UNIT 3 RULES OF INFERENCE AND THE NATURE OF VALIDITY OF ARGUMENTS

Contents

3.0 OBJECTIVES

The purpose of this module is to help students to familiarize with the Rules of Inference which can be regarded as the pivot of the study of logic. The central theme of logic consists in the classification of arguments. Through this unit this main aim is achieved. In the previous unit we learnt the technique of determining the truth of compound sentences and the decisive role played by sentential connectives. Second objective of this unit is to demonstrate the latent relation between Rules of Inference and the truth of compound sentences. This method is devised to show that the Rules of Inference are demonstratively certain. If the Rules are demonstratively certain, then scrupulous adherence to these Rules produce arguments with unquestionable validity. The aim of this unit is to discover such arguments.

3.1 INTRODUCTION– TOOLS OF TESTING ARGUMENTS

In modern logic an argument is regarded as a sequence of statements. When proof is constructed to test the argument, the proof also takes the same form, which the argument takes. In this type of proof there is correspondence between the scheme of the given argument and the scheme of the proof. Every step, which is adduced while constructing proof, is the conclusion of the preceding statements, and in turn, becomes the premise for statements, which follow it (if not all, at least to some). Rules, which govern the process of deducing hidden conclusion, constitute what are known as ‘Rules of Inference’ in modern logic. Many of these Rules have their origin in traditional logic.

There is a certain way of constructing proof in modern logic. More descriptive method, which consumes both space and time, has given way to much shorter and simpler method. Whatever conclusion can be drawn from any one or two given premises is written on the left hand side (LHS) while the Rule and the premises to which this particular Rule is applied to derive the conclusion used in further proof are written on the right hand side (RHS). Quite often, Rule of Inference is applied to one line only. As an economy measure, instead of premises, corresponding serial numbers are written. Thereby we save time. We must ensure that drawn conclusion, the respective premises and the Rule applied are always juxtaposed. This procedure is the simplest and most economical in terms of time and effort to grasp the argument.

We have learnt in the previous unit the technique of conjoining simple sentences to generate compound sentences, and also we learnt the method of fixing the truth or falsity of such sentences. This knowledge is the pre-requisite for our further study.

We make use of twenty two Rules. Out of them nine are called Rules of Inference. There are ten Rules which are called Rules of Replacement (also can be called Transformation or Equivalence Rules). Reductio ad absurdum or indirect proof, Conditional Proof and the Strengthened Rule of Conditional Proof are the other Rules. The application of Rules is not at random. The unique composition of argument determines the kind of Rule to be applied. We will begin with Rules of Inference and also we shall examine the logical status of two Rules later.

Rules of Inference:

1 *Modus Ponens* (M.P.)

$p \Rightarrow q$

p

$\therefore q$

2 *Modus Tollens* (M.T.)

$p \Rightarrow q$

$\neg q$

$\therefore \neg p$

3 *Hypothetical Syllogism* (H.S.)

$p \Rightarrow q$

$q \Rightarrow r$

$\therefore p \Rightarrow r$

4 *Disjunctive Syllogism* (D.S.)

$p \vee q$

$\neg p$

$\therefore q$

5 *Constructive Dilemma (C.D.)*

$(p \Rightarrow q) \wedge (r \Rightarrow s)$

$p \vee r$

$\therefore q \vee s$

6 *Destructive Dilemma (D.D.)*

$(p \Rightarrow q) \wedge (r \Rightarrow s)$

$\neg q \vee \neg s$

$\therefore \neg p \vee \neg r$

7 *Simplification (Simp.)*

$p \land q$

$\therefore p$

8 *Conjunction (Conj.)*

p

q

$\therefore p \land q$

9 *Addition* (Add.)

p

$\therefore p \lor q$

Copi I. M. has replaced D. D. with another Rule called Absorption.

10. *Absorption* (Abs.)

$p \Rightarrow q$

$\therefore p \Rightarrow (p \land q)$

Since it is not possible to dispense with either of these Rules, Copi's decision to replace D.D. with Absorption is not clear. Hence it is obvious that nine becomes ten. A Rule becomes indispensable only because arguments, more often than not, consist of steps with diverse structure, which demand acceptance of different Rules. Of course, law of parsimony stipulates that not a single superfluous law is admissible. Therefore in accordance with this stipulation it must be maintained that if any new Rule is added, it is presumed that the addition is necessitated by the complexity of the argument.

We may have nine or ten Rules. The number is really immaterial. This is so because they are insufficient to test arguments with every conceivable structure. Therefore Rules of Replacement are added. Before listing these Rules we must become familiar with a subtle distinction between these two sets. Suppose that a premise (also understood as line) in an argument consists of several connectives. In such a case, Rule of Inference must be applied to only *whole line.* It will be a mistake to apply to a part of the line. Why is it a mistake? Consider this example.

$$(A \land B) \Rightarrow (C \lor D)$$

According to the Rule of Inference even if A or B or both are false, 1st line remains true only irrespective of the truth-value of the second component. Suppose that the Rule of Simplification is applied to the first component. Then suppose that A Λ B becomes A. If A is false, then we are including a false proposition which does not guarantee the truth of the conclusion because when a premise is false the conclusion can as well be false though the argument is valid. Rule of M. P. or M. T. to the 1st line can be applied provided one of the lines consists of either affirmation of the 1st component or denial of the 2nd component without omitting any proposition within parentheses. However, this restriction does not apply to the second set of Rules which will be considered later.

Before we proceed further let us examine the logical status of these Rules. What applies to one Rule also applies to any Rule of Inference or Replacement. So we can restrict ourselves to just two Rules. Consider M.P. The Rule can be put in the form of an expression in this way.

Argument form compound statement

p => q [(p => q) Λ p] => q

p

∴ q

Construct truth-table for R.H.S.

Table1:

1	2	3	4	5	6	7	8	9
p	¬ p	q	¬ q	(p => q)	Λ	p	=>	q
1	0	1	0	1	1	1	1	1
1	0	0	1	0	0	1	1	0
0	1	1	0	1	0	0	1	0
0	1	0	1	1	0	0	1	0

Consider the truth-value of implication on the extreme right (column 8 which is actually called main column in propositional calculus) to determine the truth-value of compound expression. Since it takes the value 1 in all cases, the compound proposition and its corresponding Rule of Inference is a tautology which means that it is true always.

Now examine the expression for C.D.: [(p => q) Λ (r => s) Λ (p v r)] => (q v s).

The truth-table is constructed as follows. Negation of proposition is not considered in this case in order to reduce the number of columns and they are not required in this case.

Table2:

1	2	3	4	5	6	7	8	9	10	11	12
Sl. No.	p	q	r	s	{(p => q)	Λ	(r => s)	Λ	(p v r)}	=>	(q v s)
1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	0	0	1	1	1	0	0	1	0
3	1	1	1	0	1	0	0	0	1	1	1
4	1	0	1	1	0	0	1	0	1	1	1
5	0	1	1	1	1	1	1	1	1	1	1
6	1	1	0	0	1	1	1	1	1	1	1
7	1	0	0	1	0	0	1	0	1	1	0
8	1	0	0	0	0	0	1	0	1	1	0
9	0	1	0	0	1	1	1	0	0	1	1
10	1	0	1	0	0	0	0	0	1	1	0
11	0	1	1	0	1	0	0	0	1	1	1
12	0	0	1	1	1	1	1	1	1	1	1
13	0	0	0	1	1	1	1	0	0	1	1
14	0	0	1	0	1	0	0	0	1	1	0
15	1	1	0	1	1	1	1	1	1	1	1
16	0	1	0	1	1	1	1	0	0	1	1

There are sixteen rows and in all these rows the truth-value in the 11^{th} column, i.e. main column, is 1 only and hence it is a tautology. Since we have made a random selection of Rules, this conclusion must hold good for all Rules of Inference and Rules of Transformation. The student is advised to test this property in other cases to satisfy natural curiosity

3.2 METHODS OF TESTING THE VALIDITY OF ARGUMENTS

Why Look for an Alternative to the Truth-table Method?

Any attempt to test an argument by the use of logical analogy or by the method of truth-table to see whether there is any substitution instance with true premises and a false conclusion is fraught with difficulties. A given argument is proved invalid if a refuting analogy can be found for it. But discovering such refuting analogies is not always that easy. Fortunately, it is not necessary. For, there is a simpler, and a purely mechanical test for arguments of this kind based on the same principle.

Similarly, testing the validity of arguments by using the truth-table method is simple and convenient only when there are two or three variables. But the situation is very different when there are several variables. Suppose that there are two variables. We require only four rows and eight columns (see table 1). If there are three variables, then we require eight rows and eight columns (provided we do not include the negation of variables). It means that if there are 'n' number of variables, then the length of the rows is given by the formula

$$2^n$$

So if there are five variables, we have to construct thirty two rows. Then proof construction becomes highly complex. One of the priorities during construction of proof is economy in time and effort. What is brief is simple. Economy, simplicity clarity are the parameters of accepted proof construction. This is what is called law of parsimony. Therefore an alternative is required.

3.3 APPLICATION OF ELEMENTARY RULES OF INFERENCE

Let us consider an example:

p => (q v r)
¬ r
¬ q

∴ ¬ p

The meaning of proof construction should become clear now. It is a sequence of statements each of which is either a premise of that argument or follows from preceding statements of the sequence by an elementary valid argument, and the last statement in the sequence is the conclusion of the argument whose validity is being proved.

1.

1	p => (q v r)	
2	¬ r	
3	¬ q / ∴ ¬ p	
4	¬ q Λ ¬ r	3, 2, Conj.
5	∴ ¬ p	1, 4, M.T.

Since this is the first argument, let us elaborate the process.

¬ r appears in 2nd line. Therefore the Rule of 'Conj. permits us to include ¬ r . We get ¬ q Λ ¬ r.

This forms the fourth line in the sequence. Now we shall consider 1st and 4th line together.

2.

1). p => (q v r)

2). ¬ q Λ ¬ r

∴ ¬ p

In (1), (q v r) is the consequent. q v r being a disjunction, when it is denied, it becomes a conjunction with original disjuncts being replaced by their respective negation. This is a law called de Morgan's law about which more will be said later. Since consequent is denied in the second premise, the antecedent has to be denied in the conclusion and it is done. Hence the Rule followed is M. T. It is clear that the form of (1) and (2) corresponds exactly to the form of Rule 2. The conclusion, which we obtained through formal proof, is the same as the conclusion of the given argument. This is how an argument is tested for validity. This is a model of explanation, which suits any argument.

Consider an argument in natural language.

1.

If Ravi is nominated, then she will go to Delhi.
If she goes to Delhi, then she will campaign there.
If she campaigns there, then she will meet Shibu.
Ravi will not meet Shibu.
Either Ravi will be nominated or someone more eligible will be selected.
Therefore someone more eligible will be selected.'

The validity of this argument may be intuitively obvious. But we must prove it. In order to do so, we must first translate the argument into our symbolism which takes this form.

1 A => B
2 B => C
3 C => D
4 ¬D
5 A v E / ∴ E
6 A => C 1,2, H.S.
7 A => D 6,3, H.S.
8 ¬A 7.4, M.T.
9 ∴E 5, 8, D.S.

From these examples it is clear that in this method the conclusion always follows '/' and '/' Immediately succeeds the last premise. This is an important aspect because all premises are numbered whereas the conclusion is not numbered. Therefore it is written adjacent to the last premise. Steps involved in the construction of proof are also the elements of the set of premises. This method requires very few steps and Rules. On the L. H. S., steps, which are involved in the construction of proof, are written. Every line after the last premise stands in need of justification and the justification is provided by one or the other Rule listed above. It must be noted that the same Rule can be applied any number of times. This method of proof is called formal method of proof which becomes clear very shortly.

From the first two premises A => B and B => C we validly infer A => C using the Rule of Hypothetical Syllogism. From A => C and the third premise, C => D we validly infer A => D, using again the Rule of Hypothetical Syllogism. From A => D and the fourth premise, ¬D we validly infer ¬A using the Rule of *Modus Tollens*. We apply the Rule of Disjunctive Syllogism to ¬A and the fifth premise A v E and validly infer E, the conclusion of the original argument. That the conclusion can be deduced from the five premises of the argument with the help of three elementary Rules proves that this method is the most useful method. From the description given above it becomes evident that the Rule of H. S. is applied twice.

[The rest of the arguments are worked out for which explanation is not provided. The student is expected to construct the same.]

2	1	(B v N) => (K Λ L)	
	2	¬K	
	3	¬M / ∴ ¬B Λ ¬M	
	4	¬K v ¬L	2, Add.
	5	¬B Λ ¬N	1, 4, M.T.
	6	¬B	5, Simp.
	7	∴ ¬B Λ ¬ M	6, 3, Conj.

3	1	(K => A) Λ (M => D)	
	2	¬A / ∴ ¬K v ¬M	
	3	¬A v ¬D	2, Add.
	4	∴ ¬K v ¬M	1, 3 D.D.

4	1	(M v N) => (P Λ Q)	
	2	N / ∴ P Λ Q	
	3	M v N	2, Add.
	4	∴ P Λ Q	1, 3, M.P.

5	1	(A Λ B) => (C v D)	
	2	A	
	3	B / ∴ C v D	
	4	A Λ B	2, 3, Conj.
	5	∴ C v D	1,4, M.P.

6	1	(T => K) Λ (R => S)	
	2	S => D	
	3	D => T	
	4	R / ∴ T	
	5	R => S	1 Simp.
	6	S	5,4, M. P.
	7	D	2, 6, M. P.
	8	∴ T	3, 7, M.P.

7	1	(A v B) Λ(¬D ΛE)	
	2	A v B =>K / ∴ KΛ(¬D ΛE)	
	3	A v B)	1, Simp.
	4	K	2, 3, M.P.
	5	¬D ΛE	1, Simp.
	6	∴ K Λ (¬D Λ E)	4, 5, Conj.

8	1	(P =>Q) Λ(R=>S)	
	2	¬A=> ¬Q	
	3	A => B	
	4	¬B /∴ ¬P V ¬ S	

9	1	A v (B Λ C)	
	2	A=>P	
	3	¬P / ∴ C	
	4	¬A	2,3, M.T.

	5	¬A	3,4, M.T.
	6	¬Q	2,5, M.P.
	7	P=>Q	1, Simp.
	8	¬P	7, 6, M.T.
	9	∴ ¬ P v ¬ S	8 Add.

	5	B Λ C	1,4, D.S.
	6	∴ C	5, Simp.

10	1	A Λ (B v C)	
	2	A=>P	
	3	Q / ∴ PΛQ	
	4	A	1, Simp.
	5	P	2,4, M.P.
	6	∴ P Λ Q	5,3, Conj.

11	1	¬B	
	2	¬D	
	3	(A=>B) Λ(C =>D)	
	4	K / ∴ C (¬K Λ¬A)	
	5	A=>B	3, Simp.
	6	¬A	5, 1, M.T.
	7	C=> D	3, Simp.
	8	¬ C	7, 2, M.T.
	9	(¬ K Λ ¬A)	4,6, Conj.
	10	∴¬CΛ (¬KΛ¬A)	8, 9, Conj.

12	1	(B ≡ K) => (Z Λ D)	
	2	¬ (Z Λ D) / ∴ ¬ (B ≡ K)	
	3	∴ ¬ (B ≡ K)	1,2, M.T.

13	1	(K Λ T) => (A v B)	
	2	(A v B) => (P Λ ¬ L)	
	3	(P Λ¬L)=>D	
	4	¬(D) / ∴ ¬(K ΛT)	
	5	(KΛT)=>(P Λ¬L)	1,2, H.S.
	6	(K Λ T) => D	5,3, H.S.
	7	∴ ¬ (K Λ T)	6,4 M.T.

14	1	(K Λ A) => (¬ B v C)	
	2	M=> (K Λ A)	
	3	M / ∴ ¬ B v C	
	4	M => (¬B v C)	2,1, H.S.
	5	∴ ¬ B v C	4,3, M.P.

15	1	A=>D	
	2	B=>C	
	3	A v B / ∴ D v C	
	4	(A=>D)Λ (B=>C)	1,2,Conj.
	5	∴ D v C	4,3, C.D.

16	1	A => D	
	2	B => C	
	3	¬ D v ¬ C/ ∴ ¬A v ¬ B	
	4	(A=>D)Λ(B=>C)	1,2,Conj.
	5	∴ ¬Av ¬B	4,3, D.D.

17	1	(A=>G) => (K v ¬ D)	
	2	¬ (K v ¬ D) / ∴ ¬(A => G)	
	3	∴ ¬ (A=> G)	1,2, M.T.

18	1	J V (K Λ L)	
	2	J=>D	
	3	¬D / ∴ K Λ L	
	4	¬J	2,3, M.T.
	5	∴ (K Λ L)	1,4 D.S.

19	1	D v (A=> B)	
	2	(A=> B) => (C v K)	
	3	¬(C v K) / ∴ D	
	4	¬(A=>B)	2,3, M.T.
	5	∴ D	1,4, D.S.

20	1	A Λ (B => C)	
	2	B / ∴ C	

21	1	(A => B) Λ (C=>D)	
	2	A/ ∴ B v D	

	3	B => C	1, Simp.
	4	∴ C	3,2, M.P.

	3	A v C	2, Add.
	4	∴ B v D	1,3, C.D.

22	1	A v (B Λ C)	
	2	A=> D	
	3	¬D / ∴ B	
	4	¬A	2,3, M.T.
	5	B Λ C	1,4 , D.S.
	6	∴ B	5, Simp.

23	1	A => B	
	2	B => C	
	3	¬ C / ∴ ¬ A	
	4	¬ B	2,3, M.T.
	5	∴ ¬A	1,4, M.T.

24	1	(A v B) => C	
	2	D => ¬ C	
	3	D / ∴ ¬ (A v B)	
	4	¬ C	2,3, M.P.
	5	∴ ¬ (A v B)	1,4, M.T.

25	1	(A=> C) Λ (B => D)	
	2	K => A	
	3	K / ∴ C v D	
	4	A	2,3, M.P.
	5	A v B	4, Add.
	6	∴ C v D	1,5, C.D.

Not all arguments can be tested only with the Rules of Inference, though as shown above somewhat complex and diverse arguments succumb to these Rules. Just as modern logic tried to supplement traditional logic, within modern logic, the need was felt to supplement the Rules of Inference. Hence we have the Rules of Replacement. The structure of argument is such that it may require only the Rules of Replacement or only the Rules of Inference or both. We have ten such Rules, which are called the Rules of Replacement. The difference between these two sets of Rules is that the Rules of Inference are themselves Inferences whereas Rules of Replacement are not. This is because the Rules of Replacement are restricted to change or changes in the form of statements. For example, A or B is changed to B or A; A Λ (B v C) is changed to (A Λ B) v (A Λ C). Also, in the mode of application of Rules there is a restriction. Unlike Rules of Inference which should be applied to the whole line only any Rule of Replacement can be applied to any part of the line a difference pointed out earlier. All Rules of Replacement are, logically, equivalent.

Check your progress I

Note: Use the space provided for your answers.

Examine the following arguments.

1) Analyze the significance of Rules of Inference.

....
....
....

2) Distinguish between the application of Composition and Simplification.

....
....

Now let us list Rules of Replacement.

1 De Morgan's Law (De.M.) ¬ (p Λ q) ≡ ¬ p v ¬q
¬ (p v q) ≡ ¬ p Λ ¬q

2 Commutation Law for addition (Com.) p v q ≡ q v p
Commutation Law for multiplication p Λ q ≡ q v p
3 Double Negation(D.N.) ¬ (¬ p) ≡ p
4 Transposition (Trans.) (p => q) ≡ (¬q => ¬ p)
5 Material Implication (Impl.) (p => q) ≡ ¬ p v q
6 Material Equivalence (Equiv.) (p ≡ q) ≡ (p => q) Λ (q => p)
{(p ≡ q) ≡ {(p Λ q) v (¬ p Λ ¬q)}
≡ p Λ p
7 Exportation (Exp.) {(p Λ q) => r} ≡ {p => (q => r)}
8 Tautology (Taut.) p ≡ p v p
9 Association (Ass.) {p v (q v r)} ≡ {(p v q) v r}
{p Λ (q Λ r)} ≡ {p Λ q) r}

10 Distribution (Dist.) {p Λ (q v r)} ≡ {(p Λ q) v (p Λ r)}
{p v (q Λ r)} ≡ {p v q) Λ (p v r)}

[Note: When the expression includes both 'Λ' and 'v' only distribution law can be applied but not association law.]

If we construct truth-table for any Rule (for both sides of the equation), we will understand that they are equivalent expressions. The student is advised to construct truth-table for any Rule of Replacement to verify the same. Our immediate task is to become familiar with the technique of testing arguments. Wherever Rule of Replacement applies to a part of the line the transformed part is italicized.

26. 1 {I => (J => K)} Λ (J => ¬I) / ∴{(I Λ J) => K} Λ (J => ¬I)}

2 ∴{(I Λ J) => K} Λ (J => ¬I) 1, Exp.

27.

1 (R Λ S) => (¬R v ¬S) / ∴ (¬R v ¬S) => (R Λ S)

2 ∴ (¬R v ¬S) => (R Λ S) 1,De.M.

28.

1 (T v ¬U) Λ {*(W Λ ¬V) => ¬T*}/ ∴(T v ¬U) Λ {(W=>(¬V=>¬T)}

2 ∴(T v ¬U) Λ {(W=>(¬V=>¬T)} 1, Exp.

29.

1 X v Y) Λ (¬X v Z)/ ∴ (X v Y Λ ¬X) v {(X v Y) Λ Z}

2 ∴ (X v Y Λ ¬X) v {(X v Y) Λ Z} 1,Dist.

30.

1 Z => *(A => B)* / ∴ Z => ¬{ ¬ (A => B}

∴Z => ¬{ ¬ (A => B} 1,D.N.

31.

1 (¬F v G) Λ (F => G) / ∴F => G.).

2 (F => G) Λ (F => G) 1, Impl.

3 ∴ F => G 2, Taut.

Now we shall consider different types of arguments, which may involve both kinds of Rules.

32.

1	(O => ¬P) Λ (P => Q)		
2	Q => O	∴	
3	¬R => P	/ ∴ R	
4	¬Q v O	2,	Impl.
5	O v ¬ Q	4,	Com.
6	(O => ¬P) Λ (¬ Q => ¬P)	1,	Trans,
7	¬P v ¬P	6, 5,	C.D.
8	¬P	7,	Taut.
9	¬ ¬ R	3, 8,	M.T.
10	R	9,	D.N.

33.

1	X => (Y => Z)		
2	X => (A => B)		
3	X Λ (Y v A)		
4	¬Z	/ ∴B	
5	(X Λ Y) => Z	1,	Exp.
6	(X Λ A) => B	2,	Exp.
7	(X Λ Y) v (X Λ A)	3,	Dist.
8	{(X Λ Y) => Z} Λ {(X Λ A) => B}	5,6,	Conj.
9	Z v B	8, 7,	C.D.
10	∴B	9, 4,	M.T.

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1	C => (D => ¬ C)		
2	C ≡ D	/ ∴ ¬C v ¬D	
3	C => (¬ ¬*C* => ¬*D*)	1,	Exp.
4	C => (C => ¬D)	3,	D.N.
5	(*C Λ C*) => ¬D	4,	Exp.
6	C => ¬D	5,	Taut.
7	∴ ¬C v ¬D	6,	Impl.

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1 E Λ (F v G)

2 (E Λ G) => ¬ (H v I)

3	¬ (¬H v ¬I) => ¬ (E Λ F) / ∴ H ≡ I	
4	(E Λ G) => (¬H Λ ¬I)	2, De.M.
5	¬ (H Λ I) => ¬ (E Λ F)	3, De.M.
6	(E Λ F) => (H Λ I)	5, Trans.
7	{(E Λ F) => (H Λ I)} Λ {(E Λ G) => (¬H Λ ¬I)}	6,4,Conj.
8	(E Λ F) v (E Λ G)	1, Dist.
9	(H Λ I) v (¬H Λ ¬I)	7,8, C.D.
10	H ≡ I	9, Equiv.

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1	J v (¬K v J)		
2	K v (¬J v K) / ∴ J ≡ K		
3	(¬K v J) v J	1,	Com.
4	¬k v (*J v J*)	3,	Ass.
5	¬K v J	4,	Taut.
6	K => J	5,	Impl.
7	(¬J v K) v K	2,	Com.
8	¬J v (*K v K*)	7,	Ass.
9	¬J v K	8,	Taut.
10	J => K	9,	Impl.
11	(J => K) Λ (K => J)	10, 6,	Conj.
12	∴ J ≡ K	11,	Equi.

37

1	(E Λ F) Λ G		
2	(F ≡ G) => (H v I) / ∴ I v H		
3	E Λ (F Λ G)	1,	Ass.
4	(F Λ G) Λ E	3,	Com.
5	(F Λ G)	4,	Simp.
6	(F Λ G) v (¬ F Λ ¬ G)	5,	Add.
7	F ≡ G	6,	Equiv.
8	H v I	2, 7,	M.P.
8	∴I v H	8,	Com.

38

1	(M => N) Λ (¬ O v P)	
2	M v ¬ O / ∴ N v P	
3	∴ N v P	1, 2,C. D.

39

1	(L v M) v (N Λ O)	
2	(¬ L Λ O) Λ ¬ (¬ L Λ M) / ¬L Λ N	
3	¬ L Λ [O Λ ¬ (¬ L Λ M)]	2,Ass.
4	¬ L	3,Simp.
5	L v {(M v (N Λ O)} ∴	1, Ass.
6	M v (N Λ O)	5,4, D.S.
7	¬ (¬ L Λ M)	2, Simp.
8	Lv ¬ M	7, De. M.
9	¬ M	8, 4, D.S.
10	N Λ O	6, 9, D.S.
11	N	10, Simpl.
12	∴ ¬ L Λ N	4,11, Conj.

40

1	E => (F => G)	/ ∴ F => (E => G)	
2	(*E Λ F*) => G	1,	Exp.
3	(F Λ E) => G	2,	Com.
4	∴F => (E => G)	3,	Exp.

41

1	H => (I Λ J)	/ ∴H=>I	
2	¬ H v (I Λ J)	1,	Impl.
3	(¬ H v I) Λ (¬ H v J)	2,	Dist.
4	¬ H v I	3,	Simp.
5	∴ H => I	4,	Impl.

42

1	N => Q	/ ∴ (N Λ P) => O	
2	¬ N v O	1,	Impl.
3	¬ P v ¬ N v O	2,	Add.
4	¬ (P Λ N) v O	3,	De.M.
5	(*P Λ N*) => O	4,	Impl.
6	∴ (N Λ P) => O	5,	Com.

43

1	(Q v R) => S	/ ∴Q => S	
2	(¬ Q Λ ¬R) v S	1,	Imp.
3	(¬ Q v S) Λ (¬ R v S)	2,	Dist.
4	¬ Q v S	3,	Simp.
5	Q => S	4,	Impl.

44

1	T => ¬ (*U* => *V*)	/ ∴T => U	
2	T => ¬ {¬ *U* v *V*}	1,	Impl.
3	¬T v (U Λ ¬V)	2,	De. M.

4	(¬T v U) Λ (¬T v ¬V)	3,	Dist.
5	¬T v U	4,	Simp.
6	∴T => U	5,	Impl.

45

1	W => *(X v ¬Y)* / ∴W => (Y => X)		
2	W => (¬Y v X)	1,	Com.
3	∴W => (Y => X)	2,	Exp.

46

1	H => (I v J)		
2	¬I / ∴H => J		
3	¬H v *(I v J)*	1,	Impl.
4	¬H v (J v I)	3,	Com.
5	(¬H v J) v I	4,	Ass.
6	¬H v J	5, 2,	D.S.
7	∴H=> J	6,	Impl.

47

1	*(K v L)* => ¬ (M Λ N)		
2	(¬M v ¬N) => (O ≡ P)		
3	(O ≡ P) => (Q Λ R) / ∴ (L v K) => (R Λ Q)		
4	(L v K) => ¬ (M Λ N)	1,	Com.
5	(L v K) => (¬M v ¬N)	4,	De.M.
6	L v K) => (O ≡ P)	5, 2,	H.S.
7	(L v K) => *(Q Λ R)*	6, 3,	H.S.
8	∴ (L v K) => (R Λ Q)	7,	Com.

48

1	*(D Λ E)* => F		
2	(D => F) => G / ∴E=>G		
3	(E Λ D) => F	1,	Com.
4	E => (D => F)	3,	Exp.
5	∴E => G	4,2,	H.S.

49

1	(H v I) => {J Λ (K Λ L)}		
2	I / ∴J Λ K		
3	I v H	2,	Add.
4	H v I	3,	Com.
5	J Λ (K Λ L)	1, 4,	M.P.
6	(J Λ K) Λ L	5,	Ass.

7	∴ J Λ K	6, Simp.

50

1	(M v N) => (O Λ P)	
2	¬ O / ∴ ¬M	
3	¬ O v ¬ P	2, Add.
4	¬ (O Λ P)	3, De.M.
5	¬ (M v N)	1, 4, M.T.
6	¬M Λ ¬N	5, De.M.
7	∴¬M	6, Simp.

51

1	T Λ (U v V)	
2	T => {U => (W Λ X)}	
3	(T Λ V) => ¬ (W v X) /∴ W ≡ X	
4	(T Λ U) => (W Λ X)	2, Exp.
5	(T Λ V) => (¬W Λ ¬X)	3, De.M.
6	{(T Λ U) => (W Λ X)} Λ {(T ΛV) => (¬W Λ ¬X)}	4, 5, Conj.
7	(TΛU) v (TΛV)	1, Dist.
8	(W Λ X) v (¬WΛ¬X)	6,7, C.D.
9	∴W ≡ X	8, Taut.

52

1	Y => Z	
2	Z => {Y => (R v S)}	
3	¬ (R Λ S) / ∴ ¬Y	
4	Y => {Y => (R Λ S)}	1,2, H.S.
5	(Y Λ Y) => (R Λ S)	4, Exp.
6	Y => (R Λ S)	5, Taut.
7	∴¬ Y	6, 3, M.T.

53

1	A v B	
2	C v D / ∴{(A v B) Λ C} v {((A v B) Λ D)	
3	(A v B) Λ (C v D)	1, 2,Conj.
4	∴ {(A v B) Λ C} v {(A v B) Λ D}	3, Dist.

54

1	(I v ¬ ¬J) Λ K	
2	{¬ L => ¬ (K Λ J)} Λ {K => (I=> ¬M)} /∴ M Λ ¬ L	
3	{(K Λ J) => L} Λ {K => (I => ¬ M)}	2, Trans.
4	{(K Λ J) => L} Λ {(K Λ I) => ¬ M}	3, Exp.
5	(I v J) Λ K	1 D.N.
6	K Λ (I v J)	5 Com.
7	(K Λ I) v (K Λ J)	6 Dist.
8	(K Λ J) v (K Λ I)	7 Com.
9	L v ¬M	4, 8 C. D.

10	¬ M v L	9,	Com.
11	∴ M Λ ¬ L)	10,	De. M.

3.4 EXERCISES

A. For the following valid arguments, state the Rule of Inference by which its conclusion follows from their premise(s):

1 (A Λ B) => C
∴ ¬ ¬ (A Λ B) = > C

2 [N => (O Λ P)] Λ [Q => (O Λ R)]
N v Q
∴ (O Λ P) v (O Λ R)

3 ¬ (H Λ ¬I) => (H => I)
(I < => H) => ¬ (H Λ ¬I)
∴(I <=> H) => (H => I)

4 (J => K) Λ (K => L)
L=> M
∴J => K) Λ (K => L)] Λ (L => M)

5 [(O => P) => Q] => ¬ (C v D)
∴C v D) =>¬ [(O => P) => Q]

B. Construct a formal proof of validity for the following arguments by adding additional premise or premises.

1 M v N
¬M Λ ¬N
∴N

2 A => B
∴ A => C

3 (P => Q) Λ (R => S)
∴ Q v S

4 ¬ (H v I) v J
¬(¬H v I)

∴ J v ¬H

5 (W Λ X) => (Y Λ Z)

∴ ¬ (W Λ X)

6 Q => (R v S) (T Λ U) => R

(R v S) => (T Λ U)

∴ Q => R

Check your progress II.

Note: Use the space provided for your answers.

1) Distinguish between distributive law and commutative law.

....
....
....

2) Explain de Morgan's law with example.

....
....

3.5 LET US SUM UP

There are nine Rules of Inference. Six of them are those accepted by traditional logic. Remaining three Rules are the inventions of modern logic. Rules of Inference must be applied to the whole line only whereas the Rules of Replacement can be applied to a part of the line. Any complex argument can be examined easily with the help of these Rules.

3.6 KEY WORDS

Replacement: It means that there is internal change within a line which does not alter the meaning of the compound proposition.

3.7 FURTHER READINGS AND REFERENCES

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