UNIT 1 INTRODUCING THE QUANTIFIERS 'ALL' AND 'SOME' AND THEIR SYMBOLIC REPRESENTATION

Contents

1.0 OBJECTIVES

One of the principal objectives of this unit is to bring to the fore the limits of Aristotelian logic and the way in which modern logic succeeded in extending the boundaries of Aristotelian logic. Against this background, a brief reference to historical element in 'Introduction' becomes significant. It is not only significant but also necessary. Second objective of this unit is to show how in the absence of new set of rules the Rules discussed in the previous units do not help us to test a particular class of argument. In the third place, this unit aims at explaining the role played by quantifiers in the restructuring of traditional square of proposition. Thus by the end of this unit we should be able to have a basic understanding of predicate logic. Secondly, we must be in apposition to symbolize the proposition using quantifiers. Finally, we must be able to identify the internal structure of the proposition.

1.1 INTRODUCTION

Predicate logic, is a branch of logic, which is concerned with predicates or with predication of properties, and also with *things or objects to which the predicates may be ascribed*. This statement, in general, and the words in italics, in particular, must act as springboard for our further study and its significance becomes evident shortly. Quantification logic has its roots in 'Set Theory'. Set theory itself is of recent origin. This theory took its birth in the nineteenth century in the works Georg Cantor. However, other mathematicians, most notably, Boole, Venn and de Morgan meant it though they did not develop the theory. Nor did they use the term in the sense in which Cantor used. Such things are not uncommon in the development of science. For example, Michael Faraday did not use the word *field* in his work on physics though he meant it. This piece of fact from the History of Mathematics must be borne in mind in order to pay what is due to Aristotle. Prior to this, we must understand that the important concepts developed by

Cantor viz. subset, proper subset, the difference between the two, null set (empty set), denumerable and equivalence are some of the key concepts which play key role in our study.

Where did Aristotle err? This question needs to be addressed. As a matter of fact, Aristotle did not err. That is the reason why the word *defect* is not the right word to be used while assessing Aristotelian system. Instead, limitation is the apt word to be used in our analysis of Aristotelian system. Aristotle had an idea of class at elementary level. He evolved the concepts of class inclusion (total or partial) and class exclusion (total or partial), but could not proceed further. This explains the limits of his analysis of categorical proposition based on quality and quantity of proposition and the outcome of his analysis. Since Set theory in the sense in which Cantor developed was unknown during Aristotle's age, it, surely, would be anachronistic to criticize Aristotle for his limited perspective of predicate logic. Therefore let us first identify the loose ends in Aristotelian system. This will help us to understand the significance of 'Quantification Logic' in particular and modern logic in general.

Aristotle did not differentiate between universal proposition and singular proposition. A proposition is singular when the subject is a proper name. In this respect, singular proposition differs from particular proposition, though later we understand that both are existential propositions. In his analysis these two are, more or less, the same. An understanding of subtle difference and its consequences is quite illuminating. Any universal proposition of the form 'All S are P' or 'No S are P' reveals that S and P are merely class-names. If the concept of denotation is closely examined, then it becomes clear that all class-indicators include or exclude a certain number of elements known as members of a particular class, otherwise called sets. Therefore every set represented by a term in the proposition is very much similar to denumerable set which is a set of positive integers. A set is said to be denumerable when it is a set of positive integers because only then members are countable. If members are countable, then denotation makes sense, not otherwise. Similarly, the concept of intension reveals that to be a well-defined member the member must possess a definite set of properties without which it ceases to be a member of that particular set.

Against this background, we should try to know what the difference or differences between universal and singular on the one hand and particular and singular propositions on the other signify. First let us consider universal and singular propositions. The proposition 'All men are mortal' has both contrary and contradictory relations. However, the proposition 'Socrates is mortal' has only contradictory relation, but not contrary. It may be necessary to point out that, though it amounts to repetition, two propositions are contraries only when two conditions are satisfied; when p is true, q is false and when p is false q is doubtful. On the other hand, contradiction arises when p is true, q is false and when p is false q is true and *vice versa*. Suppose that the second proposition, 'Socrates is mortal' is negated. We get 'Socrates is not mortal'. When the first statement is true, the second statement is false. Though the first condition is satisfied, the second condition is not satisfied because when the first statement is false, the second statement is not doubtful, but turns out to be true. If logical relations matter, then the distinction between universal and singular proposition also ought to matter. This is a point which Aristotle failed to notice. Further, both particular and singular are existential propositions which make matters still worse. Like universal propositions, particular propositions also have two distinct relations which distinguish them from singular propositions. Instead of contrary, sub-contrary explains one type of relation between two particular propositions. If 'some men are mortal' is true, then 'some men are not mortal' is doubtful and if 'some men are mortal' is false, then 'some men are not mortal' is true. Of course, contradiction explains the relation between universal and particular. Here is the difference. Though both particular and singular propositions are existential, sub-contrary relation is not common to both. This means that universal and particular propositions, on the one hand, and particular and singular, on the other, deserve to be classified separately. They are called general propositions distinct from singular propositions because the subject of such propositions is a general term. A term which refers to an indefinite number of things is a general term which is called common noun in grammar. What we call quantifiers are applicable to general propositions, but not to singular propositions.

This is one difference. Second difference is crucial. In this context, the emphasis is on the word *existence.* If a certain proposition is characterized as existential, how do we understand such characterization? When we discussed Venn's diagram in connection with the distribution of terms, we learnt that universal propositions do not carry existential import whereas particular propositions carry existential import. The statement 'All men are mortal' does not affirm the existence of men whereas 'Some men are….' affirm the existence of men irrespective of the quality of proposition. Same is the case with 'No men are…' No assertion is made about the existence of men. Existence presupposes the presence of members in a given class. If existence makes sense, then in negative sense nonexistence also must make some sense. Suppose that a set does not contain a single member. Then what is its status? Till nineteenth century this question did not occur to anyone. In other words, the concept of null set paved the way for further progress in Aristotelian logic. How did it happen?

The concept of null set plays crucial role in distinguishing Aristotelian system from modern logic. Let us recall the very first statement of introduction; 'Predicate logic, is a branch of logic, which is concerned with predicates or with predication of properties, and also with *things or objects to which the predicates may be ascribed'*. In the strict sense of the term, predicate may be ascribed to only things or individuals actually existing. Otherwise, the commonplace difference between fact and fiction will be completely obliterated. Therefore in a restricted sense existential propositions make matters of fact relevant. When matters of fact become relevant, purely formal character of formal logic makes room for the relevance of content to a certain extent. But generalization, which is a characteristic of deductive inference, does not lose its significance. The only requirement is that the content of the argument must be factual, but not fictitious.

Where does null set figure in this discussion? One fundamental relation between propositions with which we are concerned, presently, is contradiction. The law of contradiction holds good when terms include members as a matter of fact. However, the situation is different when the terms represent null sets. Consider this proposition.

1. All female philosophers of Karnataka are the residents of New York.

This sentence is obviously false. Therefore according to the law of its contradiction, the statement mentioned below must be true.

2. Some female philosophers of Karnataka are not the residents of New York.

Statements 1 and 2 are supposed to be contradictories. The second statement ought to be true according to the law of contradiction since the first statement is false. But, in reality, this statement is also false. But two contradictories cannot be false. This problem arises because we are dealing with nonexistent members. Therefore in the strict sense of the word second statement also, like universal statement, does not carry existential import. Within the framework of traditional logic this problem remains unnoticed because there was no concept of set at all- whether null set or non-null set. Modern logic corrected this mistake by making null set a distinct entity. The underlying principle is that all existential propositions should include only non-null sets. This stipulation marks one difference between traditional and modern systems.

Equivalence is second major factor. Consider these propositions.

3. All triangles are plane figures.

4. All equilateral triangles are equiangular triangles.

Statements 3 and 4 assume the forms as follows.

3a. If any figure is a triangle, then it is a plane figure.

4a. A figure is equilateral triangle if and only if it is equiangular.

Let us symbolize these statements.

$3a \equiv F \Rightarrow P$

$3b \equiv F \Leftrightarrow P$

Again, traditional logic did not distinguish these propositions. The difference between 3a and 3b becomes clear only within the framework of modern logic. This is another important progress made by modern logic over traditional logic. Such differences matter in quantification logic.

This aspect has something to do with the difference between subset and proper subset. Proposition 3 discloses that the set of triangles is a proper subset of the set of plane figures. However, the set of equilateral triangles is equivalent to the set of equiangular triangles. This explains why the sentential connectives differ from 3a to 3b.

Traditional logic made one type of distinction among propositions; conditional and unconditional and within the latter four kinds of propositions. Modern logic not only discovered a new aspect in conditional proposition but also it added new kinds of propositions which were not considered by traditional logic. Hence it could evolve new set of rules. But these rules had limitations. In the absence of further augmentation of new rules they could not be applied to arguments which consisted of singular propositions. So the search for relevant rules did not stop.

Let us turn to basic difference between propositional logic and predicate logic. This difference lies in dealing with the internal structure of simple as well as compound propositions. Therefore predicate logic includes rules hitherto used and also a new set of rules. Predicate logic is concerned with the internal structure of propositions. It is not the case with propositional logic. In strict technical terms, this is also known as Quantification Theory or the Predicate Calculus. It has its own syntax, which helps us to devise statements, which are considered well-formed statements. We are now concerned with this new syntax.

1.2 SYMBOLIZATION OF PROPOSITIONS

How do we symbolize the statement 'Socrates is a philosopher? A unique method is devised which is merely a convention. The subject term is represented by first letter of the same which is always a small letter and predicate is represented by the first letter of the same which is always a capital letter. The proposition considered above now becomes

Ps

The singular terms are represented in predicate logic by the individual constants. These are small letters from 'a' to 'w', with or without numerical subscripts. Their function is to denote only one, unique individual or object from the domain of discourse. Since their reference remains fixed or constant within a given context, they are called individual constants.

Predicates are linguistic expressions of properties. In other words, predicates are words or expressions that we use to refer to properties or attributes that things have. For example, we may use the predicate term 'red' to refer to the property of 'being red' that a flower has. Predicate logic we are discussing is called the First Order Predicate Logic. Within the limits of this order only simpler predications such as properties of individuals or objects are considered. There can be complicated predications where we need to consider properties of properties and quantity of properties, and that would be the Second Order Predicate Logic. In higher order logic, we have variables standing for properties such as 'F', 'G', etc., and property constants.

There are three ways in which change can be effected; i) change S or ii) change P or iii) change both S and P. accordingly we have the following possibilities.

i) Pa, Pb, Pc,… etc.
ii) Gs, Ga, Gb… etc.
iii) Ab, Cd, Ef, … etc.

It is easy to notice that any kind of change is just indefinite in the sense that the list can be extended to include the whole of humanity though it is not intended. This process is just simplified by using the variable 'x' in place of constants. When we do so, we obtain Px. When variable is used in place of individual constants, we get what is called *propositional function.* Propositional function is neither true nor false. Truth-value can be assigned only when constants replace the variable. Consider the following replacements.

1. Pa where a stands for Aristotle
2. Pb where b stands for Berkeley
3. Ph where h stands for Hitler
4. Pc where c stands for Churchill

It is evident that 1 and 2 are true whereas 3 and 4 are false. That 1 and 2 are true is known only when we know what a and b stand for. Similar is the case with 3 and 4. Therefore in quantification logic we should ascertain the actual truth-status of propositions. If h stands for Himalayas, then the statement does not make any sense. Pa, Pb, etc. result from propositional function by an operation called instantiation. Accordingly, a, b, c, etc. are called substitution instances. Further, a and b are true substitution instances whereas h is not a true substitution instance.

We learnt the way to symbolize singular propositions. There is a different way of symbolizing general statements which are, doubtless, compound propositions. Quantifiers are used in this connection. A general proposition is of two types; universal and particular. So we have two quantifiers denoting these two. Since each of them may be affirmative or negative, we have four kinds of propositions, which are represented as follows:

1. All Indians are mortal.: $(x)\ Mx$
2. No Indians are mortal.: $(x) \neg Mx$
3. Some Indians are moral.: $(\exists x)\ Mx$
4. Some Indian are not mortal.: $(\exists x) \neg Mx$

'$\forall$' also can be used in place of (x).The symbols used on the R. H. S. need some explanation.

The symbol (x) is expanded in several ways. It can read 'for all values of x' or 'Given any x' or simply 'for every x', etc. where 'x' stands for individual constant, 'Indians' and 'M' stands for mortal. Therefore $\neg$ M x is read 'x is not mortal'. The symbol $\exists x$ is read 'there exists at least one x such that ….' ( ) is called universal quantifier and $\exists$ is called existential quantifier. If we substitute I (Indians) or P (Pakistanis) for x then we get a proposition, which may be true or false. It may be noted that universal quantifier is true only when every substitution instance of the same is true or it has only true substitutions whereas the existential quantifier is true when at least one substitution instance of the same is true.

Just as x is used as individual variable to denote the subject, two Greek letters 'Φ' (Phi) and 'Ψ' (Psi) are used to denote predicate. So they are called predicate variables. Using these variables A, E, I and O propositions can be represented as follows:

1. All Indians are mortal.: (A) $(x)\ \Phi x$
2. No Indians are mortal.: (E) $(x) \neg \Phi x$
3. Some Indians are mortal.: (I) $(\exists x)\ \Phi x$
4. Some Indians are not mortal.: (O) $(\exists x) \neg \Phi x$

Using class membership relation, general propositions are represented as follows:

1. $(x)\ \Phi x \equiv (x)\{x \in \Phi \Rightarrow x \in \Psi\}$ Where $\in$ is read 'element of'
2. $(x) \neg \Phi x \equiv (x)\{x \in \Phi \Rightarrow x \notin \Psi\}$ Where $\notin$ is read 'not an element of'
3. $(\exists x)\ \Phi x \equiv (\exists x)\{x \in \Phi \wedge x \in \Psi\}$
4. $(\exists x) \neg \Phi x \equiv (\exists x)\{x \in \Phi \wedge x \notin \Psi\}$

1.3 LOGICAL RELATIONS INVOLVING QUANTIFIERS

Our study begins with traditional square of opposition, which does not need any explanation. We know how A, E, I and O are denoted with the help of quantification theory. Let us replace A, E, I and O by these quantifiers in the square:

A	Contraries	E
$(x)\ \Phi x$		$(x) \neg \Phi x$
Subaltern	Contradictions	Subaltern
I		O
$(\exists x)\ \Phi x$	Sub-contraries	$(\exists x) \neg \Phi x$

Later we will learn that this square is altered when we make the assumption that there is only one individual. Therefore all relations are not discussed. Against this background, we shall restrict ourselves to only two important logical relations; viz., equivalence and contradiction. They are represented as follows:

1. Equivalence:
   1) $(x)\ \Phi x \quad \equiv \quad \{\neg (\exists x) \neg \Phi x\}$
   2) $(x) \neg \Phi x \quad \equiv \quad \{\neg (\exists x)\ \Phi x\}$
   3) $(\exists x)\ \Phi x \quad \equiv \quad \{\neg (x) \neg \Phi x\}$
   4) $(\exists x) \neg \Phi x \quad \equiv \quad \{\neg (x)\ \Phi x\}$

2. Contradiction:
   1) $(x)\ \Phi x \qquad (\exists x) \neg \Phi x$
   2) $(x) \neg \Phi x \qquad (\exists x)\ \Phi x$
   3) $\exists x\ \Phi x \qquad (x) \neg \Phi x$
   4) $\exists x \neg \Phi x \qquad (x)\ \Phi x$

When we use predicate variable, the propositional forms are expressed as follows:

1) $(x)\ \Phi x \quad \equiv \quad (x)\{ \Phi x => \Psi x\}$
2) $(x) \neg \Phi x \quad \equiv \quad (x)\{ \Phi x => \neg \Psi x\}$
3) $(\exists x)\ \Phi x \quad \equiv \quad (\exists x)\{ \Phi x \Lambda \Psi x\}$
4) $(\exists x) \neg \Phi x \quad \equiv \quad (\exists x \{\Phi x \Lambda \neg \Psi x\}$

When we represent A, E, I & O with this new set, their equivalent forms also undergo changes.

1) $(x)\{ \Phi x => \Psi x\} \quad \equiv \quad \neg \exists x\{\Phi x \Lambda \neg \Psi x\}$
2) $(x)\{ \Phi x => \neg \Psi x\} \quad \equiv \quad \neg \exists x\{\Phi x \Lambda \Psi x\}$
3) $(\exists x)\{ \Phi x \Lambda \Psi x\} \quad \equiv \quad \neg (x)\{\Phi x => \neg \Psi x\}$
4) $\exists x\{\Phi x \Lambda \neg \Psi x\} \quad \equiv \quad \neg (x)\{\Phi x => \Psi x\}$

If negations placed behind the quantifiers on the R. H. S. are removed, then automatically they become contradictories of respective propositions.

A predicate like mortal is called simple predicate because the propositional function which, if used, has true and false substitutions. All substitutions to variable are called ‘substitution instances’. When such predicates are negated, such formula or statement is called ‘normal form formula’.

What is the function of quantifiers? Quantifiers are expression in Predicate logic which state that a certain number of the individuals or objects have the property in question. They do not state which one of the individuals have the property. A quantifier consists of:

- A left parenthesis ‘(‘
- A quantifier symbol
- One of the individual variable symbols
- A right parenthesis ‘)’
- In predicate logic, we have two quantifier symbols: ‘(x)’ or ‘∀’ and ‘∃’

These quantifiers are in non-natural language the symbols of quantity indicators; ‘all’, ‘some’, and ‘no’, which may occur in statements about predications. Predicate Logic uses only two kinds of quantifier symbols: universal quantifier and existential quantifier.

Check Your Progress I

Note: Use the space provided for your answers.

1. Explain the quantification theory.

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2. How do we classify the quantifiers?

1.4 FALL-OUT OF UNIVERSAL AND EXISTENTIAL QUANTIFIERS

An important aspect needs to be clarified at this stage. Suppose that there is only one individual in the universe, which is something not logically impossible. There is a difference between saying that ‘there is only one individual in the universe’ and saying that ‘there is at least one individual’. In the latter case we commit ourselves to the possibility of a second or third thing which exists. Let us exclude this possibility. Then traditional square of opposition undergoes change automatically. Let us see how it happens. We shall use two predicates instead of one and consequently implication steps in. First let us draw the figure.

A (x) {Φ x => Ψx}	Contradictions	E (x){Φx => ¬ Ψx}
I (∃x) {Φ x Λ Ψx}		O (∃x){Φx Λ ¬ Ψx}

According to this figure of four relations mentioned earlier only contradiction holds good. We only know that there is x. Suppose that Φx does not have true substitution instance. Then both A and E are true because Φ x, i. e., antecedent is false and a false antecedent makes all implications in which it occurs always true. Therefore they are not contraries. Similarly, I and O are not sub-contraries. When Φ x does not have true substitution, both I and O are false whether Ψ is the predicate or not because one of the conjuncts in I and O is false. Subaltern (or superaltern) disappears in a different way.

When we assume that there is only one individual in the universe, it results in a unique implication. (x) {Φ x => Ψx}implies in a unique way the statement (∃x) {Φ x => Ψx}. This statement actually means that there is definitely one x and this x has the property Ψ or does not have the property Φ. Here care is required. Suppose that instead of making this disjunctive assertion we assert that x has Φ or it does not haveΨ. Then the implication becomes false. A true implication cannot imply a false consequence with true antecedent. Such an assertion is, evidently, different from the assertion (∃x) {Φ x Λ Ψx}.

Modern logic admits different types of general proposition with which we have already become familiar while learning the technique of testing arguments. A proposition like 'All A are B or C' is not the same as the proposition, 'All A are B or All A are C'. It only means that if x is A, then x is B or C whereas the second statement means that if x is A, then x is B or if x is A, then x is C. Again, we must construct truth-table to know how they differ. (This is left as an exercise for the student). On the other hand, the proposition, 'Some A are B or C' means the same as saying 'Some A are B or some A are C. This means that when the kind of quantification of a proposition changes, correct transformation of proposition also changes.

Check Your Progress II

Note: Use the space provided for your answers.

1. Explain the importance of universal quantifier.

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2. Expose the worth of existential quantifier.

1.5 EXAMPLES

1. Symbolize the following using universal quantifier.
   - a) All physical things are temporary.
     (x){Px =>Tx}
   - b) No system is permanent.
     (x) {Sx => ¬ Tx}
   - c) All dogs are mammal.
     (x) {(Dx => Mx])
   - d) No dogs are fish.
     (x) {(Dx => ¬ Mx)}
   - e) No men are immortal.
     (x) {(Mx => ¬ Ix)}

2. Symbolize the following using Existential quantifier.
   - a) Some jobs are temporary.
     (∃x) {Jx Λ Tx}
   - b) Some fish are not mammals.
     (∃x) {Fx Λ ¬ Mx}
   - c) Some dogs are Black.
     (∃x){Dx Λ Bx}
   - d) Some dogs are not fish.
     (∃x) {Dx Λ ¬ Fx}
   - e) Some men are not tall.
     (∃x) {Mx Λ ¬ Tx}

1.6 EXERCISES

Symbolize the following using quantifier.

1. All books are interesting.
2. No good books are useless.
3. Not every book is expensive.
4. No expensive things are good.
5. Ram is an ambitious person.
6. Ravi is a ruthless person.

7. Some horses are not black.

8. Rose is a colourful flower.

9. All horses are four legged.

10. Some good books are rare and expensive.

Check Your Progress III

Note: Use the space provided for your answer.

1. Symbolize the following using quantifier.

a) Some sheep are not white.

b) Every flower is beautiful and attractive.

c) All cattle are four legged.

d) Some good books are also not expensive.

e) All Humans are mortal.

1.7 LET US SUM UP

In this unit we have discussed the function of quantifiers. The universal quantification of a propositional function is true if and only if all of its substitution instances are true, and that the existential quantification of a propositional function is true if and only if it has at least one true substitution instance. If we grant that there is at least one individual, then every propositional function has at least one substitution instance (true or false). Under this assumption, we can say that if the universal quantification of a propositional function is true then its existential quantification must be true also. According to our discussion of quantifiers, contraries and sub-contraries do not stand; only contradiction remains acceptable.

1.8 KEY WORDS

Predicate logic: Logic of predicates or properties, and things or objects to which the predicates may be ascribed.

First order predicate logic: It is the elementary kind of predicate logic, in which only simpler predications such as properties of individuals or objects are considered.

Quantifiers: Symbols that state how many of the individuals have the property in question.

Universal quantifier: Universal quantifier is the symbol in non-natural language which represents universal proposition (both affirmative and negative)

Existential quantifier: It is the symbol in non-natural language which represents particular proposition (both affirmative and negative).

1.9 FURTHER READINGS AND REFERENCES

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