UNIT 17 TRIANGLES AND ITS APPLICATIONS TO TRIGONOMETRY

Structure

17.1 INTRODUCTION

The exact period when trigonometry began to be used is not known. The document called the Rhind Papyrus has evidence to show use of trigonometry in the construction of pyramids to maintain a uniform slope and give a flat face to the pyramid. The Egyptian surveyors used the concept of the horizontal change for each unit increase in vertical height.

The ancient Greeks developed trigonometry as an aid to their work in Astronomy. The "gnomon' or the vertical stick was the simplest astronomical instrument. if was used to produce a shadow on the ground and by finding the lengths of the shadows, the time of the day and the time of the year could be determined. Although the Greek trigonometry and the trigonometry we teach are very much same there is some conceptual difference. In modern trigonometry we are mainly concerned with either the ratios of the sides of a right-angled triangle or the coordinates of points on a certain circle or the functional relations, while Greek trigonometry was mainly centered on the lengths of chords of circles.

The Hindu mathematician Aryabhatta also used trigonometry in astronomy. Evidence of the use of Sine ratio and the result $\text{Sin}^2\theta + \text{Cos}^2\theta = 1$ is available in ancient documents.

17.2 OBJECTIVES

On successful completion of this unit the teacher should be able to

- make the students understand the importance of trigonometry;
- demonstrate to the students how the concepts of similarity form the basis of trigonometric ratios;
- develop in students the skill of manipulating trigonometric ratios and appreciate their relationship.

- help the students in using Pythagoras theorem to solve problems involving trigonometric ratios;
- develop in students the skills of proving trigonometric identities; and
- develop problem solving skills as required to solve height and distanceproblems.

17.3 SCOPE AND MEANING OF TRIGONOMETRY

The word "Trigonometry" has Greek components. 'Tri' means three, 'gonon' means angles and 'metric' means measure. The three words together mean triangle measurement. Thus in trigonometry we study relationship of sides and angles in a triangle. We also study about the incenter, circumcentre and other points of concurrence. We learn how to apply this knowledge to solve problems on heights and distances. Trigonometric ratios of angles which always have a unique value for any given angle form the basic tool for the study of these relationships. Knowledge of trigonometry is useful in many situations such as navigation of ships or movements of aeroplanes, rockets, astronomical sciences, engineering surveys etc. Thus trigonometry has wide applications in real life as well as in the study of other branches of Mathematics and Physics.

17.4 TRIGONOMETRIC RATIOS

17.4.1 Definitions of Trigonometric Ratios

Main Teaching Point : Invariance of t-ratios(t-ratios depend only on the angle and not on the size of the triangle).

Teaching-Learning Process : The concepts of an angle is fundamental in trigonometry. The angle is introduced with the idea of rotation. Thus an angle is obtained when a ray, say OP, rotates about the vertex O in the direction shown by the arrow till it reaches position OQ. The amount of rotation is the measure of the angle θ. As a convention we take anti-clock wise direction as the positive direction of rotation. When the rotation is clock wise, the angle is taken as negative (–)

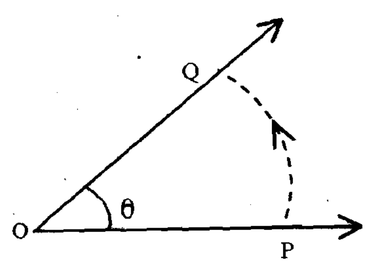

Fig. 17.1

Different systems of measurement of angle are used.

The two systems commonly used are:

1) The British System (sexagesimal system)

Here the unit is a right angle which is the amount of turn when the ray makes a quarter of a complete rotation. A right angle is divided into 90 equal parts. Each part is called a 'degree' denoted by 'o'.

$1^\circ = 60$ minutes $(60')$ and $1' = 60$ seconds $(60'')$.

Ask: When do you call two triangles similar ?

The pupils would reply that two triangles will be similar (a) if their corresponding angles are equal and (b) their corresponding sides are proportional.

Ask: What is SSS of similarity?

The teacher can bring out the condition of similarity: 'if the corresponding sides of two triangles are proportional, then they are similar'.

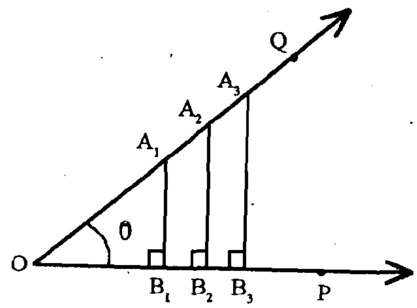

Fig. 17.2

Now teacher should ask the students to draw an angle POQ as in Fig. 17.2. Take points A_1, A_2, A_3 etc. on ray OQ and drop perpendiculars on ray OP. Let the feet of perpendiculars be B_1, B_2, B_3 etc. respectively. Ask the students to determine the ratios of different sides in different triangles like:

$$\frac{OA_1}{OB_1}, \frac{OA_2}{OB_2}, \frac{OA_3}{OB_3}$$

Ask the students to report the result. The teacher could give the definitions at this stage:

i) $\dfrac{A_1B_1}{OA_1} = \dfrac{A_2B_2}{OA_2} = \dfrac{A_3B_3}{OA_3} = \dfrac{\text{Opposite side}}{\text{Hypotenuse}} = \text{sine } \theta \text{ or } \sin\theta$

ii) $\dfrac{OB_1}{OA_1} = \dfrac{OB_2}{OA_2} = \dfrac{OB_3}{OA_3} = \dfrac{\text{Adjacent side}}{\text{Hypotenuse}} = \text{cosine } \theta \text{ or } \cos\theta$

iii) $\dfrac{A_1B_1}{OB_1} = \dfrac{A_2B_2}{OB_2} = \dfrac{A_3B_3}{OB_3} = \dfrac{\text{Opposite side}}{\text{Adjacent side}} = \text{tangent } \theta \text{ or } \tan\theta$

iv) $\dfrac{OA_1}{A_1B_1} = \dfrac{OA_2}{A_2B_2} = \dfrac{OA_3}{A_3B_3} = \dfrac{\text{Hypotenuse}}{\text{Opposite side}} = \text{cosecent } \theta \text{ or } \operatorname{cosec}\theta$

v) $\dfrac{OA_1}{OB_1} = \dfrac{OA_2}{OB_2} = \dfrac{OA_3}{OB_3} = \dfrac{\text{Hypotenuse}}{\text{Adjacent side}} = \text{secant } \theta \text{ or } \sec\theta$

vi) $\dfrac{OB_1}{A_1B_1} = \dfrac{OB_2}{A_2B_2} = \dfrac{OB_3}{A_3B_3} = \dfrac{\text{Adjacent side}}{\text{Opposite side}} = \text{cotangent } \theta \text{ or } \cot\theta$

The teacher should emphasize that the ratio is more important. The measure of a side could be obtained if one side and the corresponding ratio is known.

Methodology used: Discussion with inductive reasoning is used to show that trigonometric-ratios is independent of the size of the triangle.

17.4.2 Trigonometric Ratios for Standard Values

Main Teaching Point : Deriving values of trigonometric-ratios for standard values of angle.

Teaching Learning Process : Trigonometric Ratios for standard values could be introduced in the following manner:

Ask : State Pythagoras Theorem.

In a right triangle, the square on the hypotenuse is equal to the sum of squares on the other two sides. The teacher draws the diagram (Fig. 17.3) of an isosceles right triangle.

Ask : What is the measure, of $\angle A$ and $\angle C$ if Δ ABC is an isosceles and right angled triangle?

The student will answer that each angle would be 45°.

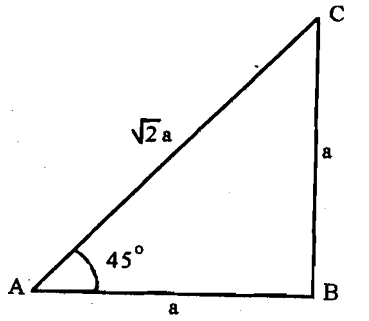

Fig. 17.3

The teacher can give the measurement of each side to be 'a', i.e. AB = BC = a

Ask : Calculate the length of hypotenuse AC

The students would answer $\sqrt{2}a$.

Ask : Work out the different trigonometric ratios for angle 45°.

$$\text{Sin } 45^\circ = \frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}} \quad , \quad \text{Cos } 45^\circ = \frac{AB}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

$$\text{Tan } 45^\circ = \frac{BC}{AB} = \frac{a}{a} = 1 \quad , \quad \text{Sec } 45^\circ = \frac{AC}{AB} = \frac{\sqrt{2}a}{a} = \sqrt{2}$$

$$\text{Cosec } 45^\circ = \frac{AC}{BC} = \frac{\sqrt{2}a}{a} = \sqrt{2} \text{ , Cot } 45^\circ = \frac{AB}{BC} = \frac{a}{a} = 1$$

For the trigonometric ratios of 30° and 60°, the following procedure could be used. Draw an

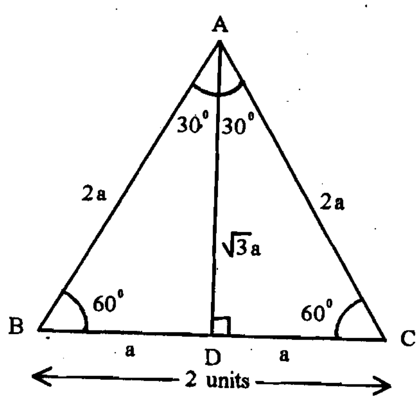

Fig. 17.4

equilateral triangle in which each side of is of 2a units.

Ask : If AD is the altitude from A to BC meeting BC at D, calculate the length of altitude AD.

By using Pythagoras Theorem the students would work out the altitude AD to be $\sqrt{3}\ a$.

Consider ΔBDA, BDA=90° & ABD=60°. (Because ABD happens to be the angle of an equilateral triangle)

$$\Rightarrow \text{BAD} = 30^\circ$$

Ask : Determine all trigonometric ratios of the angle 60°. Taking help of ΔBDA all trigonometric ratios could be worked out.

$$\text{Sin } 60^{\circ} = \frac{AD}{BD} = \frac{\sqrt{3}}{2}, \qquad \text{Cos } 60^{\circ} = \frac{BD}{AB} = \frac{1}{2}$$

$$\text{Tan } 60^{\circ} = \frac{AD}{BD} = \frac{\sqrt{3}}{1} = \sqrt{3}, \quad \text{Cosec } 60^{\circ} = \frac{AB}{AD} = \frac{2}{\sqrt{3}}$$

$$\text{Sec } 60^{\circ} = \frac{AB}{BD} = \frac{2}{1} = 2, \qquad \text{Cot } 60^{\circ} = \frac{BD}{AD} = \frac{1}{\sqrt{3}}$$

Ask : Determine all trigonometric-ratios for the angle 30°.

The teacher can help the student looking at the triangle of Fig. 17.4 in a different way as shown in Fig. 17.5.

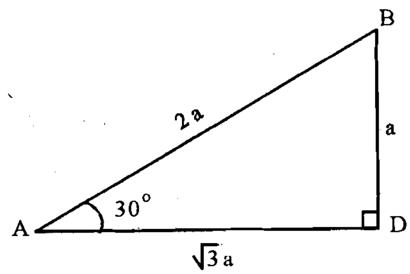

Fig. 17.5

$$\text{Sin } 30^{\circ} = \frac{1}{2}, \quad \text{Cos } 30^{\circ} = \frac{\sqrt{3}}{2}, \quad \tan 30^{\circ} = 1, \quad \text{Cosec } 30^{\circ} = \frac{2}{1} = 2$$

$$\text{Sec } 30^{\circ} = 2, \quad \text{Cot } 30^{\circ} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Next teacher can take the concept of trigonometric-ratios of 0° and 90°.

In Fig 17.6 effect of gradual decrease of angle is shown. As the angle is decreased the opposite side or the altitude keeps on decreasing. In other words, $\theta \to 0^{\circ} \to MP \to 0$ and $MP \to OP$

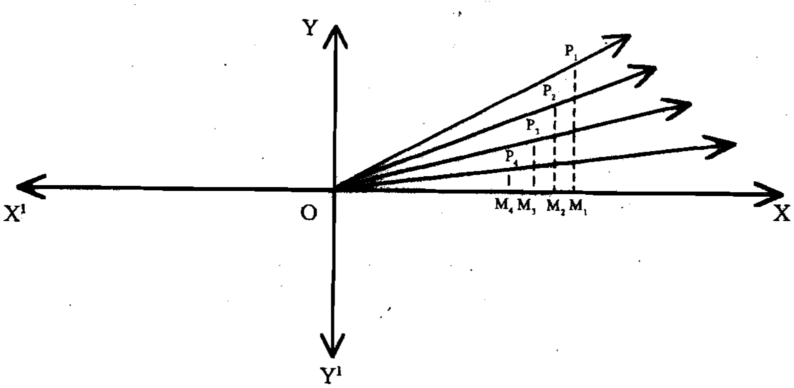

Fig. 17.6

Ask : Now, determine all trigonometric ratios of 0°.

$$\text{Sin } 0^{\circ} = \frac{MP}{OP} = \frac{O}{OP} = 0, \quad \text{Cos } 0^{\circ} = \frac{OM}{OP} = 1, \quad \text{Tan } 0^{\circ} = \frac{MP}{OP} = 0$$

$$\text{Cosec } 0^{\circ} = \frac{OP}{MP} = \frac{OP}{O} = \infty \text{ (or not defined)}, \ \text{Sec } 0^{\circ} = \frac{OP}{OM} = 1, \ \text{Cot } 0^{\circ} = \frac{OP}{MP} = \infty \text{ (or not defined)}$$

Exactly on the same lines, the students could be made to work out t-ratios for 90°. As the angle gradually increases, the adjacent side or the base of the triangle subsequently decreases.

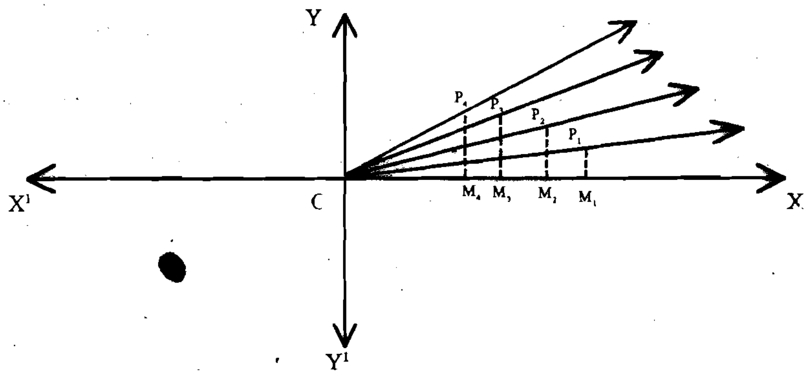

Fig. 17.7

As $\theta \to 90^o$, OM $\to 0$ and MP $\to$ OP

$\text{Sin } 90^o = \frac{MP}{OP} = 1$, $\quad$ $\text{Cos } 90^o \frac{OM}{OP} = \frac{O}{OP} = 0$

$\text{Tan } 90^o = \frac{MP}{OM} = \frac{MP}{O}\ \infty$, $\quad$ $\text{Cosec } 90^o \frac{OP}{MP} = 1$

$\text{Sec } 90^o = \frac{OP}{OM} = \frac{OP}{O}$, $\quad$ $\text{Cot } 90^o \frac{OM}{MP} = \frac{O}{MP} = 0$

Methodology used : Deductive Method is used to get the values of trigonometric-ratios for different angles.

17.4.3 Standard Values Table

Main Teaching Point : How to form the table.

Teaching-Learning Process : The teacher can help the students to prepare trigonometric-ratios table for standard angles 0^o, 30^o, 45^o, 60^o, & 90^o.

How the table is prepared could be explained easily.

Write all angles in first row and all trigonometric ratios in first column. The angles should be in ascending order. In the row where "Sin θ" is written, in each square write the following numbers 0,1,2,3 & 4. Now divide each number by 4 and take the square root as shown in the table. Now simplify each value. For "Cos θ", values to be written in reverse order as that of Sin.θ. $\text{Tan } \theta = \frac{\text{Sin } \theta}{\text{Cos } \theta}$, so divide the respective "Sine" values with "Cos" values. For Cosec θ, Sec θ and Cot θ take the reciprocal values of Sin θ, Cos θ and Tan θ.

Angle T-Ratio	0^o	30^o	45^o	60^o	90^o
Sin θ	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}} = \sqrt{\frac{3}{2}}$	$\sqrt{\frac{4}{4}} = 1$
Cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan θ	$\frac{0}{1} = 0$	$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$	$\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$	$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$	$\frac{1}{0} = \infty$
Cosec θ	$\frac{1}{0} = \infty$	$\frac{2}{1} = 2$	$\frac{\sqrt{2}}{1} = \sqrt{2}$	$\frac{2}{\sqrt{3}}$	$\frac{1}{1} = 1$
Sec θ	$\frac{1}{1} = 1$	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$\frac{2}{1} = 2$	$\frac{1}{0} = \infty$
Cot θ	$\frac{1}{0} = \infty$	$\frac{\sqrt{3}}{1} = \sqrt{3}$	$\frac{1}{1} = 1$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\infty} = 0$

Example 1 · Show that

$$\sin^2 30^\circ \ \sec^2 60^\circ \ \text{Cosec}^2 30^\circ - \tan 45^\circ \ \cos 60^\circ \ \sin 90^\circ = \frac{7}{2}$$

Solution : The teacher can solve the problem by actually using trigonometric-ratio table

LHS $= \sin^2 30^\circ \ \sec^2 60^\circ \ \text{Cosec}^2 30^\circ - \tan 45^\circ \ \cos 60^\circ \ \sin 90^\circ$

$$= \left(\frac{1}{2}\right)^2 \times (2)^2 \times (2)^2 - 1 \times \frac{1}{2} \times 1$$

$$= \frac{1}{4} \times 4 \times 4 - \frac{1}{2} = \frac{7}{2}$$

Example 2 : Verify $\cos 2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$ · When $\theta = 30^\circ$

Solution : LHS $= \cos 2.(30^\circ) = \cos 60^\circ = \frac{1}{2}$

$$\text{RHS} = \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$$

$$= \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}}$$

$$= \frac{\frac{2}{3}}{\frac{4}{3}}$$

$$= \frac{2}{3} \times \frac{3}{4}$$

$$= \frac{1}{2}$$

Hence LHS = RHS

Example 3 : If $\sin\theta = \frac{1}{3}$, determine the values of $\sec\theta$ and $\cot\theta$.

Solution : The teacher should draw, the diagram of a right angle triangle ABC on the black-board. Let the students come out with the length of opposite sides and hypotenuse as 'a' and 3a.

Ask : Determine the adjacent side i.e. AC = X

$$X^2 + a^2 = (3a)^2$$

or $$X^2 = 9a^2 - 8a^2$$

$$X^2 = 8a^2$$

$$X = 2\sqrt{2}a$$

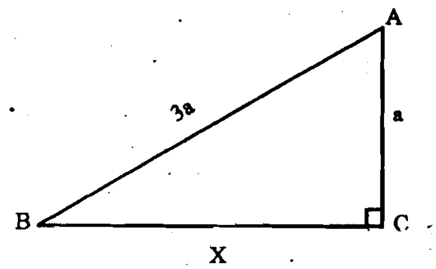

Fig. 17.8

Ask : Now, write the values of $\sec\theta$ and $\cot\theta$.

$$\text{Sec } \theta = \frac{AB}{AC} = \frac{3a}{2\sqrt{2}a} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\text{Cot } \theta = \frac{AC}{BC} = \frac{2\sqrt{2}a}{a} = 2\sqrt{2}$$

Methodology used : Deduction method is used to form the table and illustrate its use by taking different examples.

Check Your Progress

Notes a) Write your answers in the space given below.

b) Compare your answers with the one given at the end of the unit.

Illustrate how to solve the following:

1 Find the values of each of the following:

a) $\text{Cot}^2 45^\circ + \text{Sin}^2 30^\circ - \text{Cot}^2 30^\circ$

b) $2 \text{ Sin}^2 30^\circ + 2 \tan^2 60^\circ - 5 \text{ Cos}^2 45^\circ$

2. If $\tan \theta = \frac{1}{3}$, find the values of $\sin \theta$ and $\cos \theta$.

3 If $\theta = 30^\circ$, verify that $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

4 If $A = B = 60^\circ$, verify $\text{Cos } (A - B) = \text{Cos } A \text{ Cos } B + \text{Sin } A \text{ Sin } B$.

17.5 TRIGONOMETRIC IDENTITIES

An equation involving trigonometric ratios of an angle θ, which is true for all values of θ for which the given trigonometric ratios are defined is said to be a trigonometric identity

17.5.1 Standard Identities

Main Teaching Point : Derivation of basic identities.

Teaching-Learning Process : There are three basic trigonometric identities which are derived from the Pythagoras Theorem.

Ask . What is the relation between the sides of right angled triangle ABC, right angled at C?

$$AC^2 + BC^2 = AB^2 \quad \ldots\ldots\ldots\ldots(1)$$

Teacher should ask the students to divide the above equation by AB^2

$$\frac{AC^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AB^2}{AB^2}$$

or $$\left(\frac{AC}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2 = 1 \quad \ldots\ldots\ldots\ldots(2)$$

Ask . What are the trigonometric ratios represented by $\frac{AC}{AB}$ and $\frac{BC}{AB}$?

$$\frac{AC}{AB} = \text{Cos}\,\theta\,,\ \frac{BC}{AB} = \text{Sin}\,\theta$$

Substituting in (2), we get

$$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$$

Again teacher should write (1), and ask the students to divide it by BC^2

$$\frac{BC^2}{BC^2} + \frac{AC^2}{BC^2} = \frac{AB^2}{BC^2}$$

or $$1 + \left(\frac{AC}{BC}\right)^2 = \left(\frac{AB}{BC}\right)^2$$

Once again the teacher should go back to (1) and ask the students to divide (1) by AC^2 and get

$$\frac{BC^2}{AC^2} + \frac{AC^2}{AC^2} = \frac{AB^2}{AC^2}$$

or $$\left(\frac{BC}{AC}\right)^2 + 1 = \left(\frac{AB}{AC}\right)^2$$

∴ $$1 + \tan^2\theta = \text{Sec}^2\theta.$$

Methodology used : Deductive method is used to derive the three basic identities.

17.5.2 Applications

Main Teaching Point : Use of basic identities and methods of proving other identities.

Teaching-Learning Process

Example 1 : Determine whether the following equation is an identity:

$$\frac{\tan A + \text{Sin } A}{\tan A - \text{Sin } A} = \frac{\text{Sec } A + 1}{\text{Sec } A - 1}$$

Solution : The teacher can do some questions for the students. Let A lie between 0° and 90° i.e. $0^\circ < A < 90^\circ$.

L.H.S. $= \dfrac{\tan A + \sin A}{\tan A - \sin A}$

$$= \frac{\frac{\sin A}{\cos A} + \sin A}{\frac{\sin A}{\cos A} - \sin A} \qquad \left(\tan A = \frac{\sin A}{\cos A}\right)$$

$$\frac{\sin A\left(\frac{1}{\cos A} + 1\right)}{\sin A\left(\frac{1}{\cos A} - 1\right)}$$

$$= \frac{\sec A + 1}{\sec A - 1} \qquad \left(\sec A = \frac{1}{\cos A}\right)$$

= RHS

Since the equation is true for all values of A, hence it is an identity.

Exmaple 2 : Examine whether $\cos^2\theta + \cos\theta = 1$, is an identity.

Solution : $\cos^2\theta + \cos\theta = 1$

Using the quadratic equation formula

$$\cos\theta = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$\cos\theta = \dfrac{1+\sqrt{5}}{2}$ is not possible as $\cos\theta$ can not exceed 1.

(or Base can not be more than hypotenuse).

$\cos\theta = \dfrac{1-\sqrt{5}}{2}$ is true for a particular value of θ only.

$\therefore$ The given equation is not an identity.

Example 3 : If $X = a\cos\theta$, $Y = b\sin\theta$, prove that

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Solution : To prove the result, substitute the value of x and y in left hand side of the identity to be proved.

L.H.S $= \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = \dfrac{(a\cos\theta)^2}{a^2} + \dfrac{(b\sin\theta)^2}{b^2}$

$$= \frac{a^2\cos^2\theta}{a^2} + \frac{b^2\sin^2\theta}{b^2} = \cos^2\theta + \sin^2\theta = 1 = \text{RHS}$$

Hence proved

Example 4 : If $x = r\cos A\sin B$, $y = r\sin A\sin B$, $c = r\cos B$ prove that $x^2 + y^2 + z^2 = r^2$

Solution : Starting with left hand side

LHS $= x^2 + y^2 + z^2 = r^2\cos^2 A\sin^2 B + r^2\sin^2 A\sin^2 B + r^2\cos^2 B$

$= r^2\sin^2 B(\cos^2 A + \sin^2 A) + r^2\cos^2 B$

$= r^2\sin^2 B(1) + r^2\cos^2 B \qquad (\sin^2\theta + \cos^2\theta = 1)$

$= r^2(\sin^2 B + \cos^2 B)$

$= r^2 = \text{RHS}$

Methodology used: Deductive method is used to show different methods used in proving different types of identities.

Check Your Progress

Notes : a) Write your answers in tne space given below.

b) Compare your answers with the one given at the end of the unit.

How you will explain the following identities?

5. $\dfrac{\text{Cos}\,\theta}{1-\tan\theta}-\dfrac{\text{Sin}^2\,\theta}{\text{Cos}\,\theta-\text{Sin}\,\theta}=\text{Cos}\,\theta+\text{Sin}\,\theta$

6. $\left(\dfrac{\text{Cos}^2\theta}{\text{Sin}^2\theta}+1\right)\tan^2\theta=\text{Sec}^2\theta$

7. $\text{Sin}^4\,\theta-\text{Cos}^4\,\theta=\text{Sin}^2\,\theta-\text{Cos}^2\,\theta=2\,\text{Sin}^2\,\theta-1=1-2\,\text{Cos}^2\,\theta$

17.6 HEIGHT AND DISTANCE

The height of a mountain or the width of a river can-not be measured directly. This is one of the important areas where trigonometry can be used. The teacher should take up this point in detail to show the use of trigonometry.

17.6.1 Meaning of Terms Elevation and Depression

Main Teaching Point : Angle of elevation and angle of depression.

Teaching-Learning Process : The horizontal through the observer's eye is called line of sight If an object is above the line of sight the observer will have to raise his/her eyes to look at the object. The angle by which observer is raising his eye-sight is called angle of elevation i.e. the angle made by line joining the eye to the object (PO) with the line of sight (PQ) is called angle of **elevation.**

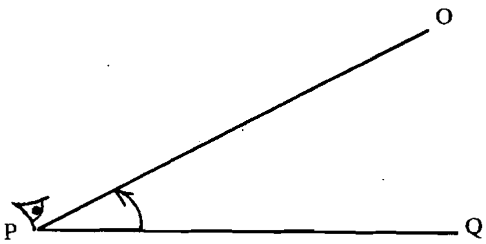

Fig. 17.9

Similarly if an object is below the line of sight the observer has to lower his/her eyes to look at the object and the angle by which he/she has to move his/her eye-sight down is called angle of depression i.e the angle made by line with line of sight (PQ) is called angle of **depression**.

Methodology used : Lecture method is used to explain the meaning of the terms.

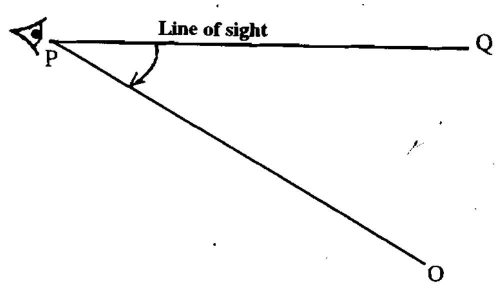

Fig. 17.10

17.6.2 Applications

Main Teaching Point : Drawing of the correct diagram and using it to find height and distances.

Teaching Learning Process : Before starting with the problems related to height and distances, the importance of drawing a correct and proper diagram should be discussed. The students should be taught to construct the suitable diagram for the given problem.

The mountain, tower or tree etc are always to be taken as straight lines since their spread is of no consequence for measurement of height The kite or the object like aeroplanes etc. or the observer are always treated as points in the diagram

The teacher must emphasize on drawing of a diagram of the situation described in the problem.

The language should be understood and diagram be drawn accordingly. It should then be labelled.

Example 1: A kite is flying at a height of 100 m. The string of the kite is making an angle of 60° with the horizontal. If there is no slack in the string, find its length.

Solution : The teacher should ask, the students to draw the diagram assuming the kite as a point The length and the width of the kite is neglected as we need to use only its height above the ground to find the length of the string.

In Fig. 17.11 P is the kite, Q is the observer. R is the point on the ground which is directly below the kite.

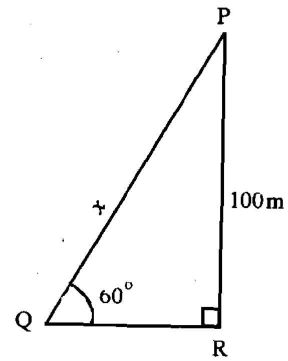

Fig. 17.11

$$\text{Sin } 60^\circ = \frac{PR}{PQ}$$

$$\sqrt{3} = \frac{PR}{100}$$

$$\therefore \quad PR = 100\sqrt{3}$$

$$= 100 \times 1.732$$

$$= 173.2 \text{ m Ans.}$$

Example 2 : Two vertical poles are fixed 60 m. apart. The angle of depression of the top of the first as seen from the top of the second, which is 150 m high, is 30°. Find the height of the first pole.

Solution : The teacher should help the students to make the diagram.

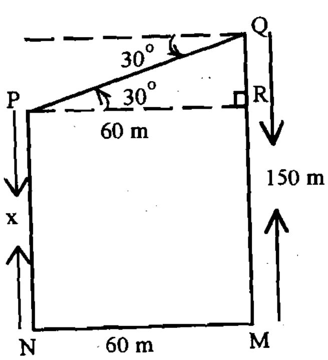

Fig. 17.12

PN & QM are two poles, the height of the longer pole is 150 m. Angle of depression (θ) is 30°.

Let PN = X. Distance between the poles = PR = 60 m

In Δ PQR, PR = 60 m QR = (150 – X) m

$$\tan 30^\circ = \frac{QR}{PR} = \frac{150 - X}{60}$$

$$\frac{1}{\sqrt{3}} = \frac{150 - X}{60} \Rightarrow \frac{60}{\sqrt{3}} = 150 - X$$

$$X = 150 - \frac{60}{\sqrt{3}}$$

$$X = 150 - \frac{60 \times \sqrt{3}}{2}$$

$$= 150 - 20 \times 1.732$$

$$= 150 - 34.640$$

$$= 115.36 \text{ m Ans.}$$

Example 3 : Two ships are approaching a light house from the same side and are moving in a straight line towards the light house. If the angle of depression of the farther ship from the top of the light house be 30^0 and that of the nearer ship be 60^0 and the height of the light house be 100 m, find the distance between the two ships.

Solution :

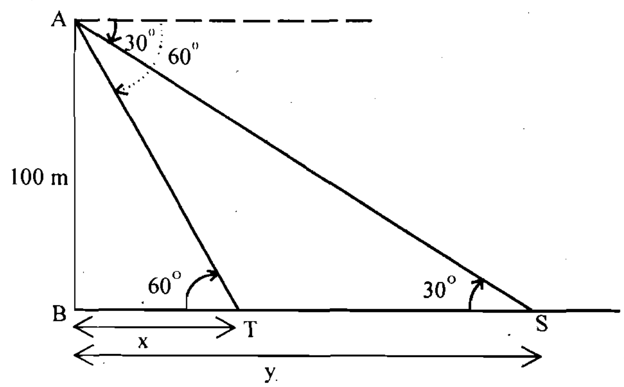

Fig. 17.13

$\angle ASB = 30^0$ and $\angle ATB = 60^0$

Let BT = x and BS = y

(We need to find y–x, hence we have to calculate x and y and then find y–x)

In ABT, $\frac{AB}{BT} = \tan 60^0 \Rightarrow \frac{100}{x} = \sqrt{3} \Rightarrow x = \frac{100}{\sqrt{3}}$

In Δ ABS , $\frac{AB}{BS} = \tan 30^0 \Rightarrow \frac{100}{y} - \sqrt{3} \Rightarrow y = 100\sqrt{3}$

$\therefore \quad ST = y - x = 100\sqrt{3} - \frac{100}{\sqrt{3}} = \frac{300 - 100}{\sqrt{3}} = \frac{200}{\sqrt{3}} = \frac{200 \times \sqrt{3}}{3} = 115.47$

Hence the distance between the two ships is 115.47 m.

Methodology used : Mostly deductive method is used. Various examples are considered and solved using deductive logic.

Check Your Progress

Notes : a) Write your answers in the space given below.

b) Compare your answers with the one given at the end of the unit.

Explain the following to the students:

8. The shadow of a vertical pole is $\frac{1}{\sqrt{3}}$ of its height. Find the altitude of the sun.

....

....

....

9. A ladder reaches a height of 8 m. when placed against a wall. If the inclination of the ladder with the ground is 60°, determine its length.

10. An observer standing on the bank of a river, observes that the angle subtended by a tower on the opposite bank is 60°, when he retires 60 m from the bank, he finds the angle to be 30°. Find the height of the tower and the breadth of the river.

11. Two pillars of same heights stand on either side of a road which is 150 m. wide. At a point on the road between the pillars the elevation of the tops of the pillars are 60° and 30° Find the heights of the pillars and the position of the point.

12. Two men on either side of a cliff 80 m high, observe the angles of elevation of the top of the cliff to be 30° and 60° respectively. Find the distance between the two men.

17.7 LET US SUM UP

In this unit, we have defined six trigonometric ratios These ratios are unique for any given angle and form the basic building blocks in the study of trigonometry. We derived trigonometric ratios of some common angles lying between 0° and 90°. We derived three fundamental identities using Pythagoras theorem and in the process showed the application of deductive method in the study of the subject Trigonometric ratios of 0° and 90° were arrived at, using a pattern of decreasing and increasing angles thereby using an inductive approach Inter-relationship of trigonometric ratios has been used to solve problems on trigonometric identities.

An important use of trigonometry has been illustrated by solving problems on heights and distances. Thus an elementary treatment of trigonometry has been provided in this unit to serve as a foundation for further study of the subject.

17.8 UNIT-END ACTIVITIES

1. Determine the value of Sin 1[illegible] using the formula :

Sin(A – B) = Sin A Cos B – Cos A Sin B

2. If $\cos A = \frac{7}{25}$ determine the value of $\tan A$.

3. If $\tan\theta = \frac{3}{4}$, detemine the value of $\cos\theta - \sin\theta$

4. If $\operatorname{Cosec}\theta = \frac{5}{3}$, verify that $\frac{\tan\theta}{1+\tan^2\theta} = \frac{\sin\theta}{\sec\theta}$

5. Prove that $4\cos^2 60^\circ + 4\tan^2 45^\circ - \sin^2 30^\circ =$ 1[illegible]

6. Prove that $\sin(90^\circ - \theta)\cos(90^\circ - \theta) = \frac{\tan\theta}{1+\tan^2\theta}$

7. Prove that $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{Cosec} 50^\circ = 2$

8. Evaluate $\sin^2 25^\circ + \sin^2 65^\circ$

9. Evaluate $\frac{\cos 20^\circ 35'}{\sin 69^\circ 25'}$

10. Solve the equation $\frac{\sin^2\theta}{\tan^2\theta - \sin^2\theta} = 3$

11. Determine whether the following equation is an identity

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta$$

12. Prove the following identities

a) $\cos^4\theta + \sin^4\theta - 2\cos^2\theta\sin^2\theta = (1-2\sin^2\theta)^2$

b) $\frac{1+\cos\theta - \sin^2\theta}{\sin\theta(1+\cos\theta)} = \cot\theta$

c) $\frac{\sec\theta + 1 + \tan\theta}{\sec\theta + 1 - \tan\theta} = \frac{1+\sin\theta}{\cos\theta}$

13. If $x = a\cos\theta - b\sin\theta$, $y = a\sin\theta + b\cos\theta$ prove that

$$x^2 + y^2 = a^2 + b^2$$

14. If $\tan\theta + \sin\theta = p$ and $\tan\theta - \sin\theta = q$ prove that

$$16pq = (p^2 - q^2)^2$$

15. A tree is broken by the wind. The top struck the ground at an angle of 30° and at a distance of 30 m from the root. Find the complete height of the tree.

16. Two ships are sailing in the sea on either side of a light house. The angles of depression of two ships as observed from the top of the light house are 60° and 45° respectively; If the distance between the ships is $\frac{200(\sqrt{3}+1)}{\sqrt{3}}$ m, find the height of the light house.

17. An observer standing 60 meters away from the building on the ground, notices that the angles of elevation of the top and the bottom of a flag-staff on the building are respectively 60° and 45°. Find the height of the flag-staff.

17.9 ANSWERS TO CHECK YOUR PROGRESS

1. a) $\frac{-7}{4}$

b) 4

2. $\text{Sin}\,\theta = \frac{1}{\sqrt{10}}$, $\text{Cos}\,\theta = \frac{3}{\sqrt{10}}$

3. $\text{Tan}\,2\theta = \text{Tan}\,60^\circ = \sqrt{3}$

$$\frac{2\,\text{Tan}\,\theta}{1-\tan^2\theta} = \frac{2\times\frac{1}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{2}{\sqrt{3}}\times\frac{3}{2} = \sqrt{3}$$

$\therefore$ R.H.S = L.H.S.

4. Cos (A – B) = Cos 0° = 1

$$\text{Cos A Cos B} + \text{Sin A Sin B} = \frac{1}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$$

$\therefore$ R.H.S = L.H.S.

5. $$\frac{\text{Cos}\,\theta}{1-\tan\theta} - \frac{\text{Sin}^2\theta}{\text{Cos}\,\theta - \text{Sin}\,\theta} = \frac{\text{Cos}\,\theta}{1-\frac{\text{Sin}\,\theta}{\text{Cos}\,\theta}} - \frac{\text{Sin}^2\theta}{\text{Cos}\,\theta - \text{Sin}\,\theta}$$

$$= \frac{\text{Cos}^2\theta}{\text{Cos}\,\theta - \text{Sin}\,\theta} - \frac{\text{Sin}^2\theta}{\text{Cos}\,\theta - \text{Sin}\,\theta} = (\text{Cos}\,\theta + \text{Sin}\,\theta) = \text{R.H.S.}$$

6. $$\left(\frac{\text{Cos}^2\theta}{\text{Sin}^2\theta}+1\right)\text{Tan}^2\theta = \frac{\text{Cos}^2\theta + \text{Sin}^2\theta}{\text{Sin}^2\theta}\times\text{Tan}^2\theta$$

$$= \frac{1}{\cancel{\text{Sin}^2\theta}}\times\frac{\cancel{\text{Sin}^2\theta}}{\text{Cos}^2\theta} = \text{Sec}^2\theta = \text{R.H.S.}$$

7. $\text{Sin}^4\theta - \text{Cos}^4\theta = (\text{Sin}^2\theta + \text{Cos}^2\theta)(\text{Sin}^2\theta - \text{Cos}^2\theta)$

$= \text{Sin}^2\theta - \text{Cos}^2\theta$

$\text{Sin}^2\theta - \text{Cos}^2\theta = \text{Sin}^2\theta - (1 - \text{Sin}^2\theta) = 2\,\text{Sin}^2\theta - 1$

$\text{Sin}^2\theta - \text{Cos}^2\theta = (1 - \text{Cos}^2\theta) - \text{Cos}^2\theta = 1 - 2\,\text{Cos}^2\theta$

8. 60°

9. 9.2 m.

10. 52 m, 30 m.

11. 64.9 m, 37.5 m

12. 185 m.

17.10 SUGGESTED READINGS

Sarna C.S. Gupta R.G. & Garg P.K. (1995) : *Mathematics for Secondary Schools*, Arya Book Depot, New Delhi.

Kalra M.R (1995) : *Comprehensive Mathematics, Class IX & X*, Laxmi Publication, New Delhi.

Loney S.L. (1917) : *Trigonometry Part – I*, McMillan India, New Delhi.

Singhal P.K. (1996) : *Planned Mathematics for Class X*, Bharti Bhawan, Patna.

A Textbook for Secondary Schools Class IX & X, National Council of Educational Research and Training, New Delhi.

Singha H.S., Singha Jatindra (1995) : *Question Bank for Class X Mathematics*, Hemkunt Press, New Delhi.