UNIT 2 CONJUNCTION, DISJUNCTION, CONDITIONAL AND BICONDITIONAL

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2. 0 OBJECTIVES

The purpose of this module is to introduce students to the classification and symbolization of sentences in symbolic logic. In this context it may be noted that we use sentences, statements, and propositions interchangeably. Recognition of compound sentence as a distinct class of proposition is a sort of line of demarcation between traditional logic and modern logic, which places symbolic logic on a different pedestal. Therefore through this unit we intend to introduce you to the elements of symbolic logic. Various species of compound proposition are introduced which serve as spring board for further study of logic. Thereby another objective is served. You will become familiar with different techniques which help you to test more complex arguments.

2.1 INTRODUCTION

We have already discussed in the previous unit the importance of argument forms in modern logic. Several classes of proposition constitute arguments. The complexity of arguments is just without bounds. A good deal of groundwork is required before we confront such arguments which is, undoubtedly, an intellectually challenging task. In order to achieve this task we have to familiarize ourselves with distinction between kinds of proposition and several ways in which sentences are combined and more importantly we have to determine the truth-conditions of such sentences accurately. These are the pre-requisites for further analysis.
Modern logic recognizes three kinds of proposition; simple, compound and general. Let us deal with the last kind first. Propositions recognized by classical logic, viz. A, E, I and O are called general in modern logic. Simple sentence in logical sense is equivalent to what is simple in grammar. In other words, a simple sentence consists of one clause only and singular term in the place of subject. Consider these statements.

1 . Rathi is neat.
2 . Rathi is neat and Rathi is sweet.

1 is a simple sentence whereas 2 is a compound sentence. A compound sentence consists of at least two components. Hence compound sentence in logical sense is equivalent to what grammar regards as compound sentence. Of course, the components of compound statement may themselves be compound.

It is important to notice one subtle distinction between compound sentence in grammatical sense and compound sentence in logical sense. This distinction has nothing to do with the structure of sentences but with our perception of sentences. In grammar we are not concerned with the truth-conditions of compound proposition. But in logic it is our primary task. The truth-value of a true compound proposition is TRUE and the truth-value of a false compound proposition is FALSE. There is a technique of determining the truth-value of compound proposition. In effect, the truth-value of a compound proposition is a function of the truth-value of its constituents. Logic which deals with this particular function is called truth-functional logic. Barring a few cases, which are exceptions, in all other cases the truth-value of compound proposition is functional. "A compound proposition is said to be truth-functionally compound if and only if its truth-value is a function of the truth-value of its components." In some exceptional cases we find compound propositions which are not truth-functionally compound. Consider this proposition, "John believes that lead is heavier than zinc." This is a *non-truth-functionally compound sentence.* Its truth-value is completely independent of its component simple sentences. Such propositions are not significant in logic. Hence we shall ignore such propositions.

There are different kinds of compound sentences, each requiring its own logical notation. Negation, conjunction, disjunction, conditional (implication), and biconditional are the kinds of compound sentence with which we are concerned. In this module an exhaustive description of the method of determining the truth-condition is attempted.

How are compound propositions formed? Sentences are conjoined using connectives called *sentential connectives.* Symbolic logic has recognized five such connectives; *not, and, or, if…then,* and *if and only if.* These five connectives generate respectively *negation, conjunction, disjunction, implication,* and *biconditional (bicondition)* or *double implication.* Propositions are replaced by lower case letters like p, q, r, etc. or simply p_1, p_2, p_3, etc. Since proposition is the central theme of our study, this is called propositional calculus or calculus of propositions. While propositions are called variables, sentential connectives are called logical constants. Later we will understand that these constants determine the truth-values of compound propositions.

2.2 NEGATION

Negation deserves our special attention because it is a compound proposition in a unique sense though grammatically it is simple only. It is also a pointer to the exact meaning of and also the sense in which we use the term *compound.* Let us consider the following sentence.

3. India is not a member of the UN Security Council.

This is a negative sentence. It is evident that only in grammatical sense it is simple. Why does logic understand this sentence as compound? This way of understanding stands in need of clarification. Negation is not merely a compound sentence. It is truth-functionally compound, i.e. the truth of 3 depends upon the truth of some other statement. We must find out what that statement is.

4. India is a member of the UN Security Council.

Suppose that 4 is true. Then 3 is false. If we suppose that 4 is false, then 3 is true. Therefore we say that 3 is truth-functionally dependent upon 4 and vice versa. Earlier we said that p, q, r,…. are variables. This is so because in their places we can insert any propositions of our choice. If a sentence replaces the variable, then such proposition is replaced by the first letter of first word or any subsequent word. While doing so, we disregard articles or verbs. Further, we must ensure that the same letter is not repeated.

This is one step in the process of symbolization. Second step is very important. We symbolize connectives too.

Connective	symbol
Not	¬

Now 3 and 4 become ¬ I and I respectively.

Since the negation of a true sentence is false and the negation of a false sentence is true, the following truth-table defines the symbol '¬' thus:

Table 1:

p	¬p
1 0	0 1

Hence the Rule of negation is:

> A negation is true if what is negated is false, and is false if what is negated is true.

In the symbolization of negation, it is important to remember that there are other words and phrases besides 'not' such as 'it is false,' 'it is untrue,' and so on. So there are different natural ways of writing negation, such as:

- It is not the case that indiscipline is tolerated.
- It is false that an honest man is a man of millions.
- It is untrue that that Indians are lazy.

2.3 CONJUNCTION

In conjunction sentences are joined by 'and'. The sentences so combined are called *conjuncts.* Sometimes propositions are misleading. The statement given below illustrates the point.

5. Shasi is intelligent and a hardworking student.

You may be tempted to think that 5 is a simple sentence. In reality, it is a compound sentence. The break-up is as follows.

5a. Shasi is intelligent.
5b. Shashi is a hardworking student.

The symbol we use for conjunction in this module is 'Λ' (and), and not the dot '.'as is customary. There are many other words besides 'and' for which the symbol 'Λ' is used. Some of these words are; but, yet, both, although, however, moreover, as well as, while, etc. Some examples are given below:

6	Hari is poor, but he is honest.	$P \Lambda H$
7	It is hot, yet tolerable	$H \Lambda T$
8	Shasi is intelligent although not very careful.	$I \Lambda \neg C$
9	Both Mohan and Mini are students of Logic.	$M \Lambda R$

Since conjunction is a truth-functionally compound sentence, its symbol **A** is a truth-functional connective. The truth-table of conjunctive proposition provides the truth-condition of conjunctive proposition.

Table 2:

p	q	$p \Lambda q$
1	1	1
1	0	0
0	1	0
0	0	0

Since this truth-table specifies the truth-value of $p \Lambda q$ in every possible case it can be taken as defining the symbol 'Λ '
We may take note of one important aspect: Conjunction has mathematical properties.
That is, a conjunctive function is *commutative, associative* (or *distributive) idempotent.*

$p \Lambda q$ if and only if $q \Lambda p$	Commutative
$p \Lambda (q \Lambda r)$ if and only if $(p \Lambda q) \Lambda r$	Associative
$p \Lambda p$ if and only if p	Idempotent

In terms of relation the properties of conjunction can be stated as follows. Symmetry, transitivity and reflexive are the relations which conjunction satisfies. It is symmetric in virtue of its commutative property. It is transitive because if the statement 'A and B' is true and the statement C and D' is true, then the statement 'A and D' is true. It is reflexive because if A is true, then A and A is also true. A relation which is symmetric, transitive and reflexive is known as equivalence relation. Therefore conjunction can be said to satisfy the parameters of equivalence relation.

2.4 DISJUNCTION

Disjunction (also called *alternation*) is a combination of two sentences with connective *or* linking the sentences. The two sentences so combined are called *disjuncts* (or *alternatives*). The symbol used for disjunction is ‘v’ (called wedge). Here are some examples and their symbolic representation:

10 . Either I will send him an email or I will telephone him. M v T
11 . Either it rains or we shall not go for an outing. R v ¬ G
12 . Either A is not honest or B is not telling the truth. ¬ H v ¬ T

The sentential connective ‘v’ can be used in two senses:
a) Weak or inclusive sense. In this sense it means *not only either-or, but can be both*. Examples:

13. Either Ramu is a cynic or he is a liar.
14. Either Sita is poor or she is sincere.

Let us make this inclusive sense more concrete: We can think of a mother asking her daughter to choose one of the two dresses A or B. The mother wants her daughter to choose one and reject the other. But we will not be surprised if the daughter says that she would take both. In contracts and other legal documents, this weak sense is made explicit by the use of the phrase ‘and/or.’

b) Strong or exclusive sense. In this sense disjunction excludes third possibility. This is possible only when the alternatives stated are mutually exclusive and totally exhaustive. Consider this example.

15. Either a line is straight or it is curved.

Let us take a concrete example to explain the exclusive sense of ‘or’. Suppose you have a guest and you ask him or her ‘what will you have, tea or coffee?’ You will be certainly surprised if your guest says ‘both.’ This means that we use both the inclusive and exclusive senses of ‘v’ not only in logic but also in our ordinary life.

Latin has two different Words corresponding to the two different senses of ‘or’: *vel* (inclusive) and *aut* (exclusive). We use ‘p v q’ in its inclusive sense. A weak disjunction is false only if both of its disjuncts are false. Therefore the Rule is:

a) At least one disjunct is true (weak).
b) One disjunct is true and the other disjunct is false (strong).

Exclusive disjunction is symbolized by A. H. Lightstone in his work ‘*Set Theory and Real Number System’* in this manner.

$$p \underline{v} q$$

This notation helps us to distinguish strong from weak.
Consider the following disjunctive syllogism:

Arg. 1

U.N.O. will be strengthened or there will be the III World War.
U.N.O. will not be strengthened.
Therefore, there will be the III World War.

It is clear that this argument is valid on *either* interpretation of 'or'. So, we symbolize 'or' by '∨' (wedge) regardless of which sense of 'or' is intended. We can therefore write 'p ∨ q' and define it as:

Table 3:

p	q	p v q
1	1	1
1	0	1
0	1	1
0	0	0

It may be noted here that, like conjunction, disjunction too has the very same mathematical properties. The disjunctive function has *commutative, associative* as well as *idempotent* properties.

p v q if and only if q v p	Commutative
p v (q v r) if and only if (p v q) v r	Associative
p v p if and only if p	Idempotent

Idempotent property is the same as reflexive property in mathematical language. Therefore disjunction satisfies the parameters of equivalence relation.

Now a word about correct *punctuation;* Punctuation becomes relevant when two or more than two sentential connectives are involved. As in mathematics, in logic also *parentheses* perform the function of punctuation. Since mathematics is more familiar to you, we shall begin with an example from mathematics.

a) (5+6) 4
b) 5+ (6) 4

It is obvious that a) and b) are not equal. This is because the positions of parentheses determine the meaning of the given expression. Similarly, in symbolic logic parentheses play a decisive role in determining the exact meaning of expressions. When two or more than two connectives are involved, we should first understand the exact meaning of expression and then use parentheses properly. Otherwise, we will go wrong.

2.5 EXERCISES

So far we have considered negation, conjunction, and disjunction. Before we go further, let us work out some exercises.

1. If A and B are true sentences and X and Y are false, discover the truth-value of the compound sentence ¬ [(¬A v X) v ¬ (B Λ Y)] .

This is how it can be worked out: Since A is true, ¬A is false, and since X is false also, the disjunction (¬A v X) is false. Since Y is false, the conjunction (B Λ Y) is false and so its negation ¬(B Λ Y) is true. Hence the disjunction (¬A v X) v ¬ (B Λ Y) is true and its negation, which is the original sentence, is false. (We always begin with the innermost component).

Check your progress I.

Note: Use the space provided for your answers.

a) If p is true and q is false, then work out the value of the following expressions:

1. ¬ (p Λ q)
2. (p v q) Λ (p Λ q)
3. ¬ (p Λ q) v (p Λ q)
4. ¬ (p v ¬q) Λ (q Λ ¬q)
5. ¬ (p Λ p) v ¬ (q v p)

b) If A and B are true sentences and X and Y are false sentences, which of the following compound sentences are true?

1. ¬ (A v X)
2. ¬A v ¬X
3. ¬B Λ ¬Y
4. ¬ (B Λ Y)
5. A v (X Λ Y)
6. (A Λ X) v Y
7. (A v B) Λ (X v Y)
8. (A Λ B) Λ (B v Y)
9. (A v X) Λ (B v Y)
10. A Λ {X v (B Λ Y)}
11. A v (X Λ (B v Y)
12. X Λ (A v (Y v B))
13. ¬ (¬ (¬ (A Λ ¬ X) v ¬A) Λ ¬X)
14. ¬ (¬ (¬ (A v ¬B) Λ ¬A) v ¬A)
15. {(X v A) Λ ¬Y)} v ¬ {(X v A) Λ ¬Y}
16. {A v (X Λ Y)} v ¬ {(A v X) Λ (A Λ Y)}
17. {(X Λ (A Λ Y)} Λ ¬ {(X v A) Λ (X v Y)}
18. {(X v (A v B)} v ¬ {(X Λ A) v (X Λ B)}
19. {X Λ (A v Y)} Λ ¬ {(X v A) Λ (X v Y)}
20. {X v (A v Y)} v ¬ {(X Λ A) v (X Λ Y)}

2.6 IMPLICATION

Conditional statement is a compound statement of the form 'if the train is late, then we will miss the connection.' In the strict sense of the term disjunction also is a conditional statement. However, for the time being we shall restrict ourselves to the form mentioned above. Other words for a conditional statements are implication and hypothetical proposition. Here we will be using them interchangeably. The component before 'then' is called *antecedent* (implicant, or rarely *protasis*) and the component after 'then' is called *consequent* (implícate, or rarely *apodosis*). In an implication, antecedent is said to imply the consequent, or the consequent is said to be implied by the antecedent.

A conditional asserts that *if* its antecedent is true, then its consequent must be true.

While symbolizing implication, it is important to identify antecedent and the consequent correctly. Some examples will be of help:

a. If it rains, then we shall go for a picnic. R => G
b. If it does not rain, then we shall not go for a picnic. ¬ R => ¬ G
c. You will get the job only if you pass the test. P => G

All the following sentence forms are symbolized as

p => q

q only if p
q if p
q provided that p
q on condition that p
q in case p
p hence q
p implies q
Since p, q
p is a necessary condition for q
p is a sufficient condition for q

There are at least four senses in which 'if-then' is used:

- *Logical:* For example, 'If all cats like liver and Dinah is a cat, then Dinah likes liver.'
- *Definitional:* In this sense, the consequent follows from by the very definition of a Word. For example, 'If the figure is a triangle then it has three sides.'
- *Causal:* In this sense, there is a causal connection between and the consequent. For example, 'If gold is placed in *aqua ragia,* then gold dissolves.'
- *Decisional*: 'If you shave your head, then I will change my name.'

How do we find out the meaning which is common to these four senses of 'if-then'? In order to find out an answer to this question, we must ask: What circumstances would suffice to establish the *falsehood* of a conditional? Hence the Rule: A conditional is false only in one case:

Any conditional with a true antecedent and a false consequent is false.

Incidentally, it is important to note that this is the chief characteristic of deductive logic. If all the premises are true and the conclusion is false, then the deductive argument is invalid. A true proposition can imply only a true proposition.

The truth - condition of implication is represented in the form of a table.

Table 4:

p	q	p => q
1	1	1
0	1	1
0	0	1
1	0	0

This table shows that implication is false under only one circumstance i. e., when a false conclusion is derived from a true premise. Under all other circumstances it is true.

Compare any truth-table with any other truth-table. You will notice that the truth-value of any two compound propositions differ though variables remain the same. Compound propositions differ only with regard to sentential connectives. This is an important aspect which proves that only sentential connectives determine the truth-value of compound propositions independent of variables.

Here are some examples worked out. In all cases assume that p is1 and q is 0.

a. (p => q) v p
= (1 => 0) v 1
= 0 v 1
= 1

b. q => (p Λ ¬ q)
= 0 => (1 Λ ¬ 0)
= 0 => (1 Λ 1)
= 0 => 1
= 1

c ¬ (p Λ q) => ¬ q
= ¬ (1 Λ 0) => ¬ 0
= ¬ 0 => 1
= 1 => 1
= 1

Material Implication

We have considered the different senses of 'if …. then.' Not all conditional statements need assert one of the four kinds of implication mentioned earlier. Material implication constitutes a fifth type that may be asserted in ordinary discourse as follows: 'If Gandhiji was a military genius, then I am a monkey's uncle.' Conditional proposition of this sort is often used as an emphatic or humorous way of denying its antecedent. So the full meaning of this conditional seems to be the denial that 'Gandhiji was a military genius' is true when 'I am a monkey's uncle' is false. What it means is simply this: since the consequent is so obviously false, the conditional must be understood as denying the antecedent.

The use of material implication (=>) as the common and partial meaning is justified on the ground that the validity of valid arguments involving conditionals is preserved when the conditionals are regarded as asserting material implication only. (It must, of course, be admitted that such symbolizing abstracts from or ignores part of the meaning of most conditional sentences. But the justification for doing so is demonstrated by some of the Rules of Inference such as Modus Ponens, Modus Tollens and Hypothetical Syllogism.

However, 'if … then' relation, like 'either or', is not as simple as it appears. For, the ordinary people and the logicians do not look at it in the same way. For an ordinary person two simple sentences are related conditionally only under two conditions: i) Their meanings must be related; and ii) the consequent must follow from antecedent. However, a logician is not particular about these conditions provided antecedent should not be true if the consequent is false. The logician's main focus is on the logical properties of an implicative relation.

Though the truth-value of implication is thus worked out indirectly, the problem is not over. For, if we look at the truth-value of implication carefully, there appears to be a paradox:
The first three rows in it are saying that any sentence, true or false, can imply a true proposition. Further, only a false conclusion from a true premise is not admissible. This is the result of the logical properties of an implication. But this is not acceptable to an ordinary person: How can any sentence, true or false, imply any true proposition? This is paradoxical. Similarly, the third row is saying that any sentence, true or false, can be implied by a false proposition. This again is paradoxical.

But we must look at the reason behind these paradoxes: In the cases of conjunction, disjunction, negation, (and, also, biconditional or equivalence, as we shall see later), the ordinary language helps the logicians in framing their truth-values. But they could establish the truth of implication only indirectly. This, however, does not fit in with the ordinary use of 'if then', and hence the paradox of material implication. This problem can be solved if we keep the logical standpoint and the ordinary standpoint separate. For logical purposes, we take the former standpoint of 'if …. then' called *material implication.*

It is quite rewarding to consider a few examples as exercises. This is the only way to learn logic.

If A and B are true sentences and X and Y are false sentences, which of the following compound sentences are true?

1. X => (X => Y)
2. (X => X) => Y
3. (A => X) => Y
4. (X => A) => Y
5. A => (B => Y)
6. A => (X => B)
7. (X => A) => (B => Y)
8. (A => X) => (Y => B)
9. (A => B) => (¬A => ¬B)
10. (X => Y) => (¬X => ¬Y)
11. (X => A) => (¬X => ¬A)
12. (X => ¬Y) => (¬X => Y)
13. ((A Λ X) => Y) => (A => Y)
14. ((A v B) => X) => (A => (B => X))
15. ((X Λ Y) => A) => (X=> (Y => A))
16. ((A Λ X) => B) => (A => (B=>X))
17. (X => (A => Y)) => ((X => A) => Y
18. (X => (X => Y)) => ((X =>X) => X)
19. ((A => B) => A) => A
20. {(X=>Y) => X} => X

2.7 BICONDITIONAL

A biconditional is a compound statement which is a combination of two sentences. The connective used to obtain this proposition is *if and only if*. For example, 'you will catch the train if and only if you reach the station on time.' is a biconditional statement. This is symbolized as C <=> R. Such sentences are true when both the components have the same truth-value. A biconditional is true in two cases only. Both the components must be either true or false, as is clear from the following truth-table:

Table 5:

p	q	p <=> q
1	1	1
1	0	0
0	1	0
0	0	1

It must be noted that bi-conditional is a conjunction of two implicative propositions.

p if and only if q is logically equivalent to (p => q) Λ (q => p). Truth-table assists us to find out how it is so.

Table 6:

p	q	p <=> q	p => q	Λ	q => p
1	1	1	1	1	1
1	0	0	0	0	1
0	1	0	1	0	0
0	0	1	1	1	1

Compare the truth-values of columns 3 and 5 in each row. Since the values have remained the same, we conclude that p if and only if q is logically equivalent to 'if p, then q and if q, then p'.

Here are some more examples that have similar but logically different forms; and therefore we have to be very careful in symbolizing them:

i) Mr. X will catch the bus if he reaches on time. R => C
ii) Mr. X will catch the bus only if he reaches on time. C => R
iii) Mr. X will catch the bus if and only if he reaches on time. C <=> R

Equivalent Forms:
It is possible to transform compound proposition without changing the meaning of proposition. Such transformation yields equivalent propositions. Let us construct a truth-table for each compound proposition to understand how this works.

Implication:

Table: 7

				Implication	Disjunction	Negation
p	q	¬ p	¬q	p => q	¬ p v q	¬ (p Λ ¬ q)
1	1	0	0	1	1	1 1 0 0
1	0	0	1	0	0	0 1 1 1
0	1	1	0	1	1	1 0 0 0
0	0	1	1	1	1	1 0 0 1

The advantage of truth-value method is obvious. Without any verbal explanation and with least effort it is possible to identify equivalence between propositions. The equivalence relation, which exists between implication and disjunction, is self-explanatory. However, relation with negation requires some clarification. There are two columns under negation, which reflect truth-values. Suppose that we ignore negation sign and corresponding truth-values and consider the last column then we are not considering negation but conjunction. The last column in the absence negation preceding first bracket is the same as the following one:

Table 8

p Λ ¬ q
0
1

0
0

However, the required form is not conjunction but negation. The truth-value of negation form, of course, truth-functionally depends upon the truth-value of conjunction form. Therefore while selecting the column, which corresponds to negation form, we should exercise a little caution.

It is necessary to consider another form of equivalence relation and this is relevant only with respect to implication. Examine this table.

Table: 9

				Implication	Contraposition
p	q	¬ p	¬ q	p => q	¬ q => ¬ p
1	1	0	0	1	1
1	0	0	1	0	0
0	1	1	0	1	1
0	0	1	1	1	1

In this transformation the components are replaced by their complements and transposed simultaneously. It will be an error to just transpose without effecting the change in quality because implication does not satisfy the symmetric property. What happens exactly when p => q becomes q => p? The reader is advised to construct truth-table and see the result.

The reader must be in a position to discover the equivalent form for disjunction. It is plain from table7 that if p, then q (p => q) is logically equivalent to ¬ p or q (¬ p v q). We shall express in the form of equations.

Disjunction Implication Negation of conjunction
(¬ p v q) ≡ (p => q) ≡ ¬ (p Λ ¬q)

The formula is simple. Replace the first disjunct by its negation and simultaneously 'v' by '=>'. We get implication. Therefore the equivalent form for 'p v q' must be '¬ p => q.' Contraposition for disjunctive proposition is superfluous because it satisfies the symmetric property. Let us construct truth-table for the equivalent forms for this proposition.

Table 10

p	q	¬ p	¬ q	p v q	¬ p => q	¬ (¬ p Λ ¬q)	
1	1	0	0	1	1	1	0
1	0	0	1	1	1	1	0
0	1	1	0	1	1	1	0
0	0	1	1	0	0	0	1

It is quite interesting to note that conjunction and biconditional do not have equivalent forms. Truth-table again comes to our rescue to know why it is so. It is sufficient if we consider any one-form, say, implication to know why it is so. If one equivalent form is absent, it is imperative that other forms are also absent.

Table 11

p	q	¬ p	¬ q	p Λ q	p => q	¬ p => q	p => ¬ q	¬ p => ¬q
1	1	0	0	1	1	1	0	1
1	0	0	1	0	0	1	1	1
0	1	1	0	0	1	1	1	0
0	0	1	1	0	1	0	1	1

Except that truth-values of conjunction do not tally with any possible arrangement in implication form, no other explanation is conceivable for the absence of equivalent forms to conjunction. The students are advised to test other forms with respect to disjunctive syllogism to convince themselves of the veracity of this statement.

Biconditional proposition also does not have any equivalent form. The reason is very simple. Biconditional is, in reality, a conjunction two implications. First we shall know why it is a conjunction.

Table 12

p	q	¬ p	¬ q	p <=> q	(p => q) Λ (q => p)
1	1	0	0	1	1 1 1
1	0	0	1	0	0 0 1
0	1	1	0	0	1 0 0
0	0	1	1	1	1 1 1

The method of computing is as follows; first, we shall compute the truth-values of first implication (p => q) and then we will compute the truth-values of second implication (q => p). These two sets of truth-values together determine the truth-value of conjunction. When we compare columns 5 and 7, we will come to know that these two expressions have identical truth-values in all instances. It shows that biconditional is also a conjunctive proposition where the conjuncts themselves are compound propositions. Therefore what applies to conjunction naturally, applies to biconditional also.

Check your progress II.

Note: Use the space provided for your answer.

1. If p is true and q is false, then work out the values of the following statements:

   1. p => (q <=> p)
   2. ¬ p <=> (p => p)
   3. (p => q) <=> (¬ p v q)

2. Which of the following sentences are true?

   a) (New Delhi is the capital of India) or (Rome is the capital of Italy).

   b) (New Delhi is the capital of Spain) and ¬ (Paris is the capital of France).

   c) (New Delhi is the capital of India) Λ ¬ (Paris is the capital of France v New Delhi is the capital of India).

   d) ¬ { ¬ (Stockholm is the capital of Norway Λ Paris is the capital Spain)} v ¬{ ¬(London is the capital of England) Λ ¬ (New Delhi is the capital of Spain)}.

   e) (Paris is the capital of France) v ¬{(New Delhi is the capital of Spain) Λ ¬ (¬Paris is the capital of France Λ ¬ New Delhi is the capital of Spain)}.

3. If A, B, and C are true statements and X, Y, and Z are false statements, which of the following are true?

   1 ¬A v B

   2. (A Λ X) v (B v Y)

   3 ¬(X Λ ¬Y) Λ (B v ¬C)

   4. ¬(X v Y) Λ (¬X Λ Y)

   5. ¬ ((A v B) v ¬ (B Λ A))

4. If A, B, and C are true statements and X, Y, and Z are false statements, determine which of the following are True.

   1. A => B
   2. (A => B) => Z
   3. X => (Y => Z)
   4. ((X => Z) => C) => Y
   5. {(A v X) ¬Y} => ((X => A) => (A => Y))

2.8 LET US SUM UP

There are five kinds of compound propositions. Compound proposition in grammatical sense is different from compound sentence in logical sense. All compound propositions do not have equivalent forms. The truth of any compound proposition is determined by the truth-values of components. Sentential connectives determine the truth-value of compound propositions.

2.9 KEY WORDS

Truth-table: It is a technique with the help of which we determine the truth- values of compound propositions.

Biconditional proposition: biconditional is a conjunction of two implicative propositions.

2.10 FURTHER READINGS AND REFERENCES

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