That means the production manager can use either the input combination (k=6 and L=1) or (k=3 and L=2) or (k=2 and L=3) or (k=1 and L=6) to produce the same amount of output (490 units). The concept of substitution is important because it means that managers can change the input mix of capital and labour in response to changes in the relative prices of these inputs.

2. In the equation given that $a = 0.50$ and $b = 0.50$. The sum of these constants is 1 (0.50+0.50=1). This indicates that there are constant returns to scale ($a+b=1$). This means that a 1% increase in all inputs would result in a 1% increase in output. For example, in Table 10.1 maximum production with four units of capital and one unit of labour is 400. Doubling the input rates to K=8 and L=2 results in the rate of output doubling to Q=800. In Table 10.1, production is characterized by constant returns to scale. This means that if both input rates increase by the same factor (for example, both input rates double), the rate of output also will double. In other production functions, output may increase more or less than in proportion to changes in inputs.

3. In contrast to the concept of returns to scale, when output changes because of changes in one input while the other remains constant, the changes in the output rates are referred to as returns to a factor. In Table 10.1, if the rate of one input is held constant while the other is increased, output increases but the successive increments become smaller. For example, from Table 10.1 it can be seen that if the rate of capital is held constant at K=2 and labour is increased from L=1 to L=6, the successive increases in output are 117, 90, 76, 67, and 60. As discussed in Unit 7, this relationship is known as diminishing marginal returns.

We will consider another empirical estimate of Cobb-Douglas production function given as:

$$Q = 10.2K^{0.194} \quad L^{0.878}$$

Here, the returns to scale are increasing because a+b=1.072 is greater than 1. The marginal product functions for capital and labour are

$$MP_K = aAK^{a-1}L^b = 0.194(10.2)K^{(0.194-1)}L^{0.878} = 0.194(10.2)K^{-0.806}L^{0.878}$$

and

$$MP_L = bAK^aL^{b-1} = 0.878(10.2)K^{0.194}L^{(0.878-1)} = 0.878(10.2)K^{0.194}L^{-0.122}$$

Based on the above MP_K and MP_L equations we can calculate marginal products of capital and labour for a given input combination. For example, suppose we are given that the input combination K=20 and L=30. Substituting these values for the constants A, a, and b gives the following marginal products:

$$MP_K = 0.194(10.2)(20)^{-0.806}(30)^{0.878} = 3.50$$

and

$$MP_L = 0.878(10.2)(20)^{0.194}(30)^{-0.122} = 10.58$$

We can interpret the above marginal products of capital and labour as follows. One unit change in capital with labour held constant at 30 would result in 3.50 unit change in output, and one unit change in labour with capital held constant at 20 would be associated with a 10.58 unit change in output.

Empirical estimates of production functions for industries such as sugar, textiles, cement etc., are available in the Indian context. We will briefly discuss some of these empirical estimates here.

There are many empirical studies of production functions in different countries. John R. Moroney made one comprehensive study of a number of manufacturing industries in U.S.A. He estimated the production function:

$$Q = AK^a \quad L_1^b L_2^g$$

Where, K = value of capital

L_1 = production worker-hours

L_2 = non-production worker-hours

A summary of the estimated values of the production elasticities (a, b, and g) and R^2, the coefficient of determination, for each industry is shown in Table 10.2.

From Table 10.2 it can be observed that R^2 values are very high (more than 0.951) for all the functions. This means that more than 95% of the variation in output is explained by variation in the three inputs. A test of significance was made for each estimated parameter, a, b, and g, using the standard t-test. Those estimated production elasticities that are statistically significant at the 0.05 levels are indicated with an asterix (*). The sum of the estimated production elasticities (a+b+g) provides a point estimate of returns to scale in each industry. Although, the sum exceeds unity in 14 of the 17 industries, it is statistically significant only in the following industries: food and beverages, apparel, furniture, printing, chemicals, and fabricated metals. Thus, only in those six industries there are increasing returns to scale. For example, in the fabricated metals industry, a 1% increase in all inputs is estimated to result in a 1.027% increase in output.

Table 10.2: Estimated production Elasticities for 17 Industries

Industry	a	b	g	a+b+g	R^2
Food and beverages	0.555*	0.438*	0.076*	1.070*	0.987
Textiles	0.121	0.549*	0.335*	1.004	0.991
Apparel	0.128	.0437*	0.477*	1.041*	0.982
Lumber	0.392*	0.504*	0.145	1.041	0.951
Furniture	0.205	0.802*	0.103	1.109*	0.966
Paper and Pulp	0.421*	0.367	0.197*	0.984	0.990
Printing	0.459*	0.045*	0.574*	1.079*	0.989
Chemicals	0.200*	0.553*	0.336*	1.090*	0.970
Petroleum	0.308*	0.546*	0.093	0.947	0.983
Rubber and Plastics	0.481*	1.033*	-0.458	1.056	0.991
Leather	0.076	0.441*	0.523	1.040	0.990
Stone and Clay	0.632*	0.032	0.366*	1.029	0.961
Primary Metals	0.371*	0.077	0.509*	0.958	0.969
Fabricated Metals	0.151*	0.512*	0.365*	1.027*	0.995
Non-electrical machinery	0.404*	0.228	0.389*	1.020	0.980
Electrical Machinery	0.368*	0.429*	0.229*	1.026	0.983
Transportation Equipment	0.234*	0.749*	0.041	1.023	0.972

Source: J.R. Moroney, "Cobb-Douglas Production Functions and Returns to Scale in U.S. Manufacturing Industry," *Western Economic Journal* 6 (December): 39-51, 1967.

Activity 1

1. Observe the following Cobb-Douglas production function:

   $Q = 50K^{0.4}L^{0.7}$

   For this production function system, are returns to scale decreasing, constant, or increasing? Explain.

   ....
   ....
   ....
   ....
   ....
   ....
   ....
   ....
   ....

2. Use the data from Table 10.1 to answer the following questions:

   a) If the rate of capital input is fixed at four and if output sells for Rs. 10 per unit, determine the total, average, and marginal product functions and the marginal revenue product function for labour and complete the following Table.

L	TP_L	AP_L	MP_L	MRP_L
1	400		—	—
2			166	
3				1270
4		200		
5				940
6	980			
7			78	
8		141.4		
9				
10	1265			650

b) Using the data from the above-completed table, if the wage rate is Rs. 675 per unit, how much labour should be employed?

....

....

....

....

....

c) If the rate of labour is fixed at 2 and the price of output is Rs. 10 per unit, determine the total, average, and marginal, product functions for capital and the marginal revenue product of capital in the following Table.

C	TP_L	AP_L	MP_L	MRP_L
1		283.0	—	—
2				117
3	490			
4				
5		126.4		
6				
7			55	
8				
9			49	
10	894			450

d) Using the data from the above-completed table, if the price of capital is Rs. 600 per unit, how much capital should be employed?

....

....

....

....

....

3. The production function for ABC company is

   $Q=50K^{0.4} \quad L^{0.6}$

   Where Q is the total output, L is the quantity of labour employed, and K is the quantity of capital.

   a) Calculate TP, AP, and MP for 10, 15, and 20 units of labour employed if capital is fixed at 30 units.

   ....

   ....

   ....

   ....

   ....

   b) To which stage of production do these quantities of labour correspond? Why?

   ....

   ....

   ....

   ....

   ....

10.4 MANAGERIAL USES OF PRODUCTION FUNCTION

There are several managerial uses of the production function. It can be used to compute the least-cost combination of inputs for a given output or to choose the input combination that yields the maximum level of output with a given level of cost. There are several feasible combinations of input factors and it is highly useful for decision-makers to find out the most appropriate among them. The production function is useful in deciding on the additional value of employing a variable input in the production process. So long as the marginal revenue productivity of a variable factor exceeds it price, it may be worthwhile to increase its use. The additional use of an input factor should be stopped when its marginal revenue productivity just equals its price. Production functions also aid long-run decision-making. If returns to scale are increasing, it will be worthwhile to increase production through a proportionate increase in all factors of production, provided, there is enough demand for the product. On the other hand, if returns to scale are decreasing, it may not be worthwhile to increase the production through a proportionate increase in all factors of production, even if there is enough demand for the product. However, it may in the discretion of the producer to increase or decrease production in the presence of constant returns to scale, if there is enough demand for the product.

Activity 2

1. Can you list some more managerial uses of production function other than those given in section 10.4?

   ....

   ....

   ....

   ....

   ....

10.5 COST FUNCTION AND ITS DETERMINANTS

Cost function expresses the relationship between cost and its determinants such as the size of plant, level of output, input prices, technology, managerial efficiency, etc. In a mathematical form, it can be expressed as,

$$C = f(S, O, P, T, E\ldots..)$$

Where, C = cost (it can be unit cost or total cost)

S = plant size

O = output level

P = prices of inputs used in production

T = nature of technology

E = managerial efficiency

Determinants of Cost Function

The cost of production depends on many factors and these factors vary from one firm to another firm in the same industry or from one industry to another industry. The main determinants of a cost function are:

a) plant size

b) output level

c) prices of inputs used in production,

d) nature of technology

e) managerial efficiency

We will discuss briefly the influence of each of these factors on cost.

a) **Plant size:** Plant size is an important variable in determining cost. The scale of operations or plant size and the unit cost are inversely related in the sense that as the former increases, unit cost decreases, and vice versa. Such a relationship gives downward slope of cost function depending upon the different sizes of plants taken into account. Such a cost function gives primarily engineering estimates of cost.

b) **Output level:** Output level and total cost are positively related, as the total cost increases with increase in output and total cost decreases with decrease in output. This is because increased production requires increased use of raw materials, labour, etc., and if the increase is substantial, even fixed inputs like plant and equipment, and managerial staff may have to be increased.

c) **Price of inputs:** Changes in input prices also influence cost, depending on the relative usage of the inputs and relative changes in their prices. This is because more money will have to be paid to those inputs whose prices have increased and there will be no simultaneous reduction in the costs from any other source. Therefore, the cost of production varies directly with the prices of production.

d) **Technology:** Technology is a significant factor in determining cost. By definition, improvement in technology increases production leading to increase in productivity and decrease in production cost. Therefore, cost varies inversely with technological progress. Technology is often quantified as capital-output ratio. Improved technology is generally found to have higher capital-output ratio.

e) **Managerial efficiency:** This is another factor influencing the cost of production. More the managerial efficiency less the cost of production. It is difficult to measure managerial efficiency quantitatively. However, a change in cost at two points of time may explain how organisational or managerial changes within the firm have brought about cost efficiency, provided it is possible to exclude the effect of other factors.

10.6 ESTIMATION OF COST FUNCTION

Several methods exist for the measurement of the actual cost-output relation for a particular firm or a group of firms, but the three broad approaches - accounting, engineering and econometric - are the most important and commonly used.

Accounting Method

This method is used by the cost accountants. In this method, the cost-output relationship is estimated by classifying the total cost into fixed, variable and semi-variable costs. These components are then estimated separately. The average variable cost, the semi-variable cost which is fixed over a certain range of output, and fixed costs are determined on the basis of inspection and experience. The total cost, the average cost and the marginal cost for each level of output can then be obtained through a simple arithmetic procedure.

Although, the accounting method appears to be quite simple, it is a bit cumbersome as one has to maintain a detailed breakdown of costs over a period to arrive at good estimates of actual cost-output relationship. One must have experience with a wide range of fluctuations in output rate to come up with accurate estimates.

Engineering Method

The engineering method of cost estimation is based directly on the physical relationship of inputs to output, and uses the price of inputs to determine costs. This method of estimating real world cost function rests clearly on the knowledge that the shape of any cost function is dependent on: (a) the production function and (b) the price of inputs.

We have seen earlier in this Unit while discussing the estimation of production function that for a given the production function and input prices, the optimum input combination for a given output level can be determined. The resultant cost curve can then be formulated by multiplying each input in the least cost combination by its price, to develop the cost function. This method is called engineering method as the estimates of least cost combinations are provided by engineers.

The assumption made while using this method is that both the technology and factor prices are constant. This method may not always give the correct estimate of costs as the technology and factor prices do change substantially over a period of time. Therefore, this method is more relevant for the short run. Also, this method may be useful if good historical data is difficult to obtain. But this method requires a sound understanding of engineering and a detailed sampling of the different processes under controlled conditions, which may not always be possible.

Econometric Method

This method is also some times called statistical method and is widely used for estimating cost functions. Under this method, the historical data on cost and output are used to estimate the cost-output relationship. The basic technique of regression is used for this purpose. The data could be a time series data of a firm in the industry or of all firms in the industry or a cross-section data for a particular year from various firms in the industry.

Depending on the kind of data used, we can estimate short run or long run cost functions. For instance, if time series data of a firm whose output capacity has not changed much during the sample period is used, the cost function will be short run. On the other hand, if cross-section data of many firms with varying sizes, or the time series data of the industry as a whole is used, the estimated cost function will be the long run one.

The procedure for estimation of cost function involves three steps. First, the determinants of cost are identified. Second, the functional form of the cost function is specified. Third, the functional form is chosen and then the basic technique of regression is applied to estimate the chosen functional form.

Functional Forms of Cost Function

The following are the three common functional forms of cost function in terms of total cost function (TC).

a) Linear cost function: $TC = a_1 + b_1Q$

b) Quadratic cost function: $TC = a_2 + b_2Q + c_2Q^2$

c) Cubic cost function: $TC = a_3 + b_3Q + c_3Q^2 + d_3Q^3$

Where, a_1, a_2, a_3, b_1, b_2, b_3, c_2, c_3, d_3 are constants.

When all the determinants of cost are chosen and the data collection is complete, the alternative functional forms can be estimated by using regression software package on a computer. The most appropriate form of the cost function for decision-making is then chosen on the basis of the principles of economic theory and statistical inference.

Once the constants in the total cost function are estimated using regression technique, the average cost (AC) and marginal cost (MC) functions for chosen forms of cost function will be calculated. The TC, AC and MC cost functions for different functional forms of total cost function and their typical graphical presentation and interpretation are explained below.

a) Linear cost function

$$TC = a_1 + b_1Q$$

$$AC = (TC)/Q = (a_1/Q) + b_1$$

$$MC = \frac{d(TC)}{dQ} = b_1$$

The typical TC, AC, and MC curves that are based on a linear cost function are shown in Figure 10.1. These cost functions have the following properties: TC is a linear function, where AC declines initially and then becomes quite flat approaching the value of MC as output increases and MC is constant at b_1.

b) Quadratic cost function

$$TC = a_2 + b_2Q + c_2Q^2$$

$$AC = (TC/Q) = (a_2/Q) + b_2 + c_2Q$$

$$MC = \frac{d(TC)}{dQ} = b_2 + 2c_2Q$$

The typical TC, AC, and MC curves that are based on a quadratic cost function are shown in Figure 10.2. These cost functions have the following properties: TC increases at an increasing rate; MC is a linearly increasing function of output; and AC is a U shaped curve.

c) Cubic cost function

$$TC = a_3 + b_3Q + c_3Q^2 + d_3Q^3$$

$$AC = (TC/Q) = (a_3/Q) + b_3 + c_3Q + d_3Q^2$$

$$MC = \frac{d(TC)}{dQ} = b_3 + 2c_3Q + 3d_3Q^2$$

The typical TC, AC, and MC curves that are based on a cubic cost function are shown in Figure 10.3. These cost functions have the following properties: TC first increases at a decreasing rate up to output rate Q_1 in the Figure 10.3 and then increases at an increasing rate; and both AC and MC cost functions are U shaped functions.

The linear total cost function would give a constant marginal cost and a monotonically falling average cost curve. The quadratic function could yield a U-shaped average cost curve but it would imply a monotonically rising marginal cost curve. The cubic cost function is consistent both with a U-shaped average cost curve and a U-shaped marginal cost curve. Thus, to check the validity of the theoretical cost-output relationship, one should hypothesize a cubic cost function.

Figure 10.1: Cost Curves Based on Linear Cost Function

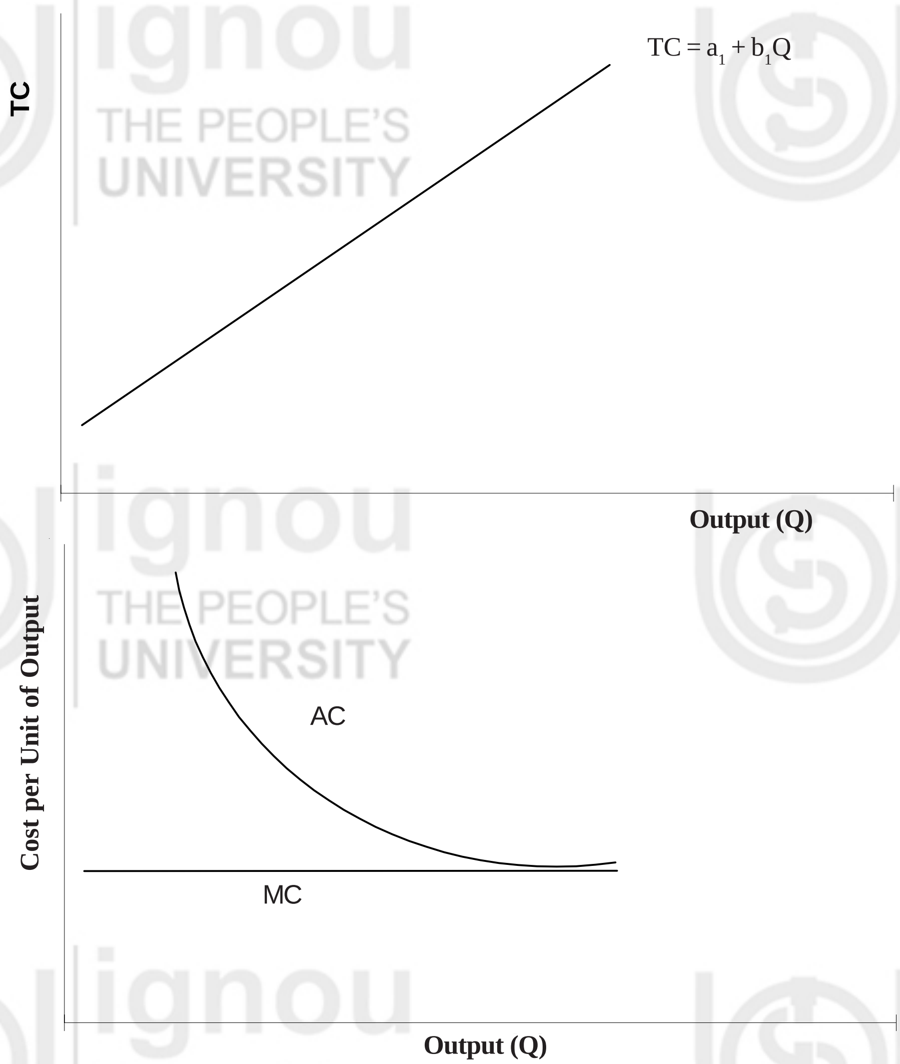
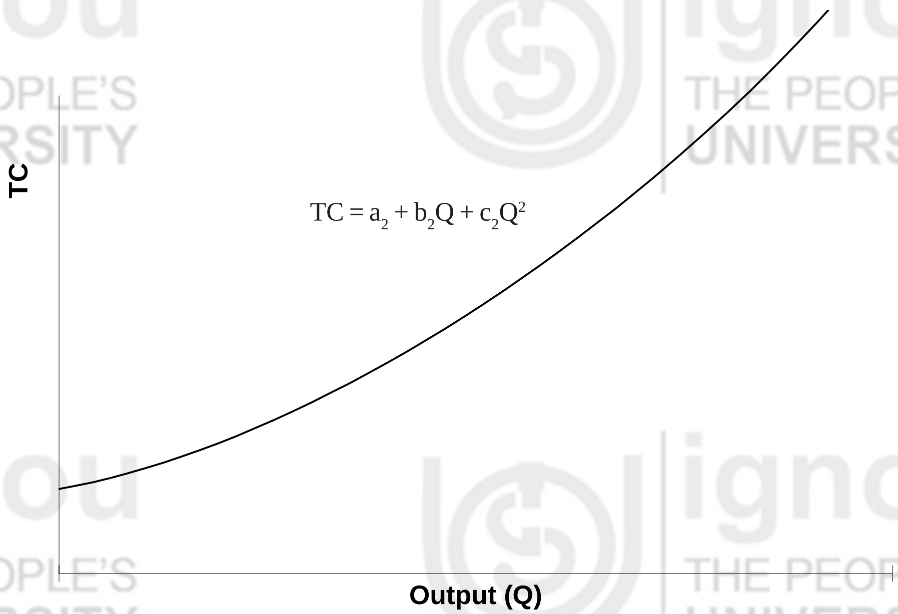
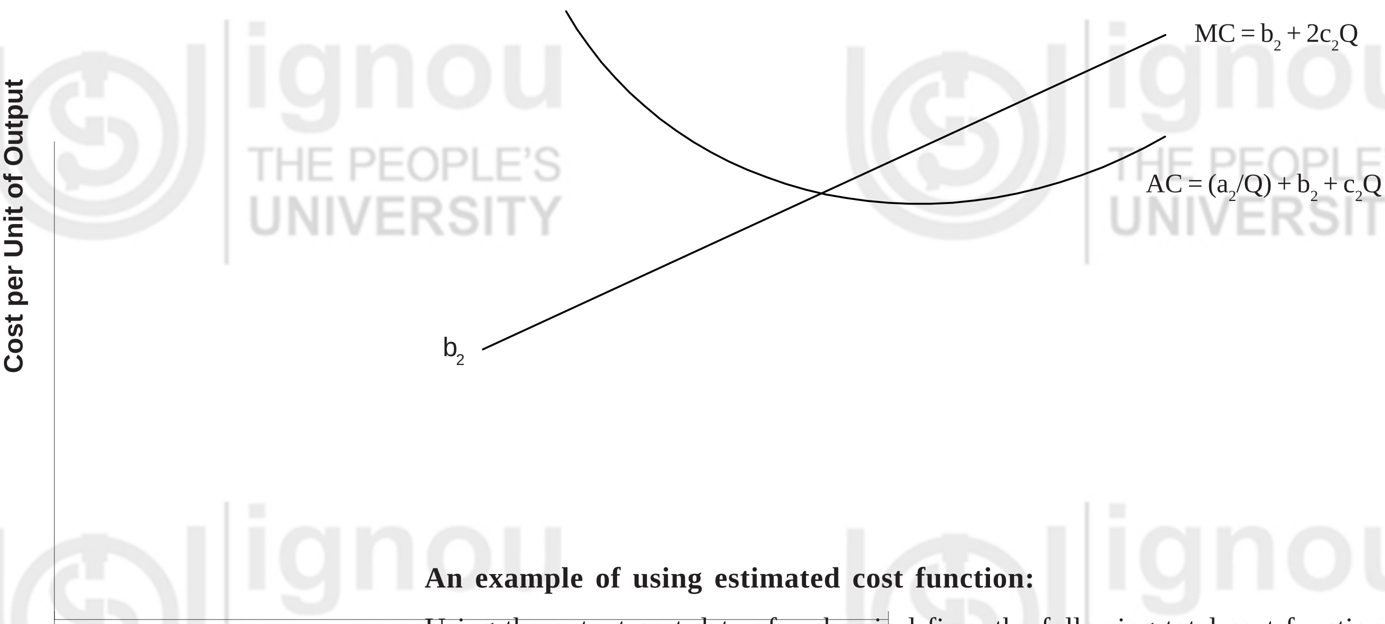


Figure 10.2: Cost Curves Based on Quadratic Cost Function





An example of using estimated cost function:

Using the output-cost data of a chemical firm, the following total cost function was estimated using quadratic function:

$$TC = 1016 - 3.36Q + 0.021Q^2$$

a) Determine average and marginal cost functions.

b) Determine the output rate that will minimize average cost and the per unit cost at that rate of output.

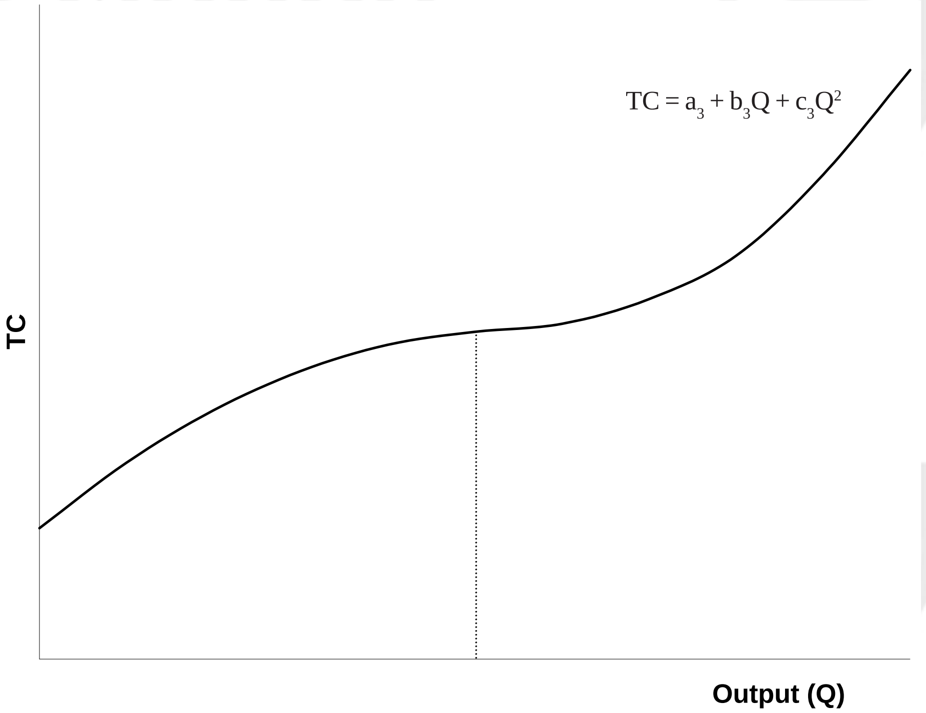
c) The firm proposed a new plant to produce nitrogen. The current market price of this fertilizer is Rs 5.50 per unit of output and is expected to remain at that level for the foreseeable future. Should the plant be built?

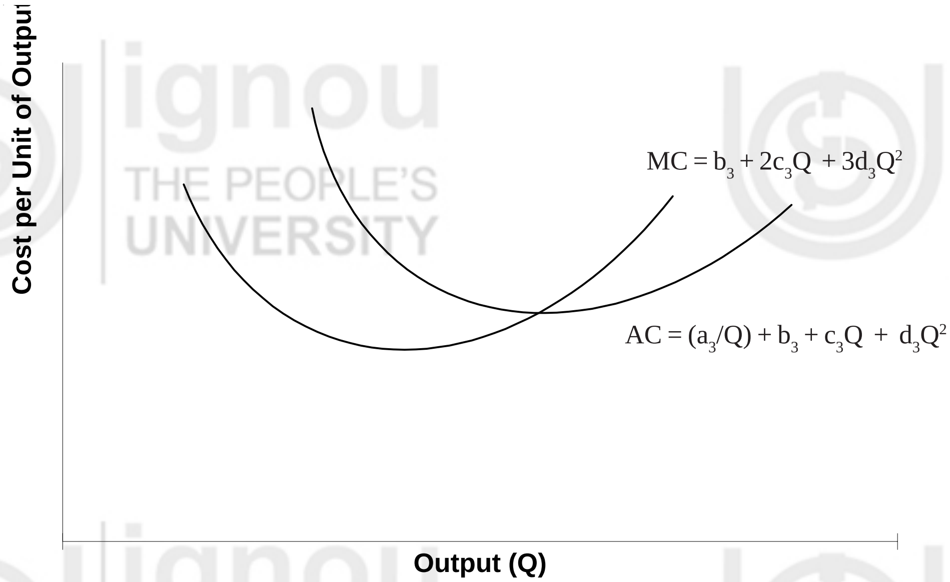
i) The average cost function is

$AC = (TC/Q) = (a_2/Q) + b_2 + c_2Q = (1016/Q) - 3.36 + 0.021Q$

and the marginal cost function is

Figure 10.3: Cost Curves Based on Cubic Cost Function





$$MC = \frac{d(TC)}{dQ} = b_2 + 2c_2Q = -3.36 + 2(0.021)Q = -3.36 + 0.042Q$$

ii) The output rate that results in minimum per unit cost is found by taking the first derivative of the average cost function, setting it equal to zero, and solving for Q.

$$\frac{d(AC)}{dQ} = \frac{a_2}{Q^2} + c_2 = -\frac{1016}{Q^2} + 0.021 = 0$$

$$\frac{1016}{Q2} = 0.021; \quad 0.021Q^2 = 1016; \quad Q^2 = 48381; \quad Q = 220$$

To find the cost at this rate of output, substitute 220 for Q in AC equation and solve it.

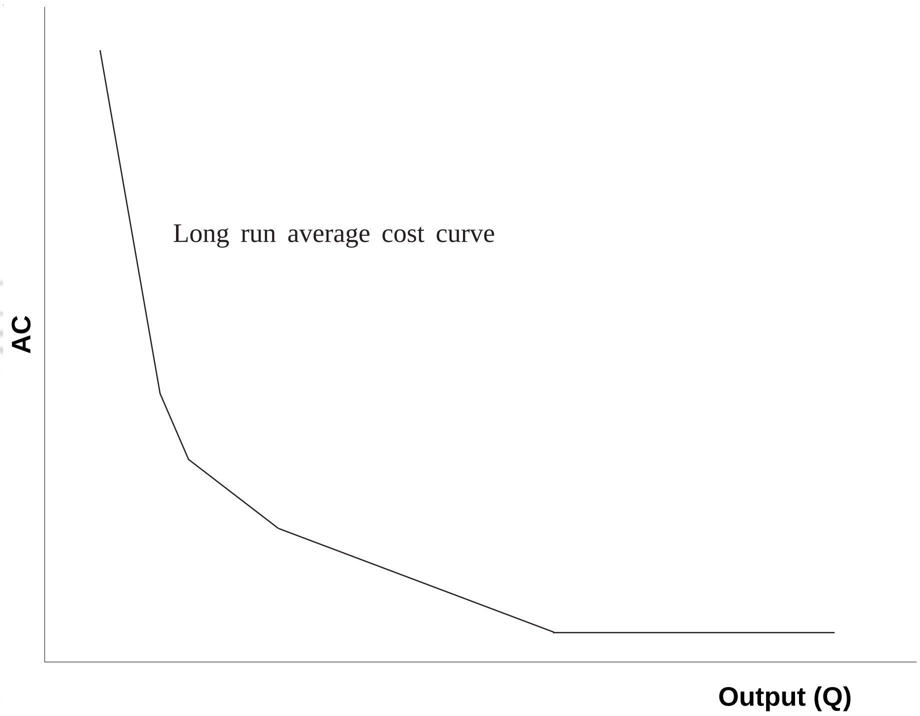
AC = (1016/Q) – 3.36 + 0.021Q = (1016/220) – 3.36 + (0.021 ∗ 220) Rs. 5.88 per unit of output.

iii) Because the lowest possible cost is Rs. 5.88 per unit, which is Rs. 0.38 above the market price (Rs. 5.50), the plant should not be constructed.

Short Run and Long Run Cost Function Estimation

The same sorts of regression techniques can be used to estimate short run cost functions and long run cost functions. However, it is very difficult to find cases where the scale of a firm has changed but technology and other relevant factors have remained constant. Thus, it is hard to use time series data to estimate long run cost functions. Generally, regression analysis based on cross section data has been used instead. Specially, a sample of firms of various sizes is chosen, and a firm's TC is regressed on its output, as well as other independent variables, such as regional differences in wage rates or other input prices.

Figure 10.4: Typical Long Run Average Cost Curve



Many studies of long run cost functions that have been carried out found that there are very significant economies of scale at low output levels, but that these economies of scale tend to diminish as output increases, and that the long run average cost function eventually becomes close to horizontal axis at high output levels. Therefore, in contrast to the U-shaped curve in Figure 9.4 shown in Unit 9, which is often postulated in micro economic theory, the long run average cost curve tends to be L-shaped, as shown in Figure 10.4.

Problems in Estimation of Cost Function

We confront certain problems while attempting to derive empirical cost functions from economic data. Some of these problems are briefly discussed below.

1. In collecting cost and output data we must be certain that they are properly paired. That is, the cost data applicable to the corresponding data on output.

2. We must also try to obtain data on cost and output during a time period when the output has been produced at relatively even rate. If for example, a month is chosen as the relevant time period over which the variables are measured, it would not be desirable to have wide weekly fluctuations in the rate of output. The monthly data in such a case would represent an average output rate that could disguise the true cost-output relationship. Not only should the output rate be uniform, but it also should be a rate to which the firm is fully adjusted. Furthermore, there should be no disruptions in the output due to external factors such as power failures, delays in receiving necessary supplies, etc. To generate the data necessary for a meaningful statistical analysis, the observations must include a wide range of rates of output. Observing cost-output data for the last 24 months, when the rate of output was the same each month, would provide little information concerning the appropriate cost function.

3. The cost data is normally collected and recorded by accountants for their own purposes and in a manner that it makes the information less than perfect from the perspective of economic analysis. While collecting historical data on cost, care must be taken to ensure that all explicit as well as implicit costs have been properly taken into account, and that all the costs are properly identified by time period in which they were incurred.

4. For situations in which more than one product is being produced with given productive factors, it may not be possible to separate costs according to output in a meaningful way. One simple approach of allocating costs among various products is based on the relative proportion of each product in the total output. However, this may not always accurately reflect the cost appropriate to each output.

5. Since prices change over time, any money value cost would therefore relate partly to output changes and partly to price changes. In order to estimate the cost-output relationship, the impact of price change on cost needs to be eliminated by deflating the cost data by price indices. Wages and equipment price indices are readily available and frequently used to 'deflate' the money cost.

6. Finally, there is a problem of choosing the functional form of equation or curve that would fit the data best. The usefulness of any cost function for practical application depends, to a large extent, on appropriateness of the functional form chosen. There are three functional forms of cost functions, which are popular, viz., linear, quadratic and cubic. The choice of a particular function depends upon the correspondence of the economic properties of the data to the mathematical properties of the alternative hypotheses of total cost function.

The accounting and engineering methods are more appropriate than the econometric method for estimating the cost function at the firm level, while the econometric method is more suitable for estimating the cost function at the industry or national level. There has been a growing application of the econometric method at the macro level and there are good prospects for its use even at the micro level. However, it must be understood that the three approaches discussed above are not competitive, but are rather complementary to each other. They supplement each other. The choice of a method therefore depends upon the purpose of study, time and expense considerations.

10.7 EMPIRICAL ESTIMATES OF COST FUNCTION

A number of studies using time series and cross-section data have been conducted to estimate short run and long run cost behaviour of various industries. Table 10.3 lists a number of well-known studies estimating short run average and marginal cost curves. These and many other studies point one conclusion: in the short run a linear total variable cost function with constant marginal cost is the relationship that appears to describe best the actual cost conditions over the "normal" range of production. U-shaped average cost (AC) and marginal cost (MC) curves have been found, but are less prevalent than one might expect.

Table 10.3: Numer of well-known studies estimating short run average and marginal cost curves

Name	Type of Industry	Findings
Dean (1936)	Furniture	Constant MC which failed to rise
Dean (1941)	Leather belts	No significant increases in MC
Dean (1941)	Hosiery	Constant MC which failed to rise
Dean (1942)	Department store	Declining or constant MC, depending on the department within the store
Ezekiel and Wylie (1941)	Steel	Declining MC but large variation
Hall and Hitch (1939)	Manufacturing	Majority have decreasing MC
Johnston (1960)	Electricity, multi product function food processing	"Direct" cost is a linear function of output, and MC is constant
Johnston (1960)	Electricity	Average total cost falls, then flattens, tending toward constant MC up to capacity
Mansfield and Wein (1958)	Railways	Constant MC
Yntema (1940)	Steel	Constant MC

Source: A.A. Walters, "Production and Cost Functions: An Econometric Survey", Econometrica, January-February 1963, PP.49-54

Table 10.4 lists a number of well known, long run average cost studies. In some industries, such as light manufacturing (of baking products), economies of size are relatively unimportant and diseconomies set in rather quickly, implying that a small plant has cost advantages over a large plant. In other industries, such as meat packing or the production of household appliances, the long run average cost curve is found to be flat over an extended range of output, there by indicating that a variety of different plant sizes are all more or less equally efficient. In some other industries such as electricity or metal (aluminum and steel) production, substantial economies of size are found, thereby implying that a large plant is most efficient. Rarely are substantial diseconomies of size found in empirical studies, perhaps because of firms recognising that production beyond a certain range leads to sharply rising costs. Therefore, they avoid such situations if all possible by building additional plants.

Table 10.4: Number of well known, long run average cost studies

Name	Type of Industry	Findings
Alpert (1959)	Metal	Economies of scale up to some level of output per month; constant returns to scale and horizontal LRAC thereafter
Bain (1956)	Manufacturing	Small economies of scale for multi-plant firms
Gribbin (1953)	Gas (Great Britain)	LRAC of production declines as output rises
Holton (1956)	Retailing	LRAC L-shaped
Johnston (1960)	Life Assurance	LRAC declines
Johnston (1960)	Road passenger transport (Great Britain)	LRAC either falling or constant
Johnston (1960)	Electricity (Great Britain)	LRAC of production declines as output rises
Lomax (1951)	Gas (Great Britain)	LRAC of production declines as output rises
Lomax (1952)	Electricity (Great Britain)	LRAC of production declines as output rises
Moore (1959)	Manufacturing	Economies of scale prevail quite generally
Nerlove (1961)	Electricity (U.S.)	LRAC (excluding transmission costs) declines and then shows signs of increasing
Gupta (1968)*	Manufacturing (India)	L-shaped in 18 industries, U-shaped in 5 industries, and linear in 6 industries

Source: A.A. Walters, "Production and Cost Functions: An Econometric Survey", Econometrica, January-February 1963, PP.49-54.

** Vinod K Gupta, "Cost Functions, Concentration, and Barriers to Entry in Twenty-nine Manufacturing Industries of India", Journal of Industrial Economics, November 1, 1968, 59-60.*

Activity 3

1. Pradeep Company's total variable function is as follows:

   $TVC = 50Q - 10Q^2 + Q^3$

   Where Q is the number of units of output produced.

   a) What is the output level where marginal cost is a minimum?

   b) What is the output level where average variable cost is a minimum?

   c) What is the value of average variable cost and marginal cost at the output specified in the answer to part (b)?

   ....

   ....

   ....

   ....

2. How would you reconcile the findings of Yntena with those of Ezekiel and Wylie?

....

....

....

....

3. How would you explain the findings of Johnston (Electricity) in short run and long run?

....

....

....

....

4. Production is related to costs. In fact, cost function can be derived from estimated production function. In view of empirical determination of production function, can you think of some limitations of statistical analysis relating to cost function?

....

....

....

....

5. Despite the above limitations listed by you, an estimated cost function is useful to a manager. Can you think of some points to support this contention?

....

....

....

....

6. Some empirical studies have suggested that the marginal cost function is approximately horizontal, but conventional cost theory suggests that the marginal cost curve is U-shaped. Provide an explanation for this apparent inconsistency.

....

....

....

....

7. The ABC Manufacturing Company's short-run average cost function in the year 2000 is $AC = 3 + 4Q$

   Where AC is the firm's average cost (in Rs. per unit of the product), and Q is the output rate.

   a) Obtain the firm's short-run total cost function.

   b) Does the firm have any fixed costs? Explain.

   c) If the price of the firm's product is Rs. 3 per unit, is the firm making profits or losses? Explain.

   d) Derive the firm's marginal cost function.

   ....

   ....

   ....

   ....

10.8 MANAGERIAL USES OF COST FUNCTION

In Unit 9 we have already discussed some of the uses and applications of cost analysis in the production process of managerial decision-making. The estimated cost function can help managers to take meaningful decisions with regard to:

1. determination of optimum plant size,
2. determination of optimum output for a given plant, and
3. determination of a firm's supply curve.

The optimum plant size, as discussed earlier, is defined in terms of minimum costs per unit of output. In other words, an optimum plant is given by that value of K (plant size) for which the average cost is minimum. If the long run total cost curve is a cubic function, the resultant long run average cost curve will be a conventional U-shaped curve. The plant level at which the long run average cost is minimum will be of optimum size.

For a given plant, the optimum output level will be achieved at a point where the average cost is the least. This condition can be easily verified from the short run total cost function.

The level of output that a firm would like to supply to the market will depend on the price that can charge for its product. In other words, a firm's supply is a positive function of the product price. To get the firm's supply schedule, one needs to know the firm's cost function and its objectives.

Activity 4

1. Can you list some more managerial uses of cost function other than given in section 10.9?

   ....

   ....

   ....

   ....

10.9 SUMMARY

Decision-making often requires a quantitative estimate of the parameters of production function. With quantitative estimates of the parameters of a production function on hand we can determine the marginal product of each input and economies of scale. Although, there are many different forms of production function we have discussed here only Cobb-Douglas production function. For this function the returns to scale are constant, increasing, or decreasing depending on whether the sum of the estimated parameters is equal to one (=1), greater than one (>1), or less than one (<1), respectively.

We have also discussed three forms of cost functions viz. linear cost function, quadratic cost function, and cubic cost function and their empirical estimates. Though, empirical estimates of both production functions and cost functions have a lot of use for managerial decision making there are conceptual and statistical problems in estimating such functions. But we understand that it will be sufficient for the manager if he knows how to interpret the estimates based on empirical research in his/her decision making process.

10.10 SELF-ASSESSMENT QUESTIONS

1. Discuss the managerial uses of production function?
2. What care should be taken while collecting the data for estimation of a production function?
3. Explain the determinants of cost function?
4. Explain the econometric method of estimating cost function? Why is this method is more popular than the other two methods (accounting and engineering) estimation costs?
5. What are the common problems you encounter while attempting to derive empirical cost functions from economic data?
6. The total cost function for a manufacturing firm is estimated as

   $C = 128 + 6Q + 2Q^2$

   Determine the optimum level of output Q to be produced?
7. Suppose that for a XYZ corporation's total cost function is as follows

   $TC = 300 + 3Q + 0.02Q^2$

   Where TC is the total cost, Q is the output.

   a) What is the corresponding fixed cost function, average fixed cost function, and variable cost function, average variable cost function?

   b) Calculate the average total cost function and marginal cost function.
8. Based on a consulting economist's report, the total and marginal cost functions for an ABC company are

   $TC = 200 + 5Q – 0.04Q2 + 0.001Q^3$

   $MC = 5 – 0.08Q + 0.003Q^2$

   The president of the company decides that knowing only these equations is inadequate for decision making. You have been directed to do the following.

   a) Determine the level of fixed cost (if any) and equations for average total cost, average variable cost, and average fixed cost.

   b) Determine the rate of output that results in minimum average variable cost.

   c) If fixed costs increase to Rs. 500, what output rate will result in minimum average variable cost?
9. Given the total cost function for Laxmi Enterprices Co.

   $TC = 100Q – 3Q^2 + 0.1Q^3$

   a) Determine the average cost function and the rate of output that will minimize average cost.

   b) Determine the marginal cost function and the rate of output that will minimize marginal cost.

10.11 FURTHER READINGS

1. Craig Peterson and W. Cris Lewis, (1994). *Managerial Economics (Chapter 6 and 7)*, Macmillian Publishing Company, USA.
2. Maddala, G.S., and Ellen Miller, (1989). *Micro Economics: Theory and Applications (Chapter 6 and 7)*, McGraw-Hill, New York.
3. Mote, V.L., Samuel Paul, and G.S. Gupta, (1977). *Managerial Economics: Concepts and Cases (Chapter 3)*, Tata McGraw-Hill, New Delhi.
4. Ravindra H. Dholakia and Ajay N. Oza, (1996). *Micro Economics for Management Students (Chapter 8 and 9)*, Oxford University Press, Delhi.