
UNIT 5 APPLICATIONS OF SYMBOLIC LOGIC

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5.0. OBJECTIVES

This unit is to limelight the application of Symbolic Logic in the Modern Era. The instrumental value of logic is well known in many disciplines, such as Philosophy, Mathematics, and Computer Science. Mathematics teaches logic almost as an extension of algebra or calculus with lemmas and proofs. And also, Computer science teaches it with more emphasis on its applicability for enriching programming power or for building ‘thinking machines’. But Philosophy, of which logic has always been a fundamental part, approaches logic somewhat differently. For then its key concerns are different kinds. We present a range of binary systems suitable for representing information in digital components. The binary number system is explained and binary codes are illustrated to show the representation of decimal and alphanumeric information. We introduce the concepts of Boolean algebra from a fundamental point of view, co-relating with Symbolic Logic. The correlation between a Boolean expressions and its equivalent interconnection of gates is emphasized. At present, in the field of communication, entertainment, medical electronics, and digital electronics has taken giant strides. Here fundamental ideas about implementing a logic circuit for a logic expression or writing a logic expression for a given logic circuit are discussed.

All possible logic operations for two variables are investigated and from that, the most useful logic gates are derived. The characteristics of digital gates available in integrated circuit form are presented. This unit supplies the diagram and tabulation methods for simplifying Boolean functions. The diagram is used to simplify digital circuits constructed with AND, OR, NAND, NOR, and wired-logic gates. The chief endeavor of this chapter is to utter that the Digital logic is not based on numbers but they are based on the sentences. More specifically, it is based on the connectivity of the propositions. Primary purpose of this unit is to facilitate education in the increasingly important areas

of multi-value logic suitable for representing information in Fuzzy logic. The role of symbolic logic is decorated in the multi-value logic. Truth status of propositions is challenging and is not restricting the future events. The fundamental of fuzzy propositions is also discussed in this chapter.

5.1. INTRODUCTION

In the history of western logic, Symbolic logic is relatively recent development. It is the study of human thoughts through symbols. It is learning towards mathematics and symbolization. It would be a well high hopeless task to discuss modern considerations of logic by the use of only ordinary language. A symbolic language has become necessary in order to achieve the required exact scientific treatment of the subject. Because of the presence of such symbolism, the resulting treatment is known as symbolic logic.

5.2. APPLICATION OF SYMBOLIC LOGIC WITH DIGITAL LOGIC

The word ‘digit’ is resultant from the Latin term “*digitus*” which means finger or toe. The fact that this system has ten digits is commonly attributed to the ten fingers of a human being. The most commonly used number system is the decimal number system; it is composed of digits ‘0’ to ‘9’. Digital logic is concerned with the interconnection among digital components and modulus. Digital computer has made possible many scientific industrial and commercial advances that would not be attainable otherwise. Our space program would have been unfeasible without real-time, continuous computer monitoring and many business enterprises functions efficiently only with the aid of automatic data processing. Computers are used in scientific calculations, commercial and business data processing, air traffic control, space guidance, the educational field, and many other areas. A digital computer is a programmable machine that processes binary data which is represented by binary number system. The simplest number system employing positional notation is the binary system. A binary number system uses only two symbols or digits namely 0 and 1. They are the binary number system which has a base or radix of 2. A binary digit 0 and 1 are called “*bits*”. 4-bit binary word is called “*nibble*”, and 8-bit binary word is called “*byte*”.

Check Your Progress I

Note: Use the space provided for your answer

1. What do you mean by “*digitus*”? Explain.

2. Explain the application of Symbolic Logic in the digital world.

5.3. BOOLEAN ALGEBRA

George Boole urbanized Boolean algebra in 1847 and used it to crack in Mathematical logic or Symbolic logic. In 1938 Claude Shannon introduced a two-valued Boolean algebra called “Switching Algebra”, in which he demonstrated that the properties of bi-table electrical circuits can be represented by this algebra. Claude Shannon previously applied Boolean algebra to design of switching network in 1939. The values used in Boolean algebra are representing the symbols ‘0’ and ‘1’. They have no numeric values. A binary ‘1’ will represent a High level, and a binary ‘0’ will represent a Low level in Boolean Equations.

Basic Operation: Boolean algebra has only three operators AND (\bullet), OR (+) and NOT ($\bar{}$) or Complement or Inverse.

AND Operator: The logical operation of AND can be articulated with symbols as follows. Let one input variable is A, the other input variable is B and the output variable is C. Subsequently the Boolean expression of this basic operator function is $C = A \wedge B$ (or) $C = AB$. The table for the Boolean expression $C = A \wedge B$, is as follows.

A	B	$A \wedge B$	C
0	0	$0 \wedge 0$	0
0	1	$0 \wedge 1$	0
1	0	$1 \wedge 0$	0
1	1	$1 \wedge 1$	1

OR Operator: The logical operation of OR can be articulated with symbols as follows. Let one input variable is A, the other input variable is B and the output variable is C. In that case the Boolean expression of this basic operator function is $C = A + B$. The table for the Boolean expression $C = A + B$, is as follows.

A	B	$A + B$	C
0	0	$0 + 0$	0
0	1	$0 + 1$	1
1	0	$1 + 0$	1

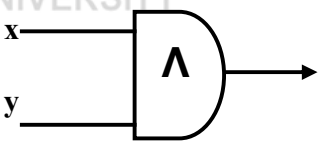

1	1	1+1	1
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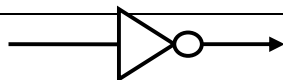
NOT Operator: The Complement of '0' is '1' and the Complement of '1' is '0'. Symbolically, we write $0' = 1$ and $1' = 0$. The logical operation of an inverter (NOT) can be expressed with symbols as follows: If the input variable is 'A' and the output variable is called X, then $X = A'$. This expression states that the output is the complement of input, so if $A=0$ then $X=1$ and $A=1$ then $X=0$. The table for the Boolean expression $A = X$, is as follows.





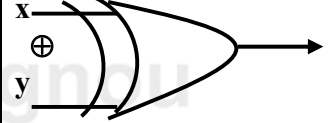
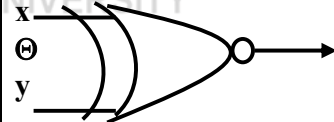
A	X
0	1
1	0

5.4 LOGIC GATES

A logic gate is an electronic circuit, which takes numerous inputs and produces a single output. Logic gates form the fundamental building blocks for all the digital circuits. AND gate, OR gate and NOT gate are called "basic gates". NAND gate and NOR gate are called "Universal Gates". All the gates are obtainable in Integrated Circuit (IC) form. The different IC families differ in their speed, power dissipation, propagation delay, etc. There are eight functions to be considered as candidates for logic gates: AND, OR, NAND, NOR, XOR, XNOR, INVERTER, BUFFER. The graphic symbols and truth tables of the eight gates are show in below.

Name	Graphic Symbol	Algebraic Function	Truth Table	
AND		$f = x \cdot y$	x y	f
			0 0	0
			0 1	0
			1 0	0
			1 1	1
OR		$f = x + y$	x y	F
			0 0	0
			0 1	1
			1 0	1
			1 1	1
			x	F



INVERTER		$f = x'$	0 1	1 0
BUFFER		$f = x$	x 0 1	F 0 1
NAND		$f = (xy)'$	x y 0 0 0 1 1 0 1 1	F 1 1 1 0
NOR		$f = (x+y)'$	x y 0 0 0 1 1 0 1 1	F 1 0 0 0
XOR		$f = x \oplus y$	x y 0 0 0 1 1 0 1 1	F 0 1 1 0
XNOR		$f = x \oplus y$	x y 0 0 0 1 1 0 1 1	F 1 0 0 1

Each gate has one or two inputs nominated by x, y, etc and the out put is designated by 'f'.

Modern Classification of Propositions and Digital Logic Gates: Modern logicians categorize propositions into three types. They are Simple, Compound and General. Compound proposition is classify further into:

1. Conjunctive
2. Implicative (Conditional)
3. Disjunctive
 - (a) Inclusive, (b) Exclusive
4. Equivalence (Bi- Conditional)
5. Negation

Conjunctive: A conjunctive proposition is a compound proposition containing two or more simple propositions, conjoined by the word 'and'. Two or more propositions so

conjoined are called conjuncts. 'Sankara is philosopher and Ramanuja is a philosopher' is one such conjunctive proposition. The symbol ' * ' or ' \wedge ' (dot) is used to represent the function of conjunctive proposition. If we substitute the variable 'p' for 'Sankara is philosopher' and 'q' for 'Ramanuja is a philosopher' then the two conjuncts are symbolized as (p \wedge q). The truth value and truth functions of these conjuncts will be as such:

p	\wedge	q	
T	1	T	
T	0	F	
F	0	T	
F	0	F	_____ Equi - 1

Among the Logic gates, AND gate is one of the basic gates; its truth table is stated below and let the equation be "2". The evaluation of this equation says that output 'f' of an AND gate is High (1) only both inputs are High (1)

p	\wedge	q	
0	0	0	
0	0	1	
1	0	0	
1	1	1	_____ Equi - 2

This gate is compared with the equation "1", Conjunctive proposition of the compound proposition in Symbolic Logic which states that when the antecedent and consequent of the proposition is True the validity of the whole proposition will be true, if not it is invalid. This equation (1) is rewritten without affecting the truth values and truth-functions for our better understanding and is stated as equation "3".

p	\wedge	q	
F	0	F	
F	0	T	
T	0	F	
T	1	T	_____ Equi - 3

Let F=0, and T=1, substitute in Equation "3"

p	\wedge	q	
0	0	0	
0	0	1	
1	0	0	
1	1	1	_____ Equi -4
\Rightarrow Equi -4 = Equi - 2			
\Rightarrow [Equi -4 = Equi - 3] = Equi - 1			
\Rightarrow Equi -4 = Equi - 1			
$\Rightarrow \therefore$ Equi - 1 = Equi - 2			

Equitation “1” interprets the basic truth table of a conjunctive proposition of a compound proposition containing two or more simple proposition, conjoined by the word ‘and’. The Equitation “2” just reveals us that output of an AND gate is High (1) only when both inputs are High (1). The above truth table specifies the out put values of every possible combination of values of the variables in the expression. In this way other gates can be proved.

Check Your Progress II

Note: Use the space provided for your answer

1. Define Logic Gate.

2. Write a short note on Boolean operators.

5.5 ROLE OF SYMBOLIC LOGIC IN MULTI – VALUE LOGIC

In 1936 K.Michalski discovered that three-valued logics had actually been anticipated as early as the 14th century by the medieval schoolman, William of Occam. The possibility of a three-valued logic has also been considered by the Philosophers Hegel and, in 1896, Hugh MacColl. It will be recalled that new geometries primary came about through the refutation of Euclid’s parallel postulate, and that new algebras first came about through the rejection of the commutative law of multiplication. Similarly, the new so-called “many-valued logics” first came about by denying Aristotle’s law of excluded middle.

In crisp logics, such as binary logic, variables are either true or false, black or white, 1 or 0. An extension to binary logic is multi-value logic, where variables may have many crisp values. In 1921, in a short two-page paper, J.Lukasiewicz considered a three-valued logic, or a logic in which proposition ‘p’ may possess any one of the three possible truth values. Very shortly after and independent of Lukasiewicz’s work, E.L.Post considered m-Valued logics, in which proposition ‘p’ may possess any one of ‘m’ possible truth values, where ‘m’ is greater than 1. If ‘m’ exceeds 2, the logic is said to be many-valued.

Bi-Value and Multi-Value Logic: We shall utilize the method of truth table, preliminary with a Basic truth table for conjunction. We, first of all, replicate the truth table for conjunction in the two-valued logic.

	q		
	\wedge	T	F
p	T	T	F
	F	F	F

Figure-1

Down the left-hand column emerge the possible truth values for proposition 'p' and across the top row show the possible truth values for the proposition 'q'. Now, knowing the truth value of 'p' and of 'q', one can find the truth value of 'p \wedge q'. 'p \wedge q' is to be true when and only when both 'p' and 'q' are true, a T appear in the top left box of the table and F's come out in all the other boxes. We now ensue to the three-valued logic and again agree to take the conjunction 'p \wedge q' to be true when and only when both 'p' and 'q' are true. Denoting the three possible truth values of a proposition by T, ?, and F. We start to build a truth table.

	q			
	\wedge	T	?	F
p	T	T		
	?			
	F			

Figure-2

By our array 'p \wedge q', the top left box in the above table must contain as T, and no other box in the table is allowed to contain a T. Since there are eight remaining boxes and each may be filled in either of two possible ways, namely, with either F or ?, in sum $2^8 = 256$ possible ways of filling the eight boxes. It follows that there are 256 different ways of developing a truth table for conjunction in a three-valued logic. To illustrate two of the possible 256 truth tables for conjunction in a three-valued logic.

	q			
	\wedge	T	?	F
p	T	T	?	F

	?	?	?	F
	F	F	F	F

Figure-3a

In paradox: Let a proposition be 's' and the Negation of it is 'not-s', the proposition and its negation are same.

∴ "s=not-s" ----- Equi.1

The truth function of this Equitation is:

⇒ $t(s) = t(\text{not-s})$ ----- Equi.2

In a Bivalent Logic

⇒ $[t(s) = t(\text{not-s})] = 1$ ----- Equi.3

⇒ $t(\text{not-s}) = 1 - t(s)$ ----- Equi.4

Substitute Equi.2 in Equi.4

⇒ $t(s) = 1 - t(s)$ ----- Equi.5

⇒ If s= true, $t(s) = 1$ then $1 = 0$

⇒ If s= false, $t(s) = 0$ then $0 = 1$

Contradiction

When we test the same Equitation in Multi-value logic from Equi.5

⇒ $t(s) = 1 - t(s)$ ----- Equi.5

⇒ $t(s) + t(s) = 1$ ----- Equi.6

⇒ $2t(s) = 1$ ----- Equi.7

⇒ $t(s) = 1/2$ ----- Equi.8

The degree of truth lies in $1/2$ (Half-Truth)

∴ An extension to binary logic is multi-value logic.

Multi-Value Logic: The fundamental assumption, upon which classical logic (or two-valued logic) is based, that each proposition is either true or false – had been questioned since Aristotle. It is understood that propositions truth status is problematic are not restricted to future events. As an end result of the Heisenberg principle of uncertainty, for example, it is acknowledged that truth values of certain propositions in quantum mechanics are innately indetermined due to fundamental boundaries of measurements. In order to treat with such propositions, we must relax the true/false dichotomy of classical two-valued logic by allowing a third truth value, which may be called indetermined. The classical two-valued logic can be extended into three-valued logic in different ways. Several three-valued logics, each with its own rationale, are now well established. It is frequent in these logics to designate the truth, falsity, and indeterminacy by 1, 0, and $1/2$, respectively.

Once the range of three-valued logics was acknowledged as meaningful and useful, it became pleasing to explore generalization into n-valued logics for an arbitrary number of

truth values ($n \geq 2$). Several n -valued logics were, in fact, urbanized in the 1930s. For any given 'n', the truth values in these generalized logics are usually labeled by rational numbers in the unit interval $[0,1]$. These values are obtained by uniformly dividing the interval between 0 and 1 exclusive. The set T_n of truth values of an n -valued logic is thus defined as:

$$T_n = \{0 = 0/n-1, 1/n-1, 2/n-1, \dots, n-2/n-1, n-1/n-1 = 1\}$$

These values can be interpreted as degrees of truth.

Fuzzy Logic: Fuzzy Sets as well as Fuzzy Logic is a factual magnum work. Fuzzy Logic addresses practically every significant topic in the broad expanse of fuzzy set theory. To us Fuzzy Sets and Fuzzy Logic is an astonishing achievement; it covers its immeasurable territory with impeccable authority, deep insight and a meticulous concentration to detail. To view Fuzzy Sets along with Fuzzy Logic in an appropriate perspective, it is compulsory to clarify a point of semantics which relates to the meanings of fuzzy sets and fuzzy logic. More exclusively, in a broad sense, fuzzy logic is a logical system which is an extension and generalization of classical multi-valued logics. However in a wider sense, fuzzy logic is almost identical with the theory of fuzzy sets.

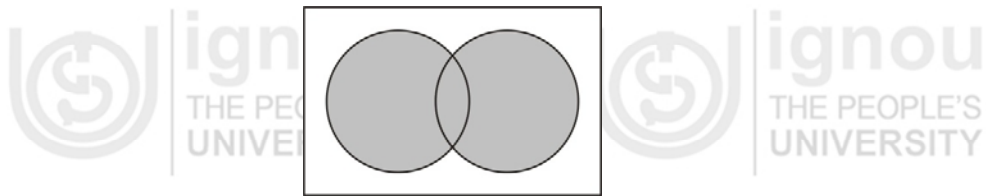
Fuzzy Propositions: The primary disparity between classical proposition and fuzzy propositions (Rule Base) is in the range of their truth values. At the same time as every classical proposition is mandatory to be either true or false, the truth or falsity of fuzzy proposition is a matter of degree. In this slice, we focus on simple fuzzy propositions, which we categorize into the subsequent four types:

1. Unconditional and unqualified proposition.
2. Unconditional and qualified proposition.
3. Conditional and unqualified proposition.
4. Conditional and qualified proposition.

Classical Set: The most vital and most fundamental term to be found in modern mathematics and logic is that of set, or class. Even though some modern studies make a technical dissimilarity between set and class, we do not do so in this treatment; it is recognized, however, that mathematicians are inclined to use the word set, whereas logicians universally refer to a class. The theory of sets, we shall see, forms one of the connecting links between mathematics on the one hand and logic on the other hand. We shall think of a set as simply a collection of well defined objects. The objects which make up a set will be called elements of the set or the items that are entered or considered as in the set are called the elements or members of the set. The important set operations can be defined as below:

- Complement of A is $A' = \{x \mid x \notin A\}$
- Intersection of A and B, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- Union of A and B, $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

The basic set operations, 'intersection', 'union' and 'complement', are illustrated using



therefore, it can be treated as one of the most fundamental activities. Decision-making is

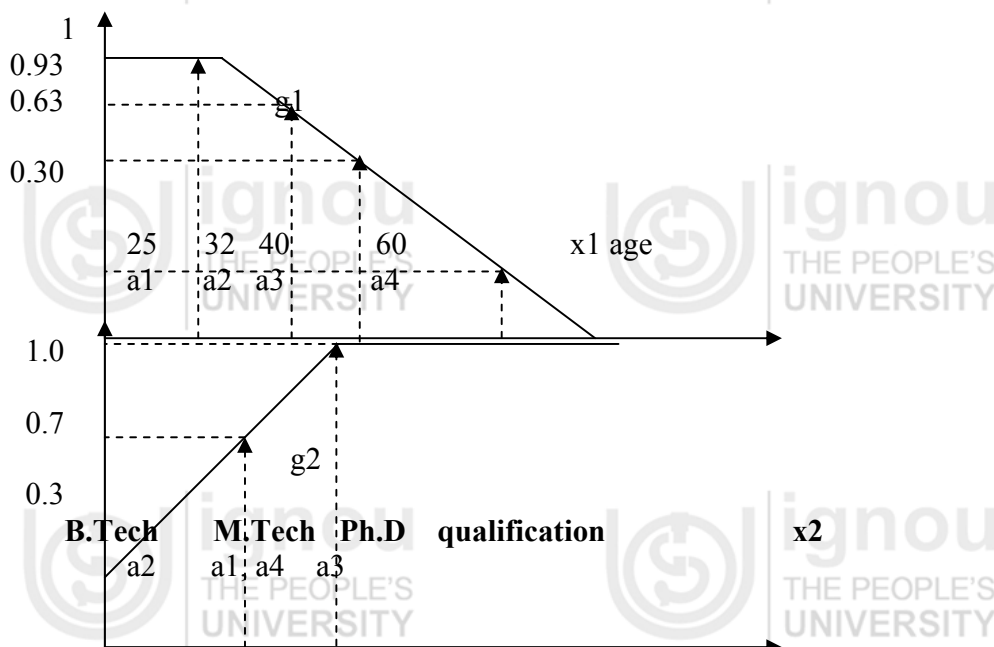
an area which studies about how decisions are actually made and how better they can be made successfully.

Problem: A research institute wants to recruit a young, dynamic and talented scientific officer to assist a team comprising three experts to work in the field of high voltage engineering. There are four applications and the details are listed below:

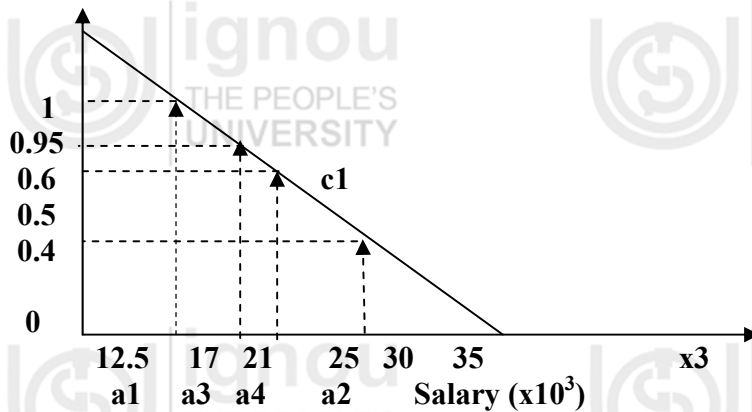
NAME	AGE	QUALIFICATION	SALARY DEMAND
a1	25	M.Tech	Rs.12,500/-
a2	55	B.Tech	Rs.25,000/-
a3	32	Ph.D	Rs.17,000/-
a4	40	M.Tech	Rs.21,000/-

The institute is in financial constraints. If the institute wishes to call only one person for interview, develop a fuzzy decision-making algorithm and give the result for the present problem.

Solution: In this problem, the institute is looking for young, dynamic (basically age decides) and talented (better qualification) candidates. Therefore, it is appropriate to consider age and qualification as goals, namely g_1 and g_2 . The Institute has financial impediments. Therefore, salary is the constraint c_1 . Thus the problem has two goals and a single constraint. Let us build simple fuzzy sets for g_1 , g_2 and c_1 suitably. It may be noted that an increase in age is less preferred. Similar is the case with salary too. As qualification is higher, it is better. Hence the three fuzzy sets are sketched as shown in the figures.



It is important to note that qualification cannot be expressed numerically. However, since a choice has to be made, this variable is assigned a suitable membership grade.



From the above fuzzy sets, we can find

$$\begin{aligned} &\mu(g1/a1), \mu(g1/a2), \mu(g1/a3), \mu(g1/a4), \\ &\mu(g2/a1), \mu(g2/a2), \mu(g2/a3), \mu(g2/a4), \\ &\mu(c1/a1), \mu(c1/a2), \mu(c1/a3), \mu(c1/a4), \\ &\mu(g1/a_i) = [1/a1, 0.3/a2, 0.93/a3, 0.63/a4] \\ &\mu(g2/a_i) = [0.7/a1, 0.3/a2, 1.0/a3, 0.7/a4] \\ &\mu(c1/a_i) = [0.95/a1, 0.4/a2, 0.6/a3, 0.5/a4] \end{aligned}$$

Using the equation $D(a_i) = \text{Min} \{a_1, a_2, a_3, a_4\}$ ie (i=total)

$$\Rightarrow D(a_i) = \text{Min} \{0.7/a1, 0.3/a2, 0.6/a3, 0.5/a4\}$$

Now, applying the equation $D = \text{Max} \{D(a_i)\}$

$$\text{We get Fuzzy decision, } D = \text{Max} \{D(a_i)\} = 0.7/a1$$

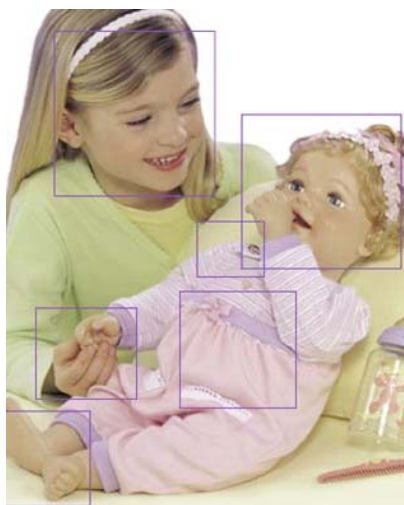
Thus, a_1 is the suitable candidate to be called for interview.

Fuzzy-Machines: Fuzzy logic principles are extensively employed in various consumer products and the sale of these products is increasingly going up in recent years. A typical application is the use of fuzzy logic principle to automatic washing machines. While many manufacturers provide with a variety of features, the underlying principle of fuzzy logic-based washing is explained in this section. As long as the washing of clothes is concerned, removal of dirt particles is the objective. Thus the input of the fuzzy control system is the quantum of dirt and its rate of change. The weight of clothes is another input. With these three inputs, the following are the outputs:

1. Quantity of washing powder
2. Water quantity
3. Water flow rate
4. Washing time
5. Rinsing time and
6. Spinning time

Fuzzy-Genetic Algorithms: There are a number of ways in which Genetic Algorithms





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1) How many kinds of fuzzy proposition? Explain.

2) Explain the Basic Operation on Fuzzy Sets.

5.7 LET US SUM UP

In sum the intellectual capacity is to include an explicit cram on Multi-value logic. The role of symbolic logic is decorated in the multi-value logic. In order to deal with such propositions, we must relax the true/false dichotomy of classical two-valued logic by allowing a third truth value, which may be called indeterminacy. The fundamental difference between classical propositions and fuzzy propositions and the range of their truth values are also discussed in this chapter. Fuzzy Sets and Fuzzy Logic is a true magnum work. The fundamental distinction between classical proposition and fuzzy propositions is in the range of their truth values. While each classical proposition is required to be either true or false, the truth or falsity of fuzzy proposition is a matter of degree. Assuming that truth and falsity are expressed by values 1 and 0, respectively, the degree of truth of each fuzzy proposition is expressed. In metro trains, fuzzy logic is used to determine the proper start, stop, and cruising speed of the train; in washing machines it determines the amount of water and the number of rinses; in cameras and camcorders it adjusts the colour, contrast, brightness, focus, and son on; in vacuums it determines the suction power based on the amount and size of particles; in automobiles with automatic transmission it determine the proper gear; in intelligent vehicular systems it finds the best route and automatically guides an automobile; and in communication systems it processes signals, schedules and routes channels, and controls the system. In financial engineering the performance of stocks is being predicted. As a result we strongly conclude that the role of symbolic logic in the modern era is immensely significant.

5.8 KEY WORDS

Logic Gate: A logic gate is an electronic circuit, which takes numerous inputs and produces a single output.

Set: Set is a collection of well defined objects.

5.9 FURTHER READINGS AND REFERENCES

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