UNIT 4 QUANTIFICATION

Contents

4.0 OBJECTIVES

In this unit, we propose to introduce you:

- to a new set of rules to test the validity of arguments, which consist of general and singular propositions.
- to all the rules involved in testing the validity of arguments.
- to understand Aristotle's theory of syllogism against the background of symbolic logic.
- to the application of the new class of rules.

4.1 INTRODUCTION

Broadly speaking, there are two types of arguments: arguments consisting of statements, which are truth-functionally compound and arguments, which are neither truth-functional nor compound. This Chapter deals with the latter kind of arguments. Logic, which deals with this branch, is called predicate logic or quantification logic. It is a system of deductive logic that combines the analysis of terms with the analysis of statements by making use of the logical properties of quantifiers. Generally, this type of argument consists of two kinds of statements, called *general and singular*. All propositions accepted by traditional logic belong to these two categories.

Both universal and particular propositions are called general because in these two kinds subject is general term, like men, horses, plants, etc. However, in a singular proposition, the subject refers to a definite individual. The individual may be a human being like 'Tendulkar' or an object like 'the farthest planet from the sun'. The difference between the truth-functional statements on the one hand, and general or singular propositions on the other, is that none of the techniques discussed so far, helps us when arguments consisting of general or singular statements are analyzed. Since quantifying expressions are involved in such statements, quantification is another technique used in our mission to subject these propositions to close scrutiny. Traditional logic or analysis of categorical proposition is the take-off point for quantification logic. Quantity of proposition and subject-predicate relation form the base.

While subject of proposition stands for any individual, predicate stands for the attributes an individual may or may not possess. These individuals and attributes are denoted by lower case letters and upper case letters respectively. With regard to lowercase letters there is one restriction. Only letters from ‘a’ to ‘w’ are used to denote individuals. These are individual constants. Generally, the practice is to choose the first letter of the term to designate the individual. Therefore term like Tendulkar, Dhoni, etc, are represented as t, d, etc. While their attributes like cricketer, swimmer, politician, etc. are designated by C, S, P, etc., by using upper case letters. However, when ‘politician’ becomes subject of a proposition it is designated by ‘p’. In logic, common noun may be subject or predicate. ‘Tendulkar is a cricketer’ is an example for common noun being used as attribute. Symbolically, it becomes ‘Ct’: it is a symbolized statement. First we write the symbol for attribute. This is followed by the symbol for subject. Such a statement can be true or false.

When ‘x’ is used for individual constant, then we have *propositional function*, which is neither true nor false. For example, ‘Bx’ would be a statement like ‘x is brave’. The process of obtaining propositions from propositional function is called ‘*instantiation*’. Thus we can say, ‘Chandran is brave’ – it is a proposition we obtain from the propositional function ‘Bx’. Accordingly, propositional function is an expression that contains one or more individual variables, such that when all its individual variables are replaced by individual constants the result is a symbolized statement. The symbol ‘y’ has a special role to play. It is used to denote an arbitrarily selected individual. In quantification, negation has the same symbol.

4.2 QUANTIFICATION: IT’S MEANING

An important aspect of quantification is the substitution of instances. There are two ways in which substitution is being made. In the case of singular proposition, substitution of any individual constant ranging from ‘a’ to ‘w’ can be made to ‘x’ which is known as individual variable; this process is, as we have just seen, instantiation. Another method is through *generalization*. Accordingly, the process of quantification takes place when the given proposition is general. A general proposition is of two types; universal and particular. So we have two quantifiers denoting these two types. Quantifiers are symbols that are used to represent quantifying expression such as everyone / everything / all or someone / something. Thus there are universal or existential quantifiers. In symbols they are respectively as follows: ‘(x)’ and ‘(∃x)’. Since they may be affirmative or negative, we have four kinds of propositions, which are represented as follows:

1. All Indians are mortal. $(x)\ Mx$
2. No Indians are mortal. $(x) \neg Mx$
3. Some Indians are moral. $(\exists x)\ Mx$
4. Some Indians are not mortal. $(\exists x) \neg Mx$

The symbols used on the right hand side need some explanation.

The symbol (x) is expanded in several ways. It can read ‘for all values of x’ or ‘Given any x’ or simply ‘for every x’, etc., where ‘x’ stands for individual constant ‘Indians’ and ‘M’ stands for ‘mortal’. Therefore $\neg$ M x is read ‘x is not mortal’. The symbol (∃x) is read ‘there exists at least one x such that ….' ( ) is called universal quantifier and ∃ is called existential quantifier. If we substitute I (Indians) or P (Pakistanis) for x then we get a proposition, which may be true or false.

Just as x is used as individual variable to denote the subject, two Greek letters 'Φ' (Phi) and 'Ψ' (Psi) are used to denote predicate. So they are called predicate variables. Using these variables, A, E, I and O propositions are represented as follows:

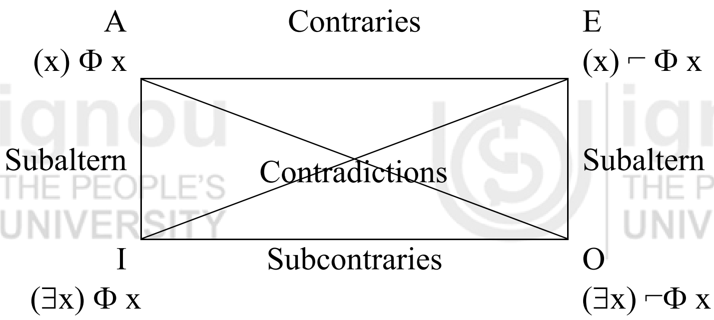
1.	All Indians are mortal	(A)	$(x)\ \Phi x$
2.	No Indians are mortal	(E)	$(x) \neg \Phi x$
3.	Some Indians are mortal	(I)	$(\exists x)\ \Phi x$
4.	Some Indians are not mortal	(O)	$(\exists x) \neg \Phi x$

Using class membership relation, general propositions are represented as follows:

1. $(x)\ \Phi x \equiv (x)\{x \epsilon \Phi \Rightarrow x \epsilon \Psi\}$ Where ϵ is read 'element of'
2. $(x) \neg \Phi x \equiv (x)\{x \epsilon \Phi \Rightarrow x \not\epsilon \Psi\}$ Where $\not\epsilon$ is read 'not an element of'
3. $(\exists x)\ \Phi x \equiv (\exists x)\{x \epsilon \Phi \wedge x \epsilon \Psi\}$
4. $(\exists x) \neg \Phi x \equiv (\exists x)\{x \epsilon \Phi \wedge x \not\epsilon \Psi\}$

4.3 LOGICAL RELATIONS INVOLVING QUANTIFIERS

Our study begins with traditional square, which does not need any explanation. We know how A, E, I and O are denoted by quantification. Let us replace A, E, I and O by these quantifiers in the square:



With this background, we represent logical relations, viz., equivalence and contradiction as follows:

1. Equivalence:
   1) $(x)\ \Phi x \equiv \{\neg (\exists x) \neg \Phi x\}$
   2) $(x) \neg \Phi x \equiv \{\neg (\exists x)\ \Phi x\}$
   3) $(\exists x)\ \Phi x \equiv \{\neg (x) \neg \Phi x\}$
   4) $(\exists x) \neg \Phi x \equiv \{\neg (x)\ \Phi x\}$

2. Contradiction:

1)	(x) Φ x	(∃x) ⌐ Φ x
2)	(x) ⌐ Φx	(∃x) Φ x
3)	(∃x) Φ x	(x) ⌐ Φ x
4)	(∃x) ⌐ Φ x	(x) Φ x

When we use predicate variable, the propositional forms are expressed as follows:

1)	(x) Φ x	≡	(x){ Φx=>Ψx}
2)	(x) ⌐ Φ x	≡	(x){ Φ x=> ⌐ Ψx}
3)	(∃x) Φ x	≡	(∃x){ Φx Λ Ψx}
4)	(∃x) ⌐ Φ x	≡	(∃x){Φx Λ⌐ Ψx}

When we represent A, E, I & O with this new set, their equivalent forms also undergo changes.

1)	(x){ Φx=>Ψx}	≡	⌐(∃x){Φx Λ⌐Ψx}
2)	(x){ Φx=> ⌐ Ψx}	≡	⌐(∃x){Φx ΛΨx}
3)	(∃x){ Φx ΛΨx}	≡	⌐ (x){Φx=> ⌐ Ψx}
4)	(∃x){Φx Λ⌐ Ψx}	≡	⌐ (x){Φx=>Ψx}

If negations inserted behind the quantifiers on the RHS are removed, then automatically they become contradictions of respective propositions.

A predicate like mortal is called simple predicate because the propositional function which, is used, has some true substitution instances and some false substitution instances. All substitutions to variable are called ‘substitution instances’. When simple predicates are negated, such formulas or statement forms ‘normal-form formula’.

4.4 QUANTIFICATION RULES

The rules of inference and replacement are augmented further with the addition of four more rules; universal instantiation (UI), universal generalization (UG), existential instantiation (EI) and existential generalization (EG). With the help of these rules and rules of inference and replacement any argument consisting of general or singular propositions or both can be tested. Before we apply these rules to test the validity of arguments, it is necessary that we know what these rules mean.

1. Universal Instantiation (UI): This rule says that any substitution instance of a propositional function can be validly deduced from a universal proposition. A propositional function always consists of variable ‘x’. Therefore any instance which is a substitution for ‘x’ must be a constant from ‘a’ through ‘w’. These letters signify subject in traditional sense, and in modern sense, an ‘instance of a form’. To transform such proposition ‘x’ is replaced by another Greek letter ‘ν’ (*nu*) when the function is universal quantifier, then ‘ν’ becomes universal instantiation.

   <u>(x) Φ x</u>
   ∴Φ ν (where ‘ν’ is any individual symbol)

2. Universal Generalization (UG): This rule helps us to proceed to generalization after an arbitrary selection is made to substitute for 'x'. In UG 'arbitrary selection' is very important because as the name itself suggests, generalization always proceeds from individual instance. And there is choice involved. In this sense, selection is 'random' or arbitrary. The letter 'y' is the symbol of 'arbitrary' selection. This process is called generalization because the conclusion is a universal proposition. If we recall the traditional rules of syllogism, universal conclusion follows from universal premise only. Therefore the process is from universal to universal through an individual. When 'y' replaces 'x' there is generalization. When universal quantifier describes the proposition, it becomes.

$\underline{\Phi\ y}$
$\therefore (x)(\Phi x)$ (where 'y' refers to any arbitrarily selected individual)

3. Existential Instantiation (EI): This rule is applicable when the proposition has existential quantifier and any symbol ranging from a through w is used as a substitute for the individual variable x. We infer the truth of any substitution instance from existential quantification. However, this rule has a clause. The constant, say 'a' which we use to substitute for x should not have occurred any where earlier in that context. It only means that in the same argument EI cannot be used twice when the substitution instance is only one.

$\underline{\Phi\ \nu}$
$\therefore (\exists x)\ \Phi x$ (where 'ν' is any individual symbol)

4. Existential Generalization (EG): This rule states that from any true substitution instance of a propositional function, an existential quantification of that function can be validly deduced. In other words, an individual constant which appears in earlier steps, is replaced by x in the conclusion.

$\underline{(\exists x)\ \Phi x}$
$\therefore \Phi\ \nu$ (where 'ν' is any individual constant other than y that has no prior occurrence in the context)

We should know why there is restriction on the use of EI. Suppose that 'a' is the constant whose existence is definite. We are not sure whether there is any other constant. In the earlier step 'a' is regarded as 'b'. The fact that 'a' is 'b' is not adequate enough to conclude that a is c in some other step when there is no reference of any to it in the premise. Since the logical constant a is used in existential mode, it is mandatory that EI should be used in the very first step of the proof. If it occupies any other position, then it is wrong.

4.5 TESTING THE VALIDITY OF SYLLOGISM

It is a matter of great interest to know that the rules of quantification project syllogism in a new perspective, which helps us to abandon the rule of distribution of terms, which is not only cumbersome in presentation but also time consuming. Further, quantification rules can be used to test non-syllogistic arguments also subject to the condition that only general and singular propositions find place in such arguments. Let us use the following arguments to illustrate these rules.

1. 1). All Indians are Asians.
   2). Tendulkar is an Indian.
   3). ∴Tendulkar is an Asian.

This is symbolized as follows: (x) (Ix=>Ax)
It
∴ At

The formal proof is constructed as follows:

1). (x)(Ix => Az)
2). It /∴At
3). It => At 1, U.I.
4). ∴At 3, 2, M.P.

In this particular argument only one premise is general. However, the argument may consist of only general propositions in which case slightly different procedure has to be followed. Consider this argument.

2. 1) All politicians are voters.
   2) All ministers are politicians.
   3) ∴All ministers are voters.

When symbolized it becomes:

1) (x)(Px => Vx)
2) (x)(Mx => Px) /∴ (x)(Mx => Vx)

The formal proof is as follows:

1) (x)(Px=>Vx)
2) (x)(Mx=>Px) /∴ (x){Mx=>Vx}
3) Pa => Va 1, U.I.
4) Ma => Pa 2, U.I.
5) Ma => Va 4, 3, H.S.
6) ∴ (x)(Mx => Vx) 5, U.G.

When the individual variable x is instantiated by any constant, then quantifier goes. We do not quantify individual or individuals. Now coming to the 6th step, it may be mentioned that if one substitution instance is true for a given structure then all substitution instances must be true for that structure. Further the universal quantification of a propositional function is true if and only if all substitution instances are true. (The 6th line is not a part of the proof)

In the third and the fourth steps we have applied universal instantiation because two premises are universal and we have substituted the constants for variables.

UG can be applied in the following manner. Add the sixth line to the proof system after we replace x by y at all stages. Then we have the application of UG

1)	(x){Px => Vx}		
2)	(x){Mx => Px}	/∴ (x){Mx => Vx}	
3)	Py => Vy	1,	U.I.
4)	My => Py	2,	U.I.
5)	My => Vy	3, 4,	H.S.
6)	∴ (x){Mx => Vx}	5	U.G.

These two examples suggest that while testing the validity of arguments UI has to be used necessarily though EI may not be necessary. The situation is similar to the traditional formation of rules of syllogism, which hint that without particular propositions it is possible to construct a valid argument, but not without universal propositions.

Now consider an argument, which has a particular proposition. Since one proposition is particular, it is imperative that the conclusion must be particular.

3. 1) All politicians are Voters.
   2) Some ministers are politicians.

   ∴Some ministers are Voters.

By now the method of symbolization should be familiar.

1)	(x){Px => Vx}		
2)	(∃x{Mx Λ Px}	/∴ (∃x{Mx Λ Vx}	
3)	Ma Λ Pa	2,	E.I.
4)	Pa => Va	1,	U.I.
5)	Pa Λ Ma	3,	Com.
6)	Pa	5,	Simp.
7)	Ma	5,	Simp.
8)	Va	4, 6,	M.P.
9)	Ma Λ Va	7, 8,	Conj.
10)	∴ (∃x) (Mx Λ Vx)	9,	I.G.

Let us examine why the restriction of EI must be honoured. Consider a fallacious argument.

1) Some animals are herbivorous.
2) Some animals are men.

∴Some men are herbivorous.

When symbolized the argument becomes:

1)	(∃x){Ax Λ Hx}			
2)	(∃x){Ax Λ Mx}	/∴(∃x)(Mx Λ Hx)		
3)	Aa Λ Ha	1,	E.I.	
4)	Aa Λ Ma	2,	E.I.	(Error)

4th Step is erroneous. The second premise tells us that there is at least one thing that is both an animal and herbivorous. It does not permit us to conclude that it should also be regarded as man. Therefore a second use of EI leads to error.

4.6 MULTIPLY GENERAL PROPOSITIONS

There are two types of general proposition; singly general and multiply general. If a general proposition has only one quantifier, then it is called *singly general*. Until now, we considered only propositions of former kind. If a general proposition consists of two or more than two quantifiers, then such a proposition is called *multiply general propositions*. Consider, for example, this proposition:

"If all Indians play cricket, then there are at least some Asians who play cricket."

Its symbolization is as follows:

1) All Indians play cricket: (x){Ix => Px}

2)There are at least some Asians who play cricket:(∃x){Ax Λ Px}

Now the symbolization of the whole sentence is as follows:

{(x)(x => Px)} => {(∃x)(Ax Λ Px)}

Depending upon the complexity of the given statement quantifiers may occur any number of times.

4.7 THE STRENGTHENED RULE OF C.P. AND QUANTIFICATION

In the previous unit, we learnt that assumption is different from conditional proof and that assumption does not include the conclusion, which depends solely on the premise. A few examples will illustrate how an argument can be tested using these techniques.

1. 1) (x)[Cx => Dx]
   2) (x)[Ex => ⌐ Dx]
   ∴ (x)[Ex => ⌐ Cx]

The argument is written in standard form;

1)	(x)[Cx => Dx]		
2)	(x)[Ex => ⌐ Dx]	/ ∴ (x)[Ex => ⌐ Cx]	
→3)	Ey		
4)	Cy => Dy	1,	U.I.
5)	Ey => ⌐ Dy	2,	U.I.
6)	⌐Dy	5, 3,	M.P.
7)	⌐Cy	4, 6,	M.T.
8)	Ey => ⌐ Cy	3, 7,	C.P.
9)	(x)[Ex => ⌐ Cx]	9,	U.G.

From (1) two aspects become clear. The limit of assumption ends, when CP is used. So next step does not depend upon this assumption. Second, since we are making an assumption, in place of 'x' only 'y'; an arbitrary chosen symbol can be used. This explanation holds good whenever the strengthened rule of CP is used.

2.

1)	(x)[Nx => Ox]		
2)	(x)[Px => ⌐ Ox]	/∴ (x) {(Nx Λ ⌐Px) => Ox}	
→ 3)	Ny		
4)	Ny => Oy	1,	U.I.
5)	Py => ⌐ Oy	2,	U.I.
6)	Oy	4, 3,	M.P.
7)	⌐ Py	5, 6,	M.T.
8)	Ny Λ ⌐ Py	3, 7,	Conj.
9)	(Ny Λ ⌐ Px) => Oy	8, 6,	C.P.
10)	(x){ (Nx Λ ⌐ Px) => Ox}	9, U.G.	

4.8 PROVING INVALIDITY

The cardinal principle underlying the classification of arguments into good and bad is that true premises do not yield false conclusion. The easiest way of identifying the false conclusion in association with true premises is the method of assigning the truth-values to the components of statements. When the method of truth-values is extended to arguments with quantifiers one requirement has to be satisfied. We have to consider a nonempty model which is similar to a nonempty set. This model is the locus of our discussion. An argument involving quantifiers is valid if and only if to every nonempty model corresponds a logically equivalent and valid truth-functional argument. Similarly, an argument is invalid if there is a nonempty model to which corresponds a logically equivalent and invalid truth-functional argument. The crux of the matter is only this; an argument consisting of quantifiers is valid if and only if its truth-functional mode is valid and invalid if and only if its truth-functional mode is invalid. Since there is recourse to truth-functional mode, it is necessary to know how statements with quantifiers can be reduced to truth-functional compound statements. The very same truth-conditions which determine the truth-value of compound propositions also determine the truth-conditions of corresponding propositions with quantifiers.

In the beginning of this section, we mentioned that an argument with quantifiers is valid if there is 'at least' one individual. It only means that there can be any number of individuals in a nonempty model. Suppose that there are only three men in the model of men, viz. a, b and c. In such a case the proposition 'A' can be represented in the following manner.

1. (x) (Φx) ≡ (Φa Λ Φb Λ Φc)

The LHS is true if and only if Φa is true, Φb is true and Φc is true. If any one of them is false, then the LHS is false. Similarly, the proposition 'E' becomes

2. (x) (⌐ Φx) ≡ (⌐ Φa Λ⌐ Φb Λ⌐ Φc)

   If a, b and c are the only men in the model of men, then as in the previous case, in the present case also the LHS is true if and only if everyone of the three components is true. If any one of them is false then LHS also is false.

While the propositions with universal quantifiers are translated to the conjunction mode, those with existential quantifiers are reduced to the disjunction mode. If we persist with the same model, then

3. $(\exists x)(\Phi x) \equiv (\Phi a \vee \Phi b \vee \Phi c)$
4. $(\exists x)(\neg \Phi x) \equiv (\neg \Phi a \vee \neg \Phi b \vee \neg \Phi c)$

From these four equations, it is clear that the truth status of propositions with quantifiers is determined by the truth-conditions of compound proposition. For example, consider (1). Even if one component on the RHS is false, then the LHS also turns out to be false. This is because conjunction is false when any component is false and in disjunction when any one component is true, the LHS is true. This type of relation is in perfect consonance with the definition of universal and existential quantifiers.

Suppose that there is only one individual. Then two corollaries follow from this supposition, which are as follows.

1. $(x)(\Phi x) \equiv \Phi a \equiv (\exists x)(\Phi x)$
2. Since there is only one true substitution instance (SI) to x, viz. a, we do not derive Φa from $(x)(\Phi x)$

When there is only one individual any logical difference between universal and existential quantifiers also ceases to operate.

Logically, there is a qualitative difference between a model containing only one individual and another model containing two or more than two individuals. (For the sake of convenience let us call the first model monadic and the second one polyadic model. If there are two individuals then the model is dyadic and if there are more than two then triadic and so on). There is a qualitative difference because in a monadic model an invalid argument may correspond to a valid truth-functional argument whereas the very same argument in any other model may correspond to an invalid truth-functional argument. Let us consider an argument which is invalid from traditional angle.

1.

All politicians are lawyers.
All judges are lawyers.
∴ All judges are politicians.

1. $(x)[Px \Rightarrow Lx]$
2. $(x)[Jx \Rightarrow Lx]$ / ∴ $(x)\, Jx \Rightarrow Px$

Since there is only one SI, this argument is logically equivalent to

3. p1:$[Pa \Rightarrow La]$
4. p2:$[Ja \Rightarrow La]$ / ∴ $Ja \Rightarrow Pa$

   In a monadic model $(x)(\Phi x) \equiv \Phi a \equiv (\exists x)(\Phi x)$

   ∴ The argument is logically equivalent to

5. $Pa \wedge La$
6. $Ja \wedge La$ / ∴ $Ja \wedge Pa$

   If we assign the value 0 to any one of the components of the conclusion then not only the conclusion is false but also one of the premises becomes false. However, according to definition, the premises must be true. It is logically impossible to derive a false conclusion from true premises. Therefore in this case the argument is valid. However, the same argument is invalid in a dyadic model. Before we consider an example for an argument in a dyadic model, let us consider the structure of the model.

[(x) (Φx)] ≡ [Φa Λ Φb]

[(x) ⌐ (Φx)] ≡ [⌐ Φa Λ⌐ Φb]

(∃x) (Φx) ≡[Φa v Φb]

(∃x) ⌐ (Φx) ≡[⌐ Φa v ⌐ Φb]

Where a and b are two individuals who (or which) are the members of a dyadic model

2. Now let us symbolise the previous argument

1. p1: (x) [Px => Lx]
2. p2: (x) [Jx => Lx] / ∴ (x) Jx => Px

Since we are considering a dyadic model the symbolic presentation is logically equivalent to

3. (Pa => La) Λ (Pb => Lb)
4. (Ja => La) Λ (Jb => Lb) / ∴ (Ja => Pa) Λ (Jb => Pb)

Assign 0 to Pa and 1 to the rest. The result can be computed as follows

5. (Pa => La) Λ (Pb => Lb)
   0 1 1 1 1 1 1
6. (Ja => La) Λ (Jb => Lb) / ∴ (Ja => Pa) Λ (Jb => Pb)
   1 1 1 1 1 1 1 1 0 0 0 1 1 1

The conjunction of the truth-values which are boxed in 5 and 6 yields true premises whereas the conclusion is false. Hence the argument is invalid. This result can be generalised to include other polyadic models with 3 or more than 3 members. Whatever holds good to a dyadic model in this case also holds good to any other polyadic model. To become familiar with this method let us work with some more problems.

3. (x) (Dx => ⌐ Ex)

(x) (Ex => Fx) / ∴(x) (Fx => ⌐ Dx)

Let us restrict this argument to a dyadic model. If this argument is invalid in this model, then it is invalid in all other polyadic models. The logically equivalent form of 3 is as follows.

1. (Da => ⌐ Ea) Λ (Db => ⌐ Eb)
2. (Ea => Fa) Λ (Eb => Fb) / ∴(Fa => ⌐ Da) Λ (Fb => ⌐ Db)

Assign 0 to ⌐ Da. In accordance with the law of contradiction Da = 1. Similarly, ⌐ Db is assigned 0. Therefore Db = 1. Assign 1 to ⌐ Ea. Ea takes 0. Assign 1 to ⌐ Eb. Eb takes 0. Assign 1 to Fa and Fb. The result can be computed as follows.

3. (Da => ⌐ Ea) Λ (Db => ⌐ Eb)
   1 1 1 1 1 1 1
4. (Ea => Fa) Λ (Eb => Fb) / ∴(Fa => ⌐ Da) Λ (Fb => ⌐ Db)
   0 1 1 1 0 1 1 1 0 0 0 1 0 0

In this argument also the conjunction of the truth-values boxed in 3and 4 yields true premises whereas the conclusion is false. Hence the argument is invalid. This result can be generalised to include other polyadic models with 3 or more than 3 members. Whatever holds good to a dyadic model in this case also holds good to any other polyadic model.

4.

1. (∃x) (Jx Λ Kx)

2. (∃x) (Kx Λ Lx) / ∴ (∃x) (Lx Λ Jx)

We shall consider this argument also in a dyadic model. This is logically equivalent to

3.(Ja Λ Ka) v (Jb Λ Kb)

4. (Ka Λ La) v (Kb Λ Lb) / ∴ (La Λ Ja) v (Lb Λ Jb)

There is a difference between this argument and the previous arguments. In this argument the premises and conclusion are disjunctive unlike the previous arguments which have conjunctive statements. The difference is due to quantifiers. In case of universal quantifiers conjunction is the connective whereas in case of existential quantifiers disjunction is the connective.

Assign the truth-values as follows; 0 to La and Jb and 1 to the rest. The result is computed as follows.

5.(Ja Λ Ka) | v | (Jb Λ Kb)
1 1 1 | 1 | 0 0 1

6. (Ka Λ La) | v | (Kb Λ Lb) / ∴ (La Λ Ja) v (Lb Λ Jb)
1 0 0 | 1 | 1 1 1 0 0 1 0 1 0 0

In this argument also the conjunction of the truth-values which are boxed in 5 and 6 yields true premises whereas the conclusion is false. Hence the argument is invalid. This result can be generalised to include other polyadic models with 3 or more than 3 members. Whatever holds good to a dyadic model in this case also holds good to any other polyadic model.

4.9 NONSYLLOGISM

All arguments need not be syllogistic even though they consist of two premises and a conclusion. Relational argument is one such example.

1. Bangalore is to the west of Chennai.
   Mangalore is to the west of Bangalore.
   ∴ Mangalore is to the west of Chennai.

Aristotelian system does not regard this class of arguments as syllogistic though this can be shown to be valid in symbolic representation, but it results in the distortion of the meaning of statements. If we try to retain the meaning, then it becomes impossible to demonstrate the validity or invalidity, as the case may be.

Apart from relational arguments, there is another class of arguments which consists of more than three terms and propositions. Consider this argument.

Men (1) are both stupid (2) and dishonest (3).
Some men are irritable (4).
∴ Some dishonest persons (3) are irritable (4).

Terms are numbered so there is no confusion. However, the statements are misleading. If we regard a conjunctive proposition as one proposition, then in this argument there are three propositions. Even if the previous statement is accepted the argument cannot be syllogistic because there are four terms. If we give priority to simple propositions then the first premise has two simple propositions. Then we will have four propositions. Therefore this type of argument is classified as nonsyllogistic. To test this kind of argument we do not require any additional rule. Proper symbolization of this class of argument is important. The symbolization is as follows:

1. (x) [Mx => (Sx Λ Dx)]
2. (∃x) [Mx Λ Ix] / ∴ (∃x) (Ix Λ Sx). Its formal proof:
3. [Ma Λ Ia] 2, E. I.
4. Ma => (Sa Λ Da) 1, U. I.
5. Ma 3, Simp.
6. (Sa Λ Da) 4, 5, M. P.
7. Sa 6, Simp.
8. Ia 3, Simp.
9. Ia Λ Sa 8, 7, Conj.
10. (∃x) (Ix Λ Sx) 9, E.G.

The status of (1) calls for our attention. Had the first premise been regarded as a conjunctive proposition, then (1) ought to have been symbolized as
11. Sm Λ Dm
It is a well known fact that conjunction does not have any equivalent form. Therefore (1) is not equivalent to (11).

Consider another statement, which has a very different structure.

Americans and Germans are pioneers in science.

This statement actually means that a pioneer in science may be an American or a German. Surely, it does not mean at a pioneer in science is both an American and a German. Hence when this innocuous statement is translated into logical language, it becomes a disjunctive proposition with exclusive 'Or'. Nor is it a conjunctive proposition of the form
Americans are pioneers in science and Germans are pioneers in science.
This is so because a conjunctive proposition of this form means the same as saying that a pioneer in science is both an American and a German, which is absurd. Consider this argument:
Americans and Germans are scientists.
Some white men are Americans.
Therefore, some white men are scientists.
This argument is symbolized as follows:

1. (x) [(Ax v Gx) => Sx]
2. (∃x) [Wx Λ Ax] / ∴ (∃x)[Wx Λ Sx]
3. Wa Λ Aa 2, E.I.
4. Aa 3, Simp.
5. (Aa v Ga) 4, Add.
6. (Aa v Ga) => Sa 2, U.I.
7. Sa 6, 5, M.P.
8. Wa 3, Simp.
9. Wa Λ Sa 8, 7, Conj.
10. (∃x)[Wx Λ Sx] 9, E.G.

In one particular sense, nonsyllogistic arguments are more significant than traditional syllogism for the simple reason that in any debate, whether based in science or politics, syllogism is seldom used. Application of nonsyllogistic arguments is widespread and more useful. Therefore there is greater need to become familiar with nonsyllogistic arguments.

Check Your Progress

Note: a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

I. Construct formal proofs of validity.

1. 1) (x)[Qx => Rx]
2) (∃x)[Qx v Rx]
∴ (∃x) Rx

----

2. 1) (x)[Sx => (Tx =>Ux)]
2) (x)[Ux=>(Vx ΛWx)]
∴ (x) [Sx => (Tx => Vx Λ Wx)]

----

3. 1) (x)[Dx => ⌐ Ex]
2) (x)[Fx => Ex]
∴ (x) [Fx => ⌐ Dx]

----

4. 1) (∃x) [Jx Λ Kx]
2) (x) [Jx => Lx]
∴ (∃x) [Lx Λ Kx]

4.10 LET US SUM UP

Quantification is another set of rules, which augments the logical tools of test. It applies to arguments, which consist of general and singular propositions. Quantification rules must be used in conjunction with the rules of inference and replacement.

4.11 KEY WORDS

Dyadic: Dyadic is that which is composed of two sets of objects say *A* and *B;* if three sets or elements, then it is known as *triadic*; if four, then *tetradic* and if five, then *pentadic*.

Polyadic: Polyadic is that which comprises of many elements.

4.12 FURTHER READINGS AND REFERENCES

Basson, A.H. & O'connor, D.J. *Introduction to Symbolic Logic*. Calcutta: Oxford University Press, 1976.

Copi, I.M. *Introduction to Logic*. New Delhi: Prentice Hall India, 9th Ed., 1995.
Hughes, G.E. and Londey, D.G. *The Elements of Formal Logic*. Bombay: B.I.Publications, 1966
Joseph, H.W.B. *An Introduction to Logic*. Oxford: 1906.
Kalish, Donald et al. *Logic: Techniques of Formal Reasoning*. New York: Harcourt Brace Jovanovich Publishers, 1980.
Lewis, C.I. & Longford, C.H. *Symbolic Logic*. New York: Dover Pub. Inc.,1959.
Suppes, Patrick. *Introduction to Logic*. New Delhi: Van Nostrand Reinhold/Affiliated East-West Press, 1969.

4.13 ANSWERS TO CHECK YOUR PROGRESS

1)	(x) [Qx => Rx]	
2)	(∃x) [Qx v Rx]	/∴ (∃x) (Rx)
3)	Qa v Ra	2, E.I.
4)	Qa => Ra	1, U.I.
5)	Ra	4, 3, M.P.
6)	(∃x) Rx	5, E.G.

2

1)	(x) [Sx => (Tx => Ux)]	
2)	(x) [Ux => (Vx Λ Wx)]	/∴ (x) [Sx => {Tx => (Vx Λ Wx)}]
3)	Sa => (Ta => Ua)	1, U.I.
4)	Ua => (Va Λ Wa)	2, U.I.
5)	(Sa Λ Ta) => Ua	3, Exp.
6)	(Sa Λ Ta) => (Va Λ Wa)	5, 4, H.S.
7)	Sa => (Ta => (Va Λ Wa))	6, Exp.
8)	∴ (x) [Sx => {Tx => (Vx Λ Wx)}]	7, U.G.

3

1)	(x) [Dx => ⌐ Ex]	
2)	(x) [Fx => Ex]	/∴ (x) [Fx => ⌐ Dx]
3)	Da => ⌐ Ea	1, U.I.
4)	Fa => Ea	2, U.I.
5)	Ea => ⌐ Da	3, Trans.
6)	Fa => ⌐ Da	4, 5, H.S.
7)	∴ (x) [Fx => ⌐ Dx]	6, U.G.

4

1)	(∃x) [Jx Λ Kx]	
2)	(x) [Jx => Lx]	/∴ (∃x) [Lx Λ Kx]
3)	Ja Λ Ka	1, E.I.
4)	Ja => La	2, U.I.
5)	Ja	3, Simp.
6)	Ka	3, Simp.
7)	La	4, 5, M.P.
8)	La Λ Ka	7, 6, Conj.
9)	(∃x) [Lx Λ Kx]	8, E.G.