UNIT 6 RIVETED AND WELDED JOINTS

Structure

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6.1 INTRODUCTION

In engineering practice it is often required that two sheets or plates are joined together and carry the load in such ways that the joint is loaded. Many times such joints are required to be leak proof so that gas contained inside is not allowed to escape. A riveted joint is easily conceived between two plates overlapping at edges, making holes through thickness of both, passing the stem of rivet through holes and creating the head at the end of the stem on the other side. A number of rivets may pass through the row of holes, which care uniformly distributed along the edges of the plate. With such a joint having been created between two plates, they cannot be pulled apart. If force at each of the free edges is applied for pulling the plate apart the tensile stress in the plate along the row of rivet hole and shearing stress in rivets will create resisting force. Such joints have been used in structures, boilers and ships.

The development welding technology in 40s of last century has considerably reduced the riveted joint applications. Welding is the method of locally melting the metals (sheets or plates – overlapping or butting) with intensive heating along with a filler metal or without it and allowing to cool them to form a coherent mass, thus creating a joint. Such joints can also be created to make structures, boilers, pressure vessels, etc. but are more conveniently made in steel. The progress has been made in welding several types of steels but large structure size may impede the use of automatic techniques and heat

treatment which may become necessary in some cases. Welded ships were made in large size and large number during second world ware and failures of many of them spurted research efforts to make welding a better technology.

Objectives

After studying this unit, you should be able to

- describe the types of riveted joint,
- explain how to calculate strength of riveted joints,
- explain how many different ways the riveted joints can fail,
- describe the types of welded joints,
- explain the manner of loading welded joint,
- define the failure modes of welded joint, and
- describe how to design a welded joint.

6.2 HEAD FORMING

You know that the riveted joint is created by passing the stem of a rivet through holes in two plates as we shown in Figure 6.1(a). The creation of head by process of upsetting is shown in Figure 6.1(b). The upsetting of the cylindrical portion of the rivet can be done cold or hot. When diameter of rivet is 12 mm or less cold upsetting can be done. For larger diameters the rivet is first heated to light red and inserted. The head forming immediately follows. The rivet completely fills the hole in hot process. Yet it must be understood that due to subsequent cooling the length reduces and diameter decreases. The reduction of length pulls the heads of rivet against plates and makes the joint slightly stronger. The reduction of diameter creates clearance between the inside of the hole and the rivet. Such decrease in length and diameter does not occur in cold worked rivet.

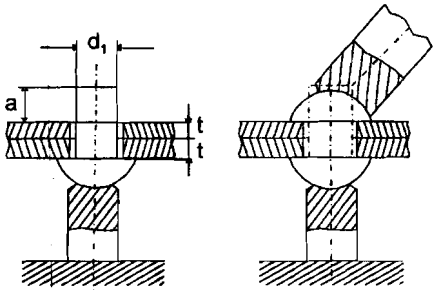


Figure 6.1 : Typical Head Forming of Rivet

6.3 TYPES OF RIVETS

For steel plates the rivets are normally made in low carbon steel. However, the rivets in copper add to resistance against corrosion and aluminum rivets can be used to reduce the overall weight of the structure. The low carbon steel is standardized in composition particularly for boiler applications.

Figure 6.2(b) and oval counter sunk rivets shown in Figure 6.2(c) are not as strong and button head rivets. They are used only when protrotruding rivet heads are objectionable. Pan heads are conical heads, Figures 6.2(d) and (e) are less frequently used and are difficult to form. Tubular rivets, Figures 6.2(f) and (g) are special deviation from solid rivet shank. These rivets are used in aircrafts.

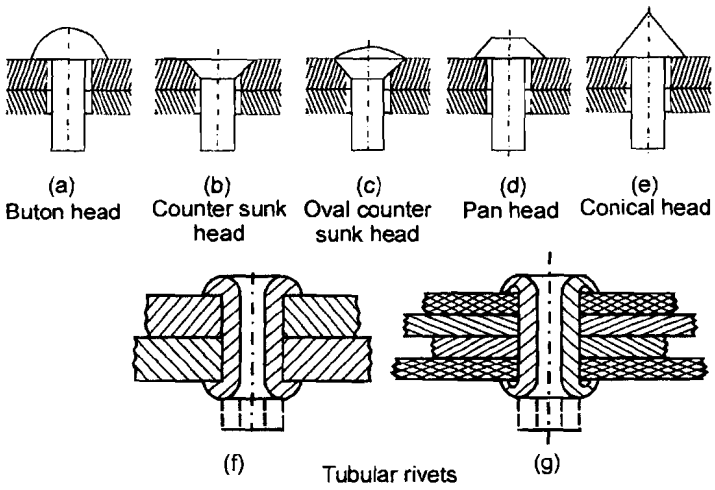


Figure 6.2 : Different Types of Rivet Heads

6.4 TYPES OF RIVETED JOINTS

The classification of riveted joints is based on following :

(a) According to purpose,

(b) According to position of plates connected, and

(c) According to arrangement of rivets.

According to purpose the riveted joints are classified as :

Strong Joints

In these joints strength is the only criterion. Joints in engineering structure such as beams, trusses and machine frames are strong joints.

Tight Joints

These joints provide strength as well as are leak proof against low pressures. Joints in reservoirs, containers and tanks fall under this group.

Strong Tight Joints

These are joint applied in boilers and pressure vessels and ensure both strength and leak proofness.

This classification has no sound basis and is arbitrary. However, it helps understand the basis of design and manufacturing. The hot working of rivets is one-way of making intimate contact between plates in the areas of joint. Further, the holes are drilled and reamed to required tolerances and burrs removed for good contact before rivets are placed in the holes. The edge of the plate is upset by means of a hammer and a caulking tool so that edge is strongly pressed against the plate surface to help lead proofing (Figure 6.3).

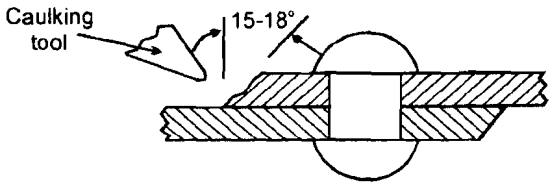


Figure 6.3 : Caulking of Riveted Joint

The riveted joints are classifed as (i) lap joint and (ii) butt joint according to position of plates. In a lap joint the edges of plates are simply laid over each other and riveted. Figures 6.4(a) and (d) show lap joints. If we pull the plates by application of tensile forces, they do not fall in the same line and hence cause the rivets and plates to bend.

Plates placed end-to-end and jointed through cover plates form single cover butt joint. Such joints are shown in Figures 6.4(b) and (e). You can see that pulling plates apart by colinear tensile forces may still cause bending of rivets. Figures 6.4(c) and (f) show the butting plates covered by two straps and then riveted. Such joints are called double cover butt joint. Plate bending and rivet bending are eliminated.

According to arrangement of rivets, the joints are called single riveted. (Figures 6.4(a), (b) and (c)) It may be noted that in a single riveted lap joint there is only one row of rivets passing through both plates while in a single riveted butt joint either of single cover or double cover type one row of rivets will pass through each of the plates. Similarly as shown in Figures 6.4(d) and (e) when two rows of rivets pass through both plates of lap joint it is called double riveted lap joint and two rows of rivets pass through each of butting plates the joint is a double riveted single cover butt joint. A double riveted double cover butt joint is shown in Figure 6.4(f).

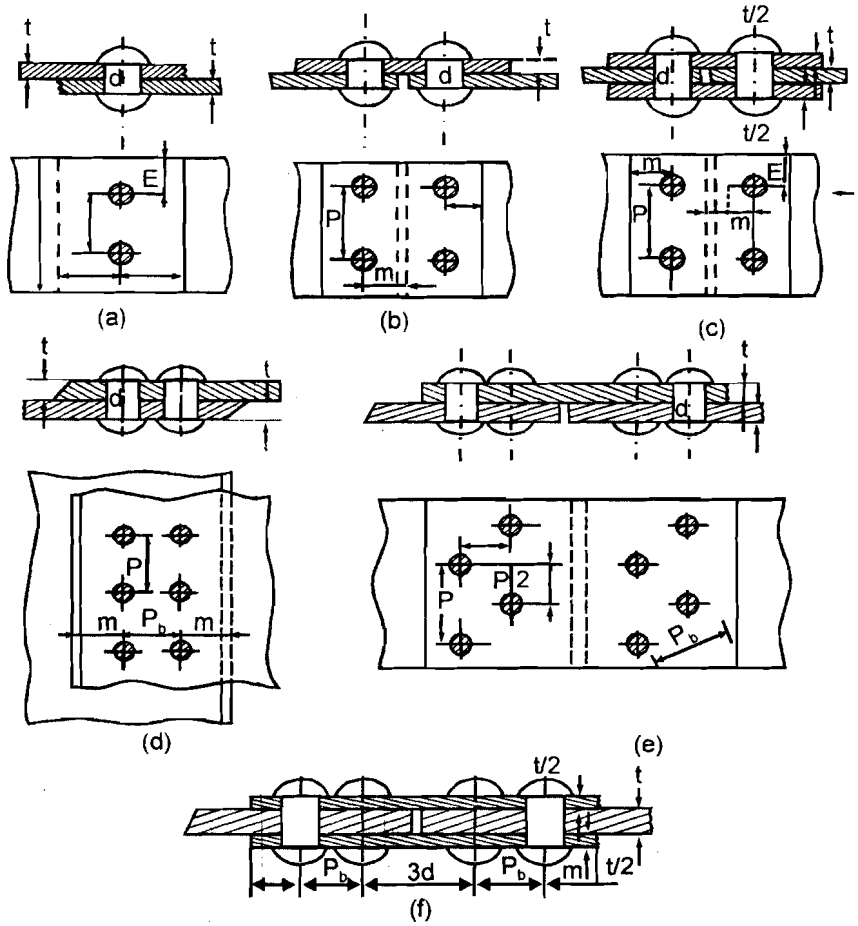


Figure 6.4 : Types of Riveted Joints : (a) Single Riveted Lap Joint; (b) Single Riveted-single Cover Butt Joint; (c) Single Riveted Double Cover Butt Joint; (d) Double Riveted Lap Joint; (e) Double Riveted Single Cover Butt Joint; and (f) Double Riveted Double Cover Butt Joint

The arrangement of rivets in Figure 6.4(d) can be described that in both the rows the rivets are opposite to each other while in Figure 6.4(e) the rivets in the adjacent rows are staggered. The joint in Figure 6.4(d) is said to be chain riveted while that in Figure 6.4(e) is zig-zag riveted joint. In zig-zag riveting the rivet in one row is placed at the middle level of the two rivets in the adjacent row.

6.5 NOMENCLATURE

Several dimensions become obviously important in a riveted joint and a design will consist in calculating many of them. These dimensions and their notations as to be used in this text are described below.

Pitch

As seen from Figures 6.4(a), (b) and (c) pitch, denoted by p, is the center distance between two adjacent rivet holes in a how.

Back Pitch

The center distance between two adjacent rows of rivets is defined as back pitch. It is denoted by P_b and is shown in Figures 6.4(d) and (e).

Diagonal Pitch

The smallest distance between centres of two rivet holes in adjacent rows of a zig-zag riveted joint is called diagonal pitch. Denoted by p_d, the diagonal pitch is shown in Figure 6.4(e).

Margin

It is the distance between centre of a rivet hole and nearest edge of the plate. It is denoted by m as shown in Figures 6.4(b), (c) and (d).

The plates to be jointed are often of the same thickness and their thickness is denoted by t. However, if the thicknesses are different, the lower one will be denoted by t_1. The thickness of the cover plate (also known as strap) in a butt joint will be denoted as t_c.

The rivet hole diameter is denoted by d. This diameter is normally large than the diameter of the rivet shank which is denoted by d_1.

A problem of designing of a riveted joint involves determinations of P, P_b, P_d, m, t, t_c and d, depending upon type of the joint.

6.6 MODES OF FAILURE OF A RIVETED JOINT

A riveted joint may fail in several ways but the failure occurs as soon as failure takes place in any one mode. Following is the description of modes of failures of a riveted joint. These modes are described with the help of a single riveted lap joint, which is subjected to tensile load P. In general the description will be applicable to any other type of joint. Reference is made in Figure 6.5 in which a single riveted lap joint is shown loaded.

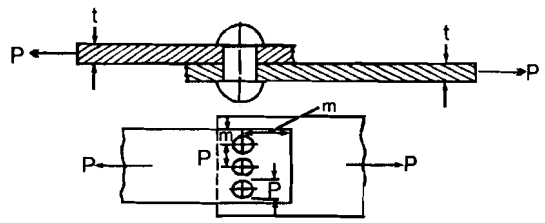


Figure 6.5 : Single Riveted Lap Joint

(a) **Tearing of Plate at the Section Weakened by Holes**

Figure 6.6 shows this mode of failure. The plate at any other section is obviously stronger, and hence does not fail. If tensile force P is to cause tearing, it will occur along weakest section, which carries the row of rivets. If only one pitch length p is considered; it is weakened by one hole diameter d. The area that resists the tensile force is

$$A_t = (p - d)\, t$$

If the permissible stress for plate in tension is σ_t, then tensile strength of the joint or tensile load carrying capacity of the joint

$$Pt = \sigma_t\, (p - d)\, t \qquad \ldots (6.1)$$

If P is the applied tensile force per pitch length then the joint will not fail if

$$Pt \geq P \qquad \ldots (6.2)$$

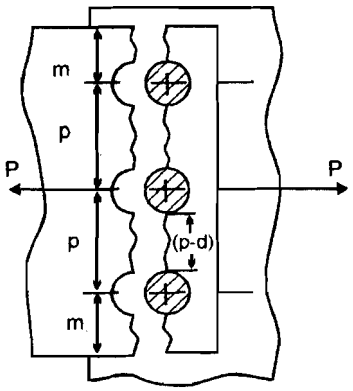


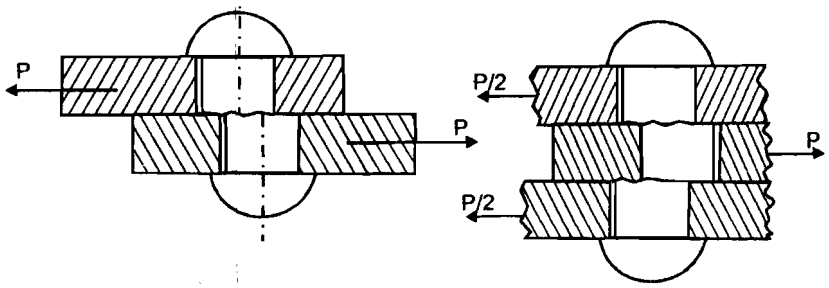
Figure 6.6 : Tearing of Plate at the Section Weakened by Holes

(b) **Shearing of Rivet**

Figure 6.7 shows how a rivet can shear. The failure will occur when all the rivets in a row shear off simultaneously. The considers the strength provided by the rivet against this mode of failure, one consider number of rivets in a pitch length which is obviously one. Further, in a lap joint failure due to shear may occur only along one section of rivet as shown in Figure 6.7(a). However, in case of double cover butt joint failure may take place along two sections in the manner shown in Figure 6.7(b). So in case of single shear the area resisting shearing of a rivet,

$$A_s = \frac{\pi}{4} d^2$$

(Since the difference between diameter of hole and diameter of rivet is very small, diameter of hole is used for diameter of the rivet).



(a) (b)

Figure 6.7 : Shearing of Rivet : (a) Single Shear; and (b) Double Shear

If permissible shearing stress in single shear of rivet is τ_s, then the shearing strength or shearing load carrying capacity of the joint.

$$P_s = \tau_s \frac{\pi}{4} d^2 \qquad \ldots (6.3)$$

The failure will not occur if

$$P_s \geq P \qquad \ldots (6.4)$$

We may also write if n is the number of rivets per pitch length,

$$P_s = n\ \tau_s \frac{\pi}{4} d^2 \qquad \ldots (6.5)$$

If the rivet is in double shear as in Figure 6.7(b) the effective area over which failure occurs in $2\ A_s$. The permissible stress in double shear is 1.75 times that in single shear. Hence in double shear

$$P_s = n \times 1.75\ \tau_s \frac{\pi}{4} d^2 \qquad \ldots (6.6)$$

(c) **Crushing of Plate and Rivet**

Due to rivet being compressed against the inner surface of the hole, there is a possibility that either the rivet or the hole surface may be crushed. The area, which resists this action, is the projected area of hole or rivet on diametral plane. The area per rivet is (see Figure 6.8).

$$A_c = dt$$

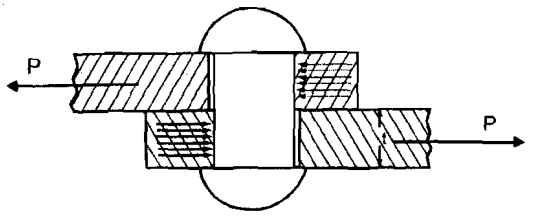


Figure 6.8 : Crushing of Rivet

If permissible crushing or bearing stress of rivet or plate is σ_c the crushing strength of the joint or load carrying capacity of the joint against crushing is,

$$P_c = dt\ \sigma_c \qquad \ldots (6.7)$$

The failure in this mode will not occur if

$$P_c \geq P \qquad \ldots (6.8)$$

where P is applied load per pitch length, and there is one rivet per pitch. If number of rivets is in a pitch length then right hand side in Eq. (6.7) is multiplied by n.

(d) **Shearing of Plate Margin near the Rivet Hole**

Figure 6.9 shows this mode of failure in which margin can shear along planes *ab* and *cd*. If the length of margin is m, the area resisting this failure is,

$$A_{ms} = 2Mt$$

If permissible shearing stress of plate is τ_s then load carrying capacity of the joint against shearing of the margin is,

$$P_{ms} = 2_{mt}\ \tau_s \qquad \ldots (6.9)$$

The failure in this case will not occur if

$$P_{ms} \geq P \qquad \ldots (6.10)$$

where P is the applied load per pitch length.

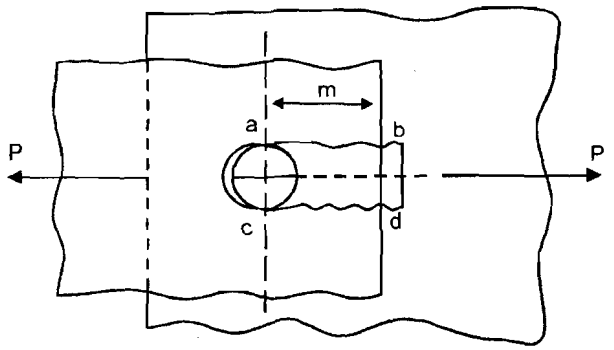


Figure 6.9 : Shearing of Margin

The modes of failure discussed above are primary in nature and in certain cases they have to be considered uniquely. One such case is when rivets are arranged in lossenge form or diamond shape. This case will be discussed at proper stage.

In writing down the above equations for strength of the joint certain assumptions have been made. It is worthwhile to remember them. Most importantly it should be remembered that most direct stresses have been assumed to be induced in rivet and plate which may not be the case. However, ignorance of actual state of stress and its replacement by most direct stress is compensation by lowering the permissible values of stresses σ_t, τ_s and σ_c, i.e. by increasing factor of safety.

The assumptions made in calculations of strengths of joint in Eq. (6.1) through (6.10) are :

(a) The tensile load is equally distributed over pitch lengths.

(b) The load is equally distributed over all rivets.

(c) The bending of rivets does not occur.

(d) The rivet holes do not produce stress concentration. The plate at the hole is not weakened due to increase in diameter of the rivet during second head formation.

(e) The crushing pressure is uniformly distributed over the projected area of the rivet.

(f) Friction between contacting surfaces of plates is neglected.

6.7 EFFICIENCY OF RIVETED JOINTS

If only a pitch length of solid or hole free plate is considered then its load carrying capacity will be

$$P_1 = pt\ \sigma_t \qquad \ldots (6.11)$$

P_1 will apparently be greater then P_t, P_s, P_c or P_{ms}. The ratio of any of P_t, P_s, P_c or P_{ms} to P_1 is defined as the efficiency of the joint in that particular mode. Ideally P_t, P_s, P_c and P_{ms} all must be equal, but actually it may not be the case. The efficiency of the joint will be determined by least of P_t, P_s, P_c, and P_{ms}. Thus efficiency of the joint is,

$$\eta = \frac{\text{Least of } P_t, P_s, P_c \text{ and } P_{ms}}{P^t\ \sigma_t} \qquad \ldots (6.12)$$

The ideal that strengths in different modes of failure are equal is not achieved in a design because the rivet hole diameters and rivet diameters are standardized for technological convenience. Table 6.1 describes the average and maximum efficiencies of commercial boiler joints.

Table 6.1 : Efficiencies of Commercial Boiler Joints

Type of Joint	Average Efficiency %	Maximum Efficiency %
Lap Joints		
Single riveted	45-60	63.3
Double riveted	63-70	77.5
Triple riveted	72-80	86.5
Butt Joints		
Single riveted	55-60	63.3
Double riveted	70-83	86.6
Triple riveted	80-90	95.0
Quadruple riveted	85-94	98.1

6.8 CALCULATION OF HOLE DIA AND PITCH

For an ideal joint the rivet should be equally strong against shearing and crushing. Hence from Eqs. (6.3) and (6.7), making $P_s = P_c$

$$\frac{\pi}{4} d_2 \tau_s = dt \sigma_c$$

$$\therefore \quad d = 1.274 \frac{\sigma_c}{\tau_s} \text{ (in single shear)} \quad \ldots (6.13)$$

If rivet is in double shear,

$$d = 0.637 \frac{\sigma_c}{\tau_s} t \quad \ldots (6.14)$$

Generally $\tau_s = 60$ Mpa or N/mm^2

$\sigma_c = 130$ Mpa or N/mm^2

giving $d = 2.75\ t$ in single shear $\quad \ldots (6.15)$

$d = 1.37\ t$ in double shear

Also equating right hand sides of Eqs. (2.1) and (2.3),

$$(p - d)\ t\ \sigma_t = \frac{\pi}{4} d^2 \tau_s$$

or $$p = \frac{\pi d^2}{4t \sigma_t} \tau_s + d$$

Substituting $\tau_s = 60$ MPa

$\sigma_t = 75$ MPa

$$P = 0.628 \frac{d^2}{t} + d$$

Using Eq. (6.15) in above equation

$P = 2.73\ d$ (in single shear)

$P = 1.86\ d$ (in double shear) $\quad \ldots (6.16)$

Equating right hand sides of Eqs. (6.7) and (6.9)

$$2mt\ \tau_s = dt\ \sigma_c$$

or $$m = \frac{d}{2} \frac{\sigma_c}{\tau_s}$$

Substituting $\sigma_c = 130$ MPa

$\tau_s = 60$ MPa

$M = 1.08\ d$ $\quad \ldots (6.17)$

There are several practical considerations due to which the design dimensions are modified. Most important of these is the pressure tightness of the joint, which is mainly achieved by caulking of the plate edges. The caulking becomes easier with short pitches and smaller rivets. It also makes it desirable that margin should be $1.5\ d$ but not greater. The results in this section are indicative of calculation procedure and by no means be treated as standard formulae. These results are valid only for particular case and permissible stresses adopted. As a common practice for plate thickness greater than 8 mm the diameter of rivet hole is determined by

$$d = 6\sqrt{t} \quad \ldots (6.18)$$

This is known as Unwin's formula.

It has been pointed out in the last sections that no attempt was made to derive formulae. The expressions various load carrying capacities were written by examining the geometry. Therefore, you must see that in each problem the geometry is understood and then the expressions for forces are written. In the examples here we would see how we can approach to design a riveted joint.

Example 6.1

Design a double riveted lap joint for MS plates 9.5 mm thick. Calculate the efficiency of the joint. The permissible stresses are :

$\sigma_t = 90$ MPa, $\tau_s = 75$ MPa, $\sigma_c = 150$ MPa

Solution

The joint to be designed is shown schematically in Figure 6.10.

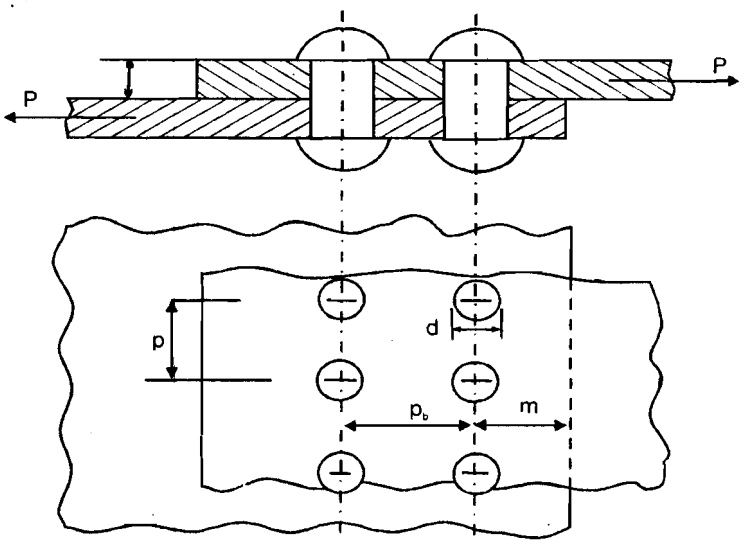


Figure 6.10

(a) **Dia. of Rivet Hole d :** It is determined by Unwin's formula, Eq. (6.18)

$$d = 6\sqrt{t}$$

or $$d = 6\sqrt{9.5} - 18.5 \text{ mm} \quad \ldots \text{(i)}$$

(b) **Pitch of the Joint, p :** In a double riveted joint there are 4 rivets in a pitch length. The rivet dia. will be taken as dia. of the hole as difference between them is small. The rivets can fail in shear or due to crushing. We will first determine the shearing and crushing strength of a rivet and equate the smaller of two to the plate tearing strength of determine p.

Shearing strength of one rivet

$$= \frac{\pi}{4} d^2 \tau_s = \frac{\pi}{4} (18.5)^2 \ 75 = 20.16 \text{ kN} \quad \ldots \text{(a)}$$

Crushing strength of one rivet

$$= \sigma_c \ dt = 150 \times 18.5 \times 9.5 = 26.36 \text{ kN} \quad \ldots \text{(b)}$$

From (a) and (b) it is seen that the rivet is weaker in shear.

$\therefore$ We will equate tearing strength of plate with shearing strength of rivets in a pitch length. There are two rivets in the pitch length.

$$\therefore \quad \sigma_t \ (p - d) \ t = 2 \times \frac{\pi}{4} d^2 \tau_s$$

or $$p = \frac{\pi}{2}\frac{d^2}{t}\frac{\tau_s}{\sigma_t} + d = \frac{\pi}{2}\frac{(18.5)^2}{9.5}\frac{75}{90} + 18.5$$

or $$p = 65.55 \text{ mm} \quad \text{say} \quad 65.7 \text{ mm} \qquad \ldots \text{(ii)}$$

The pitch should be such that head forming operation is not hindered. The practice dictates that $p \geq 3\,d$ so that head forming is permitted.
$3\,d = 55.5$ mm, and hence the value of p obtained in (ii) is acceptable.

(c) **The back Pitch p_b :** It must be between 2.5 d *ti* 3.0 d. For chain riveting the higher value is preferred for the reason of head forming

$$P_b = 3d = 3 \times 18.5 = 55.5 \text{ mm} \qquad \ldots \text{(iii)}$$

(d) **Margin, m :** m is determined by equating shearing strength of rivet (smaller of shearing and crushing strengths of rivet). Remember that there are two rivets per pitch length :

$$\therefore \quad 2mt\,\tau_s = 2\,\frac{\pi}{4}\,d^2\,\tau_s$$

$$\therefore \quad m = \frac{\pi}{4}\frac{d^2}{t} = \frac{\pi}{4}\frac{(18.5)^2}{9.5} = 28.3 \text{ mm} \qquad \ldots \text{(iv)}$$

The minimum acceptable value of m is 1.5 d = 27.5 mm hence

M = 28.3 mm is acceptable.

Thus the design is completed with

d = 18.5 mm, p = 65.7 mm, p_b = 55.5 mm, m = 28.3 mm

The diameter is standardized, apparently based on drill size. Normally fractions like 18.5 mm may not be accepted. The rivet diameters are less than hole diameter by 1 mm. Yet the head formation process increases rivet diameter. We are not yet describing standard hole and rivet diameters. We postpone it for the time being.

(e) **Efficiency of Joint**

Tensile strength of plate without holes, per pitch length

$$P_1 = \sigma_t\ pt = 90 \times 65.7 \times 9.5 = 56.2 \text{ kN} \qquad \ldots \text{(c)}$$

Shearing strength of rivets in a pitch length

$$P_s = 2 \times \tau_s \times \frac{\pi}{4}\,d^2 = 2 \times 75 \times \frac{\pi}{4}\,(18.5)^2 = 40.3 \text{ kN} \qquad \ldots \text{(d)}$$

Crushing strength of rivets in a pitch length

$$P_c = 2 \times \sigma_c \times d \times t = 2 \times 150 \times 18.5 \times 9.5 = 52.7 \text{ kN} \qquad \ldots \text{(e)}$$

The tearing strength of plate with one hole in a pitch length

$$Pt = \sigma_t\,(p - t)\,t = 90\,(65.7 - 18.5)\,9.5 = 40.36 \text{ kN} \qquad \ldots \text{(f)}$$

The shearing strength of margin

$$P_{ms} = 2\,\tau_s\,mt = 2 \times 75 \times 28.3 \times 9.5 = 40.32 \text{ kN} \qquad \ldots \text{(g)}$$

Out of all P_s, P_c, P_t and P_{ms}, the lowest is P_{ms}

$$\eta = \frac{P_{ms}}{P_1} = \frac{40.32}{56.2} = 71.74\% \qquad \ldots \text{(h)}$$

The design values are

d = 18.5 mm, p = 65.7 mm, p_b = 55.5 mm, m = 28.3 mm, η = 71.74%

6.9 RIVETED JOINTS IN STRUCTURES

For trusses, bridges or girders, etc. where the width of the plates is known in advance lozenge type or diamond shaped joints are preferred. These joints have uniform or equal strengths in all modes of failure. Marginal adjustments in calculated dimensions may slightly reduce or increase strength in any particular mode. The joints are usually of double cover butt type with rivets so arranged that there is only one rivet in the outermost row and their number increases towards inner row. (See Figure 6.11). Since the plate width is known in advance, its strength in tension can be determined. Thus the load carrying capacity of the joint

$$P = \sigma_t \, (b - d) \, t$$

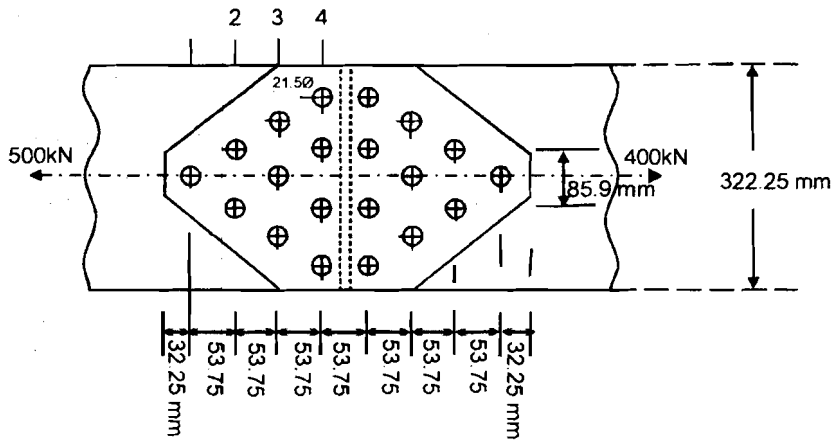


Figure 6.11

Here b is the plate width, t is thickness and d, the diameter of the hole. σ_t is the permissible tensile stress. The rivet diameter is determined by Unwin's relationship. The determination of number of rivets is the main task for the required force that is carried by the member. Of course we would first determine whether the strength of rivet is less in shear or in crushing which would depend upon relative magnitudes of τ_s and σ_c as well as on the cross sectional area of the rivet and its projected area. The next step would be to arrange the rivets in diamond shape as shown in Figure 6.11. Then we decide upon the pitch, back pitch and margin.

The joint is designed not to tear in the outer most row, i.e. row 1 in Figure 6.11. Then the row 2, which is the next inner row and weakened by two rivet holes, is subjected to tearing. Note that this tearing is possible if rivet in the outermost row (or row 1) shears or is crushed at the same time. That means this type of joint has one more possible mode of failure which comprises tearing along on inner row accompanied by shearing or crushing of rivets in all outer rows. The strength in this mode is denoted by P_{ts} or P_{tc} and one more suffix may be used to denote the row in which tearing will occur.

Thus $$P_{ts2} = \sigma_t \, (b - 2d) \, t + \tau_s \frac{\pi}{4} d^2$$

or $$P_{tc2} = \sigma_t \, (b - 2d) \, t + \sigma_c \, dt$$

And $$P_{ts3} = \sigma_t \, (b - 3d) \, t + 3\tau_s \frac{\pi}{4} d^2$$

or $$P_{tc3} = \sigma_t \, (b - 3d) \, t + 3\sigma_c \, dt$$

Example 6.2

Two steel plates 12.5 mm thick are required to carry a tensile load of 500 kN in a double cover butt joint. Calculate the width of the plate if it is not to be weakened by more than one rivet hole. Design the butt joint completely and show dimensions on a sketch. The ultimate values of strengths are as follow :

Plates in tension – 600 MPa

Rivet in Shear – 490 MPa

Plate and rivet in crushing – 920 MPa

Use a factor of safety of 4.5.

Also use following standards:

Rivet Holes

From 13.5 mm to 25.5 mm in steps of 2 mm and from 27 mm to 42 mm in steps of 3 mm.

Rivets

1.5 mm less than rivet hole diameter upto 24 mm in steps of 2 mm and 2 mm less than rivet hole diameter from 25 mm to 39 mm in steps of 3 mm.

Solution

The permissible stresses are :

$$\tau_t = \frac{600}{4.5} = 133 \text{ N/mm}^2$$

$$\tau_s = \frac{490}{4.5} = 109 \text{ N/mm}^2$$

$$\sigma_c = \frac{920}{4.5} = 204 \text{ N/mm}^2$$

Diameter of the Rivet Hole : Use 'Unwin's Formula

$$d = 6\sqrt{t} = 6\sqrt{12.5} = 21.21 \text{ mm}$$

From standards

$d = 21.5$ mm

Hence the rivet dia, $d_1 = 20$ mm ... (i)

Compare shear strength and crushing strength of one rivet.

$$P_s = \tau_4 \frac{\pi}{4} d_1^2 = 109 \times \frac{\pi}{4} \times (20)^2 = 34 \text{ kN}$$

$$P_c = \sigma_c d_1 t = 204 \times 20 \times 12.5 = 51 \text{ kN}$$

Thus rivet in crushing is stronger than in shear.

Width of the Plate

Consider the tensile strength of the weakest section of the plate i.e. the row which is weakened by one rivet hole.

$$P = \sigma_t (b - d) t$$

or $$500 \times 10^3 = 133 (b - 21.5) 12.5 \quad \text{... (ii)}$$

$$b = 300.75 + 21.5 = 322.25 \text{ mm}$$

Number of Rivets

The rivets are in double shear in double cover butt joint. The strength in double shear is 1.75 the strength in single shear. Also we assume that the head formation does not change rivet dia.

$$P_s = 1.75\ \tau_s \frac{\pi}{4} d_1^2 = 1.75 \times 34 = 59.5 \text{ kN}$$

$$\eta P_s = P \text{ i.e. } \eta\ 59.5 = 500 \text{ kN}$$

Also crushing strength

$$\eta P_c = P \text{ i.e. } \eta \times 51 = 500$$

or $$\eta = \frac{500}{51} = 9.8 \text{ say } 10 \quad \ldots \text{(iii)}$$

We will see that 10 rivets are better arranged.

Rivet Arrangement

Ten rivet can be easily arranged in four rows : 1, 2, 3 and 4 can be arranged as 1, 2, 3, and 3 which will not be a good arrangement. We should ensure that 10 rivets should not weaker the plate. The arrangement is shown in Figure 10.11. The pitch of the rivets is determined by geometric consideration. The innermost or 4th row should have a margin of $1.5\ d = 1.5 \times 21.5 = 32.25$ mm from the edge.

Thus the centers of two extreme rivets in row 4 is

$$b - 2\ m = 322.25 - 2 \times 32.25 = 257.75 \text{ mm}$$

Obviously this distance is equal to $3\ p$

$$p = \frac{257.75}{3} = 85.9 \text{ mm} \quad \ldots \text{(iv)}$$

The distance between the rows, P_b should be between 2.5 to 3 d

$$p_b = 2.5 \times 21.5 = 53.75 \text{ mm} \quad \ldots \text{(v)}$$

Cover Plate

Theoretically the cover plate may have thickness of $t/2$ but practically the thickness $T_c = 0.62\ t$

$$T_c = 0.625 \times 12.5 = 7.8 \text{ mm} \quad \ldots \text{(vi)}$$

The cover plates are given the diamond shape so as to accommodate all the rivets (See Figure 6.11).

Efficiency

Shearing strength of 10 rivets is double shear

$$P_s = 1.75 \times 10 \times \frac{\pi}{4} d_1^2\ \tau_s$$

$$= 1.75 \times 10 \times \frac{\pi}{4} (20)^2 \times 109 = 17.5 \times 34 = 595 \text{ kN} \quad \ldots \text{(a)}$$

Crushing strength of 10 rivets

$$P_c = 10 \times \sigma_c\ d_1 t = 10 \times 51 = 510 \text{ kN} \quad \ldots \text{(b)}$$

Tearing strength of plate along weakest section, i.e. along row 1

$$P_{t1} = (b + d)\ t\ \sigma_t$$

$$= 133\ (322.25 - 21.5)\ 12.5 = 500 \text{ kN} \quad \ldots \text{(c)}$$

Strength for tearing along second row and crushing of one rivet in row 1

$$P_{tc2} = 133\,(322.25 - 2 \times 21.5)\,12.5 + \sigma_c\, d_1\, t$$

$$= 133 \times 279.25 \times 12.5 + 204 \times 20 \times 12.5$$

$$= 464.25 + 51 \text{ kN}$$

or $P_{tc2} = 515.5$ kN ... (d)

Strength for tearing along third row and crushing of 3 rivets in row 1 and 2

$$P_{tc3} = 133\,(322.25 - 3 \times 21.5)\,12.5 + \sigma_c \times 3d_1\, t$$

$$= 133 \times 257.75 \times 12.5 + 3 \times 51 \times 10^3 = 428.5 + 3 \times 51 \text{ kN} \quad \ldots (e)$$

or $P_{tc3} = 581.5$ kN

Strength for tearing along fourth and crushing of rivets in other rows

$$P_{tc4} = 133\,(322.25 - 4 \times 21.5)\,12.5 + 6 \times 51 \times 10^3$$

or $P_{tc4} = 699$ kN ... (f)

(e) and (f) were expected to be more than (d), yet the calculations have been made for the sake of completeness.

Strength of solid plate without hole

$$P_t = \sigma_t\, bt = 133 \times 322.25 \times 12.5 = 535.74 \text{ kN} \quad \ldots (g)$$

The least of all strengths from (a) through (f) is $P_{t1} = 500$ kN

$$\eta = \frac{P_{t1}}{P_t} \times 100 = \frac{500}{535.74} \times 100$$

$$= 93.3\% \quad \ldots (vii)$$

SAQ 1

(a) Describe types of riveted joints.

(b) What are different modes of failure of riveted joints?

(c) Define pitch, back pitch, margin and diagonal pitch.

(d) Define efficiency of a riveted joint and write expression for various strengths.

(e) A structural joint of double cover butt type has an efficiency of 85% with the condition that the lowest strength is in tearing mode of 10.5 mm thick plate in outer most row which is weakened by one hole. Find the tensile force carried by the joint and the number of rivets of permissible stresses for plate and rivet are; $\sigma_t = 105$ N/mm^2, $\tau_s = 75$ N/mm^2 and $\sigma_c = 150$ N/mm^2. Design the joint completely and show d, d_1, p, p_b and m on the sketch. Also give dimension of cover plate.

6.10 JOINTS FOR BOILERS AND PRESSURE VESSELS

The boiler and pressure vessels are cylindrical in shape and with stand internal pressure. The vessels are required to be leak proof. The maintenance of pressure and safety of boilers have prompted several standards. ASME boiler code, Board of Trade (BOT) Rules, Indian Boiler Regulations (IBR) and ISI standards are available for design of boilers and pressure vessels.

The cylindrical pressure vessel is identified by two dimensions, viz., the length and diameter. The cylinders are made from plates and whole length may not be obtained from single sheet hence cylindrical sections are obtained bending sheets and joining edges by riveted joint. The sections are then joined together by another riveted joint along circumference. Thus there are two types of joint longitudinal and circumferential (see Figure 6.12). The longitudinal joint will bear hoop stress (σ_h) and circumferential joint bears longitudinal stress (σ_l). Since $\sigma_h = 2\sigma_l$, the longitudinal joint will have to be two times are strong as circumferential joint. Therefore, longitudinal joint are always made butt joint whereas the circumferential joints are mode as lap joints.

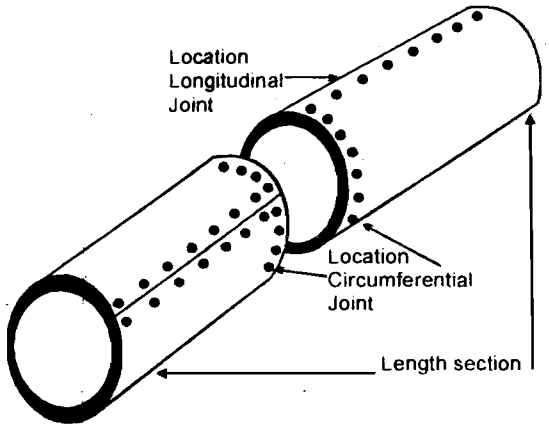


Figure 6.12 : Longitudinal and Circumferential Location for Riveted Joints

The steps followed in design of boiler riveted joints are same as followed in any joint design. They are mentioned here as described in IBR.

6.11 DESIGN PROCEDURE FOR LONGITUDINAL BUTT JOINT

Determine Thickness of Boiler Shell (t)

The efficiency of the joint is chosen from Table 6.1 and for pressure σ_r, inner diameter D and permissible tensile stress σ_t, the thickness is calculated from,

$$t = \frac{\sigma_r D}{2\sigma_t \eta} + 1 \text{ mm} \qquad \ldots (6.19)$$

The diameter and thickness will further guide in respect of rivet arrangement. Table 6.2 can be used for this purpose.

Table 6.2 : Suggested Rivet Arrangement

Dia. of Shell (mm)	Thickness of Shell (mm)	Rivet Arrangement
610-1830	6-12.5	Double riveted
915-2130	8-25.0	Triple riveted
1525-2740	9.5-31.75	Quadruple riveted

Determine Rivet Hole Diameter (d) and Rivet Diameter (d)

Unwin's formula, giving $d = 6\sqrt{t}$ is used if $t = \geq 8$ mm. In very race case if $t < 8$ mm, d is calculated by equating shearing strength and crushing strength of rivet. The diameter of hole must be rounded off to the nearest standard value with the help of Table 6.3, and the diameter of rivet also established.

Table 6.3 : Standard Rivet Hole and Rivet Diameters

D (mm)	13	15	17	19	21	23	25	28.5	31.5	34.5	37.5	41	44
D_1 (mm)	12	14	16	18	20	22	24	27	30	33	36	39	42

Determine Pitch of the Rivet (p)

The minimum pitch is 2 d to accommodate the dies to form head. The pitch is calculated by equating tearing strength with shearing or crushing strength of rivet(s). However, the pitch should not exceed certain value for beak proof nature of the joint. The maximum value of p is given by following equation.

$$P_{max} = C \times t + 41.28 \text{ mm} \quad \ldots (6.20)$$

The value of C is given in Table 6.4. If by calculation p turns out to be less than P_{max}, it will be acceptable.

Table 6.4 : The Value of Constant for Maximum Pitch

Number of Rivets	Lap Joint	Butt Joint Single Cover	Butt Joint Double Cover
1	1.31	1.53	1.75
2	2.62	3.06	3.50
3	3.47	4.05	4.63
4	4.17	–	5.52
5	–	–	6.00

Determine Back Pitch (p_b)

(a) For both lap and butt joints having equal number of rivets in different rows p_b is given as

$$p_b = (0.33\ p + 0.67\ d) \text{ mm for zig-zag} \quad \ldots (6.21)$$

and $$p_b = 2\ d \quad \ldots (6.22)$$

(b) For joints in which number of rivets in outer rows is half of that in inner rows which are chain riveted p_b should be greater of the values calculated from Eqs. (2.19) and 2.20). However, the value of p_b for rows having full number of rivets will not be less than 2 d.

(c) The third case arises for joints having inner rows zig-zag riveted and outer rows having half the number of rivets as inner rows where

$$p_b = (0.2\ p + 1.15\ d) \text{ mm} \quad \ldots (6.23)$$

The back pitch for zig-zag riveted inner rows will be

$$p_b = (0.165\ p + 0.67\ d) \text{ mm} \quad \ldots (6.24)$$

The pitch p in above equations is the one in outer row, i.e. away from butting edges.

Determine Thickness of the Cover Plate (t_c)

(a) For single butt cover with chain riveting

$$t_c = 1.125\ t \quad \ldots (6.25)$$

(b) For single cover with pitch in the outer row being twice that in the inner row

$$t_c = 1.125\left(\frac{p-d}{p-2d}\right)t \qquad \ldots(6.26)$$

(c) For double cover of equal width and chain riveting

$$t_c = 0.625\,t \qquad \ldots(6.27)$$

(d) For double cover of equal width with pitch in the outer row being twice that in the inner row

$$t_c = \left(\frac{p-d}{p-2d}\right)t \qquad \ldots(6.28)$$

(e) For double cover of unequal width (wider cover on the inside)

$$\left.\begin{aligned} t_{c1} &= 0.75\,t \quad \text{(for cover on the inside)} \\ t_{c2} &= 0.625\,t \quad \text{(for cover on the outside)} \end{aligned}\right\} \qquad \ldots(6.29)$$

Determine margin, $m = 1.5\,\alpha$ $\qquad \ldots(6.30)$

Determine Caulking Pitch, p_t

The pitch of rivets in the row nearest to the edge must be as small as possible to avoid leakage. This pitch is called caulking pitch and helps edges to be caulked effectively (see Figure 6.3). A rough rule is that this pitch should not be greater than S_{lc}. The caulking pitch is, however, calculated from following :

$$p_c = d + 13.8\,\frac{t_c}{(\sigma_{12})} \qquad \ldots(6.31)$$

This is an empirical relation in which σ_r the pressure is used in N/mm^2.

6.12 DESIGN PROCEDURE FOR CIRCUMFERENTIAL LAP JOINT

The thickness of the shell, the diameter of the rivet hole, back pitch can margin are calculated in the same way as for longitudinal butt joint. The other quantities are presented under.

Number of Rivets (n)

The rivets are in single shear and all of them are subjected to shear when pressure, σ_r acting on the circular section of the cylindrical space tends to separate two length sections of the vessel.

$$n\,\tau_s = \frac{\pi}{4}\,d_1^2 = \sigma_r\,\frac{\pi}{d}\,D^2$$

$$n = \frac{\sigma_r}{\tau_s}\,\frac{D^2}{d_1^2} \qquad \ldots(6.32)$$

Pitch, (p)

Efficiency of the lap joint η can be taken as half of the efficiency of the longitudinal butt joint. The efficiency of the lap joint is calculated on the basis of tearing load capacity of the joint which turns out to be least of strengths in all modes.

Thus, $$\eta = \frac{p-d}{p} \qquad \ldots(6.33)$$

Number of Rows, (N)

The rivets are placed all along the circumferences of the shell. Hence number of rivets in one row.

$$n_1 = \frac{\pi (D + t)}{p}$$

Hence total number of rivets in N $n_1 = n$.

$$N = \frac{n p}{\pi (D + t)} \qquad \ldots (6.34)$$

Whether the joint will be single riveted or multiple riveted will be decided by N. If N turns out to be less than 1, a single riveted will serve the purpose. In any case the pitch will have to satisfy the condition of caulking.

Overlap of Shell Length Section (l)

$$l = (N - 1)\ p_b + 2m \qquad \ldots (6.35)$$

Example 6.3

Inner diameter of a boiler is 1500 mm and the steam pressure is 2 N/mm^2. Use a proper joint along the length and design it completely. Use following permissible values of stress.

Tension $\sigma_t = 90$ MPa

Shear $\tau_s = 75$ MPa

Crushing $\sigma_c = 150$ MPa

Solution

Thickness of the Shell (t)

From Table 6.2 for shell diameter of 150° mm a double riveted butt joint is recommended and from Table 2.1 were can use an efficiency of 80%.
From Eq. (6.17).

$$t = \frac{\sigma_t D}{2\sigma_t \eta} + 1 \text{ mm}$$

$$= \frac{2 \times 1500}{2 \times 90 \times 0.8} + 1 = 20.8 + 1$$

or $t = 21.8$ mm say 22 mm

Rivet Hole Diameter (d)

From Eq. (6.18)

$$d = 6\sqrt{t} = 6\sqrt{22} = 28.14 \text{ mm}$$

The nearest standard value of hole dia is 28.5 mm, and corresponding rivet dia is 27 mm. $d_1 = 27$ mm.

Pitch (p)

In one pitch length there are two rivets which may shear or crush (Figure 6.13).

The shear strength of one rivet in double shear

$$P_{s1} = 1.75 \times \frac{\pi}{4} d_1^2\ \tau_s = 1.75 \times \frac{\pi}{4} (27)^2 \times 75 = 75.2 \text{ kN}$$

The crushing strength of one rivet

$$P_{c1} = t d_1 \sigma_c = 22 \times 27 \times 150 = 89.1 \text{ kN}$$

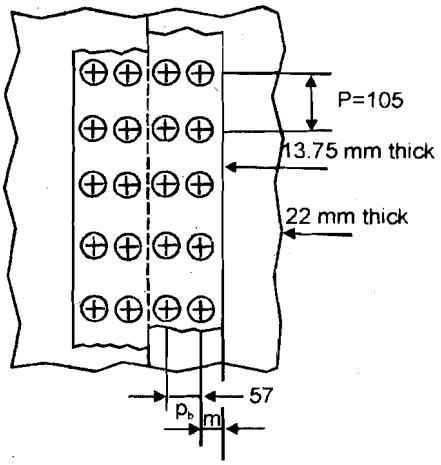


Figure 6.13

The rivet is weaker in shearing. Equating tearing strength of plate with shearing strength of 2 rivets in a pitch length,

$$(p - d)\, t\, \sigma_t = 2 \times 1.75 \times \frac{\pi}{4} d_1^2\, \tau_s$$

$$p = 3.5 \times \frac{\pi}{4} (27)^2 \times \frac{75}{22 \times 90} + 28.5 = 104.4 \text{ mm}$$

Check for maximum value of pitch from Eq. (6.20). From Table 6.4 for 2 rivets in a pitch length for a double cover double riveted joint the value of $C = 3.5$.

$$\therefore \qquad p_{max} = C \times t + 41.28 = 3.5 \times 22 + 41.28 = 118.28 \text{ mm}$$

The min. pitch is 2 d. Hence calculated value of $p = 104.4$ mm is acceptable. We may choose $p = 105$ mm.

Back Pitch (p_b)

$$p_b = 0.33\, p + 0.67\, d = 0.33 \times 105 + 0.67 \times 28.5$$

$$p_b = 34.65 + 19.1 = 53.75 \text{ mm}$$

However, p_b should not be less than 2 d or 57 mm

$$p_b = 57 \text{ mm}.$$

Thickness of Cover Plate (t_c)

The joint has two equal cover plates. From Eq. (6.27)

$$t_c = 0.625\, t = 13.75 \text{ mm}$$

Margin (m)

$$M = 1.5\, d = 42.75 \text{ mm}$$

Efficiency (η)

The shearing strength of the joint

$$P_s = 2 \times 75.2 = 150.4 \text{ kN}$$

The crushing Strength of the joint

$$P_c = 2 \times 89.1 = 178.2 \text{ kN}$$

The tearing strength of plate with holes

$$P_t = (p - d)\, t\, \sigma_t = (105 - 28.5)\, 22 \times 90$$

or $P_t = 151.47$ kN

The tensile strength of plate without holes

$$P = pt\,\sigma_t = 105 \times 22 \times 90 = 208 \text{ kN}$$

P_s is least of P_s, P_c and P_t

$$\therefore \quad \eta = \frac{P_s}{P} = \frac{150.4}{208} = 72.3\%$$

Example 6.4

Design a circumferential lap joint for boiler shell of Example 6.3.

Solution

The thickness of the shell t and rivet hole dia. d (and rivet dia. d_1) will remain same, i.e.

$$t = 22 \text{ mm}, d = 28.5 \text{ mm}, d_1 = 27 \text{ mm}$$

Number of rivets (n) : Use Eq. (6.32)

$$\eta = \frac{\sigma_r}{\tau_s} = \frac{D^2}{d_1^2} \qquad \sigma_r \text{ is pressure is boiler}$$

$$\eta = \frac{2}{75}\left(\frac{1500}{27}\right)^2 = 82.3 \text{ say } 83$$

Pitch (p)

These rivets (83 in number) have to be placed along circumference and preferably in two rows for better leak proofing. However, arranging rivets in two rows will alter the number of rivets. We will first determine pitch from efficiency, making efficiency in plate tearing mode as the least. At best it is required that efficiency of the circumferential joint should be 50% of the efficiency of the longitudinal joint. So in the case

$$\eta = \frac{0.8}{2} = 0.4$$

$$\eta = \frac{p - d}{p} = 0.4 \text{ so that } p - 0.4p = d$$

or $$p = \frac{d}{0.6} = \frac{28.5}{0.6} = 47.5 \text{ mm}$$

Number of Rows (N)

Apparently n number of rivets are to be distributed with $p = 47.5$ mm around a circumference of $\pi\,(D + t)$

$$\therefore \quad N = \frac{n\,p}{\pi\,(1500 + 28.5)} = \frac{83 + 47.5}{\pi \times 1528.5} = 0.82$$

To make $N = 2$ and keeping $p = 47.5$ mm, n will increase,

$$2 = \frac{n \times 47.5}{\pi \times 1528.5} \text{ giving } n = 202$$

Now choosing $n = 202$ will alter p

$$P = \frac{2 \times \pi \times 1528.5}{202} = 47.54 \text{ mm}$$

With number of rivets 2 per pitch length the constant C from Table 6.4 is 3.06 and using Eq. (6.20)

$$p_{max} = Ct + 41.28 = 3.06 \times 22 + 41.28 = 108.6 \text{ mm}$$

But for convenience of caulking p should be at least 2 d.

$$\therefore \quad p = 2 \times 28.5 = 57 \text{ mm}$$

This will further alter number of rivets as

$$n = \frac{2\pi \times 1528.8}{57} = 168.5 \text{ say } 168$$

So that $$p = \frac{2\pi \times 1528.8.5}{168} = 57.16 \text{ mm}$$

$$\therefore \quad n = 168, p = 57.16 \text{ mm}, N = 2$$

$$\eta = \frac{p - d}{p} = \frac{57.16 - 28.5}{57.16} = 50.44\%$$

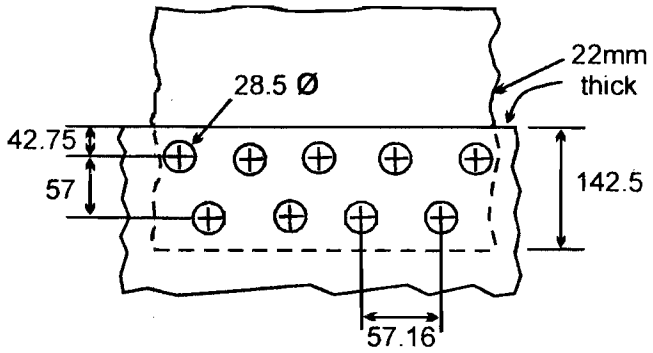


Figure 6.14

Back Pitch (P_b)

The rivets can be arranged in zig-zag rows so that

$$p_b = 2\ d = 57 \text{ mm}$$

Margin (m) : $m = 1.5\ d = 1.5 \times 28.5 = 42.75$ mm

Overlap (l) : $l = (N - 1)\ p_b + 2$ m

or $$l = 57.0 + 2 \times 42.75 = 142.5 \text{ mm}.$$

The joint is shown in Figure 6.14.

6.13 TORSIONAL LOADING AND ECCENTRIC LOADING OF RIVETED JOINT

Plate A is riveted to structural element B. A torque is applied to the Plate A. The plate will rotate, of course by slight elastic amount, about some point as o in Figure 6.15(a). It is wrong to assume that any straight line such as oc which passes through the centre of a rivet, remains straight before and after application of the torque. Then the deformation, hence strain and so the average shearing stress across the section of the rivet will be proportional to the distance between o and the centre of the rivet. Since the average shearing stress is equal to the shearing force divided by area of cross section of the rivet, the shearing force on the rivet will be proportional to the distance between o and centre of the rivet. The direction of this force will be perpendicular to the joining line.

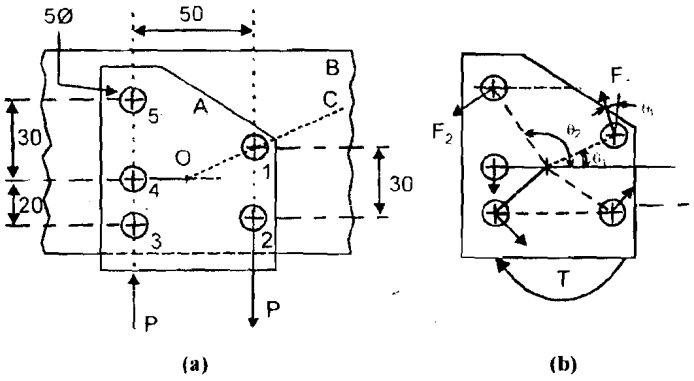


Figure 6.15

The forces F_1, F_2, etc. on individual rivets are shown in Figure 6.15(b). For satisfying condition of equilibrium, components of forces in vertical direction should sum up to zero. If the forces F_1, F_2, etc. make angles θ_1, θ_2, etc. respectively with Y-axis, then

$$F_1 \cos \theta_1 + F_2 \cos \theta_2 + \ldots + F_n \cos \theta_n = 0$$

or
$$\sum_{i=1}^{n} F_i \cos \theta_i = 0 \qquad \ldots (i)$$

But
$$F_i = \tau_i A_i = \frac{\tau_i}{r_i} r_i A_i$$

And since
$$\tau_i \propto r_i \quad \text{or} \quad \tau_i = k r_i$$

$\therefore$
$$F_i = k R_i A_i \qquad \ldots (ii)$$

Here K is a constant, τ_i = shearing stress in i^{th} rivet whose area of cross section is A_i and its centre is at a distance r_i from o.

Use (ii) and (i) to obtain

$$k \sum_{1}^{n} r_i A_i \cos \theta_i = 0$$

See from Figure 6.15(b) that $r_i \cos \theta_i = x$

$\therefore$
$$k \sum_{1}^{n} x A_i = 0$$

Which is same as $k \bar{x} A_t = 0$.

Where $\bar{x}$ is the x-coordinate of centroid of all the rivet and sum of their areas of cross sections is A_t. And since neither k nor A_t is zero therefore $\bar{x} = 0$. If then we consider sum of forces along x-axis we would arrive at the result $\bar{y} = 0$. This means that o is the point coinciding with the centroid of the rivet area system.

Example 6.5

In Figure 16.15(a) the distances between columns and rows of rivets are shown. Each rivet is 5 mm in diameter and force P = 1 kN. Calculate the maximum shearing stress in rivet.

Solution

The five rivets have been numbered as 1, 2, . . . , 5. Take centre of rivet 3 as origin and x and y axes along 32 and 35 respectively. Areas of all rivets is

$A = \frac{\pi}{4} (5)^2 = 19.64$ mm^2. If $\bar{x}$ and $\bar{y}$ are the coordinates of the centioid, then

$$50A + 50A = 5A \bar{x}$$

Hence $\bar{x} = 20$ mm

Also $50A + 20A + 30A = 5A\,\bar{y}$, $\bar{y} = 20$ mm

Hence centroid is on the horizontal line through rivet 4. We can calculate various distances of rivet centres from centroid.

$$r_1 = \sqrt{10^2 + 30^2} = 10\sqrt{10}$$

$$r_2 = \sqrt{20^2 + 30^2} = 10\sqrt{13}$$

$$r_3 = \sqrt{20^2 + 20^2} = 10\sqrt{8}$$

$$r_4 = \sqrt{20^2 + 0} = 10 \times 2$$

$$r_5 = \sqrt{30^2 + 20^2} = 10\sqrt{13}$$

Now $F_1 = k\,r_1 = k\ 10\sqrt{10}$, moment of F_1 about O,

$$M_1 = k\,r_1^2 = 100\,k$$

$$F_2 = k\,r_2 = k\,10\sqrt{13} \quad M_2 = 1300\,k$$

$$\frac{F_1}{F_2} = \frac{r_1}{r_2} = \frac{\sqrt{10}}{\sqrt{13}} \quad \text{or} \quad F_1 = \sqrt{\frac{10}{13}}\,F_2$$

We can find each of F_1, F_2, F_3, F_4 and F_5 in terms of k or we can find each force in terms of F_2. We may like to choose F_2 because F_2 is grater than all other forces because r_2 is larger than all other r.

$$F_3 = \sqrt{\frac{8}{13}}\,F_2,\ F_4 = \frac{2}{\sqrt{13}}\,F_2,\ F_5 = F_2$$

Taking moments of all forces about O and equating with the applied moment of $50\,P = 50000$ N mm.

$$\sqrt{\frac{10}{13}}\,F_2\ 10\sqrt{10} + F_2\ 10\sqrt{13} + \sqrt{\frac{8}{13}}$$

$$F_2\ 10\sqrt{8} + \frac{2}{\sqrt{13}}\,F_2\ 10 \times 2 + F_2\ 10\sqrt{13} = 5 \times 10^4$$

$$\frac{100F_2}{\sqrt{13}} + \frac{130F_2}{\sqrt{13}} + \frac{80F_2}{\sqrt{13}} + \frac{40F_2}{\sqrt{13}} + \frac{130F_2}{\sqrt{13}} = 5 \times 10^4$$

$$\therefore \quad F_2 = \frac{500\sqrt{13}}{4.8} = 375.6 \text{ N}$$

$$\therefore \text{ Maximum shearing stress} = \frac{F_2}{A} = \frac{375.6}{19.64} = 19.12 \text{ N/mm}^2$$

This stress will be in rivets 2 and 5.

Example 6.6

Figure 6.16(a) shows a plate riveted on to a vertical column with three rivets placed at three corners of an equilateral triangle of size 75 mm. A load of 37 kN acts on the plate at a distance of 125 mm from vertical line through a rivet as shown in Figure 6.16(a). If the permissible stress in rivet is 60 N/mm^2 calculate the diameter of the rivet.

The rivets are at corners of equilateral triangle hence their centroid will be at the centroid of the triangle, C. Each of rivets 1, 2 and 3 will be at the same distance from C.

$$r_1 = r_2 = r_3 = \frac{2}{3}\sqrt{75^2 - 37.5^2} = \frac{2}{3}\sqrt{5625 - 1406.25} = 43.3 \text{ mm}$$

The force of 37 kN is acting at a distance of 125 mm from vertical line through the centroid as shown in Figure 6.16(b). Apply two forces, each equal to 37 kN but in opposite direction at C. Combining two 37 kN forces as shown in the figure, we are left with a couple 37 kN by 125 mm and a vertical force 37 kN acting downward. This force will be distributed equally on three rivets, i.e. if $F = 37$ kN, then $\frac{F}{3}$ will be a direct shearing force acting downward on each rivet as shown in Figure 6.16(b). The moment of the couple, $T = 37 \times 125 = 4625$ kN mm will be balanced by moments of forces F_1, F_2 and F_3 about C. F_1, F_2 and F_3 are perpendicular to r_1, r_2 and r_3, respectively and each is proportional to its r. incidentally due to symmetry of equilateral triangle r_1, r_2 and r_3 are mutually equal and hence $F_1 = F_2 = F_3$.

Equate moments $F_1 r_1 + F_2 r_2 + F_3 r_3 = T$

$$3F_1 r_1 = T$$

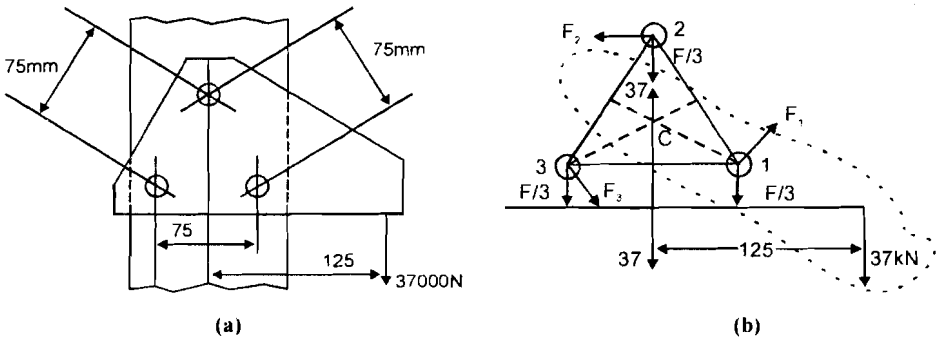


Figure 6.16

Or by putting $r_1 = 43.3$ mm

$$F_1 = \frac{4625}{3 \times 43.3} = 35.6 \text{ kN}$$

The rivet 1 is loaded by two forces $F_1 = 33.6$ kN and $\frac{F}{3} = \frac{37}{3} = 12.33$ kN at an angle of 150°. All three rivets with their forces are shown in Figure 6.17. The net force on each rivet will be the resultant of $\frac{F}{3}$ and F_1, F_2 or F_3. Apparently the largest magnitude will occur where the angle between two forces is minimum. The angle is minimum $\theta = 30°$ in rivet 3.

The resultant of two forces R_1 and R_2 with angle between them being θ is given by

$$R = \sqrt{R_1^2 + R_2^2 + 2R_1 R_2 \cos\theta}$$

∴ Resultant force on rivet 3

$$R = \sqrt{\left(\frac{F}{3}\right)^2 + F_3^2 + 2\frac{F}{3}F_3 \cos\theta}$$

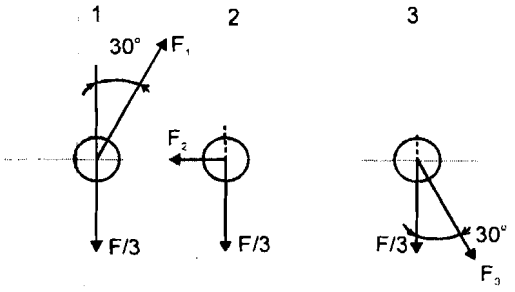


Figure 6.17

Put $F = 37$ kN, $F_3 = 35.6$ kN, $\theta = 30°$

$$R = \sqrt{(12.33)^2 + (35.6)^2 + 2 \times 12.33 \times (35.6) \cos 30}$$

$$= \sqrt{152 + 1267.36 + 761} = \sqrt{21.80.495} = 46.7 \text{ kN}$$

With τ as permissible shearing stress and d as diameter of rivet

$$R = \frac{\pi}{4} d^2 \tau$$

Use $R = 46.7 \times 10^3$ N, $\tau = 60$ N/mm^2

$$d^2 = \frac{4 \times 46.7}{\pi\, 60} \times 10^3 = 991$$

$\therefore$ $d = 31.5$ mm

We had assumed that all rivets have same diameter, that is how we had determined centriod. Hence all three rivets will have the diameter of 31.5 mm each.

SAQ 2

(a) What types of joint are used in a boiler along its length and circumference and why?

(b) Describe the quantities that are required to calculate for riveted joints and show them on sketch.

(c) What do you understand by eccentric loading of a riveted joint? Explain with the help of sketch.

(d) A boiler shell 1 m in diameter is subjected to steam pressure of 2.7 MPa gauge. It is proposed to have a longitudinal double riveted double cover butt joint with number of rivets twice in the inner row. Assume following ultimate strength values.

Ultimate tensile strength $\sigma_u = 352$ MPa

Ultimate shearing strength $\tau_u = 256$ MPa

Ultimate crushing strength $\sigma_{uc} = 512$ MPa

For plate and rivet materials.

With machine riveting assume a factor of safety of 4. Sketch the joint and show dimensions. Calculate the efficiency of the joint.

(e) Determine diameter of rivets if permissible shearing stress is 65 N/mm^2. Assume all rivets have same diameter in Figures 6.18 and 6.19.

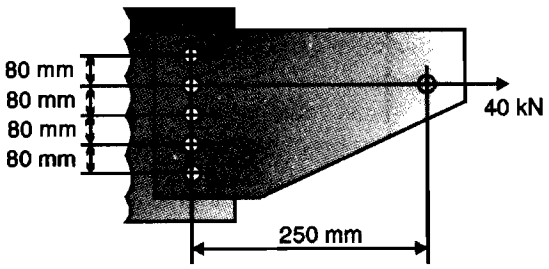


Figure 6.18

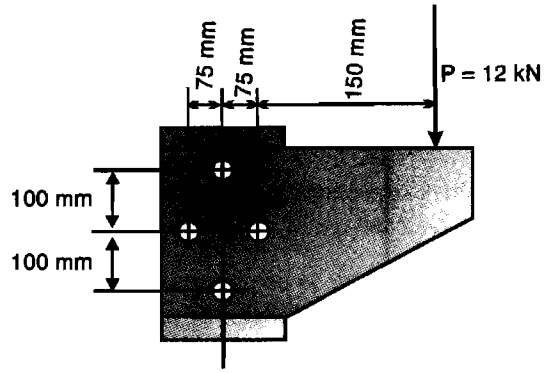


Figure 6.19

6.14 WELDED CONNECTIONS

Welding is a process of joining two or more pieces of metals. The process is of course adopted to obtain specific shapes and sizes to perform specific function. In the process of welding the temperature of metal to be jointed is raised to a level so that the metal becomes plastic or fluid. When metal is just plastic then pieces to be welded are pressed together to make the joint. When metal is melted to fluid state another metal is filled in the region of the joint and allowed to cool to solidify.

Connections between metal plates, angles, pipes, and other structural elements are frequently made by welding. Fusion welds are made by melting portions of the materials to be joined with an electric arc, a gas flame, or with thermit. In fusion welds, additional welding material is usually added to the melted metal to fill the space between the two parts to be joint or to form a fillet. A gas shields is provided when welding certain metals to prevent rapid oxidation of the molten metal. A good welded joint will usually develop the full strength of the material being joined unless the high temperature necessary for the process changes the properties of the materials.

Metals may also be joined by resistance welding in which a small area or spot is heated under high localized pressure. The material is not melted with this type of welding. Other joining methods for metals include brazing and soldering, in which the joining metal is melted but the parts to be connected are not melted. Such connections are usually much weaker than the materials being connected. Fusion welding is the most effective method when high strength is an important factor, and it will be discussed more in detail.

At present time welding has become a powerful technology and almost all joining of steels is done by welding. Welding has replaced riveting, particularly in the manufacturing of boilers and ships, and in many cases is being preferred in construction of structure. Some of the advantages of welding over riveting are as follows :

(a) The plates and sections to be joined are not weakend as happens in case of riveting. For riveting drilling or punching removes the metal from working sections thus making them weaker. The net weight of metal making the joint is less in case of welded joints. The weight added due to filling of metal is much less then the weight added by way of riveting. The butt welded joint do not require nay cover keeping the weight low.

(b) The riveted joints require a great deal of labour in marking and making holes. There is no possibility of making the riveting process automated whereas welding has become fully automatic particularly when long seams, such as in boiler are to be produced.

(c) Tight and leak proof joints are ensured by welding.

(d) Welding is a noiseless operation whereas riveting can never be noiseless

(e) Curved parts are easily joined by welding

However, following difficulties in producing good welded joints must be kept in mind. Some of these may become disadvantage of welding unless special care is taken.

(a) The parts to be joined have to be prepared carefully along the seam and arranged to have sufficient clearance so that filler metal can easily be filled.

(b) Since metal is heated to a very high temperature (to melting point in most cases) there exists a strong possibility of metallurgical changes taking place in parent metals, particularly in the close vicinity of the joint. These changes may deteriorate the mechanical properties. The loss of ductility is a major problem.

(c) Since the metal to be joined is held by clamping, residual stresses develop in the region of weld. These residual stresses are often tensile in nature and greatly affect the behaviour of metal under fatigue loading.

(d) The quality of weld is highly depended upon the welder if automated process is not used.

(e) The residual stresses may be removed and metallurgical changes reversed by heat treatment (annealing and normalising). But very large structures are difficult to heat.

(f) Stress concentration is produced where filler metal joins with the parent metal. Care must be taken in post welding clearing and grinding of joint to eliminate such stress concentration.

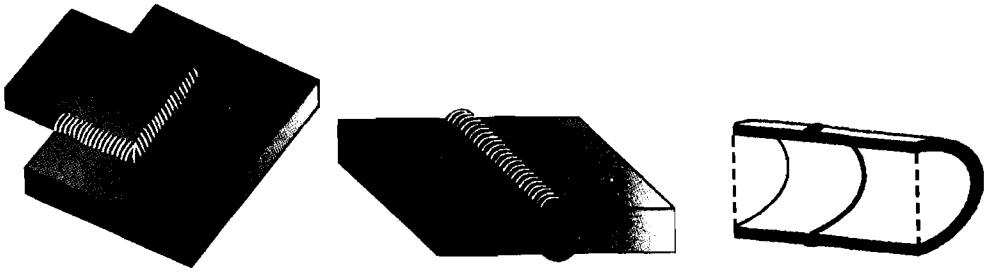
(g) The welded joints are particularly found to loss their ductility at low temperature. Combination of possible existence of defects, stress concentration and loss of ductility has been the reason of various structural failure in ships, reservoirs, pressure vessels and bridge structure.

The designer has to keep above points in mind while designing welded joins. Recommendation about treatment, grinding welds and inspection for defects must be thoroughly incorporated in design. Further, in non-automated processes the welder should be made to undergo rigorous skill tests before he is put on a job.

6.15 TYPES OF WELDING JOINTS, STRENGTH

Two types of welding joints are clearly recognized viz. joints between two plates that overlap and joints between two plates that butt with each other. Figure 6.20(a) shows a fillet joint and Figures 6.20(b) and (c) show example of butt joints. Four types of fillet joints commonly used are illustrated in Figure 6.21.

It may be of interest to note that if a weld is analyzed elastically the shearing stress distribution in the weld turns out to be distributed as shown in Figure 6.22(a). The stress is much higher at ends but quickly reduces to constant minimum. But in actual practice the ends of the weld deform plastically making distribution almost uniform. This is true if weld is ductile which is normally true. Thus, in the design of a welded joint it is reasonable to assume uniform distribution of shearing stress. However, the fillet welds are designed on the assumption that failure will occur by shearing the minimum section of the weld. This minimum section is called the throat of the weld and is shown as section AB in Figure 6.22(b). This is true in case of both parallel and transverse welds as shown in Figure 6.21 and is supported by experiments.

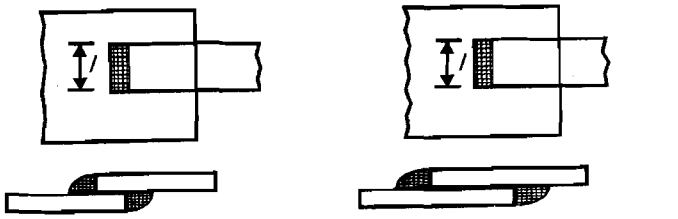


(a) Fillet Weld

(b) Belt Joint Double Vee Groove Weld

(c) Butt Joint in Pipe Single Vee Groove Weld

Figure 6.20 : Welded Joints



(a) Single Fillet Joint (Transverse)

(b) Double Fillet Joint (Transverse)

(c) Parallel Fillet Joint

(d) Compound Fillet Joint

Figure 6.21 : Types of Fillet Welded Joints

Although it is desirable to make fillet weld slightly concave, yet a reinforced weld obtained from welding is ground to obtain a triangular section with two sides equal to the thickness of plates jointed. Therefore, the width of the section AB in Figure 6.22(b) will be 0.707 t and area over which shearing will occur, for a length l, will be 0.707 $t\,l$ and force of resistance will be 0.707 $t\,l\,\tau$ as shown in Figure 6.22(c). Here τ_s is the permissible shearing stress. The permissible shearing stress is chosen as 50% of permissible tensile stress of parents metal for manual welding. In case of automated process the permissible shearing stress in the weld is assumed as 70% of permissible tensile stress of parent metal. If the load on the joints varies between P_{min} and P_{max}, the permissible stress is multiplied by a factor r, where

$$\gamma = \frac{1}{\frac{4}{3}} - \frac{1}{3}\frac{P_{min}}{P_{max}} \qquad \ldots (6.36)$$

Thus for a load P acting on a fillet welded joint of length l, is

$$P = 0.707\, t\, l\, \tau_s \qquad \ldots (6.37)$$

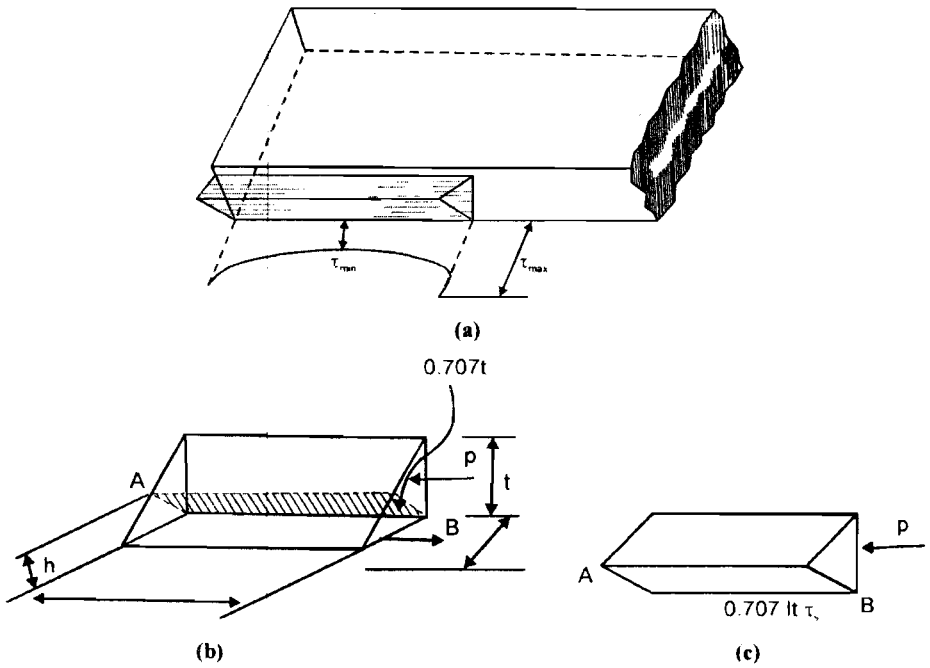


Figure 6.22

This equation will be used to calculate the design dimension l

$$l = \frac{P}{0.707\, t\, \tau_s} \qquad \ldots (6.38)$$

This length is increased by 12.5 mm to take care of starting of weld in each segment.

6.16 T-JOINT

It is a case of fillet weld but a plate is welded at right angle to another. The joint may be subject a tension, P or bending due to P acting parallel to weld, as can be seen in Figure 6.23. Two loads are shown in this figure for convenience but they will be analysed separately.

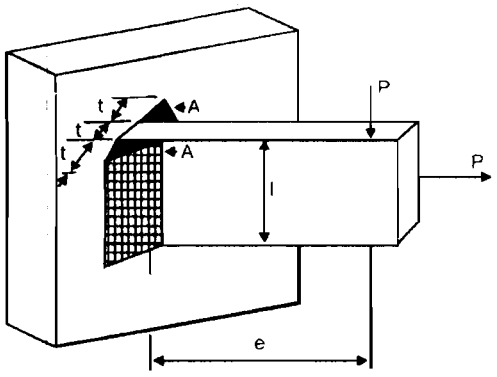


Figure 6.23 : A T-Joint under Axial and Eccentric Load

Axial Tension

It is a case of fillet weld. The leg of the weld is equal to thickness of the plate, t and the cross-section of fillet is an isosceles triangle. Thus the depth of the throat of the weld is 0.7 t as in last section. The length of the weld is l and hence the areas to resist shearing failure

$$A = 0.7\, tl \times 2$$

$$\therefore \qquad P = 1.4\, tl\, \tau \qquad \ldots (6.39)$$

where τ_s is the permissible shearing stressing the weld.

Eccentric Load

Figure 6.23 shows an eccentric load P with an eccentricity of e which is measured as distance between line of action P and the line joining the centers of gravity of triangular sections of fillet welds on two sides. The load P will have two actions on the fillet, viz. :

(a) induce shearing along the throat plane, and

(b) causes bending of throat plane of the weld.

The shearing stress due to P acting as a shearing force will be induced on the area

$$A = 0.7\, t l \times 2$$

$$P = 1.4\, t l\, \tau \qquad \ldots (6.40)$$

Fore calculating bending stress, one has to consider modulus of the section of the throat plane. This section has a width of $0.7\, t$ and depth equal to l. There are two such sections. Through both these sections are not perpendicular to the axis of the joint this fact is disregarded and modulus of section is calculated as if these sections were perpendicular to the axis.

Calling modulus of section, Z

$$Z = 2 \times \frac{1}{6} 0.7\, t\, l^2$$

Bending moment $M = P \,.\, e$

Bending stress $$\sigma = \frac{M}{Z} = \frac{6Pe}{1.4 t l^2} \qquad \ldots (6.41)$$

The value of bending stress σ occurs at top of the fillet at point A in Figure 6.23. At these points the shearing stress is given by Eq. (6.40). The maximum shearing stress can be found by using the formula,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Here $\sigma_x = \sigma$, $\tau_{xy} = \tau$ and $\sigma_y = 0$ (Eqs. (6.40) and (6.41))

$$\therefore \qquad \tau_{max} = \sqrt{\left(\frac{3P \,.\, e}{1.41 t l^2}\right)^2 + \left(\frac{P}{1.4 t l}\right)^2}$$

or $$\tau_{max} = \frac{P}{1.4 t l} \sqrt{\left(\frac{3e}{1}\right)^2 + 1} \qquad \ldots (6.42)$$

τ_{max} should be equal to permissible shearing stress τ_s for design purposes which will be taken as 50% of permissible tensile stress in parent material for manual welding and 70% of permissible tensile stress in parent material for automatic welding.

6.17 UNSYMMETRICAL SECTION LOADED AXIALLY

Figure 6.24 presents an angle section welded to a plate. If a tensile force P is applied so as to pass through the centre of gravity of the section then the length of the fillet nearer to CG (l_b) will take greater proportion of the force P than the length of fillet wild which is away from CG. The lengths l_a and l_b are to be so proportioned that the forces carried by two fillet welds exert no moment about centre of gravity axis. The two fillets are at distances of a and b respectively from CG (see Figure 6.24) and if S is the force per unit length carried in the welds, then

$$s l_a \; a - s l_b \; b = 0$$

or
$$\frac{l_a}{l_b} = \frac{b}{a}$$

or
$$\frac{l_a + l_b}{l_b} = \frac{l}{l_b} = \frac{a+b}{a}$$

$$l_b = \frac{al}{a+b} \text{ and } l_a = \frac{bl}{a+b} \qquad \ldots (6.43)$$

where $l = l_a + l_b$.

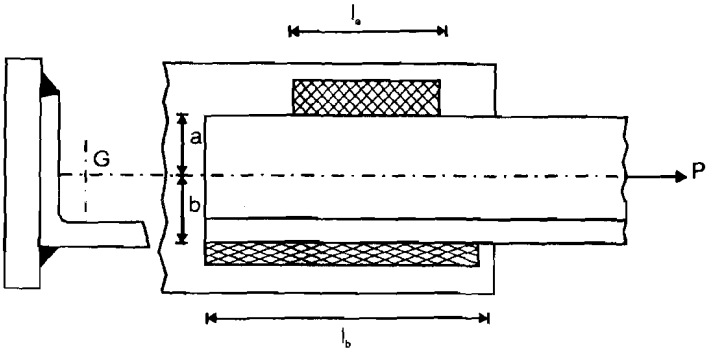


Figure 6.24 : An Unsymmetrical Section Welded to Plate and Welded along Line of CG

6.18 ECCENTRICALLY LOADED WELDED JOINT

We have earlier talked of a *T*-joint under axial load. Another example eccentric loading is shown in Figure 6.25. In this case the force applied at a distance from the CG of the weld group is in the plane of the fillet weld. This will cause the tensional effect in the same way as was considered for riveted joint in Figure 6.16. Two equal and apposite force are assumed at CG, each being equal to P will result in a couple and a single force downward at G. This force P at G will cause what we term as primary shearing stress denoted by τ_1

$$\tau_1 = \frac{P}{0.707t\,(2a+b)} \qquad \ldots (6.44)$$

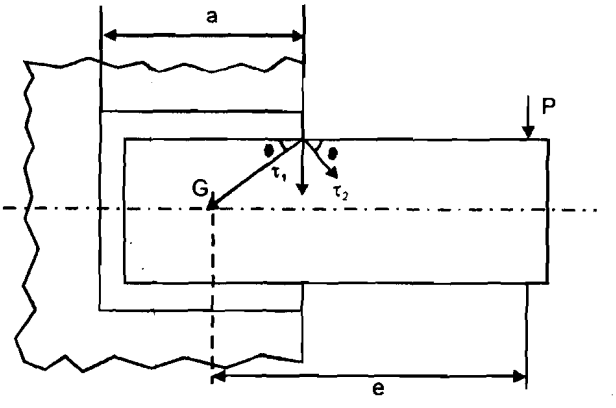


Figure 6.25 : Welded Joint Loaded Eccentrically

Here t is the thickness of the plate. The torque $P \,.\, e$ will cause secondary shearing stress τ_2 at the weld end. A, which will be greater than shearing stress at any other point in the weld. A is at a distance of R from CG. It can be shown that

$$\tau_2 = \frac{P \,.\, e \,.\, R}{J_G} \qquad \ldots (6.45)$$

Here, J_G is the polar moment of inertia of fillet weld about G. AS shown in Figure 6.25, τ_1 and τ_2 at A act at an angle Q. Since nature of both these stresses is same, they can be added vectorially. The resultant stress τ_A is

$$\tau_A = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau\ 1\ \tau_2\ \cos\theta} \qquad \ldots(6.46)$$

To solve a problem of eccentric loading such as one depicted in Figure 6.25 one would require computing the value of J_G. It is generally required to find the size of the weld, t, whereas the width involved in calculation of J_G is that of throat of the weld, say h. It is also known that $h = 0.707\ t$ so that the section of weld is an isosceles triangle. In a given problem the length a is given which is the distance from point A to the inner side of the vertical fillet while distance b will be equal to the width of the plate or distance between inner edges of horizontal fillets. To compute J_G following procedure is adopted.

(a) Determine the position of CG of weld group with reference to inside edges of vertical and horizontal fillets.

(b) Determine second moment of inertia I_{xx1} and I_{yy1}, respectively first with respect to horizontal and vertical axes passing through their CG and then transferring them to horizontal and vertical axes passing through CG of weld group.

(c) Similarly for vertical fillet determine and transfer I_{xx2} and I_{YY2} to the horizontal and vertical axes passing through CG of weld group.

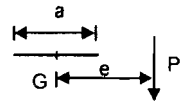
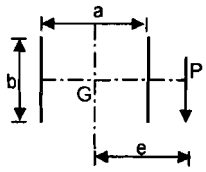
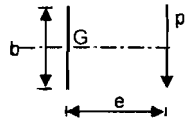
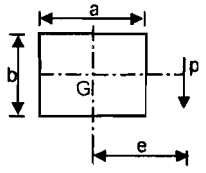
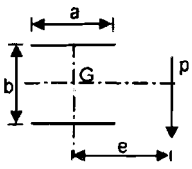
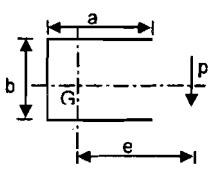
(d) By adding the four moments of inertia obtain J_G, i.e.

$$J_G = I'_{xx1} + I'_{yy1} + I'_{xx2} + I'_{yy2}$$

where quantities on right hand side refer to horizontal and vertical axes through CG of weld group.

One important point if considered during calculation will greatly help achieve solution. The expressions for I'_{xx1}, I'_{yy1}, etc. will contain terms like ah^3 and ha^3, etc. Since h is much smaller than a or b, its cube can be neglected. This will greatly reduce the efforts on computations. Finally h may be replaced by $0.707\ t$. The procedure will be illustrated in one of the following solved problems. Table 6.5 gives formulae for J_G in different weld groups.

Table 6.5 : Polar of Inertia of Weld Groups about Axis Passing through Centre of Gravity

Weld Group	J_G	Weld Group	J_G
	$\frac{ha^3}{12}$		$\frac{hb\ (3a^2 + b^2)}{6}$
	$\frac{hb^3}{12}$		$\frac{h\ (a + b)^3}{6}$
	$\frac{ha\ (3b^2 + a^2)}{6}$		$h\frac{8a^3 + 6ab^2 + b^3}{12} - \frac{ha^4}{2a + b}$

Example 6.7

A steel plate strip of 150 mm width and 10 mm thickness is welded by a compound fillet weld to another plate. The strip is required to carry an axial load P such that P is equal to tensile load capacity of the strip with a factor of safety of 2.5 on ultimate tensile strength of strip. Calculate the length of the fillet weld and show on diagram. Ultimate tensile strength of strip material is 380 MPa. Find fillet length if $P_{min} = \frac{P}{2}$ and $P_{max} = P$.

Solution

This is the type of the joint shown in Figure 6.21(d). The permissible tensile stress in strip material is $\frac{380}{2.5} = 152$ MPa. Hence permissible shearing stress in the weld, with the assumption of manual welding,

$$\tau_s = \frac{1}{2} \times 152 = 76 \text{ MPa}$$

The axial load on the joint as shown in Figure 6.26

$$P = (150 \times 10)\, \sigma_t = 1500 \times 152 = 228000 \text{ N}$$

Also $$P = 228000 = 0.707 \times 10\, (2l_1 + l_a) \times 76$$

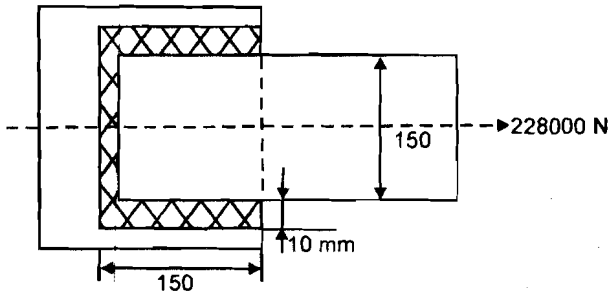


Figure 6.26

Apparently the vertical weld length, $l_2 = 150$ mm

$$\therefore \quad l_1 = \frac{228000}{2 \times 0.07 \times 76} - \frac{150}{2} = 137.16 \text{ mm}$$

Correct l_1 by adding 12.5 mm, $l_1 = 137.16 + 12.5 = 149.66$ say 150 mm when P varies between $P_{min} = \frac{P}{2}$ and $P_{max} = P$, the factor

$$r = \frac{1}{\frac{A}{3} - \frac{1}{3}\frac{P_{min}}{P_{max}}} = \frac{1}{\frac{A}{3} - \frac{1}{3}\cdot\frac{1}{2}} = \frac{1}{\frac{8-1}{6}} = \frac{6}{7}$$

$$\therefore \quad \tau_s = \frac{6}{7} \times 76 = 65.143 \text{ N/mm}^2$$

$$\therefore \quad l_1 = \frac{228000}{2 \times 0.07 \times 65.143} - 75 + 12.5 = 185 \text{ mm}$$

Example 6.8

A bracket is welded to its support as shown in Figure 6.27. All welds are fillet welds of equal thickness. Determine the fillet size if the permissible stress in the weld is 80 N/mm^2.

Solution

The welded joint of Figure 6.27 is eccentrically loaded. The weld will be subjected to two types of shearing stress. The direct primary shearing stress τ_1 due to shearing force acting through CG. The shearing force is 25 kN, through CG of weld group. The secondary shearing stress τ_2 due to torque $P.e$ acting about CG.

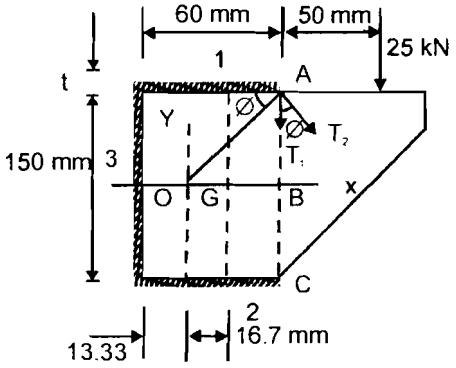


Figure 6.27

The primary shearing stress,

$$\tau_1 = \frac{P}{0.707t\,(2a + b)}$$

Use $a = 60$ mm and $b = 150$ mm, $t = ?$, $P = 25000$ N

$$\tau_1 = \frac{25000}{0.707\,(120 + 150)\,t} = \frac{131}{t}\ \text{MPa} \qquad \ldots (i)$$

Position of CG

The weld group is made up of three fillet weld, 1, 2, and 3 which are regarded as rectangles of width t and lengths respectively 60 mm, 60 mm and 150 mm. With o (middle of vertical fillet of height 150 mm) ox and oy are drawn horizontal and vertical axes. Areas of fillets 1, 2 and 3 are denoted by A_1, A_2 and A_3 so that

$$A_1 = 60\,t,\ A_2 = 60\,t,\ A_3 = 150\,t\ \text{mm}^2$$

Since the weld group is symmetrical about horizontal axis, CG will be on ox axis. Its distance from oy is $\bar{x}$. Then

$$\bar{x} = \frac{A_1\,(30) + A_2\,(30) + A_3\,(0)}{A_1 + A_2 + A_3}$$

i.e.
$$\bar{x} = \frac{60h \times 30 + 60h \times 30}{60h + 60h + 150h} = \frac{2 \times 60 \times 30}{270}$$

$$= 13.33\ \text{mm}$$

Remember $h = 0.707\,t$.

Polar Moment of Inertia of Weld Group about CG

I_{yx} and I_{yy} denote second moments of inertia of plane rectangles about horizontal and vertical axes passing through CG of respective rectangles. I'_{xx} and I'_{yy} denote moments of inertia about horizontal and vertical axes passing through CG of weld group, G. The suffixes 1, 2 and 3 will be used to distinguish between areas as 1, 2 and 3. It may also be noted that since t is much smaller than and also not known at the initial stage, it may be neglected in comparison with transfer distances 13.33 mm and 75 mm, etc.

$$I_{xx1} = \frac{60h^3}{12} = 5h^3$$

$$I_{xx2} = \frac{60h^3}{12} = 5h^3$$

$$I_{xx3} = \frac{h\,(150)^3}{12} = 281.25 \times 10^3\, h$$

$$I'_{xx1} = 5h^2 + 60h\,(75)^2 = 5h^3 + 337.5 \times 10^3\, h$$

$$I'_{xx2} = 5t^2 + 60t\,(75)^2 = 5h^3 + 337.5 \times 10^3\, h$$

$$I'_{xx3} = I_{xx3} = 281.25 \times 10^3\, h$$

$$I_{yy1} = \frac{h\,(60)^3}{12} = 18 \times 10^3\, h$$

$$I_{yy2} = \frac{h\,(60)^3}{12} = 18 \times 10^3\, h$$

$$I_{yy3} = \frac{150 \times (h)^3}{12} = 12.5\, h^3$$

$$I'_{yy1} = 18 \times 10^3\, h + 60h\,(16.1)^2 = 34.7 \times 10^3\, h$$

$$I'_{yy2} = 18 \times 10^3\, h + 60h\,(16.7)^2 = 34.7 \times 10^3\, h$$

$$I'_{yy3} = 12.5h^3 + 120h \times (13.33)^2 = 12.5h^3 + 21.32 \times 10^3\, h$$

The polar moment of inertia of the weld group about CG.

$$J_G = I'_{xx1} + I'_{xx2} + I'_{xx3} + I'_{yy1} + I'_{yy2} + I'_{yy3}$$

$$= 5h^3 + 337.5 \times 10^3\, h + 5h^3 + 337.5 \times 10^3\, h + 281.25 \times 10^3\, h$$

$$+ 34.7 \times 10^3\, h + 34.7 \times 10^3\, h + 12.5h^3 + 21.32 \times 10^3\, h$$

$$= (22.5h^3 + 1047 \times 10^3\, h)\ \text{mm}^4$$

Since h is a small quantity, contribution of its cubes, h^3, may be neglected. Hence to the first approximation,

$$J_G = 1047 \times 10^3\, h\ \text{mm}^4 \qquad \ldots \text{(iii)}$$

The secondary shearing stress at any point on the fillet at a distance of R from G is given as

$$\tau' = \frac{P.\,eR}{J_G}$$

This stress will act perpendicular to R. Obviously its value will be higher if R is higher. R at A and C is greatest. From triangle ABG.

$$R_A = \sqrt{(AB^2 + BG^2)}$$

$$= \sqrt{(75^2 + 46.7^2)} = 88.5\ \text{mm}$$

$$e = 50 + BG = 50 + 46.7 = 96.7\ \text{mm}$$

Using equation for τ' and calling its value at A as τ_2

$$\tau_2 = \frac{25000 \times 96.7 \times 88.35}{1047 \times 10^3\, h} = \frac{204}{h}\ \text{MPa} = \frac{288.5}{t}\ \text{MPa} \qquad \ldots \text{(iv)}$$

$$\cos\theta = \frac{BG}{R_A} = \frac{46.7}{88.35} = 0.53$$

Since τ_1 and τ_2 are of some type they can be added vectorially to obtain resultant stress.

$$\tau_A = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1 \tau_2 \cos\theta}$$

$$= \frac{1}{t}\sqrt{(131)^2 + (288.5)^2 + 2 \times 131 \times 288.5 \times 0.53}$$

$$= \frac{1}{t}\sqrt{17161 + 832.5 + 40067} = 374.8 \text{ MPa}$$

The stress τ_A should not exceed 80 MPa, the permissible value

$$\therefore \quad \frac{374.8}{t} = 80$$

$$\therefore \quad t = \frac{374.8}{80} = 4.685 \text{ say } 5.0 \text{ mm}$$

Example 6.9

A bracket welded to a vertical plate and geometry of weld group are shown in Figure 6.28. A load of 7.5 kN acts at a distance of 120 mm from the plane of weld group. If the permissible shearing stress in the weld is 20 MPa, calculate the width of the leg of weld.

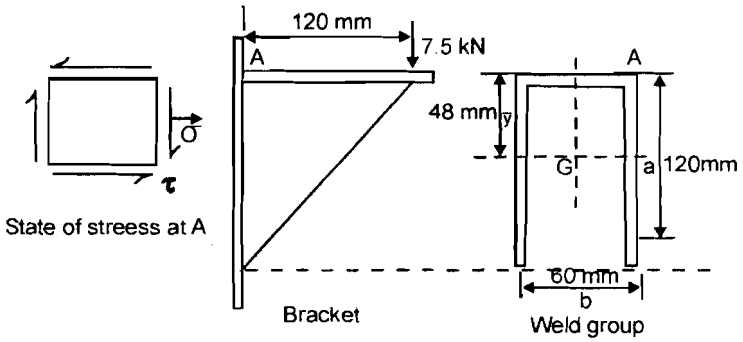


Figure 6.28

Solution

Let h the throat size of the weld and t the size of the leg. Then $h = 0.707\ t$.

It is known that failure of the weld occurs due to shearing alone throat section when the weld is in bending.

Area is horizontal fillet, $A_1 = b\ t$.

Area of vertical fillets, $A_2 = 2\ a\ t$

Distance of the CG of weld group from top or horizontal fillet

$$\bar{y} = \frac{2at\,\frac{a}{2}}{bt + 2at} = \frac{a^2}{b + 2a}$$

Substituting $a = 120$, $b = 60$ mm

$$\bar{y} = \frac{120 \times 120}{60 + 240} = 48 \text{ mm} \qquad \ldots \text{(i)}$$

Moment of inertia of horizontal fillet about horizontal axis passing through CG.

$$I_{xx1} = \frac{bh^3}{12} + bh\,\bar{y}^2$$

Moment of inertia of vertical fillets about horizontal axis passing through CG.

$$I_{xx2} = 2\frac{ha^3}{12} + 2ha\left(\frac{a}{2} - \bar{y}\right)^2$$

Hence moment of inertia of weld group about horizontal axis passing through CG.

$$I_{xx} = I_{xx1} + I_{xx2}$$

$$= bh\,\bar{y}^2 + \frac{2ha^3}{12} + \frac{ha^3}{2} + 2ha\,\bar{y}^2 - 2ha^2\,\bar{y}$$

In this expression $\frac{bh^3}{12}$ has been neglected

$$\therefore \quad I_{xx} = \frac{ha^3}{6} + \frac{ha^3}{2} + h\,\bar{y}^2\,(b + 2a) - ha^2\,\bar{y}$$

$$= \frac{2}{3}ha^3 + h\,(b + 2a)\,\bar{y}^2 - 2ha^2\,\bar{y}$$

Substituting $a = 120$ mm and $b = 60$ mm, $\bar{y} = 48$ mm

$$I_{xx} = \left[\frac{2}{3}(120)^3 + (60 + 240 - (48)^2 - 2\times(120)^2 \times 48\right] h$$

$$= [1157.8 + 691.2 - 1382.4] \times 10^3\, h$$

$$= 466.6 \times 10^3\, h \text{ mm}^4$$

The bending stress at extreme top

$$\sigma = \frac{M}{I_{xx}}\frac{a}{2}$$

i.e. $$\sigma = \frac{7.5 \times 10^3 \times 120}{466.6 \times 10^3} \times \frac{120}{2h} = \frac{115.7}{h} \text{ N/mm}^2 \qquad \ldots \text{(ii)}$$

The direct shearing stress due to shearing force of 7.5 kN

$$\tau = \frac{7.5 \times 10^3}{bh + 2ah} = \frac{7.5 \times 10^3}{h\,(60 + 240)} = \frac{25}{h} \text{ N/mm}^2$$

The maximum tensile bending stress, σ, occurs at top fillet weld where as the shearing stress τ is same at every pint of weld group. The state of stress at any point of the weld group is shown at left of Figure 6.28. The maximum shearing stress at A

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= \frac{1}{h}\sqrt{\left(\frac{115.7}{2}\right)^2 + (25)^2} = \frac{63}{h} \text{ N/mm}^2$$

τ_{max} must be equal to permissible shearing stress which is 20 MPa

$$\therefore \quad \frac{60}{h} = 20$$

or $$h = \frac{63}{20} = 31.5 \text{ mm}$$

$$\therefore \quad t = \frac{3.15}{0.707} = 4.455 \text{ say } 5 \text{ mm}$$

SAQ 3

(a) What is a welded joint? Compare welded joint with riveted joint.

(b) Describe types of welded joint.

(c) Give examples of eccentrically loaded welded joints. How are they analysed?

(d) An L 200 × 150 × 20 steel angle is to be welded to a flat plate with long side of the angle against the plate (see Figure 6.24). Determine the minimum length l_a and l_b that will cause the angle to carry maximum allowable axial load. The allowable tensile stress for the material in the angle is 124 MPa and allowable shearing stress in the weld material is 94 MPa. Each leg of the weld is 15 mm.

(e) A shaft of 20 mm diameter is welded coaxially with another shaft of much larger diameter. The shaft is to transmit a torque, which is just safe. Calculate the width of the peripheral fillet weld between two shafts. The permissible shearing stress for the shaft material is 70 MPa and that for weld is 50 MPa.

6.19 SUMMARY

The joining of plates is an important requirement of manufacturing industry. Several examples of plate joining in structure parts, ships, airplanes, boilers and pressure vessels. Welding is the process in which parent metal is almost melted by heat from electric or gas source and filler metal is filled in the gap by melting. The joint is then allowed to cool to develop strength, which is almost equal to that of parent metals. In riveting holes are drilled or punched and a rivet with head on one side of the shank is passed through two coaxial holes in two plates. A head is created on the other side to keep plates perfectly connected.

The welding is much more developed now and can be automated to enhance production rate and maintain quality. Both welded joints and riveted joints are loaded and tensile forces as well as eccentric loads, which do not pass through CG. The eccentric loads may act in the same plane as the joint in which case they tend to rotate the joint about an axis through CG and perpendicular to the plane of the joint. This produces shearing stress in the joint. The eccentric load may also act in a plane, which is parallel to the plane of the joint in which case they tend to bend the joint about an axis passing through CG in the plane of the joint. This produces bending stress in the joint.

6.20 KEY WORDS

Pitch : Pitch is the distance between the centres of adjacent rivets measured on the gauge line.

Eccentric Loading : Whenever the line of application of the load does not pass through the CG of the weld, the loading is known as eccentric loading.

Hot-riveting : In hot-riveting, the rivet with a single head is heated to a plastic state (900-1000°C) and the shank of the rivet is inserted into the hole in the members to be fastened together.

Lap Joint : In lap joint one plate overlaps the other and the rivets pass through both the plates.

Butt Joint : In butt joint, the plates are kept in alignment and a butt strap or cover plate is placed over the joint.

6.21 ANSWERS TO SAQs

SAQ 1

(e) Follow the method of Example 6.11, Section 6.11 and determine $d = 19.5$ mm, $d_1 = 18$ mm

Width of plate = 130 mm

Number of rivets = 5

Cover plate thickness = 6.6 mm

Shearing strength of 5 rivets in double shear

$$P_s = 5 \times 1.75 \times \frac{\pi}{4}(18)^2 \times 75 = 167 \text{ kN}$$

Crushing strength of 5 rivets

$$P_c = 5 \times 18 \times 10.5 \times 150 = 141.75 \text{ kN}$$

Tearing strength of plate along weakest section

$$P_{t1} = (130 - 19.5)\ 10.5 \times 105 = 121.8 \text{ kN}$$

Tearing in second row and crushing in outer most row, strength

$$P_{tc2} = (130 - 2 \times 19.5)\ 10.5 \times \frac{105}{1000} + \frac{141.75}{5} = 128.7 \text{ kN}$$

The strength of tearing in the third row will be still higher.

Strength of solid plate

$$P_t = \sigma_t bt = 105 \times 10.5 = 143.325 \text{ kN}$$

$$\therefore \quad \eta = \frac{121.8}{143.325} = 85\%$$

The pitch is determined from margin, $m = 1.5$, $d = 29.25$ mm

$$p = b - 2m = 130 - 58.50 = 71.5 \text{ mm}$$

$$p_b = 2.5d = 2.5 \times 19.5 = 48.75 \text{ mm}$$

The arrangement of 5 rivets is 1, 2, 2 in rows as shown in Figure 6.29.

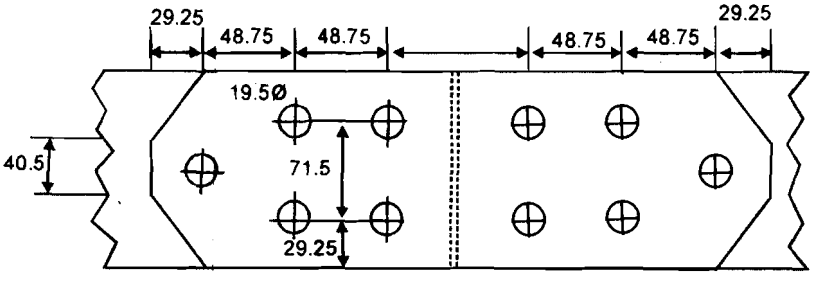


Figure 6.29

(d) Follow the procedure as in Example 6.1, Sections 6.13 and obtain

σ_t = 88 MPa, τ_s = MPa, σ_c = 128 MPa

t = 21.45 mm say 22 mm

d = 28.142 mm chose standard d = 28.5 and d_1 = 27.0 mm.

p = 139.2 mm from equating tensile strength with shearing strength of rivets in a pitch length. Check for maximum pitch

$$p_h = 65.03 \text{ mm, greater of } 0.33\,p + 0.67\,d \text{ and } 2\,d$$

$$t_c = 0.625\,t\,\frac{p-d}{p-2d} = 18.52 \text{ say } 18.5 \text{ mm}$$

$$m = 1.5\,d = 42.75 \text{ mm}$$

η = 79.5%. The least strength is in shearing of rivets. The joint is shown in Figure 6.30.

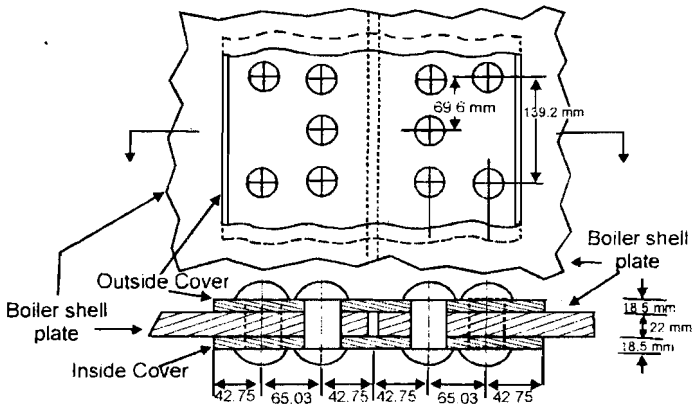


Figure 6.30

(f) Follow the method of Examples 6.1 and 6.2 of Section 6.15. The CG of rivets group is the middle one. R_{max} = 160 mm.

The primary shearing stress $\tau_1 = \frac{40}{A}$ kN/mm^2 acting horicontally to left.

The moment 40 × 250 = 10,000 kN mm will cause secondary shearing stress $\tau_2 = k\,R_2 = 160k$.

There are two forces on two extreme rivets and two others on middle two. Equating moments

$$(2k\,R_2^2 + 2K\,R_1^2)\,A = 10{,}000 \text{ kN mm, } R_1 = 80 \text{ mm, } R_2 = 160 \text{ mm}$$

$$K\,(160^2 + 80^2)\,A = 5000$$

$$K = \frac{5000}{32000\,A} = \frac{0.15625}{A}$$

$$\tau_2 = \frac{0.15625 \times 160}{A} = \frac{25}{5A} \text{ kN/mm}^2$$

Resultant stress $\tau = \tau_1 + \tau_2$

$$= \frac{5 + 40}{A} \text{ kN/mm}^2$$

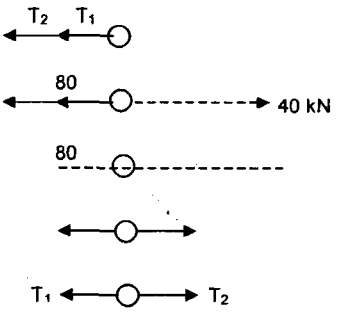


Figure 6.31

$$\therefore \quad = \frac{45 \times 10^3}{A} = 65$$

$$A = 692 \text{ mm}^2 = \frac{\pi}{4} d^2$$

$$d = 29.7 \text{ mm}$$

(g) Follow the same procedure as in SAQ 5.

The primary shear force acts at CG, which is obviously at the centre of rivet group r_1, r_3 and r_2 r_4 are equal. With $F = k\,r$

$$F_1 r_1 + F_2 r_2 + F_3 r_3 + F_4 r_4 = 12 \times 10^3 \ (150 + 75) = 27 \times 10^5 \text{ Nmm}$$

$$2k\ (r_1^2 + r_2^2) = 2k\ [(75)^2 + (100)^2] = 27 \times 10^5$$

$$1250k = 27 \times 10^5 \quad k = 2160$$

$$\therefore \quad F_1 = 2160 \times 75 = 162 \text{ kN}$$

$$F = \frac{12}{4} = 3 \text{ kN}$$

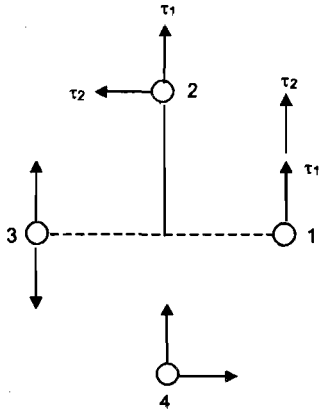


Figure 6.32

At rivet 1 shearing force = 162 + 3 = 165 kN

$$F_4 = 2160 \times 100 = 216 \text{ kN}$$

A rivet 4, shearing force $= \sqrt{F_4^2 + F^2} = \sqrt{216^2 + 3^2} = 216.02$ kN

At rivet 2 and 4 the shearing force is maximum.

$$A = \frac{\pi}{4} d^2 = \frac{216.02}{65} 10^3 = 3323.4$$

$$\therefore \quad d = 65.1 \text{ mm}.$$

(d) Refer to Figure 6.24.

Call vertical side of the angle as 1 and horizontal side as 2.

$$A_1 = 200 \times 20 = 4000 \text{ mm}^2,\ A_2 = (150 - 20)\ 20 = 2600 \text{ mm}^2$$

$$b = \frac{A_1 \times 100 + A_2 \times 10}{A_1 + A_2} = \frac{4000 \times 100 + 2600 \times 10}{4000 + 2600} = 64.55 \text{ mm}$$

$a = 200 - 64.55 = 135.45$ mm , a and b define position of G.

Assume that uniform stress is induced in the weld. Then force in the top weld $= F_a = l_a\ h\ \tau$.

A force in the bottom weld $= F_b = l_b\ h\ \tau$, h = throat size of the weld.

Taking moments about an axis through G, and perpendicular to plane of paper.

$$F_a \times a = F_b \times b$$

$$l_a\ h\ \tau\ 135.45 = l_b\ h\ \tau\ 64.55$$

$$\therefore \quad \frac{l_a}{l_b} = 0.477$$

Area of cross section of angle section, $A = A_1 + A_2 = 6600 \text{ mm}^2$

The permissible stress in the angle section 124 MPa.

The maximum load carried by angle section $= 6600 \times 124 = 818400$ N

This load has to be carried by welds

$$\therefore \quad 818400 = (l_a + l_b)\ h\ \tau_s$$

$$h = 0.707t = 0.707 \times 15 = 10.605 \text{ mm},\ \tau_s = 94 \text{ MPa}$$

$$\therefore \quad 818400 = (l_a + l_b)\ 10.605 \times 94$$

$$\therefore \quad l_a + l_b = 821$$

Use $l_a = 0.477\ l_b$

$1.477 l_b = 821$ or $l_b = 566$ mm, $l_a = 265.14$ mm

Add 12.5 mm

$l_a = 277.6$ mm, $l_b = 568.5$ mm

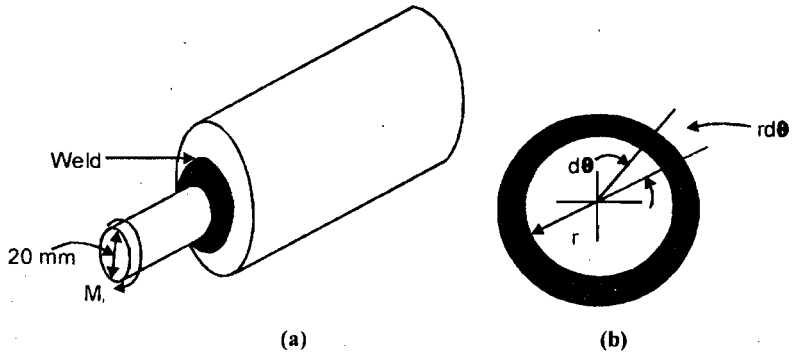


Figure 6.33

The torque that the shaft can carry safely

$$M_t = \frac{\pi}{16} d^3 \tau_s = \frac{\pi}{16} (20)^3 \times 70 = 109956 \text{ Nmm}$$

The weld has to carry the same torque.

The weld becomes a ring whose inside dia. is dia. of shaft and width is equal to the leg of the weld. The throat is h, which is effective width of the ring.

If an element is considered to subtend an angle $d\theta$ at the centre then the length of the element is $r\, d\theta$, where r = rad of the ring at the periphery of the shaft.

The shearing force in the element if stress τ is induced in the ring

$$dF = \tau\, h\, r\, d\theta$$

The moment about central axis due to dF

$$dM_t = dF\, r = \tau\, h\, r^2\, d\theta$$

$$\therefore \quad M_t = \int_0^{2\pi} \tau\, h\, r^2\, d\theta$$

$$\tau = 50 \text{ N/mm}^2,\ r = \frac{20}{2} = 10 \text{ mm},\ M_t = 109956 \text{ Nmm}$$

$$109956 = h \times 50 \times 100 \times 2\pi$$

$$\therefore \quad h = 3.5 \text{ mm} = 0.707\, t$$

$$\therefore \quad t = 4.95 \text{ say } 5 \text{ mm, width of the fillet weld.}$$