UNIT 4: PROVING INVALIDITY

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UNIT 4.0 OBJECTIVES

The central endeavor of this unit is to explicate the significance of proving invalidity. Proving invalidity is significant not in negative sense, but in positive sense. The singular objective of this unit is very clear. If we know what is wrong, we will know what is right in the *right* sense of the word and we will avoid consciously the pitfalls of illogical ways of arguing. Otherwise, we may walk into the trap of fallacies. Thus by the end of this unit one should be able to establish invalidity of seemingly valid arguments. Further, one should be able to identify the difference or differences between proving tautology and proving invalidity.

4.1 INTRODUCTION

Building up of proof system is an efficient mode to show that an argument is valid. Suppose that an argument happens to be invalid. Then it is not possible to construct proof of its invalidity using any rule applied so far. If it is not possible to show that given argument is not valid, then it should be possible to demonstrate its invalidity. Let us consider two cases to make the point clear. Suppose that we fail to demonstrate that God exists. Then half the battle is won (or half the battle is last). Next stage is to demonstrate that God does not exist. In contrast consider this case. Suppose that the prosecution fails to establish that the accused has committed the crime. The court does extend the benefit of doubt and acquits the accused. But this is not an accepted position in logic. The function of logic is two-fold; prove a certain proposition and disprove some other. Inability to prove (or disprove) is not tantamount to contradiction. At worst, it can only be a contrary. In modern logic contrary is not an accepted relation. Only contradiction is logically sound. When the logician accomplishes both the tasks, his victory is complete. Against this background, we must regard the relevance of proof of invalidity.

The foregoing discussion makes one point clear. If the method of proving invalidity is construed as a rule, then in addition to the rules we have become familiar with, we are in possession of one more tool to test arguments.

UNIT 4.2 PROVING INVALIDITY

Construction of truth-table is the foundation of propositional calculus. It is not possible to prove invalidity without its help. The method is quite simple. Irrespective of the number of propositions, the entire operation can be completed in one straight line. We devise a single, row represented by a straight line. All variables, their negations and all compound propositions are arranged horizontally above the straight line. The truth-value is entered exactly below the respective proposition. It is not necessary that the truth-value of every variable has to be entered. It depends upon the situation. The straight line separates the variables and the respective truth-values. Calculation of truth-value of propositions follows the elementary principles of propositional calculus. Since we have to prove that the argument is invalid, '0' is the truth-value to be assigned necessarily to the conclusion. This is the first step to be followed. If the conclusion is a compound proposition, then the sentential connective must be assigned the value '0'. Suppose that in the conclusion there are multiple sentential connectives. Then the sentential connective which has maximum range must be assigned the designate value. If implication links the antecedent and the consequent, then implication will have maximum range. Therefore in such case the implication must be assigned this value. In all other cases a little examination is adequate to identify the connective with maximum range. In the next step, all premises must be assigned the truth-value '1' only. Again, if any premise is a compound proposition then same method which is applicable to compound conclusion also applies to the premise or premises. A word of caution is required. It may not be possible to assign the required truth-value always in the very first attempt. We may have to take to trial-and-error method at times. This is more so when the premises are quite lengthy with multiple sentential connectives. It is also possible that more than one combination of truth-values for the variables of the premises may yield the desired result. Suppose that it is impossible to assign the truth-values in any the manner described above. Then the argument must be regarded as valid. Therefore this method can be used to prove validity also. However, we are concerned with proving invalidity at present.

At this stage one clarification is required. Unlike validity, invalidity is not governed by any rules. Of course, it is more than obvious that errors do not have any rules, which govern. On the other hand, only violation is possible. Hence the method of proving invalidity is different. The principle of inference dictates that a true premise and a false conclusion together result in invalidity. This is the reason why in order to determine invalidity we should assign truth-values in such a way that the premise or premises are true and the conclusion is false. If we succeed in doing so then the argument is invalid. This method is so simple that the test can be completed in one line (or two lines depending upon the number of variables and constants) as it happens in the case of truth-table.

So far we concentrated on the theoretical aspect. We shall apply now this method to an argument. The conclusion finds place at the end of the line always.

I

1

1. E => (F v G)
2. G => (H Λ I)
3. ¬ H /∴ E => I

4 7 5 8 6 3 1 2

E	F	G	H	I	{E => (F v G)} Λ {G => (H ΛI)} Λ ¬H	E=> I
1	1	0	0	0	1 1 1 1 0 1 0 1 0 0 0 1 1	1 0 0

While following this method '0' should be assigned to the conclusion making the premises true. If this combination cannot be achieved, then the argument is valid, i.e., after making the conclusion 0 if the conjunction of premises cannot take the value 1, then the argument is valid. There is no need to look for too many false premises. It is enough if one premise is false. The components of conclusion and the components of the premises should be paired properly to carry out the test.

We know the way of filling up the truth-values. Since the conclusion is a compound proposition the conclusion is false only when the sentential connective of the conclusion is false. Therefore this column is filled up first. Since the truth-values of the components determine the truth-value of any compound proposition, those respective columns are filled up next. Exactly on similar lines, the truth-values of premises are filled up. Accordingly, last but one column shows that the conclusion is false. 4, 5 and 6 show that first, second and third premises are true. 7 and 8 show that the conjunction of all the premises is true. We have begun our job with assigning the truth-value to the conclusion and the rest of the steps logically followed the first one. This being the case, false conclusion should not be derivable from the conjunction of true premises in the case of a valid argument. Since this has happened the argument is invalid. We shall consider some more examples. The order of filling up of truth-values is not given for the remaining arguments. The student is advised to find out the same. From now on conjunction linking the premises must be treated as implicit.

2

1. J => (K => L)
2. K => (¬ L => M)
3. (L v M) => N /∴ J => ¬N

Note that J => (K => L) ≡ (J Λ K) => L according to the Rule Material Equivalence. Therefore the truth-value is fixed for the second implication in similar cases.

J K L ¬L M N ¬N {J => (K => L)}{K => (¬L => M)}(L v M) => N} (J => ¬N)
1 0 1 0 1 1 0 1 0 1 1 0 0 1 1 1 1 1 1 1 1 0 0

Here the conclusion is '0' whereas the combination of premises is 1. Hence the argument is invalid.

3

1. (O v P) => Q
2. Q => (P v R)
3. O => (¬ S => P)
4. (S => O) => ¬ R /∴P ≡ Q

In accordance with the Rule of Material Equivalence, the conclusion can be restated as follows.

(P ≡ Q) ≡ {(P => Q) Λ (Q => P)}. In other words, equivalence relation satisfies the truth-condition of biconditional proposition. For the sake of convenience we shall use the statement on R. H. S. This expression is very long. A little care while assigning the truth-values to the propositions is required.

Suppose that the first component of the conclusion is taken as false. That is sufficient for the entire conclusion to become false since the conclusion is a conjunction. There is no need to fill up the truth-value of the last component. Let us assume, therefore, that P => Q is false. We shall work out the rest as per the procedure and find out the result. The truth-values for the individual variables are omitted for the remaining examples. The student is advised to fill up the same for the sake of practice.

{(O v P) => Q} {Q => P v R}{O=>¬S=>P}{S=>O=>¬R}(P => Q) Λ(Q => P)
1 1 1 0 0 1 0 0 0

Note that O => ¬ S => P ≡ (O Λ ¬ S) => P. Accordingly fix the truth-value.

The very first premise is false when the conclusion is false (when the premise turns out to be 0, there is no need to assign the truth-values to the remaining premises since the result remains unaltered). Therefore according to the assumption we made the conclusion must be valid. However, there is one more component which has to be examined before arriving at final judgment. Let us assume that Q => P is false and then proceed to show the invalidity.

{(O v P) => Q}{Q => (P v R)}{O => (¬S=>P)}S => (O=> ¬R)}(P => Q)Λ(Q => P)
0 0 0 1 1 1 1 0 1 1 0 0 1 0 1 0 1 0 0 1 0 0

4

1. X ≡ (Y => Z)
2. Y ≡ (¬ X Λ ¬ Z)
3. Z ≡ (X v ¬ Y)
4. Y /∴ X v Z

We should be in a position now to deal with the premises. Since components on L.H.S. and R.H.S. of all the premises become consequent all of them must have same truth-value. Keeping this point in mind let us assign truth-values.

{X≡ (Y => Z)}	{Y≡ (¬XΛ¬Z)}	{Z≡ (Xv¬Y)}	Y	(X v Z)
0 1 1 0 0	1 1 1 1 1	0 1 0 0 0	1	0 0 0

It is important to note that the truth-values entered below equivalence relations correspond to the truth-values of the premises.

The explanation remains the same in all cases.

5

1. T ≡ U
2. U ≡ (V Λ W)
3. V ≡ (T v X)
4. T v X /∴T Λ X

The conclusion is conjunction. Therefore, again, we have two options which are required to be tested. Suppose that T takes the value 0 and X takes 1. Then we will get the following result.

(T≡U)	(U ≡ V ΛW)	{V≡ (T v X)}	(T v X)	(TΛX)
0 1 0	0 1 0 0 0	0 0 0 1 1		0 0 1

The third premise is false when the conclusion is false. Hence according to this assumption the argument turns out to be valid. Let us make second assumption. Accordingly, X takes the value 0. Then we will get the following result.

(T≡U)	(U≡ VΛW)	{V≡ (T v X)}	(T v X)	(TΛX)
1 1 1	1 1 1 1 1	1 1 1 1 0	1 1 0	1 0 0

One aspect becomes clear. When the conclusion is a compound proposition, we have to try all possibilities till we get the desired result. Even after exhausting all possibilities if a premise cannot become false, only then can we conclude that the argument is valid. The student is advised to find out the result when T and X are 0.

6

1. (A => B) Λ (C => D)
2. A v C
3. (B v D) => (E Λ F)
4. E => (F => G)

5. G => (A => H) /∴H

<u>{A =>B)Λ(C =>D)}{(A v C){(B v D =>(EΛ F)}{(E=> (F =>G)}{G => (A => H)   H</u>
0 1 1 1 1 1 1   0 1 1   1 1 1 1 1 1 1   1   1 1 1   1   0 1 0   0

The argument is invalid.

7

1. I v (J Λ K)
2. (I v J) => (L ≡ ¬ M)
3. (L => ¬ M) => (M Λ ¬ N)
4. (N => O) Λ (O => M)
5. (J => K) => O   /∴O

<u>{I v (JΛK)}{(I v J)}=> (L≡¬M)}{(L=>¬M) => (MΛ¬N)}{(N => O) Λ</u>
11 0 0 0 1 1 0 1 0 1 0   0 1 0   1   1 1 1   0 1 0 1
<u>(O=>M)} {J => (K => O) O</u>
0 1 1   0   0 1 0   0

8

1. P ≡ (Q ≡ ¬ R)
2. Q => (¬R v ¬ S)
3. {R => (Q v ¬ T} Λ (P => Q)
4. {U => (S Λ T)} Λ (T => ¬ V)
5. {(Q Λ R) => ¬ U} Λ {U => (Q v R)}
6. (Q v V) Λ¬ V   /∴ ¬ U Λ ¬ V

<u>{P≡(Q≡¬R)}{Q => (¬Rv¬S)}{R => (Qv¬T)}Λ(P => Q)}[{U=>(SΛT) }</u>
1 1 1 1 1   1 1   1 1 0   0 1   1 1 0   1 1 1   1 1 1 1 1

<u>Λ(T=>¬V)] [{(Q ΛR) => ¬U} Λ{U = > (Q v R)}]{(Q v V) Λ ¬V(¬UΛ¬V)</u>
1 1 1 1   1 0 0 1 0   1 1 1 1 1 0   1 1 0 1 1 0 0 1

9

1. A=> B
2. C=> D
3. B v C /∴ A v D

{A=> B}	{C=> D}	{B v C}	{A v D}
0 1 1	0 1 0	1 1 0	0 0 0

II

It is a very good exercise to consider arguments in verbal form and then translate them to the usual symbolic form.

10

If pressure increases, then volume decreases.

Pressure has not increased.

Therefore volume does not decrease.

First we shall symbolize the variables.

a) pressure increases ---------- P

b) pressure has not increased ------ ¬ P

c) volume decreases ------- V

d) volume does not decrease ------- ¬ V

In the form of symbols the argument is represented as follows.

P = > V

¬ P

∴ ¬ V

As mentioned earlier we shall assign the truth-values as follows.

{P = > V}	¬P	¬V
0 1 1	1	0

Consider now a polysyllogistic argument.

2

1 All minister are politicians. ----------- M

2 All politicians are voters.---------- P

3 All voters are educated.-------- V

4 No educated persons are honest.-------- ¬E

Therefore no ministers are honest. ------- ¬ M

We must remember that all universal propositions are hypothetical in nature. Therefore the first proposition actually means that if there is anyone who is a minister, then he must be a politician. Accordingly, we shall symbolize the statements.

1 M = > P

2 P = > V

3 V = > ¬ E

∴ ¬
E

{M = > P}	{P = > V}	{V=>¬E}	¬E
0 1 0	0 1 0	0 1 0	0

Since all three implications are true, when the conclusion is false, the argument is invalid. It is advantageous to mark the conclusion at the end of R. H. S. and enter its truth-value first followed by the truth-values of premises. Only then we will be in a position to fix without confusion the truth-values of component propositions.

One distinct advantage of this method must now be more than obvious. There is no need to remember the distribution pattern of terms in categorical proposition. When this is abandoned the laws of syllogism or polysyllogism are rendered superfluous. It is sufficient if we are familiar with the truth-conditions of compound propositions.

Consider now a slightly complex argument.

3 If tax evaders are not punished, then either development takes back seat or the government is compelled to keep tax slab high. If the government is compelled to keep tax slab high, then common man becomes the victim of inept administration in democratic set-up. Common man does not become a victim of inept administration in a democratic set-up. Therefore either development does not take back seat or the tax evaders are punished in a democratic set-up.

This argument deserves to be split into individual components and then symbolized.

1 tax evaders are not punished --------- $\neg$ T

2 development takes back seat -------- D

3 the government is compelled to keep tax slab high ------- H

4 common man becomes the victim of inept administration in democratic set-up ---- C

5 common man does not become the victim of inept administration in democratic set-up ------- $\neg$ C

6 development does not take back seat --------- $\neg$ D

7 the tax-evaders are punished in a democratic set-up ------- T

We can now symbolize each proposition.

1st premise: If tax evaders are not punished, then either development takes back seat or the government is compelled to keep tax slab high. $\neg T => (D \vee G)$

2nd premise: If the government is compelled to keep tax slab high, then common man becomes the victim of inept administration in democratic set-up. $G => C$

3rd premise: Common man does not become a victim of inept administration in a democratic set-up. $\neg C$

Conclusion: Therefore either development does not take back seat or the tax evaders are punished in a democratic set-up. $\neg D \vee T$

Even a professional cannot discover easily fallacy in this argument. This argument can be easily shown to be invalid if the method of assigning the truth-value is followed.

1 $\neg T => (D \vee G)$

2 $G => C$

3 $\neg C$

$\therefore \neg D \vee T$

$\underline{\{\neg T \Rightarrow (D \vee G)\} \quad \{G \Rightarrow C\} \quad \neg C \quad \{\neg D \vee T\}}$

1 1 1 1 0 0 1 0 1 0 0 0

There is no other way of showing the invalidity of this argument. For confirmation let us use I. P. method.

1 ¬ T = > (D v G)

2 G = > C

3 ¬ C / ∴¬ D v T

4 ¬ (¬ D v T) I. P.

5 D Λ ¬ T 4, De. M.

6 ¬ T 5, Simp.

7 D v G 1, 6, M.P.

8 ¬ G 2, 3, M.T.

9 D 7, 8, D. S.

10 D Λ ¬ T 9, 6, Conj.

We only succeeded in returning to the fifth step. Since the argument is supposed to be invalid, instead of showing contradiction we should have succeeded in showing consistency with the help of I. P. method. We could show neither validity nor invalidity of this argument since we encountered sort of stalemate.

4 Let us slightly alter Berkeley's argument on immaterialism and find out its logical status.

'If things are material, then they are bundle of qualities. All qualities are ideas. All ideas are equivalent to mental entities. Therefore there is no matter'.

We shall split the argument into its components.

1st premise: If things are material, then they are bundle of qualities. --------- M = > Q

2nd premise: All qualities are ideas. -------- Q = > I

3rd premise: All ideas are equivalent to mental entities. ----- I = > E

Conclusion: Therefore there is no matter. ----- ¬ M

We shall put the argument in formal manner.

1 M = > Q

2 Q = > I

3 I = > E / ∴ ¬
M

To an unsuspecting mind no error is noticeable in this argument unless the argument is expressed in formal way. It is easy to conclude that the valid conclusion should have been M = > E. But if we consider what M and E stand for in Berkeley's system, then it becomes a different story altogether. That is not our concern now. It is enough if we mention that if H. S. is applied twice (to 1 and 2 and later to 2 and 3), we obtain M = > E. But this is not the conclusion which is required to be tested.

{M = > Q}	{Q = > I}	{I = > E}	¬M
1 1 1	1 1 1	1 1 1	0

Check Your Progress I

Note: Use the space provided for your answer.

1. Define and explain the proof of Invalidity.

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2. How do we confirm an argument as Invalid?

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4.3 ADVANTAGES OF THE METHOD OF PROVING INVALIDITY

Suppose that we opt for truth-table method. What will be the situation? Consider, for example, the following argument:

P => (Q v R)
S => (T v U)
¬ Q => (U v V)
(U => S) Λ (¬ T => ¬ S)
¬ V

∴P => (Sv U)

A truth-table for this argument will have 128 rows according to the formula 2^n where n= 7 and n is the number of variables and 10 columns of truth-values. Therefore if the truth-table method has to be followed, then the number of boxes to be filled up it is, incredibly, 1280. The distinct advantage of this method lies precisely here. Let us begin by assuming that P=> (Sv U) is false. Given that the only way for a conditional statement to be false is; its antecedent must be true and its consequent must be false which means that 'P' must be true and (S U) must be false, and since a disjunction is false only when both of its disjuncts are false, ‘S’ and ‘U’ must both be false on our crucial line of the truth-table. Notice how far we have come already:

P => (Q v R)
S => (Tv U)
¬ Q => (U v V)
(U => S) Λ (¬ T => ¬ S)
¬ V

P => (S v U)

P	Q	R	S	T	U	V
1			0			0

At the second stage, we consider true premises. The fifth premise is simple for our purpose. ‘¬ V’ is true if and only if ‘V’ is false. And, since ‘U’ and ‘V’ are both false, the consequent of the third premise is false; in order to make that premise true, its antecedent, ‘¬ Q’ must also be made false, which entails that ‘Q’ must be true. Thus, we have narrowed our search even further:

P => (Q v R)
S => (T v U)
¬ Q => (U v V)
(U => S) Λ (¬ T => ¬ S)
¬ V

P => (S v U)

P	Q	R	S	T	U	V
1	1		0		0	0

We have restricted ourselves only to the required propositions. That is the way to economize time and effort.

Thus, we have proved that the argument is invalid. When ‘P’, ‘Q’, ‘R’, and ‘T’ are true and ‘S’, ‘U’, and ‘V’ are false, the premises are true and the conclusion is false. We have discovered the easiest way of proving invalidity, no matter how complex is the argument.

4.4 ASSUMPTIONS OF PROVING INVALIDITY

Let is restate the assumptions to be followed.
i. Assume the conclusion as False and at the same time given premises as True.
ii. Using Basic Truth-Table Method to substitute the truth-values for all variables.
iii. Proceed from conclusion to premises.
iv. If we find no false premise, then the given argument is invalid.

Check Your Progress II

Note: Use the space provided for your answer.

1. Elucidate the advantages of proving invalidity.

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2. Write down the assumptions of proving invalidity.

4.5 SECOND METHOD OF PROVING INVALIDITY; EXAMPLES

We shall consider a second method of proving invalidity.

1

0 0 1 1 0
(¬A Λ B) => C
1 0 0 1 0
(A => C) => D
1 1 1
B => E

∴B => (D Λ E)
1 0 0 0 1

2

1 1 1 1 0
(A => B) v C
1 0 0 1 0
(A => C) => D
1 1 1
B => E

∴ B => (D Λ E)
1 0 0 0 1

Same type of method is adopted in this argument also and found that there is no contradiction, in the given premises. Hence the argument is invalid.

3

0 0 1 1 0
(¬ P Λ Q) => R
1 0 0 1 0
(P => R) => S
1 1 1
Q => T

∴Q => (S Λ T)
1 0 0 0 1

4

1 1 1 1 0
(P Λ ¬ Q) v R
1 0 0 1 0
(P => R) => S
1 1 1
¬ Q => ¬ T

∴¬ Q => (S Λ ¬ T)
1 0 0 0 1

4.6 EXERCISES

1. P => Q
   R => S
   Q v R   / ∴ P v S
2. E => (F v G)
   G => (H Λ I)
   ¬ H   /∴ E => I
3. J => (F => L)
   K => (¬ L => M)
   (L v M) => N   / ∴J => N
4. (A v ¬ A) => ¬ (¬ B Λ ¬ C)
   (B v C) => ¬ D   / ∴ ¬ D v A
5. [(¬ W v X) => (Y Λ W)]
   (X => Y)
   [¬ Z => (¬ W v X)] / ∴ (Z v ¬ W)
6. A => B

C => D

B v C /∴ A v D

7. ¬ E v (F v G)

¬ G v (H Λ I)

¬ H /∴ E v I

8. ¬ J v (¬ F v L)

¬ K v (L v M)

¬ (L v M) vN /∴ ¬ J v N

9. (X v ¬ X) => ¬ (¬ B Λ ¬ C)

(B v C) => ¬ Y /∴ ¬ Y v X

10. [(X v W) => (¬ W Λ Y)]

(¬ X v Y)

[¬ Z => (X v W)] /∴ (W v Z)

11. ¬ P => ¬ Q

¬ R => ¬ S

¬ Q v ¬ R /∴ ¬ P v ¬ S

12. (Fv G) v ¬ E

¬ (H Λ I) => ¬ G

¬ H /∴ ¬ I => ¬ E

13. (¬ F v L) v ¬ J

(L v M) v ¬ K

¬ (L v M) v N /∴ N v ¬ J

14. (P v ¬ P) => ¬ (¬ Q Λ ¬ R)

(Q v R) => ¬ S /∴ ¬ S v P

15. [(¬ P v Q) => (R Λ P)]

(Q => R)

[¬ Z => (¬ P v Q)] /∴ (Z v ¬ P)

4.7 LET US SUM UP

In this Unit we have tried to give an idea about proving invalidity, by giving definition, importance, and principles of this method. This new method of proving invalidity is shorter than writing out a complete truth-table, and the amount of time and work saved is proportionally greater for more complicated arguments. In proving the invalidity of more extended arguments, a certain amount of trial and error may be needed to discover a truth-value assignment, which works. But even so, this method is quicker and easier than writing out the entire truth-table. It is obvious that the present method will suffice to prove the invalidity of any argument, which can be shown to be invalid by a truth-table.

4.8. KEY WORDS

Argument: A structure composed only of statement variables and symbols such that all

its substitution instances will be arguments.
Invalid argument form: An argument form, which has at least one false substitution instance.

4.9 FURTHER READINGS AND REFERENCES

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