UNIT 14 BASIC CONCEPTS, PARALLEL LINES AND PARALLELOGRAM

Structure

14.1 INTRODUCTION

The notions of plane geometry originated in ancient times. They arose out of the necessity to solve practical problems. Egyptians used geometry to determine the lengths, areas or volumes of various objects. Ancient Hindus used it to design altars (Vedis) for worship. Later, Greeks formulated the logical or deductive aspects of geometry and developed it as a discipline.

We find ample application of geometry in daily life. A learner can be helped to appreciate the use of geometrical forms in nature and architecture all around. He/she may also use the concept of geometry to construct figures or develop designs. Geometrical propositions can be understood through practical applications and logical reasoning. These approaches are reflected in this unit.

The unit deals with the investigation and study of the salient concepts of elementary geometry. Practical methods, intuition and deduction have been freely employed to demonstrate their truth. The unit is designed to enable you to get a greater understanding of geometrical concepts and to proceed to a logical treatment of the subject.

14.2 OBJECTIVES

At the end of this unit you, the teacher, would be able to:

- explain the basic concepts of geometry;
- develop an activity-based and problem-based approach for teaching various concepts covered in this unit;
- create interest in studying parallel lines and parallelograms;
- illustrate the sequential development of various concepts starting from some basic concepts.
- establish the need to start with some axioms to study any mathematical system; and
- demonstrate geometric dissections

14.3 BASIC CONCEPTS

Main Teaching Points : a) Concepts of point, line, plane and angle.

b) Related axioms.

Teaching-Learning Process : The unit should be introduced by reviewing fundamental concepts on points, lines and planes in such a manner that the practical aspect of geometry is realised. The method outlined in the unit is:

1. Intuitive approach
2. Experimental approach
3. Deductive and axiomatic approach
4. Analytical approach

Activities which set the stage for deductive arguments are given for a few selected topics.

Activities

1. Ask : Examine the outside cover of an ordinary box of matches. Write. The box has six sides or faces, each of which is a rectangle.

   Explain : The box may be represented by drawing it in two ways:

   In the first drawing (Figure 14.1 (a)), we can see only three faces of the box. The other three faces are hidden from view.

   If we were to construct a box of the same shape using a piece of wire for the outline of each face, it will look as in Figure 14.1 (b). All faces of this box can be seen. The hidden part of Figure 14.1(a) has been shown by dotted lines.

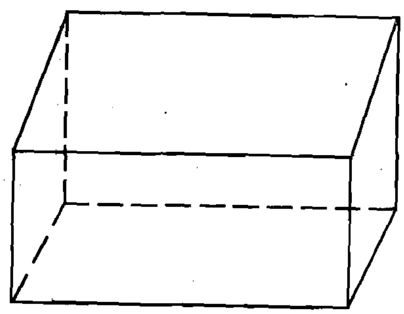

Fig. 14.1(a)

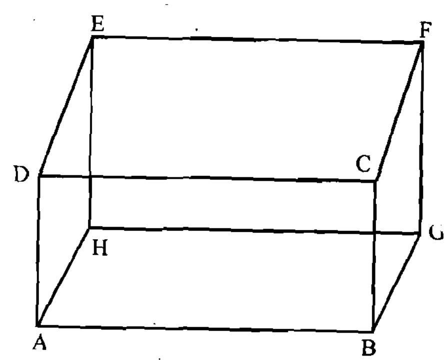

Fig. 14.1(b)

2. Ask : Which faces have the same shape and size in Figure 14.1?

   Write: Faces ABCD and HGFE are two such faces. Similarly AHED and BGFC are another pair of such faces. ABGH and CFED are a third pair. They are on opposite sides of each other.

   Explain : In geometry, we are concerned only with the study of the shape and size of objects. We are not concerned with the matter or material which they may contain.

   Explain that a solid body is conceived as occupying space and the amount of this space is called its volume.

3. Ask : Where do the two adjacent faces meet?

   Write: The two adjacent faces meet in a **straight line** which is called an edge.

   Explain : The faces ABCD and CFED meet in the straight line CD. In the whole box there are 12 straight lines or edges.

4. Ask : What is the intersection of two edges called?

   Write. The intersection of two edges is a point

Explain : The edges AD and DC meet at the point D. Explain that there are eight such points which are called corners. Each of these points indicates the meeting points of three edges. Thus D indicates the intersection of edges DE, AD and DC.

5 Ask : What, in your opinion, should be taken as basic terms which we shall accept as undefined and proceed to define other terms using these basic terms? There may be different suggestions made by learners, or may be there is no response.

Explain : The terms "point", "straight line" and "plane" are difficult to explain in simple words or in terms of the relatively simpler notions. We take them as undefined. Explain that points such as D, E, F, etc., have corresponding positions in space. We frequently mark a position on a piece of paper, or a map, or on a picture by marking a small dot which indicates some particular position. Thus **a point indicates a position in space and has no size or magnitude.**

6 Ask : How can we explain the formation of a line using points?

Mark P and Q as two points on the surface of the paper as shown in Figure 14.2. Suppose point P moves to position Q.

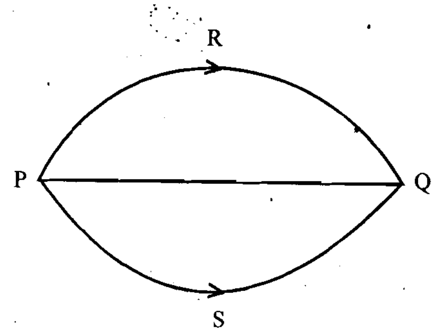

Fig. 14.2

Explain : There are a number of paths which P may take such as PRQ and PSQ. These vary in length. The most direct path will be along the straight line PQ which joins the points. Thus, a straight line or a line may be described as : **the shortest path between two points P and Q.**

Explain : The straight line which marks the path of a moving point has length but no width. Hence a line is said to have one dimension only.

Explain that from the geometrical point of view a line is a set of points and extends endlessly in both directions. The symbol PQ or QP is used for the line *l* as shown in Figure 14.3.

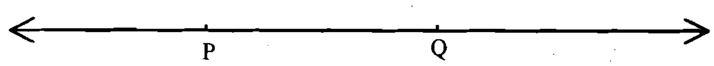

Fig. 14.3

7.	Ask	:	How many points are there on a line?
			Write : A line consists of an infinite number of points.
	Explain	:	We may consider it as the first axiom on lines and points.
8.	Ask	:	How may lines can pass through a given point?
			Write: An Infinite number of lines can pass through a given point.
	Explain	:	It is the second axiom on lines and points.
9.	Ask	:	How many lines can be drawn through two given points?
			Write: One and only one line can be drawn through two given points.
	Explain	:	It is the third axiom on lines and points.
			Explain that as a consequence of the third axiom, it is deduced that if two lines intersect, this intersection is exactly one point.
			Let us consider a line as a number line, then each point of the line corresponds to a real number. The number associated with a point is called the coordinate of the point, and the point associated with a number is called the graph (or graphical representation) of that number.
10.	Ask	:	Can you think of any characteristic of the relationship between the points of a line and real numbers?
			Write: To every real number, there corresponds exactly one point of the line and vice versa.
	Explain	:	The correspondence described here is called a one-to-one correspondence between the points of a line and the set of real numbers.
			It is called the "**Ruler axiom**".
			Explain that if the coordinate of a point P is X and the coordinate of the point Q is Y, then the distance between P and Q is $\lvert Y - X \rvert$ which is equal to $\lvert X - Y \rvert$. We will accept this as one of our basic assumptions.
11.	Ask	:	How many positive real numbers can be assigned to the distance function for a pair of distinct points?
			Show that corresponding to every pair of distinct points, there is one and only one positive real number that can be assigned for the distance function.
	Explain	:	This real number is called the distance between two points. This distance is the absolute value of the difference of the real numbers corresponding to the two points. Explain that symbol PQ is used to refer to the number which is the distance between points P and Q.
12.	Ask	:	Can you think of objects with flat surfaces?
			Bring out that table tops, mirrors, papers, walls, etc., are objects with flat surfaces.
	Explain	:	Flat surfaces are examples of planes. But these are physical models of planes. A plane is a mathematical abstraction.
			Explain that collinear points are points that lie on the same line. Similarly coplanar points are points that are on the same plane.
			Explain the following Figure 14.4 of a pyramid,

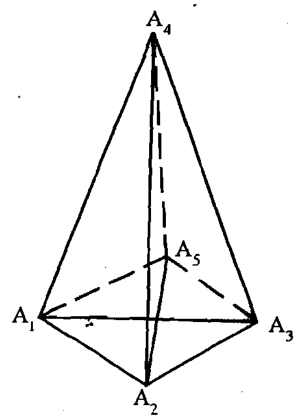

Fig. 14.4

A_2 and A_3 are collinear; A_1, A_4 and A_2 are non-collinear ; A_1, A_2 and A_4 are coplanar; A_1, A_2, A_3, A_4 are non-coplanar.

13. Ask : How many planes can pass through three non-collinear points?

Bring out that there is exactly one plane containing three non-collinear points.

Explain : This is the first axiom on lines and planes.

14. Ask : Think of any two points in a plane.

Bring out that the line containing them is contained in the plane.

Explain : This is the second axiom on lines and planes.

15. Ask : How many lines can be formed by the intersection of two planes?

Write: If two planes intersect, then their intersection is exactly one line.

Explain : It is the third axiom on lines and planes. Using these axioms many theorems can be proved.

16. Ask : Take two distinct points P and Q on line *l* (Figure 14.5) Bring out that P and Q determine a line segment, or simply a segment.

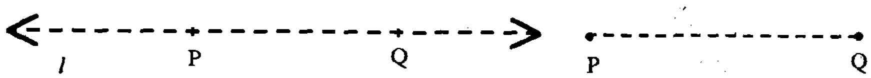

Fig. 14.5

Explain : The union of the set containing two points P and Q of line *l* and the set of all points of *l* between P and Q is called a **segment** denoted by PQ The **length** or **measure** of PQ is the distance between P and Q and is denoted by PQ. Thus

PQ = {P} ∪ {Q} ∪ **{points between P and Q}**

Explain : A line may be named by any two of its points but a segment is always named by its end-points.

17. Ask : Take point O on line *l* (Figure 14.6)

Bring out that O separates all the other points of the line into two sets of points. Point O does not belong to the set of points on **either** side of O.

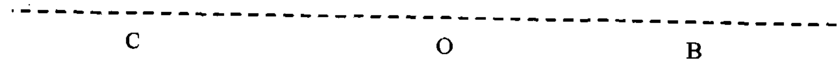

Fig. 14.6

Explain : The set of all points on either side of O, excluding O, is called a half-line. To describe the two half-lines on either side of O. consider two points B and C on either side of O, Consider P as a moving point on the line.

Denoting half-line on the side of B by $\overrightarrow{OB}$

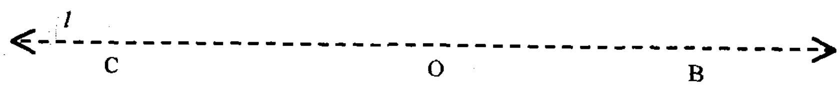

Fig. 14.7

$\overrightarrow{OB}$ = **{all points P between O and B} ∪ {B} ∪ {all points P such that B is between O and P}.**

Similarly $\overrightarrow{OB}$ = **{all points P such that P lies between O and ~ C∪ {C}∪ {all points P such that C lies between O and P}**

Explain that the half-line does not contain the end point O.

18. Ask : What is the union of the set containing the point O and a half-line $\overrightarrow{OB}$

Bring out that it is a ray denoted by $\overrightarrow{OB}$ (Figure 14.8).

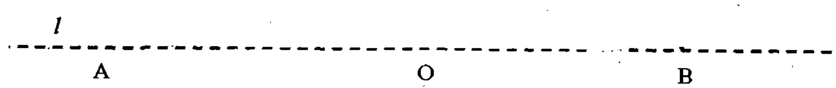

Fig. 14.8

Explain : Point O is called the end point of the ray.

Explain that the rays OA and OB are called opposite rays if and only if O is between A and B

19 Ask : Draw Figure 14.9 on the board. Identify a line, several rays, and several segments.

Ask what is $\overrightarrow{OP} \cup \overrightarrow{OQ}$

Bring out it is the union of two rays which are not subsets of the same line.

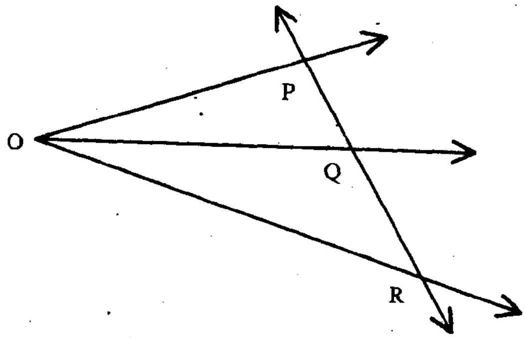

Fig. 14.9

Figure 14.10 below shows —$\overrightarrow{OP} \cup \overrightarrow{OQ}$

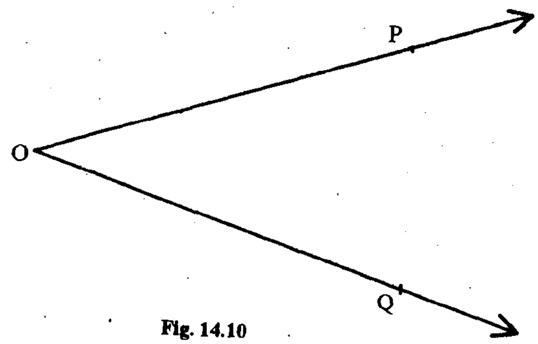

Fig. 14.10

Write: It is the picture of an angle.

Explain : An angle is the union of two non-collinear rays having a common end point. The two rays are called the sides or arms of the angle. The common end point is called the **vertex** of the angle.

Adjacent angles: Explain that two angles which have a common vertex, a common arm and which lie on opposite sides of the common arm form a pair of adjacent angles.

20. Ask : Think of a ray standing on a line.

Bring out that the sum of two adjacent angles so formed is 180°. Conversely, if the sum of two adjacent angles is 180°, the non-common arms of the angles are two opposite rays.

Explain : This is an axiom on the angle.

Explain that two angles are called supplementary angles if their sum is 180°, and two angles the sum of whose measures is 90° are called complementary angles. For example 70° and 110° are a pair of supplementary angles and 55° and 35° are a pair of complementary angles. Explain that ray $\overrightarrow{PS}$ is said to be the "bisector" of $\angle QPR$ if S is a point in the interior of $\angle QPR$, and $\angle QPS = \angle SPR$. Here, $\angle RPS = 1/2\ \angle QPR$. (Fig. 14.11).

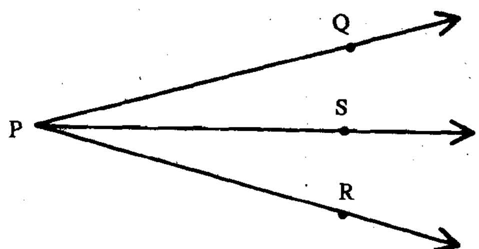

Fig. 14.11

Methodology used : Intuitive deductive analytical and experimental approaches form the essential feature of teaching geometry

Check Your Progress

Notes : a) Write your answers in the space given below

b) Compare your answers with the one given at the end of the unit.

1. Take three collinear points A, B and C. How many triangles can you draw with A, B C as the vertices ?

....

2. How many edges does a cube have? Illustrate.

....

....

....

3. Explain how to find the measure of an angle which is equal to its supplement ?

....

14.4 PARALLEL LINES

Main Teaching Point : The concept of parallel lines and related axioms.

Teaching-Learning Process : We have seen in the preceding section that when two lines intersect, we get a number of geometric figures such as rays and angles. What happens if two lines do not intersect ? In this section, we shall consider certain properties of non-intersecting lines.

21. Ask : Examine the ruled printed lines on an exercise book.

Bring out two facts:

a) the distance between any pair of lines is always the same;

b) even if the lines are produced to any extent beyond the page of the exercise book, they never meet.

Explain : Such straight lines drawn in a plane are called parallel straight lines.

Explain that two lines are parallel if and only if they are coplanar and they do not intersect

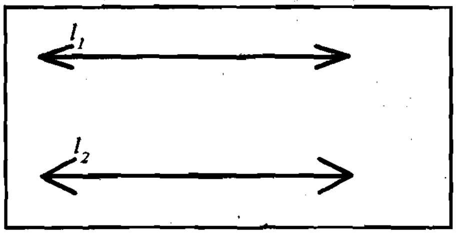

Fig. 14.12

22. Ask : Think of two coplanar lines and a line which intersects each of the two given lines.

Bring out two distinct points.

Explain : A line which intersects two coplanar lines at two distinct points is called a transversal line. Explain that in Figure 14.13, l_1 and l_2 are coplanar lines and are cut by a transversal m at two distinct points A and B. Although another line n intersects both l_1 and l_2 at the intersection point C, n is not a transversal.

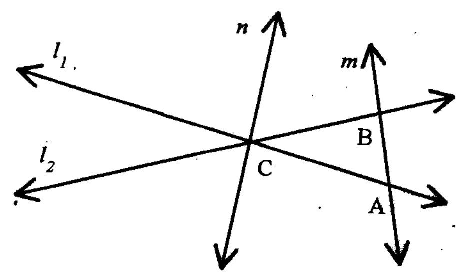

Fig. 14.13

Explain that transversals and the lines they intersect form alternate angles, interior angles, corresponding angles, etc. These pairs of angles are of special importance in our investigation of geometric properties.

23. Ask : Think of a line *l* and a point A not on *l*

Ask how many lines can be drawn through A and parallel to *l*.

Bring out that there is one and only one line which passes through A and is parallel to *l*. (Figure 14.14.)

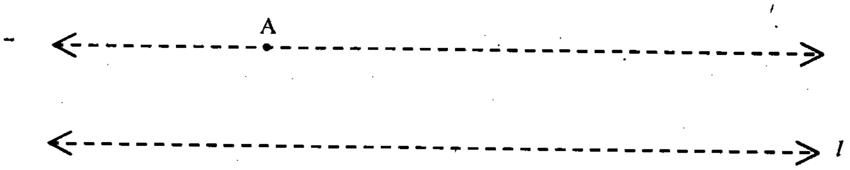

Fig. 14.14

Explain : This is the first parallel axiom.

Explain that if a transversal intersects two parallel lines, then each pair of corresponding angles are equal. Conversely, if a transversal intersects two lines making a pair of corresponding angles equal, then the lines are parallel. This is the second parallel axiom.

Explain that if two lines are intersected by a transversal such that two alternate interior angles are equal, then the lines are parallel. It can easily be proved using the parallel axioms.

Explain that, using parallel axioms, several theorems on parallel lines and constructions involving parallel lines can be proved. The theorems and their proofs may be seen in any standard book on geometry.

Distance between two parallel straight lines

24. Ask : Draw two parallel straight lines PQ and RS. (Figure 14.15). Take a point A on PQ. Take AB as a straight line which is perpendicular from A to RS.

Bring out that AB is the distance between the two parallel straight lines.

14.5 PARALLELOGRAMS

Main Teaching Point : Different types of quadrilaterals and their interrelation.

Teaching Learning Process

26. Ask : What is a plane figure?

Bring out that a part of plane surface which is enclosed by line segments or by a closed curve is called a region and the outline of the region (boundary) is called a plane figure.

Explain : If the boundary lines are all line segments, the figure is called a rectilinear figure. If four straight lines in a plane intersect in pairs, the figure formed is a quadrilateral. In Fig. 14.16 the points P, Q, R, S are four vertices of the quadrilateral.

Explain lines that join two opposite vertices are called diagonals.

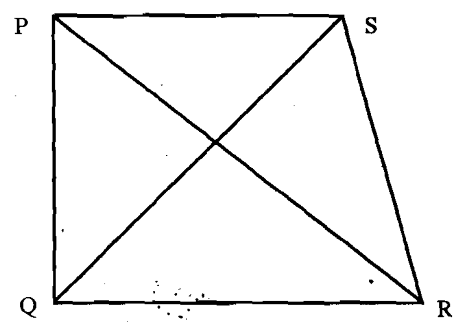

Fig. 14.16

The sum of the angles of any quadrilateral is equal to four right angles.

27. Ask : Think of a quadrilateral in which both pairs of opposite sides are parallel and one of its angles is a right angle.

Bring out that it is a rectangle. (Fig. 14.17.)

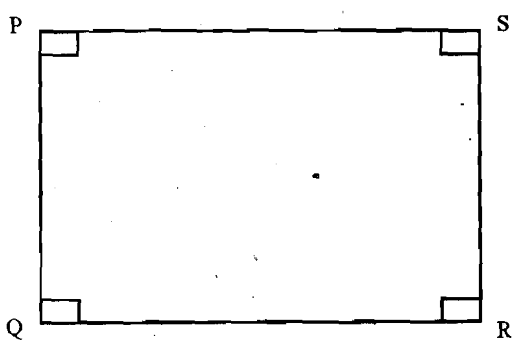

Fig. 14.17

28. Ask Take the outer cover of an ordinary match box without its inner open box. Squeeze gently so that two opposite edges come closer. The rectangular shape of open ends changes. Angle between two edges is no longer a right angle but the opposite edges are still parallel. A quadrilateral with each pair of opposite sides parallel is called a parallelogram. (Figure 14.18.)

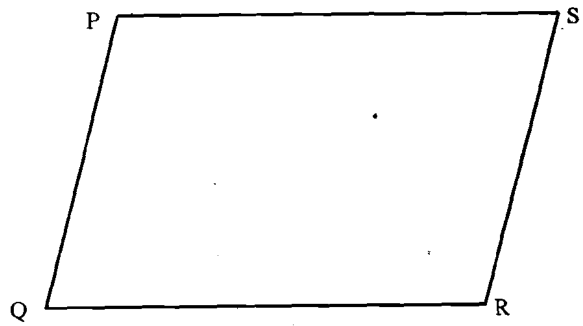

Fig. 14.18

29. Ask : What about the shape of the open end of the match box if it had been a square instead of a rectangle? Bring out that it would still be changed to a parallelogram, but its sides will all be equal. It is called a **rhombus** which is again a special form of a parallelogram.

Explain : A quadrilateral with both pairs of opposite sides parallel, and both adjacent sides equal is called a rhombus. (Fig. 14.19.)

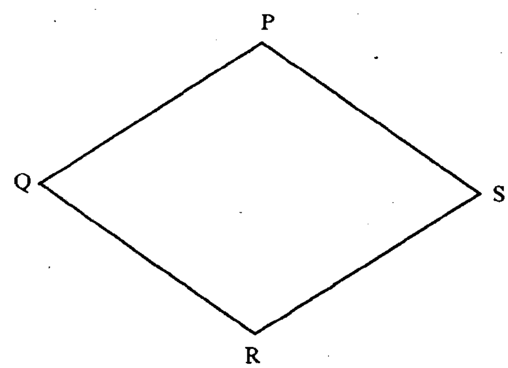

Fig. 14.19

30. Ask : Think of a quadrilateral whose both pairs of opposite sides are parallel, one of its angles a right angle and both adjacent sides equal.

Bring out that it is a square. (Fig. 14.20.)

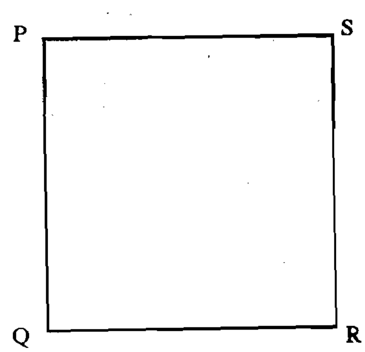

Fig. 14.20

31. Ask : What about a quadrilateral in which two opposite sides are parallel but the other sides are not parallel?

Bring out that it is a trapezium. (Figure 14.21.)

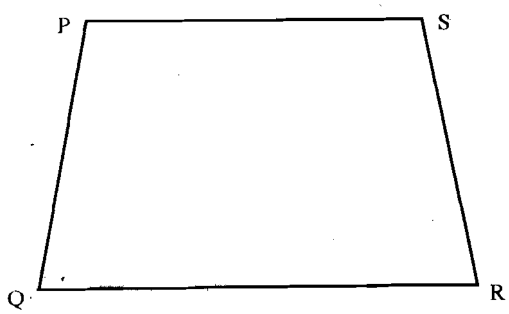

Fig. 14.21

32. Ask . Can you think of a chart in the development of the properties of quadrilaterals ?

Suggest that students consider the definitions of different figures.

Quadrilateral is a plane figure made up of four sides (line segments). **Trapezium** is a quadrilateral whose one pair of opposite sides is parallel.

Parallelogram is a quadrilateral whose opposite sides are parallel.

Rectangle is a quadrilateral whose opposite sides are parallel and one angle is a right angle.

Rhombus is a quadrilateral whose opposite sides are parallel and adjacent sides are equal (or a rhombus is a parallelogram whose all sides are equal in length.

Square is a rectangle whose all sides are equal.

Using the above definitions let them draw a chart interconnecting different types of quadrilaterals mentioned above.

Explain : Every square is a rhombus and rectangle but the converse is not true.

Every rectangle (and rhombus) is a parallelogram but converse is not true.

Every parallelogram/trapezium is a quadrilateral but the converse is not true.

33 Ask Can you think of a way of learning geometry by using a geoboard?

Explain A geoboard may be made of 6 mm or 8 mm plyboard of size 6" x 6" it is divided into 36 equal squares with one pin fixed vertically at the centre in each square (Figure 14.22).

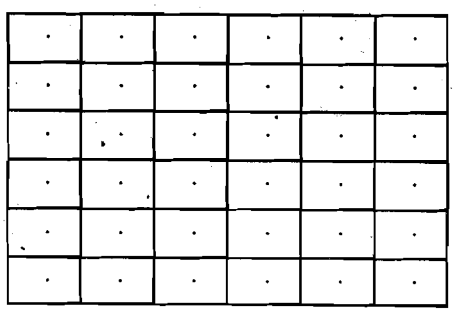

Fig. 14.22

Explain : Using rubber bands of different colours, various geometrical figures can be made on the board. Explain: Several geometrical patterns viz. , Parallelogram, rectangle, square, rhombus etc. can be made on the geoboard. (Figure 14.23).

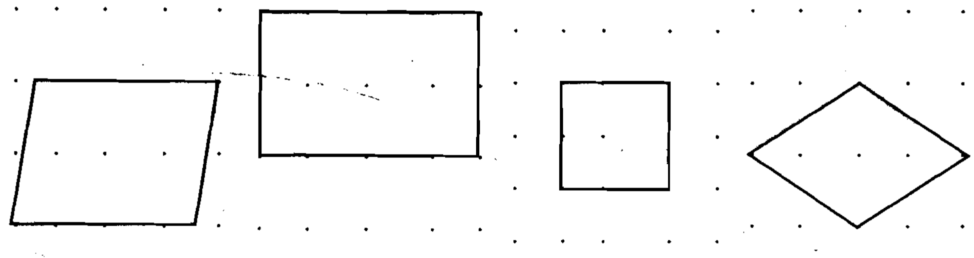

Fig. 14.23

34. Ask : Using a geoboard can you think of proving results like (a) diagonals of a parallelogram bisect each other. (b) diagonals of a rhombus bisect each other at right angles. Write yes and draw result on geoboard. (Fig. 14.24 and 14.25.)

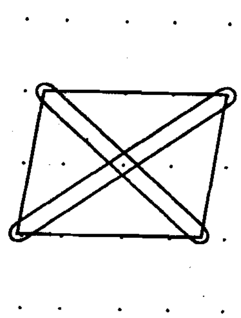

Fig. 14.24

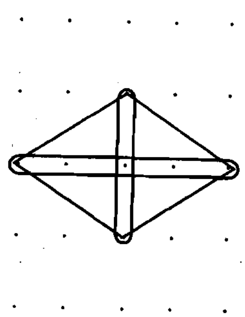

Fig. 14.25

Methodology used : While teaching geometry, more stress should be on intuitive and experimental approach. Analytic and Synthetic approach is suitable while doing proofs of standard theorems.

Check Your Progress

Notes : a) Write your answers in the space given below.

b) Compare your answers with the one given at the end of the unit.

6. State whether true of false.

   i) If $l \parallel m$ and n is a transversal, show that the bisectors of the interior angles form a parallelogram.

   ....

   ii) Show that the quadrilateral obtained by joining the mid-points of the sides of a rectangle is a rhombus.

   ....

   iii) Show that in a square, the diagonals are equal and perpendicular to each other.

   ....

14.6 ENRICHMENT MATERIALS

Non-Euclidean Geometry

1. Ask : What is the sum of angles of a triangle ? Ask for sums of several triangles, all drawn on the plane surface, and have the students write the sums.

Ask them what they notice about the sums of three angles of a triangle The conclusion is that they are all 180°.

Then ask them to give answer for the sum of the angles of a triangle drawn on a sphere or on an elliptical surface.

Have students give their own arguments. Finally, write the sum of the angles as greater than 180°.

Again ask them for the sum of the angles of a triangle drawn on hyperbolic surface. Have students give their own arguments. Finally write the sum of the angles less than 180°.

Thus the possibility of the sum of the angles of a triangle not equal to 180° occur only in Non-Euclidean geometry due to N.I. Lobachevesky and J. Bolyai.

2 Ask : How many lines can be drawn which are parallel to a given line and passing through a given point, not on the line, in Non-Euclidean space?

Have students give their arguments. Finally tell them that on hyperbolic surface, infinite number of lines can be drawn through a given point and parallel to the given line where as no line can be drawn passing through the given point and parallel to the given line if the surface is elliptical or spherical.

3 Ask : Can you begin discussion of dissections by demonstrating the area equality between a rectangle and a parallelogram with the same base? Using heavy paper or cardboard construct a rectangle PQRS.

Make a straight cut from vertex P to a point L on side $\overline{SR}$, Remove ΔPSL placing side $\overline{PS}$ along side $\overline{QR}$ to form parallelogram PQML. (Fig. 14.26.)

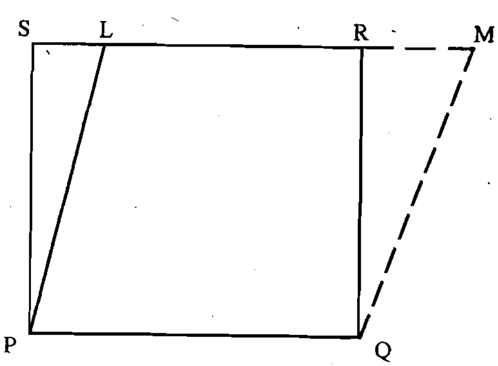

Fig. 14.26

4. Ask : Can you construct a parallelogram and a trapezoid with one of the non-parallel sides as common base such that the two figures have equal area ?

Consider any trapezoid P Q R S, find the mid-point L of side QR, and through L draw a line parallel to PS which intersects PQ at A and SR at B. Since ΔRLB and ΔQLA are congruent, the area of trapezoid PQRS and parallelogram PABS are equal (Fig. 14.27)

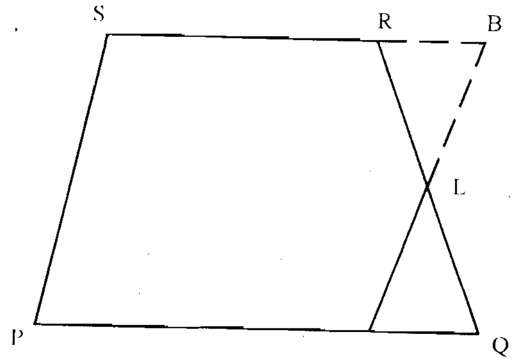

Fig. 14.27

The range of possible transformations of polygons into other polygons by means of dissections is vast. J. Bolyai, one of the founders of non-euclidean geometry, was the first to suggest that given any two polygons with equal area, either figure could be dissected a finite number of times such that upon rearrangement, it would be congruent to the other. Here we are concerned only with specific transformations that require a minimum number of dissections.

5. Ask : Can you consider the problem of dissecting a given acute triangle to form a rectangle?

Take ΔPQR. Find the mid-points of sides PR and QR and connect these points to form BD. From R construct a perpendicular to BD to meet at C. Take ΔBCR and place it so that C is now at A and $\angle BRC$ is adjacent to $\angle RPQ$. Similarly move ΔDCR so that C is now at E and $\angle DRC$ is adjacent to $\angle RPQ$. (Fig. 14.28.)

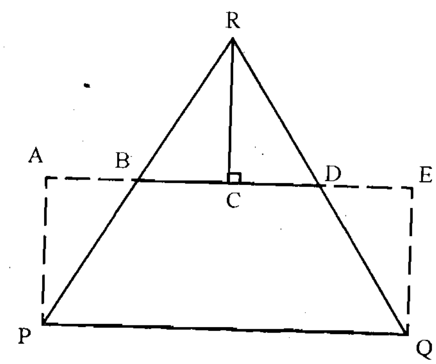

Fig. 14.28

The required rectangle is PQEA.

14.7 LET US SUM UP

This unit provides an opportunity to a teacher to demonstrate the importance of intuitive, deductive, experimental and analytic approach as an essential feature of productive thinking in geometry. To make students think intuitively, conduct experiments for understanding various concepts, it is necessary that the teacher leads students to think intuitively and conduct experiments alongwith the students. The teacher must always be fully prepared and knowledgeable. The intuitive and experimental approach develop self-confidence and courage in the students since they permit willingness to learn from mistakes. The students make conjectures verify them, discard them if found incorrect and proceed to make new conjectures.

Teachers should be encouraged to constantly, gather materials and ideas for enriching their teaching of mathematics. Regardless of the ability level of the students, appropriate enrichment activities can always be found. The enrichment material offered in section 4.7 of this unit is just an illustration of how this can be achieved.

14.8 UNIT-END ACTIVITIES

1. Draw a picture of three collinear points, P, Q and R.
2. Draw a picture of three non-collinear points A, B and C.
3. If P, Q and R are three non-collinear points
   i) How many different lines are determined by choosing a distinct pair of points each time?
   ii) How many different planes would contain the three points?
4. We proved that if two lines intersect, their intersection is a point. Consider the possible intersection of two distinct planes. Can two planes intersect in
   i) exactly one point?
   ii) exactly two points?
   iii) exactly three non-collinear points?
   iv) exactly one line?
5. If lines PQ, PR, PS, and PT are parallel to a line l, what can be said about the points P, Q, R, S, and T?
6. Given two points P and Q, how many line segments do they determine?
7. Given three collinear points P, Q and R, name all the line segments they determine.
8. Tell which of the following statements are true and which are false.
   i) If two lines intersect, then they intersect in exactly one point.
   ii) If two planes intersect, then they intersect in exactly one point.
   iii) There is exactly one plane containing a given line and a given point not on the given line.
   iv) If a point is between two other points, then the three points are collinear.
   v) If PQ = 6, QR = 8, and PR = 14, then point Q is between points P and R.
9. Discuss each of the following briefly. Support your argument by appropriate axioms or theorems.
   i) Three points P, Q, and R are in plane p. These same three points are in plane q. Is it certain that plane p is equal to plane q?
   ii) If a line and a plane not containing the line intersect, then the intersection is a unique plane containing two intersecting lines.
   iii) There is a unique plane containing two intersecting lines.
10. If S lies in the interior of $\angle QPR$,

    $\angle QPR = 80^0$ and $\angle QPS = 45^0$

    What is the measure of $\angle SPR$?
11. An angle is 28^0 more than its complement. What is its measure?
12. The measure of an angle is thrice the measure of its supplementary angle. Find its measure.

13. Which of the following statements are true and which are false? Draw a picture to illustrate each answer.

    i) No two parallel segments intersect.

    ii) Any two parallel rays are coplanar.

    iii) Any two lines that do not intersect are parallel.

    iv) Two lines are either parallel or they intersect.

    v) Any two rays that do not intersect are parallel.

    vi) Any two intersecting lines are coplanar.

14. In Fig. 14.29 L is the mid-point of side PS of a trapezium PQRS, with $\overline{PQ} \parallel \overline{SR}$

    A line through L parallel to PQ meets QR in M. Show that M is the mid-point of QR.

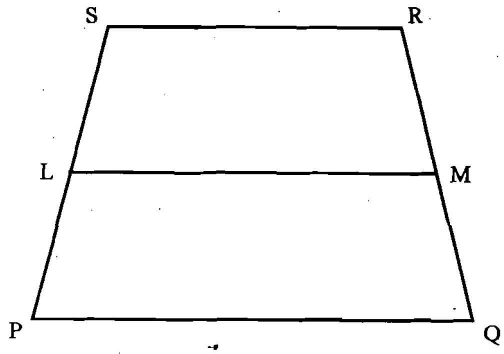

Fig. 14.29

15. In Fig. 14.30 of parallelogram PQRS if SL $\perp$ PQ and QM $\perp$ $\overline{SR}$, prove that $\overline{SL}$ is congruent to MQ.

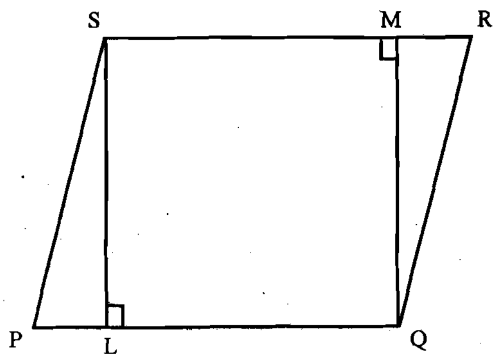

Fig. 14.30

16. Prove that in a parallelogram, the sum of the squares of the **lengths of the diagonals** is equal to twice the sum of the squares of the lengths of any two adjacent sides.

17 Write the answer in the space provided. Indicate whether always, sometimes or never true.

    i) Diagonals of a rhombus bisect each other...

    ii) If two lines are perpendicular to the same line, they are parallel ....

    iii) If the diagonals of a parallelgoram are perpendicular to each other it is a rhombus.

    ....

    iv) Opposite angles of a parallelogram are supplementary. ....

18 Given parallelogram PQRS,

$\angle QPS = 60^o$, $\angle PQS = 85^o$, find $\angle RQS$.

14.9 ANSWERS TO CHECK YOUR PROGRESS

1. None
2. 12
3. 90^o
4. $\angle 1 = 110^o$, $\angle 2 = 110^o$ $\angle 3 = 70^o$
5. True
6. i) True

   ii) True

   iii) True

14.10 SUGGESTED READINGS

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