UNIT 16 MENSURATION : AREA AND VOLUME

Structure

16.1 INTRODUCTION

Since the beginning of civilization man has explored the question of "how much"?

Man started measuring land, water or milk etc. This was the origin of **mensuration** which later became the basis of architecture and engineering.

Mensuration is a branch of mathematics which deals with lengths of lines, area of surfaces and volume of solids.

A student studying mensuration should be conversant with metric system of units and their conversion. The concepts of area and volume should be known to the student.

16.2 OBJECTIVES

After studying this unit, you should be able to:

- make the students aware of the concepts of perimeter, area and volume;
- prepare the students to identify similarities in some of the formulae;
- develop in students the skill of conversion of units;
- prepare the students to apply deductive methods for arriving at formulae for area and volume for different objects;
- enable the students to apply the concepts in solving problems.

16.3 MEASUREMENT

Main Teaching Point : What is the need of measurement?

Teaching-Learning Process : Nature has created human beings and provided them with the most universal "built-in" instruments in the body. These instruments, which continually bring information, are the senses. The eyes enable us to sense big and small objects. But the eyes can deceive too. There are innumerable examples of such 'optical illusions':

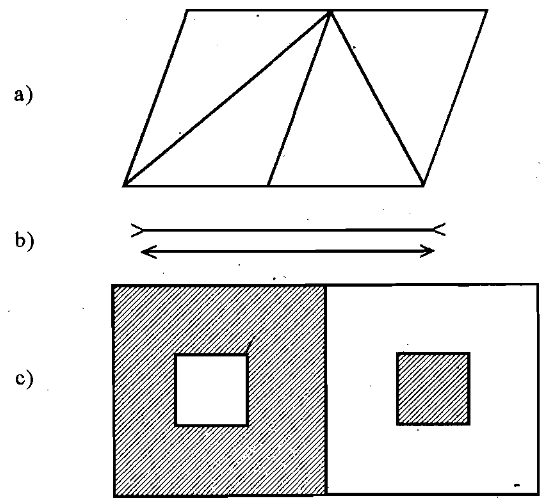

Fig. 16.1

The teacher can draw these diagrams on the blackboard and ask : Could you tell me which diagonal is bigger among the two diagonals in diagram (a)? Let them guess. Later a student could be called to the blackboard and asked to measure the lengths and conclude that both are equal.

Ask : Among the two lines in the figure (b), which is smaller? The students would make different guesses. Finally one student could be called and asked to measure and conclude that both the lines are equal.

Ask : In figure (c), compare the relative sizes of the two rectangles. Again different visual conclusions are drawn. The teacher could call a student to measure the sides and diagonals of the two smaller rectangles.

Explain : Based on these three diagrams the teacher could conclude that the sense of sight may sometimes lead us to incorrect conclusions about the size of objects. For accurate measurement, we need the help of instruments, the simplest one being a graduated scale.

Methodology used : Demonstration-cum-discussion is used to tell the importance of measurement.

16.4 MEASUREMENT OF AREA AND PERIMETER

16.4.1 Meaning of Perimeter and Area

Main Teaching Points : i) Perimeter is sum of lengths of all sides.

ii) Area is measured by comparing it with a unit area.

Teaching Learning Process

Ask : Could you tell me what is the perimeter of a triangle? There will be different versions of answers like "the sum of the lengths of three sides of a triangle", "distance going around a triangle", etc. Bring out the meaning of perimeter of a closed geometric figure.

"The sum of lengths of all sides of a closed geometrical figure or the length of the boundary of a closed geometrical figure is called its perimeter."

In the case of area, the teacher can develop the idea of comparison by drawing Fig. 16.2 and 16.3.

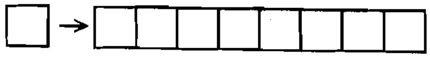

Fig. 16.2

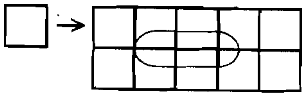

Fig. 16.3

Ask	:	In Fig. 16.2, compare the areas of the big rectangle with the area of the small square.
		The students will come out with the answer that the area of the rectangle is eight times the area of the square.
Ask	:	In Fig. 16.3 the bigger square is how many times the smaller one?

The students will count the number of subdivisions made in the bigger square, each subdivision being equal to the small square. The answer is "ten times".

Similarly, the concept of comparison can be developed with the help of equilateral triangles. Four smaller equilateral triangles can be fitted in the bigger one as in Figure 16.4.

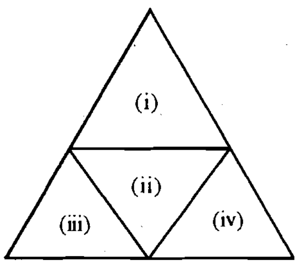

Fig. 16.4

However, it will be very confusing if each person were to use his own unit of area to measure area of other figures. To avoid this confusion, we compare the area of a plane figure with the area of a small square of side one unit.

Methodology Used : Proper demonstration will help the students in assimilating how the area is measured. Guided discussion is required.

16.4.2 Units of Measuring Area

Before beginning with the formulae for measuring area of different figures, the students should be introduced to the units of area and their conversion from one unit to another.

1 sq. cm	=	100 sq. mm
1 sq. dm	=	100 sq. cm
1 sq. m	=	100 sq. dm
1 sq. dam	=	100 sq. m
1 sq. hm	=	100 sq. dam
1 sq. km	=	100 sq. hm
1 acre	=	100 sq. m
1 hectare	=	100 acre, = 10,000 sq. m

16.4.3 Area of Quadrilaterals, Triangle and Circle

Main Teaching Point : For mulae for measuring area of different figures.

Teaching Learning Process : Starting from the area of a rectangle, areas of different rectilinear figures can be found using logical deduction. The formulae for the area and perimeter of a rectangle and hence that of a square can be deduced from the definition of the unit of area.

1) Square : a = side

   Perimeter = 4 a Area = a^2

2) Rectangle : a = length b = breadth

   Perimeter = 2 (a + b) Area = a × b

3) Triangle: Ask: Consider Fig. 16.5. ΔABC is made up of two triangles ABD and ADC. Compare area of ΔABD with rectangle AFBD and area of triangle ADC with that of rectangle ADCE. What is the area of ABC with respect to rectangle BCEF?

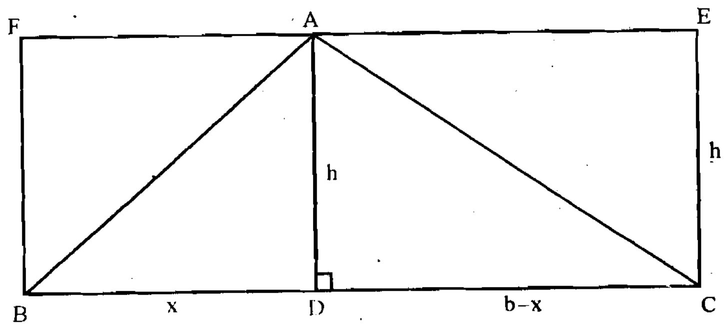

Fig. 16.5

Explain : i) Area of ΔABC = area of ΔADB + area of ΔADC

= 1/2 area (ADBF) + 1/2 area (ADCE)

= 1/2 area (BCEF) = 1/2 b.h

where h is the altitude to the base, b of ΔABC

ii) Perimeter = a + b + c

where a, b, c are the lengths of the sides of Δ ABC

iii) We also have,

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Where } S = \frac{a+b+c}{2} = \text{semi-perimeter}$$

4) Isosceles Right Triangle

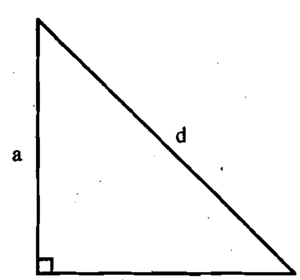

Fig. 16.6

Let a = side, d = hypotenuse.

Ask : Find a relation between a and d.

By pythagoras theorem, $a^2 + a^2 = d^2$

$$d = a\sqrt{2}$$

Perimeter = 2 a + d

Ask : Consider the two perpendicular sides as a and b.

Use the formula: area of a triangle = $\frac{1}{2}$ bh and ask what is the area of this triangle?

Area = $\frac{1}{2}a^2$

5) Equilateral Triangle

a = side, h = altitude, $h = \frac{\sqrt{3}}{2}a$

Perimeter = 3 a, Area = $\frac{1}{2}$ a.h = $\frac{\sqrt{3}}{4}a^2$

6) Parallelogram

Ask : Guess the relationship between the areas of a rectangle AEFD and the parallelogram ABCD.

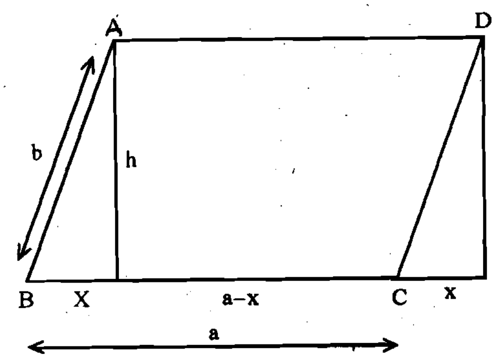

Fig. 16.7

Explain : Let a, b be lengths of different sides of the parallelogram.

h = distance between the parallel sides of length a (say). Perimeter = 2 (a + b), Area = ah

7) Rhombus

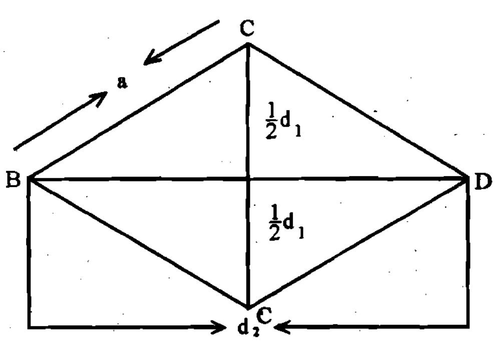

Fig. 16.8

a = side; d_1, d_2 = two diagonals

Perimeter = $4a$

Ask : What is the area of ΔABD and of ΔCBD ?

What is the area of the rhombus ABCD?

Explain : Area $= \frac{1}{2} d_1 d_2$

8) Trapezium

a, b = Lengths of parallel sides;

h = distance between the parallel sides.

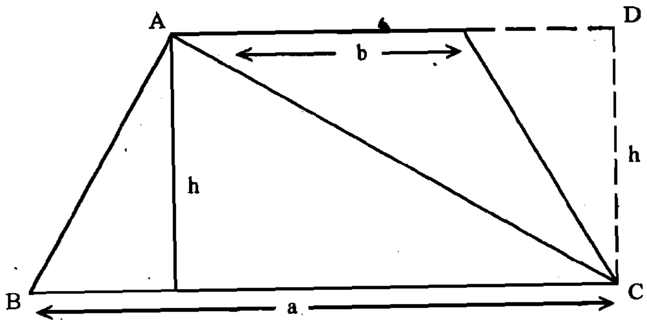

Fig. 16.9

Ask students to state the areas of ΔABC and ΔCDA. Ask them to add and obtain the area of the trapezium.

Explain : Area of a trapezium $= \frac{1}{2}(a + b)\, h$

9) Quadrilateral ABCD

Let h_1 & h_2 = Altitudes from A & C on BD.

Area $= \frac{1}{2} BD\,(h_1 + h_2)$

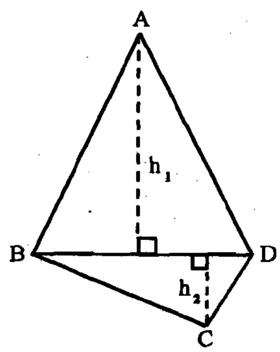

Fig. 16.10

Ask the students to state the area using the diagonal AC. Let them copy the figure, find the area in two ways and then compare.

Explain : Area of quadrilateral $= \frac{1}{2} BD\,(h_1 + h_2)$

$= \frac{1}{2} AC$ (sum of perpendiculars from B and D on AC.)

10) Circle

Ask : Define circle

Encourage the students to come out with the definitions of circle i.e.,

A circle is the path traced by a point which moves in a plane in such a way that its distance from a fixed point in the plane, remains constant. The fixed point is called the centre and the constant distance is called the radius.

or

A circle is a set of points in a plane which are at the same distance from a fixed point in the same plane.

Ask : Does the circle consist of all the points on the curve or inside the curve or both?

Explain to them that the circle consists of only those points which are on the curve. As in Figure 16.11, points A and B are on the circle but points C and D are not on the circle.

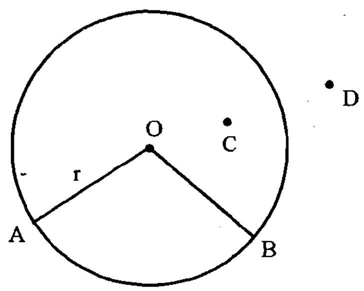

Fig. 16.11

Define Interior of a circle.

All points which lie inside the circle form the interior of the circle. P, Q, R lie in the interior of the circle. All points which lie outside the circle from the exterior of the circle. Points A, B, C lie in the exterior of the circle.

The circle alongwith its interior is called the circular region.

Ask : Define arc.

Explain : If we take any two points on a circle, the circle is divided into two parts. Each part of the circle is called an arc.

Minor Arc

An ar. ..hose length is less than that of a semicircle is called a "minor arc".

An arc whose length is more than a semicircle is called "major arc. "

Major Arc > Semi Circle > Minor Arc

Ask : Define chord and distinguish between chord and diameter.

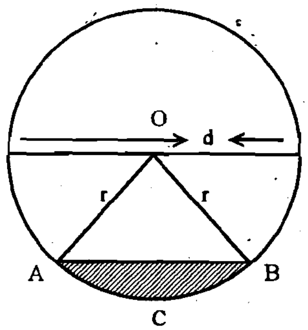

Fig. 16.12

The line-segment joining the end points of an arc is called a chord, such as AB in Fig. 16.12. Any chord which contains the center of the circle is called its diameter. The length of the chord varies with the length of the arc. The biggest chord is called the diameter (d) and its length is twice that (r) of the radius of the circle.

Ask : What is a segment of a circle?

Explain to the students that it is a part of the interior of a circle enclosed between an arc and its chord. e.g. in Fig. 16.12, the shaded region between the arc ACB and chord AB is a segment of the circle. If arc ACB is a major arc, then the segment formed is called a major segment and if the arc ACB is a minor arc, then the segment formed is called a minor segment.

Ask : Define sector of a circle.

A part of the interior of a circle enclosed by an arc and two radii from the two end points of the arc such as AOBC (Fig. 12) is called a sector of the given circle. If $\angle AOB$ is 90° (i.e. arc ACB is one-fourth of the circle,) then the sector is called a quadrant.

The teacher may then provide the following formulåe relating to a circle:

10) Circle

r = radius, $\pi = \frac{22}{7} = 3.1416$

Circumference = $2\pi r$, Area = πr^2

11) Semi-Circle

r = radius

Perimeter = $\pi r + 2r$, Area = $\frac{1}{2}\pi r^2$

12) Sector of a Circle

θ = Central angle of the sector

r = radius, l = length of the arc

Perimeter = $l + 2r$

Activity : Ask students to construct a sector of central angle 60° in a circle. Make a copy of this sector on a piece of paper, cut it out and place it on the circle, with the vertex of the sector coinciding with the center of the circle. How many times do you place it to cover the circle? Repeat with a sector of central angle 45°.

Help the students to conclude that

$$\frac{\theta}{360^\circ} = \frac{l}{2\pi r}$$

$$\text{OR} \quad l = \frac{\theta}{360^\circ} \times 2\pi r$$

Similarly, area of the sector = $\frac{\theta}{360}(\pi r^2)$

13) Segment of a Circle

r = radius

θ = angle at the center of the corresponding sector

$$\text{Perimeter} = \frac{\theta^\circ \times 2\pi r}{360^\circ} + 2r\sin\left(\frac{\theta}{2}\right)$$

$$\text{Area} = r^2\left[\frac{\pi\theta}{360^\circ} - \frac{\sin\theta}{2}\right]$$

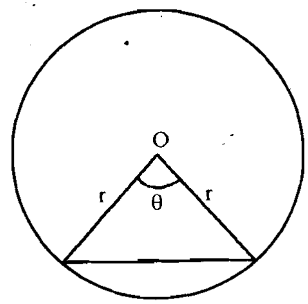

Fig. 16.13

Methodology used : The formulae for finding the area of different figures should be derived using the deductive method.

16.4.4 Applications

Main Teaching Point : How to apply the formulae learnt for finding the area.

Teaching-Learning Process

Example: Four equal circles are described about the four corners of a square so that each touches two other circles. Find the area of the space enclosed between the circumference of the circles, each side of the square measuring 14 cm.

Ask : What is given in the problem?

The students will list out details like "each circle touches the other two", "all circles are of equal radius".

If the four centres A, B, C & D are joined to form a figure ABCD, then ABCD, is a square. Lead students to discover that each side of the square will contain the point of contact.

Ask : What is required in the question?

The shaded area enclosed by the four circles inside the squares.

Ask : How will you calculate it?

Let the students come out with the statement that it is the difference between the area of the square and the area of the four sectors of the circles each of whose central angle is 90°.

Area of the square = $a^2 = 14 \times 14 = 196\,cm^2$

Area of a sector = $\frac{\theta}{360} \times \pi r^2$

Ask : How will you work out the radius of the circle?

Bring out from the students that since the point of contact lies on the side of the square, the radius will be half the side of the square.

Radius = $r = \frac{14}{2} = 7cm$

The central angle of each square will be 90° since each point A, B, C and D are the vertices of a square.

Area of four sectors $= \frac{4 \times 90 \times \pi (7)^2}{360}$

$= \frac{22}{7} \times 7 \times 7$

= 154 sq. cm

Hence area of shaded part =196 - 154 = 42 cm^2

The teacher can take a few more examples.

Methodology used : For application, the Heuristic approach is the best suited. Students should first think of what is given, then what is to be found and then guided using inductive and deductive methods to reach the answer.

Check Your Progress

Notes . a) Write your answers in the space given below.

b) Compare your answers with the one given at the end of the unit.

How will you explain the following to the students:

1. A steel wire bent in the form of a square encloses an area of 121 sq. cm. If the same wire is bent in the form of a circle, find the area of the circle. (Take $\pi = 22/7$)

   ....

2. The minute hand of a clock is $\sqrt{21}$ cm long. Find the area described by the minute hand on the face of the clock between 7.00 am and 7.05 am.

   ....

3. A path of 7 m width runs around outside a circular park whose radius is 18 m. Find the area of the path. (Take $\pi = 22/7$)

   ....

   ....

4. Find the area of the minor segment of a circle given that the angle of corresponding sector is 120° and the radius of the circle is 21 km. (Take $\pi = 3.1416$).

   ....

   ....

5. A sector is cut off from a circle of radius 21 cm. The angle of the sector is 120°. Find the length of its arc.

   ....

   ....

16.5 MEASUREMENT OF VOLUME

16.5.1 Volume and Surface Area of Cube and Cuboid

Main Teaching Point : To derive the formulae for volume and surface area of cube and cuboid.

Teaching-Learning Process : The teacher shows the students a large cube and a small cube.

Ask : Which cube occupies more space?

Explain : The larger cube occupies more space.

The teacher should bring out the definition of volume that the amount of space or measure of space which a body occupies is known as its volume".

Ask : How do you measure volume?

The teacher should come out with the idea of comparing the volume of a body with that of a unit cube.

Explain that if we were to dissect the larger cube into smaller cubes each cube being of the size of a unit cube, and if we obtain n unit cubes then the volume will be n units.

The teacher can draw the diagram and explain the figure:

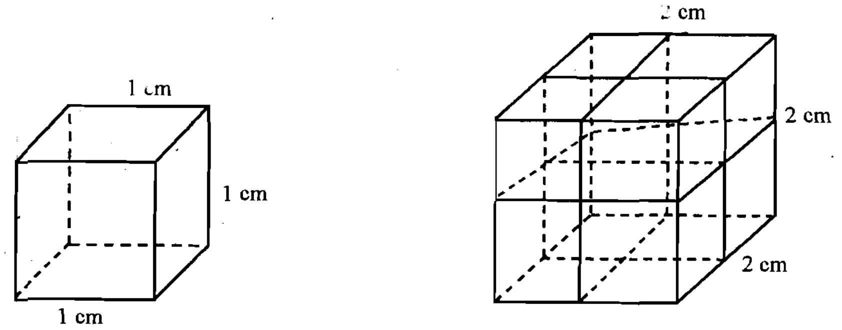

Fig 16 14

Ask : How many small cubes can be fitted in the big cube?

The students can be directed towards thinking that there are two layers of four cubes each. One layer of 4 unit cubes is below and one layer is above it.

Explain that volume of the cube = $(\text{side})^3 = a^3$ where 'a' is the measure of a side of the cube.

Ask : What is the area of the square face which forms the bottom of the cube?

The students will work out: $2 \times 2 = 4\ \text{cm}^2$

Ask : Had the height been 1 cm, how many cubes would have been there? As there will be only one layer, there would be 4 cubes.

Ask : If the height is 3 cm, how many layers of cubes are there?

In that case, there would be 3 layers.

Similarly, the teacher can increase the length or breadth or height and let the student discover that the volume of a cuboid is length multiplied by breadth multiplied by height. Volume of cuboid = $l \times b \times h$ where l, b, h are length, breadth & height of the cuboid. Once this in achieved, the focus would be shifted to surface area.

The teacher can show a cube to the student.

Ask : How many faces does this cube have? What is their shape?

Call a student and let him count the faces and tell the shape.

Ask : What is the surface area of the cube?

The student should discover that the surface area is six times the area of one face and the area of one face = square of length of a side.

Explain : Surface area of a cube = $6a^2$, where 'a' is the measure of a side of the cube.

Similarly, for cuboid, the teacher shows a cuboid.

Ask . How many faces does this cuboid have? What is their shape?

A student would be called and asked to count the number of faces and tell their shape.

Ask : What is the surface area of a cuboid?

By using the previous knowledge let them discover the formula for the surface area.

Explain : Surface area of a cuboid = 2 lb + 2 lh + 2 bh = 2 (lb + lh + bh)

Using Pythagoras theorem also illustrate that

Largest diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$

Methodology used : Deductive reasoning is used to get the desired results.

16.5.2 Units of Measuring Volume

The teacher can introduce the units of volume and their conversion.

1 cu. cm	=	1000 cu. mm
1 cu. dm	=	1000 cu. cm
1 cu. m	=	1000 cu. dm
1 cu. Dm	=	1000 cu. m
1 cu. Hm	=	1000 cu. Dm
1 cu. Km	=	1000 cu. Hm
1 litre	=	1000 cu. cm

The teacher can make the students realize why each conversion has 1000 as the number of lower units e.g.: 1 cu. cm = 1 cm × 1 cm × 1 cm = 10 mm × 10 mm × 10 mm

= 1000 cu mm.

16.5.3 Volume of Prisms, Pyramids and Sphere

Main Teaching Point : Derivation of formulae for finding volume of prisms and pyramids .

Teaching Learning Process : Prisms are those solids whose top and bottom consist of congruent parallel faces and whose lateral faces consist of a set of parallelograms (or rectangles, in case of a right prism)

If the number of lateral faces tend to become very large, the prism tends to approximate to a cylinder.

Cylinder

You may ask the students to give the example of a cylinder and can also help them define a cylinder.

A cylinder has a curved surface with two congruent circular ends parallel to each other. The axis of a cylinder is the line joining the centers of circles at either end. If the axis of this cylinder is perpendicular to the two ends, then it is known as a right circular cylinder.

The teacher can show generation of cylinder by rotating a rectangle about one of its sides. If a rectangle ABCD is rotated about AB, it will generate a cylinder with axis as AB and radius as AD or BC.

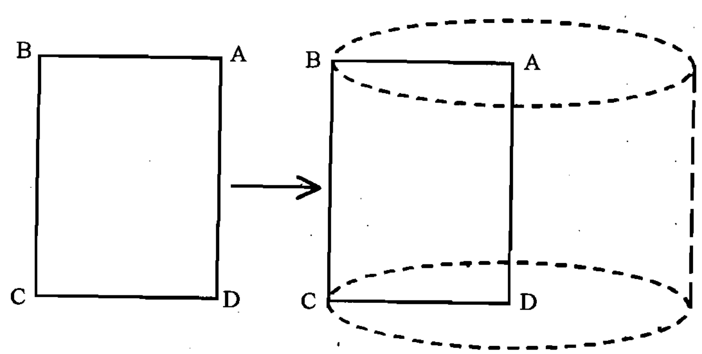

Fig. 16.15

The surface generated by rotation of line CD generates the curved surface of the cylinder.

Now the teacher can give the various formulae.

Volume of the prism = Area of Base × height

What will be the formula for the volume of right circular cylinder? Help them to come out with formula.

Area of base = πr^2, height = h, r = radius of cylinder

So the volume of the cylinder = $\pi r^2 h$

Now try to get the formula for the surface areas of the cylinder.

Ask	:	What are the area of the two faces (top and bottom) of cylinder
		Since the two faces are circles, their total area is $2\pi r^2$.
Ask	:	How will you work out the area of lateral surface of the cylinder?
		Demonstrate with a rectangular sheet of paper. Roll it into a cylindrical shape, then spread it again into a rectangle. Roll the rectangle once again into cylinder.
Ask	:	What is the relation between the lateral surface area of the cylinder and the area of the rectangle from which the cylinder has been made.
		If the students have observed the earlier activity carefully, they will come out with the correct answer.
Ask	:	What will be the breadth of this rectangle?
		Again with the help of a sheet, help them to discover that the length is the circumference of the circle of the cylinder and breadth is nothing but the height of the cylinder.
Ask	:	What is the lateral surface area?
		Now the students can work out the lateral surface area as $2\pi r \times h$ where r and h are the radius and the height of the cylinder.
Ask	:	What is the total surface area of the cylinder?
		The teacher can prompt the students to come out with the final formula.
		The teacher can take up a couple of examples.
Example	:	A cylindrical tank has a capacity of 6, 610 cu. m. Find its depth if its radius is 14 m. Calculate the cost of painting its curved outer surface at the rate of Rs. 3 per square meter (Take π = 22/7)
Ask	:	What data about the cylinder are given in the problem?
		The student will list out:
		Volume of the cylindrical tank = 6,160 cu. m.
		Radius of the tank = 14 m
Ask	:	What is to be determined?
		The students will examine the question and state that depth and the cost of painting the curved surface are to be determined.
Ask	:	What will be the steps to achieve this?
		Let the students come out. Obtain the curved surface area, multiply with the rate to get the cost.

Ask : What is the formula for the curved or lateral surface area?

The students will give the answer 2π rh

Ask : Radius is 14m. How will you obtain the height of the cylinder?

The height can be obtained by using the volume.

We know

$$V = \pi r^2 h$$

$$6160 = \frac{22}{7} \times 14 \times 14 \times h$$

$$h = 6160 \times \frac{7}{22} \times \frac{1}{14} \times \frac{1}{14} = 10 \text{ m}$$

Now using the radius as 14m, height as 10 m and rate as Rs. 3 per meter, calculate the total cost.

Curved surface area $= 2\pi r h = 2 \times \frac{22}{7} \times 14 \times 10$

$= 880$ sq. m.

Now cost of 1 sq.m = Rs. 3

Cost of painting 880 sq. m = 880×3 = Rs. 2640

Cone

All pyramids have a vertex, one plane surface as the base, and a number of triangular faces with a common vertex. When the number of faces tends to become very large, the base takes the shape of a curve and the pyramid approximates a cone. A pyramid with a circular base is called a circular cone.

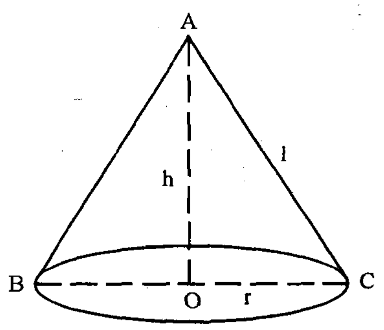

Fig. 16.16

Ask : Give an example of a cone.

The sharpened part of a pencil, an ice-cream cone, joker's cap, etc.

Now the teacher can show the generation of a cone by right angled triangle about one of its sides. (other than hypotenuse). Explain that one of the sides containing the right angle should be used as the base and rotation should take place about the other side.

Ask : Define a cone.

Help the students to come out with the definitions.

"When a right angled triangle is revolved about one of the sides containing the right angle, the solid thus generated is called a right circular cone." The side about which it is rotated is called the axis of the cone. In the case of a right circular cone, this axis is perpendicular to the base.

The volume of a pyramid is one third the volume of an equivalent prism.

Volume of pyramid = $\frac{1}{3}$ Volume of Prism

In Fig. 16.20, AO is the height of the cone. Since the cone is a right circular one, the height and axis are same. AC is the slant height.

Ask : Calculate the slant height of a cone.

Prompt the students to use Pythagoras theorem:

$$l = \sqrt{r^2 + h^2}$$

The Teacher can give the formula of cone.

1. Slant height of the cone $l = \sqrt{r^2 + h^2}$
2. Curved surface area = $\pi r l = \pi r = \sqrt{r^2 + h^2}$
3. The total surface area = Area of the base + Curved surface area

$$= \pi r^2 + \pi r l$$

Explain to students that if a right circular cone of height h and slant height l is cut by a plane parallel to the base we get a smaller cone on one side and a solid called frustum on the other side. (as show in Fig. 16.17.)

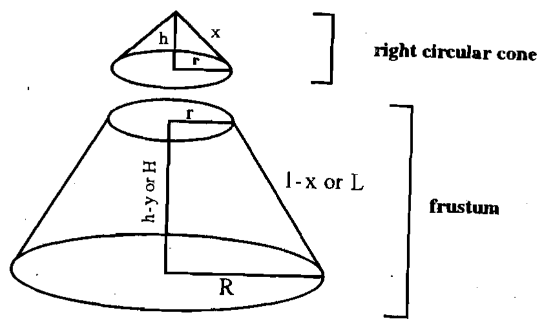

Fig. 16.17

4. Lateral surface area of frustum

$$\pi (l - x)(R + r)$$

$$\text{Where } (l - x) \text{ or } L = \sqrt{(h - y)^2 + (R - r)^2}$$

$$\text{or } L = \sqrt{H^2 + (R - r)^2}$$

5. Total surface area of frustum

= Area of Base + Area of Top + Lateral Surface Area.

$$= \pi R^2 + \pi r^2 + \pi L (R + r)$$

6. Volume of frustum = $\frac{\pi H}{3}(R^2 + r^2 + Rr)$

Methodology used: Different formulae for volume and surface area should be derived using inductive and deductive method.

16.5.4 Applications

Example : A conical tent is of the diameter 24 m at the base and its height is 16 m.

a) Find the slant height

b) The canvas required in square meters

c) At most how many persons can the tent accommodate if each person required 54 m^3 of air ? (Take $\pi = 22/7$)

Solution

Ask	:	What data is given in the problem?
		The diameter of the cone = 24 m
		The height of the cone = 16 m
		The volume of air required by each person = 54 m^3
Ask	:	List the things to be determined.
		The slant height of the cone
		The area of the canvas
		Number of persons accommodated
Ask	:	List the steps of calculation.
		To determine the slant height, use the formula. To determine the area of canvas, determine the curved surface area of the cone.
		To determine the number of persons, calculate the volume of cone and divide by the volume of air required by each person.

Calculations

a) Slant height, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{12^2 + 16^2}$$

$$= \sqrt{144 + 256}$$

$$= \sqrt{400} = 20 \text{ m}$$

b) The canvas required = Curved surface area of cone

$$= \pi r l = \frac{22}{7} \times 12 \times 20 = 754.3 \text{ sq. m}$$

c) Volume of air in the tent

$$= \frac{1}{3} \pi r^2 \times h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 16$$

$$= \frac{16896}{7} \text{ m}^3$$

Volume of air required by one person = 54 m^3

No. of persons that can be accommodated

$$= \frac{16896}{7} \times \frac{1}{54} = 44.7 = 44$$

Incidentally, explain why we should write the number of persons as 44 and not round it off to 45.

Sphere

The teacher could ask the students to give examples of a sphere after showing them a ball.

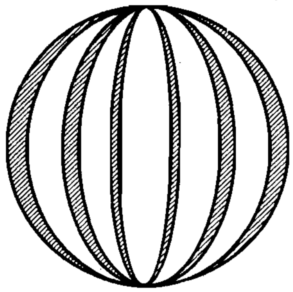

Fig. 16.18

Ask : Give more examples of a sphere.

From their experience, the students are likely to give many examples such as balls used in various games. various types of fruits, the Earth, the Sun etc All of them may not be perfect spheres, so the teacher can keep correcting wherever the figure deviates from perfection.

Ask : How would you define a sphere?

"All the points in space equidistant from a fixed point constitute a sphere".

or

"Locus of a point in space which moves in such a way that its distance from a fixed point is constant, is known as a sphere".

or

"Sphere is generated by revolving a semi-circle about its diameter which is kept fixed".

The teacher can demonstrate the justification of the last definition, by revolving a semi-circular cardboard disc about its diameter.

Ask : How will you define the center and radius of sphere?

Bring out the definitions of centre and radius of a sphere on the pattern of a circle. The fixed point in the above definitions is the centre of the sphere. The fixed distance or the radius of the revolving semi-circle (stated in the last definition) is known as the radius.

Now the teacher can give various formulae relating to a sphere of radius r.

Volume of sphere $= \frac{4}{3} \pi r^3$

Surface area of sphere $= 4 \pi r^2$

Volume of hemisphere $= \frac{2}{3} \pi r^3$

Curved surface area of hemisphere $= 2 \pi r^2$

Total surface area of hemisphere $= 3 \pi r^2$

Example : Determine the mass of a spherical shell of external and internal diameter as 10 cm and 9 cm respectively. The density of metal is 7g/cm3. $\left[\text{Take} \pi = \frac{22}{7}\right]$

Ask : List out the data given in the problem.

The students would give the following:

Internal diameter = 9 cm

External diameter = 10 cm

Density of metal = 7 g/cm^3

Ask : What is to be calculated.

The mass of the sphere is to be calculated.

Ask : What is the formula for mass in terms of density and volume?

The students may require prompting.

Mass = Volume × Density

Ask : Since the volume need to be determined in the above formula, how will you determine the volume of the shell?

Let the students come out with the idea that the required volume can be obtained by subtracting the volume of the inner sphere from the that of the outer one.

Ask the students to calculate the volume of the shell.

Internal radius = $\frac{9}{2}$ cm

External radius = 5 cm

Volume of outer sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 5 \times 5 \times 5$

$= \frac{11000}{21}$ cm^3

Volume of inner sphere

$= \frac{4}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{9}{2} \times \frac{9}{2}$

$= \frac{8019}{21}$ cm^3

Volume of shell = Volume of external sphere – volume of internal sphere

$= \frac{11000}{21} - \frac{8019}{21}$

$= \frac{2981}{21}$ cm^3

Now the students are in a position to calculate the mass of the shell.

Mass of the shell = Volume × Density

$= \frac{2981}{21} \times 7 = 993.67$ g

Methodology used : Heuristic approach should be used when dealing with applications.

Check Your Progress

Notes : a) Write your answers in the space given below.

b) Compare your answers with the one given at the end of the unit.

How will you explain the following to the students :

6. The other dimensions of a box are 42 cm × 30 cm × 27 cm. If the box is made of wood 1 cm thick, determine the capacity of the box?

....

....

7. The area of three adjacent faces of a cuboid are x, y and z. If its volume is V, then prove that

$$V = \sqrt{xyz}$$

....

8. Find the volume of a cylinder generated by rotating a rectangular sheet of dimensions 10 cm by 7 cm about the shorter side.

....

9. Find the slant height and curved surface area of a cone whose volume is 12,936 cm^3 and diameter of the base is 42 cm.

....

16.6 LET US SUM UP

In this unit, we have discussed briefly how the need for mensuration arose as civilization progressed. We have discussed the perimeter and area of simple plane figures and shown the usefulness of deductive approach in arriving at areas of different plane figures starting from a square. We have also shown the need for convenient units of measurement of length, area and volume and their conversion into sub-units and bigger units. We have illustrated through examples and exercises the use of these measurements in daily life. We have also demonstrated the analytic approach of problem-solving by linking up what is required to be found with the given data through a number of sequential steps. Finally, we have given a large number of unit-end activities for reinforcement and recapitulation.

16.7 UNIT-END ACTIVITIES

1. A wire when bent in the form of an equilateral triangle encloses an area of $121\sqrt{3}$ cm^2. If the same wire is bent into the form of a circle, find the area of the circle

2. A wire is in the form of a circle of radius 28 cm. Find the area of the square into which it can be bent.

3. A chord is 10 cm long and drawn into a circle whose radius is $5\sqrt{2}$ cm . Find the area of both the segments.

4. Find the length of the longest rod that can be placed in a room 12 m long, 9 m broad and 8 m high.

5. A cube of side 4 cm is cut into cubes of side 1 cm each. Calculate the surface area of big and small cubes What is the ratio of surface area of the smaller cubes to that of the bigger cube?

6. If V denotes the volume of a cuboid of dimensions a, b, c, and S is its surface area, then prove that

$$\frac{1}{V} = \frac{2}{S}\left[\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right]$$

7. If V is the volume of a cube and T its surface, show that

$$V = \frac{T}{36}\sqrt{6T}$$

8. Two cubes each of side 15 cm are joined end to end. Find the surface area of the resulting cuboid

9. The dimensions of a rectangular box are 2:3:4 and the difference between the cost of covering it with the sheet of paper at the rate of Rs 8 and Rs 10 per sq. meter is Rs 416. Find the dimensions of the box.

10. Three metal cubes of sides 6 cm, 8 cm and 10 cm are melted to form one new cube. Find the edge of the new cube.

11. Determine the weight of an empty metallic cistern, open at the top having dimensions $3m \times 2.08m \times 1.80m$ and having thickness of 0.04 m. The density of metal is $7 g/cm^3$.

12. The inner radius of a pipe is 2.5 cm. How much water can 10 m of this pipe hold?

13. The volume of a cylinder is $448\ cm^3$ and height is 7 cm. Find the lateral surface area and the total surface area.

14. A rectangular piece of paper is 22 cm long and 12 cm wide. A cylinder is formed by rolling the paper along its length. Find the volume of the cylinder

15. A metallic cylindrical pipe has an inside radius r, and outside radius R and length 1. Prove that the volume of metal is $\pi l (R^2 - r^2)$

16. The diameter of a roller 120 cm long is 84 cm. If it takes 500 complete revolutions to level a ground, find the cost of leveling the ground at the rate of 30 paise per square meter.

17. A solid cylinder has total surface area of 462 sq. cm. Its curved surface area is one third of its total surface area. Find the volume of the cylinder. $\left(\text{Take } \pi = \frac{22}{7}\right)$

18. A metallic wire of diameter 0.6 cm is wound round a cylinder of length 60 cm and diameter 4.9 cm to cover th whole surface. Find the weight of wire used if the density of wire is $10\ g/cm^3$.

19. The diameter of a right circular cone is 12 m and its slant height is 10 m. Find its total surface area.

20. Find the ratio of curved surface area of two cones if the diameter of their bases are equal and slant heights are in the ratio 4:3.

21 The ratio of volumes of two cones is 4:5 and the ratio of the radii of their bass if 2:3. Find the ratio of their vertical heights.

22. If 'h', 'c' and 'v' be height, curved surface and volume of a cone, show that $3\pi h^3 - c^2 h^3 + 2v^3 = 0$

23. If the radii of the ends of a bucket 40 cm high are 32 cm and 8 cm, determine its capacity.

24. A sector of a circle of radius 6 cm has the central angle 120°. It is rolled in such a way that the two bordering radii are joined together to form a cone. Find the radius of the base of the cone so made and its volume.

25. A right circular cone is 3 6 cm high and radius of its base is 1.6 cm. It is melted and recast into a right circular cone with radius of its base as 1.2 cm. Find the height.

26. Find the surface area and volume of a sphere whose diameter is 21 cm.

$\left(\text{Taking } \pi = \frac{22}{7}\right)$

27. Find the diameter of a sphere whose surface area is $5544\ cm^2$.

27. Find the diameter of a sphere whose surface area is 5544 cm^2.

28. How many lead balls, each of radius 1 cm, can be made from a sphere whose radius is 8 cm.

29. A spherical ball of iron has been melted and made into smaller balls. If the radius of the smallers balls is one fourth of the original one, how many such balls can be made? How does the total surface of the smaller balls compare with that of the original one?

30. Find the volume and the total surface area of a hemisphere of radius 3.5 cm.

$$\left(\text{Use } \pi = \frac{22}{7}\right)$$

31. A vessel is in the form of a hemispherical bowl, mounted by a hollow cylinder. The diameter of the hemisphere is 25 cm and the total height of the vessel is 25 cm. Find the capacity of the vessel. ($\pi = 3.14$).

32. A cube and a sphere have equal surface area. Find the ratio of their volumes.

33. A sphere of diameter 5 cm is dropped into cylindrical vessel partly filled with water. The diameter of the vessel is 10 cm. If the sphere is completely submerged, find the height to which the surface of the water will rise in the cylindrical vessel.

34. Prove that the total surface area of the sphere is equal to the curved surface of the cylinder that circumscribes it all around.

35. The radius of the internal and external surfaces of a hollow spherical shell are 3 cm and 5 cm respectively. It is melted and recast into a solid cylinder of height $2\frac{2}{3}$ cm. Find the diameter of the cylinder.

36. An iron pillar consists of cylindrical portion 240 cm high and 16 cm in diameter and a cone of 36 cm in height surmounting it. Find the weight of the pillar, given that a cubic cm of iron weights 7.8 g. $\left(\text{Take } \pi = \frac{22}{7}\right)$

37. A tent is of the shape of right circular cylindrical portion upto a height of 3 m and then becomes a right circular cone with a maximum height of 13.5 m. above the ground. Calculate the cost of painting the inner side of the tent at the rate of Rs 2 per sq. meter if the radius of the base is 14 m.

38. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 108 cm, and the diameter of the hemispherical ends is 36 cm, find the cost of polishing the surface at the rate of 7 paise per sq. cm.

41. A cylinder whose height is two-third of its diameter, has the same volume as sphere of radius 4 cm. Calculate the radius of the base of the cylinder.

16.8 ANSWERS TO CHECK YOUR PROGRESS

1. 154 sq. cm
2. 5.5 sq. cm
3. 946 sq. m
4. 270.86 sq. km
5. 44 cm
6. 28000 cm^3
7. Prove $V = \sqrt{x\,y\,z}$
8. 2200 cm^3
9. 35 cm, 2310 cm^2

16.9 SUGGESTED READINGS

Hogben, Lancelot; (1960) : *Mathematics for the Million,* W.W. Norton & Company Inc., New York.

NCERT, *Mathematics, A Textbook for Class X.*