UNIT 2 RULES OF UNIVERSAL INSTANTIATION AND GENERALIZATION, EXISTENTIAL INSTANTIATION AND GENERALIZATION, AND RULES OF QUANTIFIER EQUIVALENCE

Contents

2.0 OBJECTIVES

The first objective of this unit is to emphasize the need to have a new set of rules. Before we apply these rules a brief explanation of these rules is needed. Therefore there is another important objective. This second objective is to provide a theoretical basis for the application of these rules. However, these rules only supplement the rules discussed in detail in the preceding unit. Hence the third objective is to demonstrate how the integration of the new set of rules with the rules with which we are already familiar is necessary. Thus by the end of this unit we should be able to have a basic understanding of the significance of quantifiers. Secondly, we must succeed in identifying the quantified statements. Finally, we must succeed in testing successfully the validity of arguments with the help of quantifiers

2.1 INTRODUCTION

A quantifier is a tool with the help of which we will be in a position to measure the *magnitude* of the subject of a proposition. What do we mean by 'magnitude'? The term magnitude applies to any physical quantity which is measurable. In this particular case the measurable physical quantity is *things* to which the proposition in question refers. If we translate this term to traditional mould, then it just signifies the concept of the distribution of terms. The difference, however, is that in traditional analysis this concept turns out to be quite clumsy and ambiguous. Various synonymous words make matters further worse. The use of quantifiers in non-natural language resolves all these difficulties at one stroke. It is also possible to do away with mathematical interpretation of distribution of terms if we wish so. Further, this technique renders the application of distribution to the predicate term superfluous. The significance of the rules of quantification must be understood against this background.

There are four rules of quantification. They are as follows; Universal Instantiation (UI), Universal generalization (UG), Existential Instantiation (EI.) and Existential generalization (EG). The first two rules involve the quantifier which is called Universal quantifier which has definite application. Whenever an affirmative proposition contains words like ‘all’, ‘every’ ‘each’, etc. and propositions contain words like ‘No’, ‘None’, etc., universal quantifier replaces all such words. This sort of economy also achieves simplicity. This is the distinct advantage of the use of quantifiers. On the other hand, whenever propositions irrespective of quality contain words like ‘someone’, ‘many things’, ‘a few, etc., existential quantifier is used. In symbolic logic these quantifiers are symbolized as follows. ‘(x)’ or ‘(∀x)’ is the symbol for universal quantifier and ‘(∃x)’ is the symbol for existential quantifier. The symbolic representation of these quantifiers removes ambiguity in addition to achieving economy and simplicity. The difference between the instantiation and generalization rules with respect to both the quantifiers is that for universal quantifier UI allows the elimination of the universal quantifier whereas UG allows us to introduce a universal quantifier and similarly, for existential quantifier EI allows the elimination of an existential quantifier and EG allows us to introduce the same.

Every quantifier has a certain range. The range of ‘(x)’ or ‘(∀x)’ is indefinite whereas the range of ‘(∃x)’ is definite in the sense that in the latter case we are definite that there is at least one member whereas in the former case we are not.

2.2 RULES OF QUANTIFICATION

In predicate calculus the letter ‘x’ signify individual variable. The aforementioned four rules permit the transformation of non-compound into equivalent compound propositions to which the Rules of Inference and Equivalence are applicable. They also permit the transformation of compound propositions into equivalent non-compound propositions. These additional rules thus make it possible to construct formal proofs of validity for arguments whose validity depends upon the inner structure of some non-compound statements contained in those arguments. These rules stand in need of brief explanation.

Universal Instantiation (UI)

This rule says that any substitution instance of a proposition function can be validly deduced from a universal proposition. A universal proposition is true only when it has only true substitution instances. This is the necessary and sufficient condition for any true universal proposition. Therefore any true substitution instance can be validly deduced from the respective universal proposition. A propositional function always consists of variable ‘x’. At times z also is used as a variable and y has a definite role to play other than that of constant. Therefore any instance which is a substitution for x is regarded as a constant and letters from ‘a’ through ‘w’ are symbols for constants. These letters signify subject in traditional sense, and in modern sense, an ‘instance of a form’. To obtain such an instance of a form ‘x’ is replaced by another Greek letter ‘ν’ (*nu*) which is another symbol for an individual constant. It is also an example for universal instantiation because the universal quantifier is instantiated here.

This rule is symbolically represented as follows:

$$\underline{(x)\ \Phi x}$$
$$\therefore \Phi\ \nu$$

This rule requires a little elaboration. Let us replace ν by a more familiar constant, say, a, and Φ by F. When Fa is inferred from (x) Fx, the quantifier (x) is dropped. The reason is that the universal quantifier has indefinite extension whereas constant is restricted to one particular individual. Therefore in this context it is wrong to use universal quantifier. The rule of UI allows such of those instances where we replace all variables bound by a universal quantifier with individual constant. Thus (x) (Sx => Px) will yield (Sa => Pa) where a is the constant used in place of the variable. The application of UI goes with a few stipulations. The quantifier (x) in (x) Fx should not be within the range of a negation ($\neg$). It should not also be within the extent of another quantifier. The span of (x) in (x) Fx must extend to the complete expression. A violation of any one of these limitations will lead to an incorrect utilization of UI. In other words, if we say that $\neg$ (x) Fx implies $\neg$ Fa, then it must be viewed as a wrong understanding of UI.

We may use the UI rule in the following way:

a) First, remove the universal quantifier.

b) Next, replace the resulting free variable by a constant.

Universal Generalization (UG)
This rule helps us to proceed to generalization after an arbitrary selection is made to substitute for x. In UG, 'arbitrary selection' is very important because as the name itself suggests, generalization always proceeds from individual instances. Arbitrary selection always means 'any'. And there is no specific choice involved. In this sense, selection is 'random' or arbitrary. The letter y is the symbol of 'arbitrary' selection. This is the reason why 'y' is not regarded as a constant. This process is called generalization because the conclusion is a universal proposition. The underlying principle is that what holds good in the case of any variable selected at random must hold good in all instances. In other words, the variable y is equivalent to saying 'any'. If we recall the traditional rules of syllogism, universal conclusion follows from universal premises only. It only means that we need prior universal proposition. Let us club UI with this step. Then we are allowed to say that if universal proposition is true then any variable selected at random must be true. Therefore it must be understood that in this case the process is from universal to universal through an individual. When 'x' replaces 'y' there is generalization. When universal quantifier describes the proposition it becomes UG. The procedure is as follows.

$$\frac{\Phi y}{\therefore (x)\, \Phi x}$$

In the above given rule the letter 'y' in Φy (or Fy) stands for any arbitrarily selected individual. It is only a pseudo name and not the name of a particular individual. This letter 'y' in UG is not a constant but an individual variable only. But it is different from x in the sense that it is an indefinite replacement for x. In UG we substitute first all pseudo names with variables and then bind them with universal quantifiers.

We apply UG to the statement in the following manner:

a) First add the universal quantifier.

b) Then ensure that in the conclusion the variable is bound by this newly introduced universal quantifier.

Existential Instantiation (EI)

This rule is applicable when the proposition has existential quantifier and in this case any symbol ranging from a through w is used as a substitute for the individual variable x. We can infer the truth of any substitution instance from existential quantification because existential quantification is true only when there is at least one true substitution instance. However, this rule has a clause when it is applied to an argument. The constant, say 'a' which we use to substitute for x should not have occurred anywhere earlier in that argument. It only means that in the same argument EI cannot be used twice when it is assumed that there is only one true substitution instance. The rule is represented as follows.

$$\frac{(\exists x)\ \Phi x}{\therefore \Phi\ \nu}$$

This formula says that there is at least one or some unspecified number of members in the domain in question have a certain property, say, 'Φ'. The letter 'ν' in Φν stands for that unspecified number of members and hence it is in a sense sort of pseudo name and not the name of any particular individual. Therefore this may be called an existential pseudo name. It is necessary to adhere to certain stipulations while implementing this rule. In the first place, the formula (∃x) in (∃x) Φx should not be within the range of negation (¬). Secondly, (∃x) in (∃x) Φx should not be within the extent of another quantifier. Thirdly, the quantifier (∃x) must cover the complete expression. We may use the rule EI in a statement in the following way:

a) First, remove the existential quantifier.

b) Next, replace the resulting free variable with a constant.

Existential Generalization (EG)

This rule states that from any true substitution instance of a propositional function, the existential quantification of that function can be validly deduced. Only then the existential quantification can become true. When the existential quantification is so deduced, the individual constant which appeared in earlier steps is replaced by x in the conclusion. The unique feature of this rule is that though there is generalization, the conclusion continues to be existential. The rule is represented as follows.

$$\frac{\Phi\ \nu}{\therefore (\exists x)\ \Phi\ x}$$

In the rule EG the letter 'ν' may be the name of a particular individual or again a pseudo name. (∃x) Φx states that there is at least one x such that x is Φ. When we apply EG, we have to follow some conditions. Each happening of 'ν' in Φ x must be substituted by 'x' and the scope of (∃x) must extend to the entire expression.

The method of application of EG is simple:

a) Insert an existential quantifier.

b) Ensure that at least one occurrence of the individual variable which we have generalized is bound by the newly introduced existential quantifier.

Earlier, it was stated that the EI should not have occurred earlier in any argument. But no explanation was given for this stipulation. We should know why there is this particular restriction on the use of EI. Suppose that 'a' is the constant whose existence is definite. We are sure of the existence of a, but we are not sure whether there is any other constant. In an argument in an earlier step a constant, say, 'a' is regarded as 'b'. The fact that 'a' is 'b' is not adequate enough to conclude in some other step that 'a is c' when there is no reference of any kind to 'c' in the premise. Since the logical constant 'a' is used in existential mode, it is mandatory that EI should be used in the very first step of the proof. If it occupies any other position, then it is a mistake.

Check Your Progress I

Note: Use the space provided for your answers.

1. Explain UI and EG.

----

2. Explain the significance of random variable.

2.3 RULES OF QUANTIFIER EQUIVALENCE

In the previous unit we learnt the technique of framing the equivalence for quantifiers. In this unit we shall consider examples for equivalence forms. The standard forms of categorical statements are A, E, I and O. Let us start with A statement: "Every cat is a mammal." In this statement there are two individuals, viz., 'a cat' and 'a mammal. We also know that these propositions are symbolized as follows.

Cx: *x* is a cat.
Mx: *x* is a mammal.

We should also note that the predicate of 'being a mammal' is predicated to every member which has the property of being a cat. We need universal quantifier in this case. On Boolean interpretation, universal statements are actually conditional statements with no existential commitment. We may paraphrase it as: For every *x*, if *x* is a cat then x is a mammal.

The expression 'if x is a cat, then x is a mammal' is translated with a truth-functional symbol '=>'. Thus translated, it becomes (x) (Cx => Mx). Using the same symbolization key, we symbolize the 'E' statement 'No cat is a mammal' as, (x) (Cx => ¬ Mx). It must be remembered that "No cat is a mammal" can be paraphrased as: For every x, if x is a cat, then x is not a mammal. This proposition does not deny the predication of the property of 'being a cat', but it denies the property of 'being a mammal' to any cat. This is the reason why the negation sign is placed before 'Mx' in the statement '(x) (Cx => ¬ Mx)'.

Let us consider the 'I' and 'O' statements. 'Some cats are mammals' is an illustration for 'I'. When paraphrased, we have to admit the existential import and also the property predication. It has to be stronger in assertion than the conditional we state for the universal statements. Considering these aspects, we translate I and O propositions to symbolic form.

There is (exists) at least one x such that x is a cat and x is a mammal. This is symbolized as follows:

$$(\exists x)\ (Cx\ \Lambda\ Mx)$$

Note that a truth-functional symbol 'Λ' from propositional logic has been used to denote that both properties belong to 'x'.

We translate the 'O' statement, 'Some cats are not mammals' in the following way.

There is (exists) at least one x such that x is a cat and x is not a mammal; and this expression is symbolized as follows:

$$(\exists x)\ (Cx\ \Lambda\ \neg\ Mx)$$

As a matter of convention these propositions are represented symbolically as follows:

1. (A) $(x)\ \Phi x$
2. (E) $(x) \neg \Phi x$
3 (I) $(\exists x)\ \Phi x$
4 (O) $(\exists x) \neg \Phi x$

Using class membership relation, these propositions are represented as follows:

1. $(x)\ \Phi x \equiv (x)\{x \in \Phi => x \in \Psi\}$ Where ϵ is read 'element of'
2. $(x) \neg \Phi x \equiv (x)\{x \in \Phi => x \notin \Psi\}$ Where ∉ is read 'not an element of'
3. $(\exists x)\ \Phi x \equiv (\exists x)\{x \in \Phi\ \Lambda\ x \in \Psi\}$
4. $(\exists x) \neg \Phi x \equiv (\exists x)\{x \in \Phi\ \Lambda\ x \notin \Psi\}$

Where Φ and Ψ are the symbols for attributes. These are the four rules of quantifier equivalence. They are also known as Quantifier Negation Rules because if negation is placed behind quantifier, then it becomes the contradiction of the original statement.

2.4 APPLICATION OF THE QUANTIFICATION RULES

Before we proceed further let us recapitulate what we learnt in the previous unit. In order to relearn we shall apply these rules to statements. This will be a good introduction to the next stage of our learning.

I. We shall apply 'UI' for the statements mentioned below and remove the quantifier.

a) $(x)\ (Hx => Mx)$

$\overline{\underline{(Ha => Ma)}}$

b) $(x)\ (Hx => \neg Mx)$

$\overline{\underline{(Ha => \neg Ma)}}$

c) $(x)\ (Mx => \neg Ix)$

$\overline{\underline{(Ma => \neg Ia)}}$

II. We shall now apply 'UG' for the statements mentioned below to add quantifier and then generalize. Very soon we ought to discover that this is really the reverse process.

d) $(Ha => Ma)$

$\overline{\underline{(x)\ (Hx => Mx)}}$

e) $(Ha => \neg Ma)$

$\overline{\underline{(x)\ (Hx => \neg Mx)}}$

f) $(Ma => \neg Ia)$

$\overline{\underline{(x)\ (Mx => \neg Ix)}}$

III. We shall apply 'EI' for the statements mentioned below to remove the quantifier.

g) $(\exists x)\ (Hx\ \Lambda\ Mx)$

$\overline{\underline{(Ha\ \Lambda\ Ma)}}$

h) (∃x) (Hx Λ ¬ Mx)

____

(Ha Λ ¬ Ma)

i) (∃x) (Mx Λ ¬ Ix)

____

(Ma Λ ¬ Ia)

IV. We shall apply EG for the statements mentioned below to add quantifier and to generalize:

j) (Ha Λ Ma)

____

(∃x) (Hx Λ Mx)

k) (Ha Λ ¬ Ma)

____

(∃x) (Hx Λ ¬ Mx)

l) (Ma Λ ¬ Ia)

____

(∃x) (Mx Λ ¬ Ix)

[Note: Whenever universal quantifier has to be symbolized instead of (x) we can also use (∀x). However, the former is more extensively used.]

Check Your Progress II

Note: Use the space provided for your answers.

1. Explain the significance of rules quantifier equivalence.

----

2. Briefly explain the uses of instantiation and generalization.

2.5 EXAMPLES

1. Symbolize the following using universal quantifier.

a) Every Human is a mammal
$(x)\ (Hx \Rightarrow Mx)$

b) No Horse is a mammal.
$(x)\ (Hx \Rightarrow Mx)$

c) All dogs are four legged.
$(x)\ (Dx \Rightarrow Fx)$

d) No donkeys are birds.
$(x)\ (Dx \Rightarrow \neg Bx)$

e) No men are immortal.
$(x)\ (Mx \Rightarrow \neg Ix)$

2. Symbolize the following using existential quantifier.

a) Some flowers are red.
$(\exists x)\ (Fx \wedge Rx)$

b) Some flowers are not red.
$(\exists x)\ (Fx \wedge \neg Rx)$

c) Some birds are White.
$(\exists x)\ (Bx \wedge Wx)$

d) Some fish are not snake.
$(\exists x)\ (Fx \wedge \neg Sx)$

e) Some men are tall.
$(\exists x)\ (Mx \wedge Tx)$

2.6 EXERCISES

Symbolize the following using quantifiers.

1. Some animals are dogs.
2. All cats are mammals.
3. No donkeys are blues.

4. All crows are black.

5. All parrots are not black.

6. Some philosophers are not Indians.

7. All Indians are not Tamilians.

8. All Tamilians are Indians.

9. All horses are not four legged.

10. Some good books are not expensive.

(Note: when a statement contains the words All…. not the corresponding proposition is 'O'.)

2.7 QUANTIFICATION RULES AND ARGUMENTS

It is quite interesting and also rewarding to learn how quantification rules can be applied to various arguments studied by traditional logic. This will also help us to discover the limits of traditional logic.

As mentioned earlier, out of five relations under square of opposition only contradiction survives and this has already been explained. Therefore let us concentrate on conversion and obversion among equivalence relation and categorical syllogism among mediate inference. While doing so, let us remind ourselves of the restriction imposed by modern logic which stipulates that from universal quantifier alone existential quantifier cannot be deduced and *vice versa*. Therefore A – I and I – A are excluded. However, a special form of A is considered where both S and P are equivalent sets and is examined with the help of quantification rules.
Consider the following proposition.

1. All spinsters are unmarried female persons.
   ∴ All unmarried female persons are spinsters.

Since the rule of distribution is adhered to, the argument is valid. Let us see how the rule of quantification can be applied to this example.

There are two ways of symbolizing. They are as follows

a) $(x)\{x \in S \Rightarrow x \in U\}$

b) $(x)\{ x \in S \Leftrightarrow x \in U\}$

a) does not completely convey the meaning of 1. Therefore we have to consider b). It can be reformulated as follows:

c) $(x)\{ x \in S \Rightarrow x \in U\} \wedge \{ x \in U \Rightarrow x \in S \}$

For the sake of simplicity let us drop the quantifier. Applying commutative law, we get

d) $\{ x \in U \Rightarrow x \in S \} \wedge \{ x \in S \Rightarrow x \in U\}$

This is an instance of simple conversion. Now apply simplification law.

e) $\{ x \in U \Rightarrow x \in S \}$

Translate 'e' to natural language. We get

All unmarried female persons are spinsters.

It may be noted that if this method is followed, we do not get the existential conclusion. Therefore conversion by limitation does not find place in this interpretation. (It is possible to get the identical result if commutative law is not used. But then it will not be clear to an untrained mind that the proposition is converted. It must be noted that commutative law is nothing but conversion). Examination of E proposition is left for the student as an exercise.

The case of existential proposition is simple. Examine this statement.

2. Some bananas are sweet.

Translate this statement to symbolic form.

a) $(\exists x) \{(x \in B) \wedge (x \in S)\}$

Again, drop the quantifier for the sake of simplicity and apply commutative law. We get

b) $\{(x \in S) \wedge (x \in B)\}$

When b) is translated to natural language, we get conversion in traditional sense.

Some sweet objects are bananas.

The case of O is a special case. It is quite illuminating to apply the quantification rules to know why it does not have conversion.

3. Some bananas are not sweet.

Translate this statement to symbolic form.

a) $(\exists x) \{Bx \wedge \neg Sx\}$

(For the sake of simplicity class-membership is not considered in this particular case).

In this case simple conversion is possible in a different way altogether. We can only say

$(\exists x) \{\neg Sx \wedge Bx\}$

If we reflect for a while we easily discover that this is, in reality, the symbolic form of partial contraposition. Suppose that we restrict ourselves to conversion. We are only entitled to convert (or commute) Bx and Sx. The negation sign ought to remain unaffected. For further clarity, let us compare the scene with algebra. $a + b = - c$ becomes $c = - (a + b)$. Just as negative sign is not disturbed in algebra while interchanging, so also in modern logic the negation sign remains undisturbed when we interchange the terms. But this is not conversion. In the case of algebraic equation, signs on both sides change. Therefore it is not equivalent to commutation. Commutation and conversion are technically identical. The upshot of the argument is that when there is negation on any one side or term, conversion or commutation is not possible. In other words, commutation holds good only when the relation is symmetric.

On similar lines, obversion can be explained. If x is not an element of S, then it means that x is an element of the complement of S.

The position of equivalence relation has now become clear. We have learnt that along with quantification rules we also require Rules of Inference and Rules of Replacement. We use the same technique to test the validity of syllogism. It is a matter of great interest to know that the rules of quantification project syllogism in a new perspective, which helps us to abandon the rule of distribution of terms, which is not only cumbersome in presentation but also time consuming. Further, quantification rules can be used to test non-syllogistic arguments also subject to the condition that general and singular propositions find place in such arguments. Let us use the following arguments to illustrate these rules.

1.

1) All Indians are Asians.
2) Tendulkar is an Indian.
3) ∴Tendulkar is an Asian.

This is symbolized as follows: (x) {Ix => Ax}
It
∴At

The formal proof is constructed as follows:

1)	(x){Ix => Ax}		
2)	It / ∴At		
3)	It => At	1,	UI
4)	∴At	3, 2,	M.P.

In this particular argument only one premise is general. However, the argument may consist of only general proposition in which case slightly different procedure has to be followed. Consider this argument.

2.

1) All politicians are voters.
2) All ministers are politicians.
3) ∴All ministers are voters.

When symbolized it becomes:

1) (x){Px => Vx}
2) (x){Mx => Px} / ∴ (x){Mx => Vx}

The formal proof is as follows:

1)	(x){Px => Vx}		
2)	(x){Mx => (Px} / ∴ (x){Mx => Vx}		
3)	Pa => Va	1,	UI
4)	Ma => Pa	2,	UI
5)	Ma => Va	4, 3,	H.S.
6)	∴ (x){Mx => Vx}	5,	UG

When the individual variable x is instantiated by any constant, then quantifier goes and we do not quantify individual or individuals. Now coming to the 6th step, it may be mentioned that if one substitution instance is true for a given structure then all substitution instances must be true for that structure. Further the universal quantification of a propositional function is true if and only if all substitution instances are true. (The 6th line is not a part of the proof).

In the third and the fourth steps we have applied universal instantiation because both premises are universal and therefore we have substituted the constants for variables.

UG can be applied in the following manner. Add the sixth line to the proof system after we replace x by y at all stages. Then we have the application of UG

1)	(x){Px => Vx}		
2)	(x){Mx => Px} / ∴ (x){Mx => Vx}		
3)	Py => Vy	1,	UI
4)	My => Py	2,	UI
5)	My => Vy	3, 4,	H.S.
6)	∴ (x){Mx => Vx}	5	UG

These two examples suggest that while testing the validity of arguments in general, UI has to be used necessarily though EI may not be necessary. The situation is similar to the traditional formation of rules of syllogism, which hint that without particular propositions it is possible to construct an argument, but not without universal propositions.

Now consider an argument, which has a particular proposition. Since one proposition is particular, it is imperative that the conclusion must be particular.

3.
1) All politicians are voters.
2) Some ministers are politicians.
∴Some ministers are voters.

By now the method of symbolization should be familiar.

1)	(x){Px => Vx}		
2)	(∃x{Mx Λ Px} / ∴ (∃x{Mx Λ Vx}		
3)	Ma Λ Pa	2,	E.I.
4)	Pa => Va	1,	UI
5)	Pa Λ Ma	3,	Com.
6)	Pa	5,	Simp.
7)	Ma	5,	Simp.
8)	Va	4, 6,	M.P.
9)	Ma Λ Va	7, 8,	Conj.
10)	∴ (∃x)(Mx Λ Vx)	9,	UG

Let us examine why the restriction of EI must be honoured. Consider a fallacious argument.
1) Some animals are herbivorous.
2) Some animals are men.
∴Some men are herbivorous.

When symbolized the argument becomes:

1) $(\exists x)\{Ax \wedge Hx\}$
2) $(\exists x)\{Ax \wedge Mx\}$ / $\therefore$ $(\exists x)(Mx \wedge Hx)$
3) $Aa \wedge Ha$ 1, E.I.
4) $Aa \wedge Ma$ 2, E.I. (Error)

4th Step is erroneous. The second premise tells us that there is at least one thing that is both an animal and herbivorous. It does not permit us to conclude it should also be regarded as man. Therefore a second use of EI leads to error.

Check Your Progress III

Note: Use the space provided for your answers.

Employ proper method to remove the quantifiers in the following statements.

1. (x) (Hx => Mx)

----

2. (x) (Ax => ¬ Lx)

2.8 LET US SUM UP

In this unit we have presented a new set of rules called quantification rules which supplement the existing rules. UI, UG, EI and EG are these rules. We learnt the art of testing arguments which consist of singular and general propositions and also the art of translating general statements to truth- functionally compound statements. Rules of quantifier equivalence are also discussed. These rules will allow us to replace a quantified expression by its equivalent expression whenever there is need. Application of Rules of Instantiation and Generalization were discussed. Contradiction and conversion were presented in terms of the theory of quantification. Syllogism is another type of argument tested within the frame-work of this theory.

2.9 KEY WORDS

Quantified statement: A statement which does it refer to a particular person or a object. It refers to the quantity or magnitude of subject term.

Scope of a quantifier: The extent of the interpretive power of a quantifier.

Rules of Quantifier Equivalence: It is between *A, E, I, O,* which follow from the Square of opposition, but phrased as quantified expressions and their negations in predicate logic.

Bound variable: If the variable is either part of the quantifier or lies within the scope of a quantifier, then it is called bound variable.

Free variable: Any variable is free if and only if the variable is not bound.

2.10 FURTHER READINGS AND REFERENCES

Balasubramanian, P. *An Invitation to Symbolic Logic.* Madras: Sri Ramakrishna Mission Vivekananda College, 1977.

_________________ *Symbolic Logic and Its Decision Procedures.* Madras: University of Madras, 1980.

Chhanda, Chakraborthi. *Logic: Informal, Symbolic and Inductive.* Second Edition. New Delhi: Prentice-Hall of India Pvt., Ltd. 2007.

Copi, M. Irving. *Symbolic Logic.* Fourth edition. New York: Macmillian Publishing Co., 1965.

______________ *Introduction to Logic.* Third Edition. New York: Macmillian Publishing Co., 1968.

______________ and James A. Gould. *Readings on Logic.* Second Edition. New York: The Macmillian Co., 1972.

______________ and Carl Cohen. *Introduction to Logic.* Tenth Edition. New Delhi: Pearson Education, 2001.

_____________________________ *Introduction to Logic.* Eleventh Edition. New Delhi: Pearson Education, 2004.