UNIT 2 ELECTROMAGNETISM, ELECTROSTATICS AND ELECTRICAL INSTRUMENTS

Structure

2.1 INTRODUCTION

In this unit, you will study the laws of electrostatics which will give the details of static electricity with importance of capacitors in storage of energy. Further the study of magnetism occurred at the time of the ancient Greeks. In 1820, Oersted discovered that current passing through a conducting wire produced a magnetic field in the space surrounding it. After this discovery, the physicists were curious to know whether a magnetic field could produce an electric current? Later in 1831, Faraday discovered that current can also be produced if a magnet is made to move near a coil. Since then, rapid advances in this field have occurred and now it is a well established fact that electricity and magnetism are deeply linked. Every electric current creates a magnetic field around it, and moving a magnet past an electric wire will induce an electric current in it. This gave rise to a branch of physics called electromagnetism.

The study of electromagnetism is important because of its numerous and varied applications. The principles of electromagnetism are responsible for the operation of most of electrical measuring instruments and electrical machinery resulting in innumerable commercial and industrial applications. This necessitates a thorough understanding of electromagnetism.

In this unit, we shall study the concept of magnetism and also, the laws of magnetism with the other basic concepts like magnetic flux, permeability etc. shall be discussed very briefly. Biot-Savart law and its simple applications have been discussed in this unit. Electromagnetic induction and their types are also given in detail. In view of the significance of electromagnetism in electrical measuring instruments, it is imperative to mention some of the electrical instruments used frequently in laboratories. Therefore, in this unit, we have discussed the working of the galvanometer, ammeter and voltmeter and their utility in electrical circuits. At last, there is a brief discussion about Hysteresis.

Objectives

After studying this unit, you should be able to

- explain the laws of electrostatics,
- calculate the capacitance of series and parallel combination capacitors,
- explain the basic concepts related to magnetism,
- state laws of magnetism and their applications,
- define self and mutual induction,
- explain eddy currents and their significance,
- distinguish between galvanometers, ammeters and voltmeters, and
- explain the phenomenon of hysteresis.

2.2 MAGNETISM

2.2.1 Magnetic Field

We have discussed about the electric charges. The charges are of two types: positive and negative charges. These charges exert force on another charge when it is kept in space around these charges. Similarly, a magnet has two ends known as North Pole and South Pole. If we bring any other magnetic pole near to it, either a force of attraction or repulsion is experienced. Later it was demonstrated that a current carrying wire behaves like an ordinary magnet. Any magnetic pole near this conductor also experiences a force. Hence, the space around a magnet in which magnetic effect or force can be experienced by any other magnetic pole is called **magnetic field**. Usually, the symbol used for magnetic field is $\vec{B}$.

Suppose the current is passing through a conductor, there would be the lines of magnetic field. These magnetic lines of force can be observed by spreading the iron fillings around the magnet. These fillings are arranged in a line pattern from one pole to another. Collectively, these lines of force are known as **magnetic flux,** which shall be discussed in detail later.

The SI unit of magnetic field is Tesla. It is represented by *T*.

$$1 \text{ Tesla} = 1 \text{ Newton Ampere}^{-1} \text{ meter}^{-1} = NA^{-1}m^{-1}$$

Magnetism was explained by atomic theory of magnetism. This theory gave some facts about magnetism. Here, we shall discuss very briefly about this theory.

In atomic theory of magnetism, both the orbital and spin motions of electrons give rise to tiny circular currents. These tiny current loops behave like small magnets that are known as "atomic magnets". When the material is not magnetised, these magnets form closed chains thereby annulling each other's effect (Figure 2.1). On the other hand when the material is magnetized, the elementary magnets are aligned in the same direction (Figure 2.2). Besides other facts, it is also explained by this theory that when a magnet is heated, the thermal energy of the elementary magnets increases. These again form closed chains and magnetism is lost.

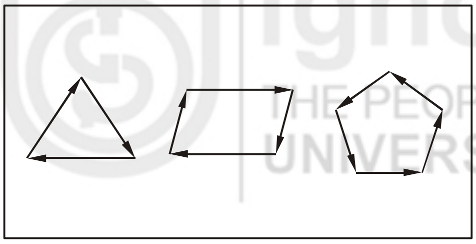

Figure 2.1

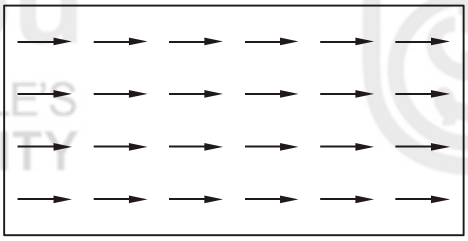

Figure 2.2

As it is noted that whenever an electric current flows through the conductor (straight conductor, coil etc.), the lines of magnetic field is produced around the conductor. The question arises that what would be the direction of these lines of magnetic field? The direction of these magnetic field lines can be determined by Right hand thumb rule or Maxwell's cork screw rule.

The right hand thumb rule states that if we hold the straight conductor in the palm of the right hand, the thumb points towards the direction of flow of current in the conductor and the direction of magnetic field is given by the direction in which four-fingers curl (Figure 2.3). On the other hand, the Maxwell's cork screw rule states that if the screw is rotated so that it progresses in the direction of flow of current through the conductor, then the circular motion of the screw shows the direction of magnetic field lines (Figure 2.4).

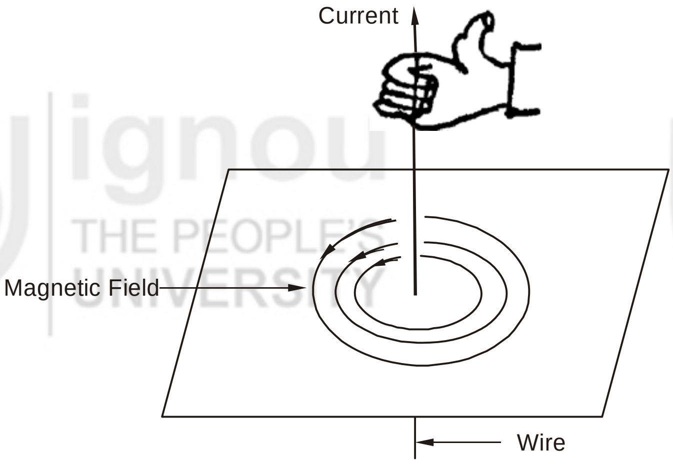

Figure 2.3 : Right Hand Thumb Rule

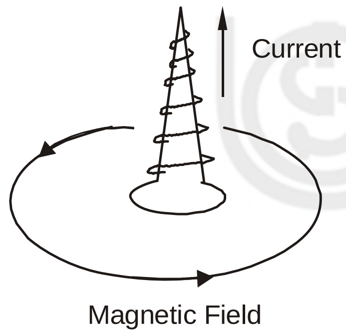

Figure 2.4 : Maxwell's Cork Screw Rule

By convention, field directed inward with reference to paper is shown with tail of an arrow as a cross (×), while that directed outwards with head or dot (.).

2.2.2 Magnetic Flux

When a coil is placed inside a magnetic field, a number of magnetic field lines pass through the coil. The number of magnetic field lines crossing the surface perpendicularly is known as **magnetic flux**. It is also measured as the product of the component of magnetic field along the normal, B_n, to the surface area, ΔA, held inside a magnetic field. It is represented by ϕ and it is a scalar quantity. The SI unit of magnetic flux is Weber (Wb) and its dimensional formula is $ML^2 T^{-2} A^{-2}$.

Mathematically, the magnetic flux can be expressed as

$$\phi = B_n \Delta A \qquad \ldots (2.1)$$

If θ is the angle between the direction of the magnetic field $\vec{B}$ with the normal to the surface area ΔA as shown in Figure 2.5, then the component

$$B_n = B \cos \theta \qquad \ldots (2.2)$$

$$\therefore \quad \phi = B \cos \theta \, \Delta A$$

$$\text{or} \quad \phi = \vec{B} \,.\, \Delta \vec{A} \qquad \ldots (2.3)$$

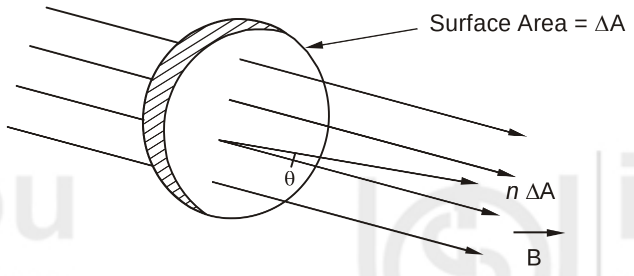

Figure 2.5 : Magnetic Field Lines Crossing the Surface

Therefore, when the direction of magnetic field is normal to the surface area then $\phi = B \Delta A$. It means that magnetic flux linked with a surface is maximum and the magnetic flux is zero, when the direction of magnetic field is parallel to the surface area.

The relation between Tesla and Weber is

$$1 \text{ Telsa} = \frac{1 \text{ Wb}}{1 \text{ m}^2} = 1 \text{ Wb m}^{-2}.$$

2.2.3 Laws in Magnetism

The Coulomb's law in magnetism is similar to the law in electrostatics between two charges. The force between two magnetic poles of strength m_1 and m_2 placed distance r apart is given by this law. According to this law, the force of attraction or repulsion between the two poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.

Mathematically, it can be written as

$$F \propto \frac{m_1\ m_2}{r^2}$$

or

$$F = \frac{k\ m_1\ m_2}{r^2} \qquad \text{. . . (2.4)}$$

In this, k is the constant of proportionality. Its numerical value is

$$k = \frac{\mu_0}{4\pi} = 10^{-7}\ \text{WbA}^{-1}\ \text{m}^{-1}$$

Here μ_o is called the absolute permeability of free space. The measure of the degree to which the lines of force of the magnetising field can penetrate or permeate the medium is called the absolute permeability of the medium.

Suppose the strength of magnetic pole $m_1 = m_2 = m$, $r = 1$ m and $F = 10^{-7}$ N, then using the Coulomb's law in magnetism, we have

$$10^{-7} = \frac{10^{-7} \times m \times m}{1^2}$$

$$m = \pm\ 1 \text{ ampere meter} \qquad \text{. . . (2.5)}$$

Therefore, from Eq. (2.5), the strength of a magnetic pole is said to be one ampere metre, if it repels an equal and similar pole with a force of 10^{-7} N, when placed in vacuum (or air) at a distance of 1m from it. The magnetic permeability of a magnetic substance gives the measure of its conducting power towards the passage of magnetic lines of induction. In general, magnetic permeability of a magnetic substance is defined as the ratio of magnetic induction (magnetic field lines inside the material crossing per unit area normally through the magnetic substance) to the magnetic intensity.

$$\mu = \frac{B}{H} = \frac{\text{Tesla}}{\text{Ampere metre}^{-1}} = \text{Tesla metre Ampere}^{-1}$$

$$= \text{Tm A}^{-1}$$

Also $\frac{\mu}{\mu_0} = \mu_r$, the relative permeability of the magnetic substance.

$$\therefore \qquad \mu = \mu_0\ \mu_r \qquad \text{. . . (2.6)}$$

Example 2.1

Calculate relative permeability of steel sheet at a magnetic field strength of $250\ \frac{\text{AT}}{\text{m}}$ when flux density is 0.8 T.

Solution

$$\mu = \frac{B}{H}$$

$$\mu_0\, \mu_r = \frac{B}{H} \Rightarrow \mu_r = \frac{B}{\mu_0\, H} = \frac{0.8}{4\pi \times 10^{-7} \times 250}$$

$$= \frac{8 \times 10^7}{3140 \times 10}$$

$$= \frac{80000 \times 10^3}{31400} = 2548$$

With the same analogy, the **ohm's law** for magnetic circuits will be
Magneto motive force = Flux × Reluctance

$$\text{mmf} = \phi \times S \qquad \ldots (2.7)$$

where ϕ is the magnetic flux and S is the reluctance.

The magnetomotive force is that force which generates a magnetic flux. This also depends on current and number of coil turns. If a coil of N turns carrying a current of I A, then mmf = NI ampere turns. The unit of mmf is Ampere turns (AT). The reluctance in magnetic materials is similar to resistance in electrical circuits, which offered opposition. Therefore

$$\text{Reluctance} = \frac{1}{\text{Permeability}} \times \frac{\text{Length}}{\text{Area}}$$

$$S = \frac{1}{\mu} \times \frac{l}{A} \qquad \ldots (2.8)$$

The permeability, μ, in magnetic material is similar to conductivity, σ, in electrical circuits. In terms of relative permeability

$$S = \frac{1}{\mu_0\, \mu_r} \times \frac{l}{A} \qquad \ldots (2.9)$$

Another equation related to magnetic field is the divergence of magnetic field. Maxwell gave an equation, which describes the behaviour of electromagnetic fields. The equation is

$$\text{Div}\, \vec{B} = \vec{\nabla} \,.\, \vec{B} = 0 \qquad \ldots (2.10)$$

This is one of the four equation of Maxwell, which describes the behaviour of electromagnetic fields. In Cartesian coordinates, the above equation can be written as

$$\vec{\nabla} \,.\, \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \qquad \ldots (2.11)$$

The divergence of electric field is a measure of the density of the sources of the electric field. The divergence of magnetic field can be similarly interpreted as the source density for field $\vec{B}$. What is the physical significance of Eq. (2.10)? If its source density is zero everywhere, how is the field generated at all? The physical interpretation of this equation is that one cannot find sources of $\vec{B}$ of single sign, i.e. magnetic monopoles do not exist. This implies that the lines of $\vec{B}$ are closed loops. One cannot predict from where a given line starts or terminates.

SAQ 1

An iron ring having mean diameter 25 cm and cross-sectional area 2 cm^2 is uniformly wound with 400 turns and carries a current of 5A. The permeability of iron is 450. Calculate

(a) Magnetomotive force flux,

(b) Reluctance, and

(c) Flux.

2.3 BIOT-SAVART'S LAW AND ITS APPLICATIONS

When the current flows through the conductor, the magnetic field is produced around it. Biot-Savart's law gives the magnetic field produced at a point due to a current carrying conductor.

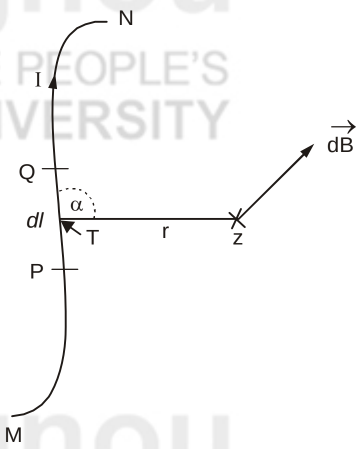

Figure 2.6

Consider a conductor *MN* in which current *I* is flowing and as a result magnetic field is produced around the conductor. Let *z* is the point at a distance *r* from the centre (*T*) of the length *PQ* (*dl*). The magnetic field produced due to this small length element *dl* at a point *z* is

(a) directly proportional to the current

$$dB \propto I \qquad \ldots (2.12)$$

(b) directly proportional to the elementary length

$$dB \propto dl \qquad \ldots (2.13)$$

(c) directly proportional to the sine of the angle α between the direction of the flow of current and the line joining the elementary length to the observation point *z*,

$$dB \propto \sin \alpha \qquad \ldots (2.14)$$

(d) inversely proportional to the square of the distance *r* between the point *z* and central point *T* of the elementary length,

$$dB \propto \frac{1}{r^2} \qquad \ldots (2.15)$$

Combining Eqs. (2.12), (2.13), (2.14) and (2.15), we get

$$dB = \frac{K\, I\, dl \sin \alpha}{r^2} \qquad \ldots (2.16)$$

where *K* is a constant of proportionality.

In SI unit, the value of *K* for vacuum is

$$K = \frac{\mu_0}{4\pi} = 10^{-7}\,\text{Wb}\,\text{A}^{-1}\,\text{m}^{-1} \qquad \ldots (2.17)$$

where μ_0 is called absolute permeability of vacuum or free space.

$$\therefore \quad dB = \frac{\mu_0}{4\pi}\frac{I\,dl \sin\alpha}{r^2} \qquad \ldots (2.18)$$

Eq. (2.18) gives the magnitude of the magnetic field produced due to a small current element.

The magnetic field due to the whole conductor *MN* at a point *z* can be obtained by integrating *dB* over the entire length of the conductor as

$$B = \int dB = \frac{\mu_0}{4\pi}\int \frac{I\,dl \sin\alpha}{r^2} \qquad \ldots (2.19)$$

2.3.1 Applications of Biot-Savart's Law

Magnetic Field due to Current Flowing in a Straight Conductor

The magnetic field in a straight conductor *MN* in which current *I* is flowing can be determined using Biot-Savart's law. The point *z* is at a perpendicular distance *a* from the conductor at which magnetic field is to be determined.

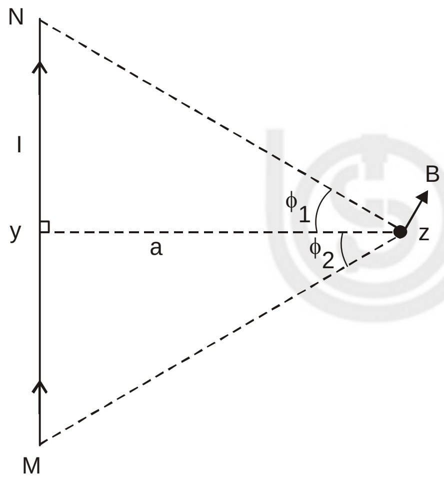

Figure 2.7

The expression for magnetic field due to current flowing through the whole conductor *MN* is given here without proof :

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a}(\sin\phi_1 + \sin\phi_2) \qquad \ldots (2.20)$$

Eq. (2.20) is the expression of magnetic field due to a current carrying straight conductor of finite length. For infinitely long conductor the magnetic field at point *z* can be obtained by putting $\phi_1 = \phi_2 = \frac{\pi}{2}$ in Eq. (2.20) as

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a}\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right) \qquad \ldots (2.21)$$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a} \qquad \ldots (2.22)$$

The direction of magnetic field can be obtained by right hand thumb rule as discussed earlier.

Magnetic Field at a Point on the Axis of a Loop

The magnetic field due to a circular loop of radius *a* in *yz* plane can also be determined by using the Biot-Savart's law. Let *P* is the point on the axis of the loop at distance *x* from its center *O*.

The expression for magnetic field at point *P* is again given here without proof :

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I\ \pi\ a^2}{(a^2 + x^2)^{\frac{3}{2}}} \qquad \ldots (2.23)$$

And, the magnitude of magnetic field at point *P* due to a circular loop of *n* turns is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi\ n\ I\ a^2}{(a^2 + x^2)^{\frac{3}{2}}} \qquad \ldots (2.24)$$

If the point *P* is taken at the centre of the loop, then $x = 0$. Therefore the magnitude of magnetic field due to circular loop of *n* turns at its centre is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I n}{a} = \frac{\mu_0\ n\ I}{2a} \qquad \ldots (2.25)$$

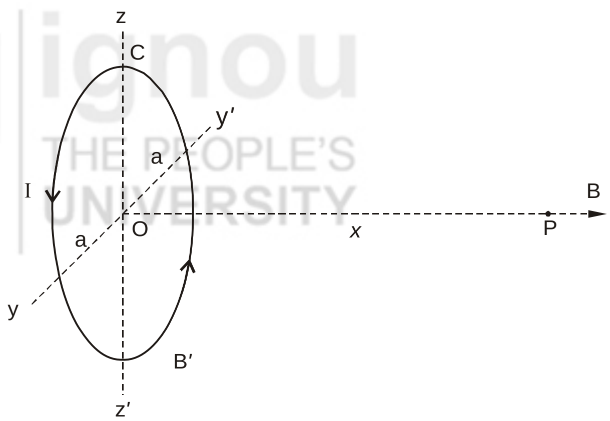

Figure 2.8

SAQ 2

(a) Calculate the magnitude of the magnetic field at a point 10 cm away from the straight wire which carries a current of 5 A in it.

(b) A circular coil is of 1000 turns and of diameter 0.50 m. Calculate the magnetic field due to this coil if it is carrying a current of 7 A at a point on the axis of the coil at a distance of 0.15 m. What would be the magnetic field if the point is taken at the center of the coil.

In the next section, we will study about the concept of electromagnetic induction and their types.

2.4 ELECTROMAGNETIC INDUCTION

As mentioned earlier, whenever the current flows through the conductor, magnetic field is produced around the conductor and this phenomenon is known as magnetic effect of electric current. Later it had been discovered by performing various experiments that the change in magnetic flux linked with a coil leads to another effect known as **electromagnetic induction**. Hence, electromagnetic induction is a phenomenon of the production of electric current in a coil, when the magnetic flux linked with the coil changes. This is converse to the magnetic effect of electric current.

There are many ways of changing the magnetic flux through a coil that results to setup an induced emf. Some of ways are :

(a) varying the magnetic field,

(b) changing the area A of the loop, and

(c) varying the relative orientation of the coil w. r. t. magnetic field.

Faraday conducted many experiments by changing the flux linked with the coil either by moving the coil (Figure 2.9) or the source of magnetic field (Figure 2.10) toward or away from coil. It is noted that when the magnetic flux linked with the coil changes, an emf is produced across the two ends of the open coil (an electric current flows in the coil provided the coil is closed). The emf (electric current) so produced is called induced emf (induced current). Some of the results obtained by him in the form of Laws are known as **Faraday's laws of electromagnetic induction**.

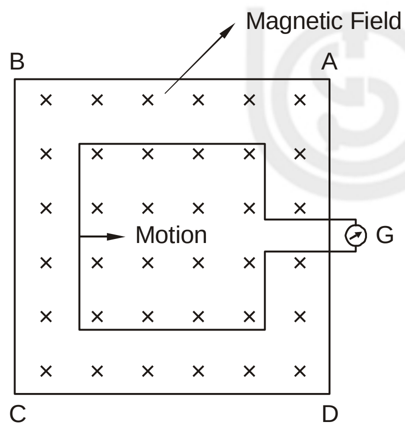

Figure 2.9 : Coil Moving in a Stationary Magnetic Field

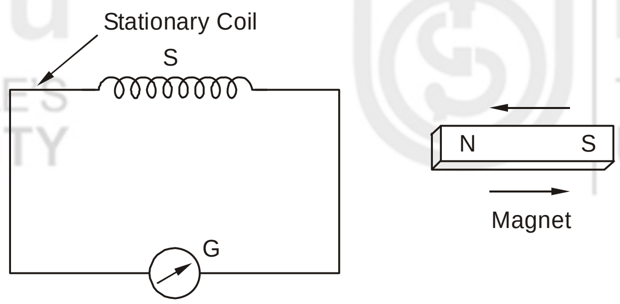

Figure 2.10 : Magnet Moving Towards/Away from Stationary Coil

2.4.1 Faraday's Laws

First Law

Whenever there is a change in magnetic flux linked with a coil, an emf and consequently a current is induced in the circuit. However, it lasts as long as the change in magnetic flux takes place.

Second Law

The magnitude of the induced emf is directly proportional to the rate of change of magnetic flux linked with the circuit. Mathematically, it can be written as

$$\text{Rate of change of magnetic flux} = \frac{\phi_2 - \phi_1}{t}$$

where ϕ_1 is the initial flux ($t = 0$), ϕ_2 is the flux after time t, then $(\phi_2 - \phi_1)$ is the change in flux in time t.

If e is the induced emf, then

$$e = k\,\frac{\phi_2 - \phi_1}{t} \qquad \text{. . . (2.26)}$$

where k is a constant of proportionality. In SI unit, the constant of proportionality is unity. Therefore,

$$e = \frac{\phi_2 - \phi_1}{t} \qquad \text{. . . (2.27)}$$

If $d\phi = \phi_2 - \phi_1$ is small change in magnetic flux in small time dt, then

$$e = \frac{d\phi}{dt} \qquad \text{. . . (2.28)}$$

For a coil, which consists of N turns, emf induced is

$$e = N\,\frac{d\phi}{dt} \qquad \text{. . . (2.29)}$$

2.4.2 Lenz's Rule

It may be noted that the Faraday's laws have not mentioned about the direction of induced emf and the direction of current. The direction of induced emf was taken care of by Lenz's rule. **This rule states that the induced current produced in a circuit always flow in such a direction that it opposes the change or the cause that produces it.** It is taken into account by inserting negative sign in the expression of induced emf in Eq. (2.29).

$$e = -\,N\,\frac{d\phi}{dt} \qquad \text{. . . (2.29(a))}$$

Here the negative sign indicates that the induced emf is such that it opposes the change in magnetic flux.

This law is in accordance with the principle of conservation of energy. It can be explained further as we have noted that induced emf opposes the change that produces it. It is this opposition against which one performs mechanical work in causing the change in magnetic flux. The electrical energy in the form of induced emf or induced current, which is converted at the expense of mechanical energy.

2.4.3 Eddy Currents

Faraday found that when magnetic flux linked with coil changes, induced emf or induced current is produced due to conductor of the coil providing a path for current to flow. These induced currents are set up in the conductor in the form of closed loops and look like '**eddies**' and, therefore, known as **eddy currents**. They are also known as Foucault's currents. In simple words, eddy currents are the currents induced in a conductor, when placed in a changing magnetic field.

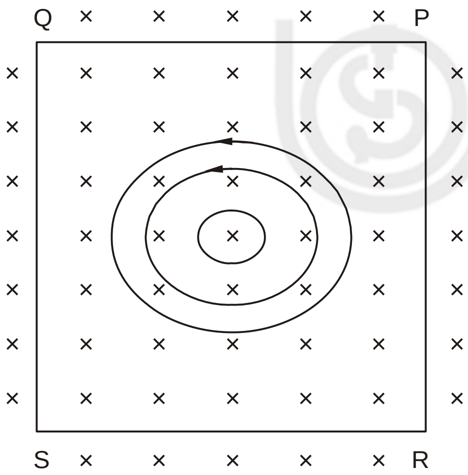

Figure 2.11 : Eddy Currents Flowing in a Metal Plate in Increasing Magnetic Field

Eddy currents, as shown in Figure 2.11, are flowing in a metal plate placed in an increasing magnetic field which is perpendicular to the plane of the paper and directed inwards.

Applications of the eddy currents are made in instruments such as deadbeat galvanometer, induction motors, speedometers, electric brake etc. Here some of them will be discussed briefly.

Applications of Eddy Currents

In automobiles, the speedometer is fixed. In the speedometer, a small magnet that is mounted on an aluminum cylinder with the help of hairspring is geared to the main shaft of the vehicle. As the magnet rotates with the speed of the vehicle, it produces eddy currents in the drum that tries to oppose the motion of the rotating magnet. As a result, the drum experiences a torque and gets deflected through certain angle. A pointer attached to the drum moves over a calibrated scale, which directly indicates the speed of the vehicle.

Electric brakes is another application of eddy currents. These brakes are usually employed in wheels of the trains. A metallic drum is coupled to the wheels of the train. As the train runs, the attached drum also rotates. A strong magnetic field is applied to the rotating drum to stop the train. The large eddy currents produced oppose the motion of the drum. Since the drum is connected to the wheels of the train, the latter comes to halt.

The oscillations of moving coil galvanometers generally take a long time to die out. But by winding its coil on a metallic frame that is made up of aluminum or copper, the galvanometer can be made deadbeat. Because of the production of eddy current in the metallic frame, the coil of the galvanometer comes to rest very soon.

Besides various applications of eddy currents such as speedometers, electric brakes etc., eddy currents produce a large amount of heat. Therefore, it is also important to know how these eddy currents can be minimized? This is because heating effect of eddy currents is undesirable in some cases like transformers, dynamos, etc. where the coil is wound on an iron core. In the core of a transformer, the eddy currents are reduced by dividing the solid iron core into number of thin sheets, which are electrically insulated from each other by coating them with varnish. The resistance of the iron strip is much larger than the core as its effective cross-sectional area is very small. Therefore, eddy currents are considerably reduced and transformer is saved from getting heated.

After studying about eddy currents and their applications, now we will discuss the types of induction.

2.4.4 Types of Induction

Self Induction

Let us consider a coil, *L*, connected to a battery B through a tapping key K as shown in Figure 2.12. As the key is pressed, the current starts flowing through the coil. The current in the coil begins to grow and hence magnetic flux linked with the coil also begins to increase. This in turn produces induced emf in the coil in accordance with the Faraday's laws of

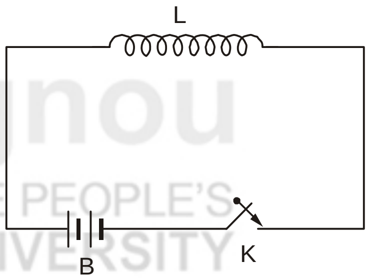

Figure 2.12 : Self-Inductances

electromagnetic induction. The direction of the induced emf in the coil is such that it opposes the growth of current in the coil according to Lenz's law. Thus, the current in the coil attains maximum value. When the key is released, the current in the circuit starts decaying from maximum to zero value as a result of which the magnetic flux linked with the coil also starts decreasing. The emf is induced in the coil such that it opposes the decay of current. As a result, the current does not become zero instantaneously but takes some time to become zero. Hence **self-inductance is the phenomenon of production of induced emf in a coil as a result of change in current or magnetic flux in the coil itself.**

Let current I flows through the coil when key K is pressed and ϕ is the magnetic flux linked with the coil, then at any instant ϕ is directly proportional to the current passing through it at that instant.

$$\phi \propto I$$

or

$$\phi = LI \qquad \ldots (2.30)$$

where L is a constant of proportionality called coefficient of self-induction or self-inductance of the coil.

If the current $I = 1$, then according to Eq. (2.30), $L = \phi$. Hence, the self-inductance of a coil is numerically equal to the magnetic flux linked with the coil when unit current flows through it.

Self-inductance of a coil depends upon the number of turns, cross-sectional area and permeability of the core on which the coil is wound.

On differentiating Eq. (2.30) both sides w. r. t. time t and suppose e is the induced emf produced in the coil due to self-inductance at the instant, then

$$-N\frac{d\phi}{dt} = -\frac{d}{dt}(LI) \qquad \ldots (2.31)$$

Using Eq. (2.29(a)), it becomes

$$e = -L\frac{dI}{dt} \qquad \ldots (2.32)$$

Here, the negative sign shows the opposing nature of emf and if we take $\frac{dI}{dt} = 1$, where $\frac{dI}{dt}$ is the rate of change of current in the coil, then considering only magnitude, $L = e$.

Hence, self-inductance of a coil is numerically equal to the induced emf produced in the coil when the rate of change of current in it is unity.

Eq. (2.32) can be rewritten

$$L = -\frac{e}{\frac{dI}{dt}}$$

The unit of self-inductance can be found as :

$$L = -\frac{e}{\frac{dI}{dt}} = \frac{1\ \text{Volt}}{1\ \text{ampere/second}} = \frac{V}{As^{-1}} = 1\ \text{Henry.}$$

Therefore, SI unit of self-inductance is Henry (H). The self-inductance of a coil is said to be of one henry if the rate of change of current of one ampere per second induced an emf of one volt in the coil.

The physical importance of self-inductance in electrical circuit is that it slows down the change in current. This is because the induced emf opposes the growth of current and hence the current cannot attain its maximum value instantaneously but takes some time to do so. This time may be of several seconds if the inductance is large.

Example 2.2

A uniform magnetic flux density of 0.2 Wbm^{-2} extends over a plane circuit of area 2 m^2 and is normal to it. How quickly must the field be reduced to zero if an emf of 100 V is to be reduced in the circuit?

Solution

Flux through the circuit is given by

$$\phi = BA$$

Using Faraday's Law, an induced emf $e = -\frac{\text{Change in flux}}{\text{Time}}$

$$e = -\frac{0 - BA}{t}$$

As negative sign indicates that the emf (e) induced is in opposition to the change causing it but has no other significance. Therefore time is

$$= \frac{BA}{e} = \frac{0.2 \times 2}{100} = 4 \times 10^{-3}\ s = 4\ \text{ms}$$

Energy Stored in an Inductor

We know $\qquad e = -L\frac{dI}{dt} \qquad \ldots (2.33)$

Assuming I grows linearly from 0 to I_0 in time t

$$\frac{dI}{dt} = \frac{I_0 - 0}{t} = \frac{I_0}{t},$$

Average voltage $= \frac{LI_0}{t}$ and average current $= \frac{\frac{1}{2} I_0 t}{t} = \frac{1}{2} I_0$.

The total work done in maintaining a current I_0 in the coil can be obtained by,

$$w = \text{Average voltage} \times \text{Average current} \times t$$

or $$w = \frac{L I_0}{t} \times \frac{I_0}{2} \times t$$

$$\therefore \quad w = \frac{1}{2} L I_0^2 \qquad \text{. . . (2.34)}$$

Mutual Induction

In mutual induction, two coils P and S are placed close to each other in contrast to only one coil in the case of self-induction. These two coils are known as primary and secondary coil respectively. As shown in Figure 2.13, the battery (B) and key (K) are connected to the primary coil (P) and galvanometer (G) is connected with the secondary coil (S). The two circuits are said to be coupled circuits. As soon as the key K is pressed, an induced emf is produced in the P coil due to self-induction which opposes the growth of current in the P coil. The magnetic flux set up by current flowing in coil P starts linking with the S-coil being closer to P-coil and an induced emf is set up in S-coil. This induced emf in S-coil also produces current in the P-coil as its circuit is closed by the galvanometer G. Therefore, the pointer of the galvanometer gives deflection in one direction (say to left).

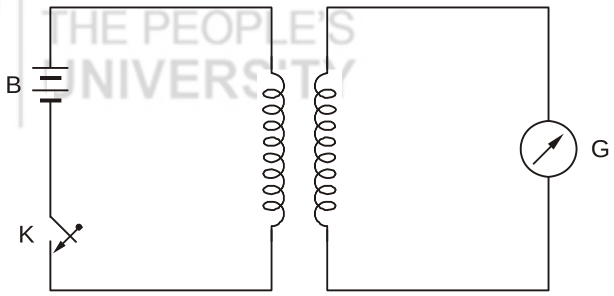

Figure 2.13 : Mutual Inductance

But when the key is released, the magnetic field of *P*-coil collapses and induced emf is produced in the primary as well as in the secondary coil. The sign of the induced emf produced in this case will be opposite to that produced when the key is pressed. The galvanometer gives deflection in other direction (say to right). Therefore, it is observed that during both growth and decay of the current in the primary coil, an opposing induced emf is produced in the secondary coil. It is observed that self-induction and mutual induction occur simultaneously. Hence, **Mutual Induction is a phenomenon of production of induced emf in one coil due to the varying current or magnetic flux in another coil.**

Suppose I is the current flowing in the P-coil and ϕ is the magnetic flux of P-coil linked with the S-coil, therefore

$$\phi \propto I$$

$$\phi = M I \qquad \ldots (2.35)$$

where M is a constant of proportionality called coefficient of mutual inductance or mutual inductance. If $I = 1$, then

$$M = \phi \qquad \ldots (2.36)$$

Thus, mutual inductance of two coils is equal to the magnetic flux linked with one coil when unit current flows through the other coil.

Mutual induction between two coils depends upon the shape and size of the two coils, separation between them and the permeability of the material of the core on which two coils are wound. It also depends upon the manner in which the coils are oriented relative to each other.

If e is the induced emf in the S-coil, then induced emf can be related with the mutual inductance. Differentiating Eq. (2.35) w. r. t. time 't', and using Faraday's law, we get

$$e = \frac{d\phi}{dt} = M \frac{dI}{dt} \qquad \ldots (2.37)$$

But applying Lenz's law also, we get

$$\therefore \qquad e = -M \frac{dI}{dt} \qquad \ldots (2.38)$$

If the rate of change of current in the P-coil, $\frac{dI}{dt} = 1$,

Then $\quad e = M \quad$ (numerically) $\qquad \ldots (2.39)$

Therefore, mutual inductance of two coils is equal to the induced e.m.f. set up in the one coil when the rate of change of current in the other coil is unity.

From Eq. (2.37), it can be rewritten as

$$M = \frac{e}{\frac{dI}{dt}} = \frac{1 \text{ Volt}}{1 \text{ Ampere/second}^{-1}} = 1 \text{ Henry.}$$

The SI unit of mutual inductance is Henry (H).

Hence, the mutual inductance of two coils is said to be of 1 Henry, if a current changing at the rate of 1 Ampere per second in one coil induces an emf of one volt in other coil.

2.4.5 Solenoid

A solenoid can be obtained when a current carrying wire is wound tightly on the surface of a cylindrical tube. Its length is very large as compared to its diameter.

Consider a solenoid of length l and area of cross-section A. If N is the total number of turns over a length l, then n, the number of turns per unit length of the solenoid, equals $\frac{N}{l}$. Let I is the current passing through the solenoid, then the magnetic field inside a solenoid is given by

$$B = \mu_0 \, n \, I \qquad \ldots (2.40)$$

The magnetic field given by the above formula is only at points well inside the solenoid. The magnetic flux passing through each turn of the solenoid

$$\phi_1 = B \times \text{Area of each turn} = \mu_0\, n\, IA$$

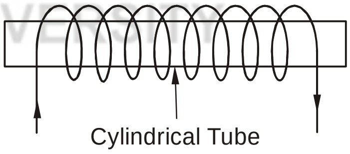

Figure 2.14 : A Solenoid

Total magnetic flux linked with the solenoid, ϕ, is

$$\phi = \text{Magnetic flux linked with one turn} \times \text{Total Number of turns}$$

$$\phi = \mu_0\, n\, IA \times N$$

$$\phi = \mu_0\, n\, IA \times n\, l$$

$$\therefore \quad \phi = \mu_0\, n^2\, I\, A\, l \qquad \ldots (2.41)$$

Comparing Eq. (2.41) with Eq. (2.30) ($\phi = LI$), we get

$$L = \mu_0\, n^2\, Al$$

This is the expression of self-inductance of a long solenoid in terms of length l, area of cross-section A having n number of turns per unit length.

SAQ 3

(a) Find the magnitude of emf induced in a 100 turns coil with cross-sectional area of 0.16 m^2, if the magnetic field through the coil changes from 0.10 Wbm^{-2} to 0.70 Wbm^{-2} at a uniform rate over a period of 0.02 second.

(b) The air coil solenoid has l = 15 cm and inside diameter D = 1.5 cm. Calculate inductance if coil has 900 turns with total flux 1.33×10^{-7} Wb, when coil current is 100 mA.

2.5 STATIC ELECTRICITY

After studying the Kirchhoff's law now we focus on static electricity. Everybody is neutral under normal conditions, as it is containing equal number of protons (+ ve charge) and electrons (– ve charge). A body contains + ve charge if some electrons are removed from the neutral body. Similarly, a body contains – ve charge when some electrons are added to it. Thus, a body is said to be charged when it possesses excess or deficit of electrons from its normal share.

When two charged bodies are separated by some insulating medium, the electrons (i.e. charges) do not move but remain static on the body. This is called **Static Electricity** and the branch of engineering which deals with static electricity is called **Electrostatics**.

2.5.1 Laws of Electrostatics

Laws of electrostatics, called as Coulomb's laws of electrostatics, are stated as under :

First law

Like charges repel each other whereas unlike charges attract each other.

Second law

The force of repulsion or attraction between two point charges Q_1 and Q_2 is

(a) directly proportional to the product of their charges,

(b) inversely proportional to the square of distance between them, and

(c) depends in the nature of medium in which charges are placed.

Mathematically,

$$F \propto \frac{Q_1\, Q_2}{d^2}$$

or,
$$F = K\, \frac{Q_1\, Q_2}{d^2} \qquad \text{. . . (2.42)}$$

where Q_1 and Q_2 = Strength of two point charges in Coulomb,

d = Distance between two charges in metres, and

K = A constant whose value depends upon the medium in which the charges are placed.

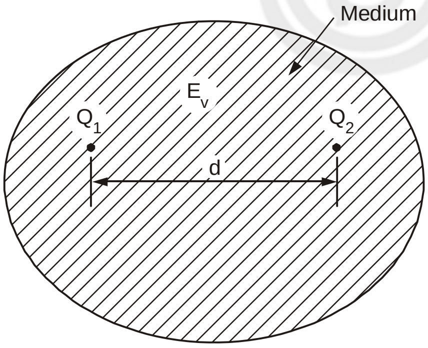

Figure 2.15

In SI units, the value of K is given by

$$K = \frac{1}{4\pi\, \varepsilon_0\, \varepsilon_r} \qquad \text{. . . (2.43)}$$

where ε_0 = Absolute permittivity of free space or vacuum or air. Its value in SI units is 8.854×10^{-12} F/m, and

ε_r = Relative permittivity of the medium, between the charges, with respect to free space or air.

For air, ε_r =1. If Q_1 = 1 Coulomb, Q_2 = 1 Coulomb

D = 1 metre

then, $$F = \frac{1 \times 1}{4\pi \times 8.854 \times 10^{-12} \times 1 \times (1)^2} \text{ (in air } \varepsilon_r = 1)$$

$$= 8.99 \times 10^{9} \text{ Newtons}$$

$$\approx 9 \times 10^{9} \text{ Newtons.}$$

Hence, a unit charge, i.e. one Coulomb is defined as that charge which when placed at a distance of 1 metre in air from an equal and similar charge, experiences a force of repulsion of 8.99×10^9 Newton or approximately
9×10^9 Newton.

2.5.2 Electric Field

Earlier it was explained that a force exists between charged bodies – for like charges repulsion and for unlike charges attraction force. Similar forces occur between magnetic poles and similar to magnetism a force field exists around electrically charged bodies. The existence of electrical lines of force, however, cannot be demonstrated as in magnetism. These lines of force are called electric flux.

Electric charge (Q) and electric flux (ψ) are both measured in Coulomb (C). A body that is charged to Q Coulombs emits a total electric flux of Q Coulombs. The symbol generally used for electric flux is ψ, so,

$$\text{Electric flux, } \psi = Q \text{ Coulomb} \quad \ldots (2.44)$$

The electric flux density (D) is flux per unit area. So, electric flux density

$$D = \frac{\Psi}{A} \quad \ldots (2.45)$$

or $$D = \frac{Q}{A} \quad \ldots (2.46)$$

Its unit is Coulomb/m^2 or C/m^2.

The electric field strength (E) between the two plates across which a voltage V is applied is given by,

$$E = \frac{V}{d} \quad \ldots (2.47)$$

here V is applied voltage and d is distance between plates. Its unit is volt/meter (V/m).

2.5.3 Permittivity

While discussing electrostatics a certain property of the medium called permittivity plays an important role. Permittivity is the property of the medium which affects the magnitude of force exerted between two point changes. The greater the permittivity the lesser the force. It is a property of dielectric material which assists to pass the electric flux through it. Its analogous is permeability in magnetic circuits. Like permeability, it can be divided in two parts, permittivity of free space ε_0 and relative permittivity ε_r. It is known that in a medium the ratio of electric flux density D and electric field strength E is constant and this constant is called permittivity of the medium.

Thus, $$\varepsilon = \frac{D}{E} \quad \ldots (2.48)$$

or, $$\varepsilon_0 \, \varepsilon_r = \frac{D}{E} \quad \ldots (2.49)$$

Here $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m . . . (2.50)

The absolute permittivity of air or vacuum, epsilon$_0$ (ε_0), is minimum and is equal to 8.854×10^{-12} F/m. The value of absolute permittivity of all other insulating materials is more than ε_0.

The ratio of absolute permittivity of some insulating material, ε, to the absolute permittivity of air or vacuum, ε_0, is called the relative permittivity of that material.

Thus, $\varepsilon_r = \dfrac{\varepsilon}{\varepsilon_0}$ or $\varepsilon = \varepsilon_0 \, \varepsilon_r$ . . . (2.51)

where ε = Absolute or whole permittivity of material,

ε_0 = Absolute permittivity of air or vacuum = 8.854×10^{-12} F/m, and

ε_r = Relative permittivity of material.

For example, for air, $\varepsilon_r \text{ (air)} = \dfrac{\varepsilon}{\varepsilon_0} = \dfrac{\varepsilon_0}{\varepsilon_0} = 1$ . . . . (2.52)

2.5.4 Capacitor

A device which stores electrical charge is called a **capacitor**.

Actually, a capacitor is a component that consists of a layer of insulating material sandwiched between two metal plates.

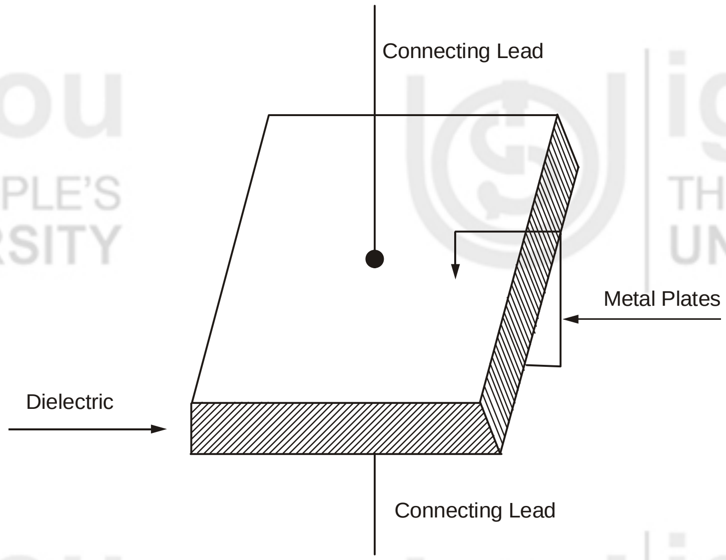

Figure 2.16 : Parallel Plate Capacitor

In Figure 2.17, a capacitor is connected across a battery. It stores charge as shown in Figure 2.17. The potential across capacitor plates increases and as soon as the plates are charged to the same potential as the battery terminals, it can be disconnected from battery and discharged by short circuiting its terminals.

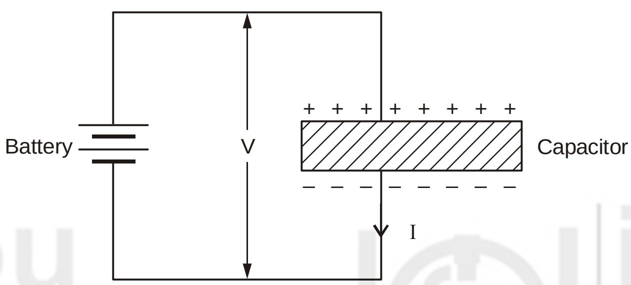

Figure 2.17

If V is applied voltage across capacitor, then charge on capacitor is proportional to the applied voltage V, i.e.

$$Q \propto V$$

or, $$Q = C\,V \qquad \ldots (2.53)$$

where C is constant of proportionality and is called capacitance. The SI unit of capacitance is Farad (F).

The quantity of charge that can be stored by a capacitor with a given terminal voltage is termed its capacitance. One Farad is the capacitance of a capacitor that contains a charge of 1 Coulomb when the potential difference between its terminals is 1 volt.

Generally, capacitance is expressed in milliFarad (mF), microFarad (μF) and picoFarad (pF).

We know that

$$D = \frac{Q}{A}$$

and $$E = \frac{V}{d}$$

so $$\varepsilon_0\,\varepsilon_r = \frac{Q}{A} \times \frac{d}{V}$$

∵ For a capacitor, $$Q = CV$$

so $$\varepsilon_0\,\varepsilon_r = \frac{CV}{A} \times \frac{d}{V}$$

$$\varepsilon_0\,\varepsilon_r = \frac{Cd}{A}$$

or $$C = \frac{\varepsilon_0\,\varepsilon_r\,A}{d} \qquad \ldots (2.54)$$

Here A is area of plates (m^2) and d is thickness of dielectric in m.

Thus, the quantity of charge stored in capacitor depends on the dimension of the plates and insulating material as well as on the battery voltage.

Capacitance of any capacitor is directly proportional to area of plate. For a large area the flow of electrons will be more, and it is inversely proportional to thickness of dielectric.

So, $$\text{Capacitance} \propto \frac{\text{Plate area } (\text{m}^2)}{\text{Thickness of dielectric (m)}}$$

or, $$C \propto \frac{A}{d}. \qquad \ldots (2.55)$$

2.5.5 Capacitors in Series

In series connection, as shown in Figure 2.18, the total thickness of capacitor is increased.

If $d_1, d_2, \ldots, d_n$ are the thicknesses of same dielectrics for n capacitors which are connected in series then

From $C = \frac{\varepsilon_0 \; \varepsilon_r \; A}{d}$,

we get $C_s = \frac{\varepsilon_0 \; \varepsilon_r \; A}{d_1 + d_2 + d_3 + \ldots + d_n}$

or $\frac{1}{C_s} = \frac{d_1 + d_2 + d_3 + \ldots + d_n}{\varepsilon_0 \; \varepsilon_r \; A}$

or $\frac{1}{C_s} = \frac{d_1}{\varepsilon_0 \; \varepsilon_r \; A} + \frac{d_2}{\varepsilon_0 \; \varepsilon_r \; A} + \ldots + \frac{d_n}{\varepsilon_0 \; \varepsilon_r \; A}$

or $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$ . . . (2.56)

The charging current in series connection is same and the charge supplied to each capacitor is equal to the charge supplied to all capacitors.

Since $Q = I \cdot t$ (I and t being same in series connection)

$Q = Q_1 = Q_2 = \ldots = Q_n$ . . . (2.57)

We know $V = \frac{Q}{C}$

In Figure 2.18, $V_1 = \frac{Q}{C_1}$ and $V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}$ and $V = \frac{Q}{C_s}$

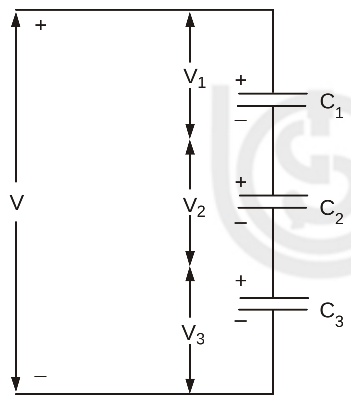

Figure 2.18 : Capacitors in Series

Using $V = V_1 + V_2 + V_3$,

we get $\frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

or $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ . . . (2.58)

2.5.6 Capacitors in Parallel

When capacitors are connected in parallel, as shown in Figure 2.19, the resultant area increases. If the capacitors have plate area $A_1, A_2, \ldots, A_n$, then

from $C = \frac{\varepsilon_r \; \varepsilon_0 \; A}{d}$,

we get $C_p = \frac{\varepsilon_0 \; \varepsilon_r}{d} (A_1 + A_2 + \ldots + A_n)$

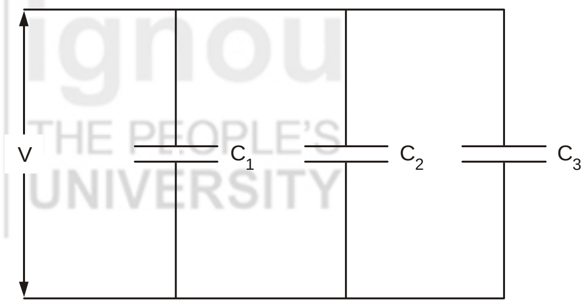

Figure 2.19 : Parallel Capacitors

$$= \frac{\varepsilon_0\ \varepsilon_r}{d} A_1 + \frac{\varepsilon_0\ \varepsilon_r}{d} A_2 + \ldots + \frac{\varepsilon_0\ \varepsilon_r}{d} A_n$$

$$C_p = C_1 + C_2 + C_3 + \ldots + C_n \qquad \ldots (2.59)$$

In Figure 2.19, applied voltage across capacitors is same, so

$$Q_1 = C_1 V, \qquad Q_2 = C_2 V \qquad \text{and} \qquad Q_3 = C_3 V$$

Then total charge $\quad Q = Q_1 + Q_2 + Q_3.$

$$= C_1 V + C_2 V + C_3 V$$

$$= (C_1 + C_2 + C_3) V$$

or, $\quad Q = C_p V$

where $\quad C_p = C_1 + C_2 + C_3 \qquad \ldots (2.60)$

2.5.7 Energy Stored in Charged Capacitor

Energy is stored in the capacitors and the expression is derived as follows :

We know $\quad C = \frac{Q}{V}$ and $Q = I \cdot t$

So, $\quad C = \frac{It}{V}$

or, $\quad I = \frac{CV}{t}$

For a charged capacitor the average input voltage is $\frac{1}{2} V$, when it gets charged to V linearly in time t.

So, electrical energy w = Average voltage × Average current × t

$$w = \frac{1}{2} V \times I \times t$$

$$= \frac{1}{2} V \times \frac{CV}{t} \times t$$

Hence, the expression for energy stored in a capacitor is as follows :

$$w = \frac{1}{2} CV^2 . \qquad \ldots (2.61)$$

SAQ 4

(a) Three capacitors of capacity 10, 20 and 40 μF are placed in series across a 350 V source. Determine

(i) Equivalent capacitance of the combination,

(ii) Charge on each capacitor,

(iii) Voltage drop across each capacitor, and

(iv) Total stored energy.

(b) Two capacitors of 4 μF and 8μF are connected in parallel and this combination is connected in series with a capacitor of 24 μF. Determine

(i) Total capacitance,

(ii) Total charge, and

(iii) Charge on each capacitor

if applied voltage is 32 volts.

(c) A 50 μF capacitor is charged from 200V supply. After being disconnected it is immediately connected in parallel with a 30 μF capacitor. Find

(i) PD across the combination,

(ii) Energy stored before connection, and

(iii) Energy stored after connection.

(d) Determine the capacitance of a parallel plate capacitor composed of thin foil sheet of 25 cm and separated by a glass dielectric 0.5 cm thick with relative permittivity 6.

2.6 ELECTRICAL INSTRUMENTS

2.6.1 Galvanometer

Galvanometer is an instrument which is used to detect and measure small currents or small potential difference in an electrical circuit. It is based on the principle that when a current carrying coil is placed in a magnetic field, it experiences a torque.

A galvanometer is denoted by the symbol –(G)– in an electrical circuits. It is also known as moving coil galvanometer because it consists of a coil OPQR wound on a soft iron core placed between the poles of a permanent magnet NS. This coil moves when current flows through it.

The coil is suspended from the torsion head with a suspension wire and the other end of it is attached to a spring. A concave mirror M is attached to the suspension wire. A pointer is attached to the coil that moves on a galvanometer scale for measuring current. The deflection of the pointer on the scale is proportional to the current in the coil. The zero of the galvanometer is usually in the middle of the scale. Thus, we can measure current flow in either direction.

As shown in Figure 2.20, the whole arrangement is enclosed in a non-magnetic case to avoid disturbance due to air. The levelling screws are used to level the galvanometer so that the coil can rotate freely without touching the poles of the magnet or iron core. T_1 and T_2 are the binding screws used to connect the galvanometer in the electrical circuit.

Before passing the current through the galvanometer, the plane of the coil is parallel to the magnetic field. When current is passed through the coil, it rotates due to the torque acting on it. As the coil rotates under the effect of torque, the restoring torque is developed in the spring. The coil will rotate, till the deflecting torque acting on the rectangular coil due to the flow of current is balanced by the restoring torque developed in the spring, i.e. at final rest position,

Deflecting torque = Restoring torque

If K is the restoring torque per unit twist and α is the angle of twist (the coil comes to rest after rotating through an angle α), then

$$N\,B\,I\,A \sin\theta = K\alpha \qquad \text{. . . (2.62)}$$

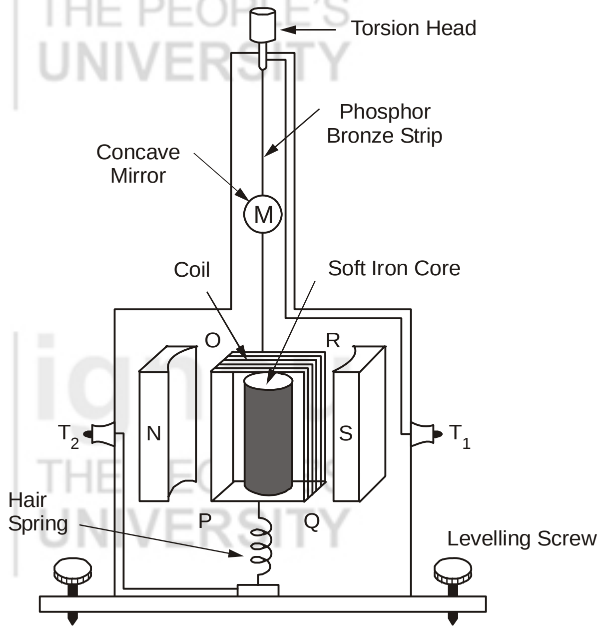

Figure 2.20 : Galvanometer

The poles of the magnet are made concave to produce radial magnetic field. For radial magnetic field, the deflection produced in the coil of the galvanometer may be directly proportional to the current passed. Therefore, in case of radial field $\theta = 90^{o}$.

Therefore, $$N\,B\,I\,A = K\,\alpha \qquad \text{. . . (2.63)}$$

$$I = \left(\frac{K}{NBA}\right)\alpha \qquad \text{. . . (2.64)}$$

where $G = \dfrac{K}{NBA}$ is called the **galvanometer constant**.

Hence, $$I = G\,\alpha \qquad \text{. . . (2.65)}$$

The current flowing through galvanometer is proportional to the deflection produced in it. Such a galvanometer will have a linear scale. Now we will use this concept to define **sensitivity of a galvanometer**.

If a galvanometer gives a large deflection even when a small current is passed through it or a small voltage is applied across the coil of the galvanometer it is said to be highly sensitive. **Sensitivity** is of two types : current sensitivity and voltage sensitivity.

Current Sensitivity is defined as the deflection produced in the galvanometer on passing unit current through its coil.

$$\text{Current Sensitivity} = \frac{\alpha}{I} = \frac{NBA}{K} \quad \ldots (2.66)$$

where α is the deflection produced when a current I is passed.

Voltage Sensitivity is defined as the deflection produced in the galvanometer when a unit voltage is applied across its coil.

$$\text{Voltage sensitivity} = \frac{\alpha}{V} \quad \ldots (2.67)$$

Using $V = IR$ in the above equation, we get

$$\text{Voltage sensitivity} = \frac{\alpha}{IR} = \frac{NBA}{KR} = \frac{\text{Current sensitivity}}{R} \quad \ldots (2.68)$$

From this equation, one can say that the galvanometer will be highly sensitive if N, B and A is large and K and R is small. B is made as large as possible and K is as small as possible. To increase B, a very strong horse-shoe magnet is used. To decrease K, the suspension wire is made of phosphor bronze or quartz as for these materials, the value of K is very small. The value of N and A cannot be increased beyond a certain limit. If it was so, the size of the galvanometer and electrical resistance of the galvanometer will increase.

If N is the number of scale divisions in the galvanometer and K is the current for one scale deflection in the galvanometer, called **figure of merit**, and I_g is the current, which produces full-scale deflection in the galvanometer, then

$$I_g = NK \quad \ldots (2.69)$$

2.6.2 Ammeter

An ammeter is an instrument that is used to measure direct current in electrical circuits. Ammeter is essentially a galvanometer having a known low resistance known as shunt (S) in parallel. An ammeter is denoted by the symbol Ⓐ in an electrical circuit.

Suppose S is a shunt, G is the galvanometer and G_R is the galvanometer resistance. Galvanometer is a low current instrument and as such it cannot be directly used to measure relatively higher current in the circuit. Shunt resistance is used to protect the galvanometer from strong currents so that the coil does not damage. Large amount of current produces a large amount of heat also.

Let I is the total current in the circuit, I_g is the current passed through the galvanometer for which it gives full-scale deflection. $(I - I_g)$ is the current passed through the shunt resistance. To convert the galvanometer into an ammeter of range 0 to 1, a small shunt resistance S is connected in parallel with the galvanometer as shown in Figure 2.21.

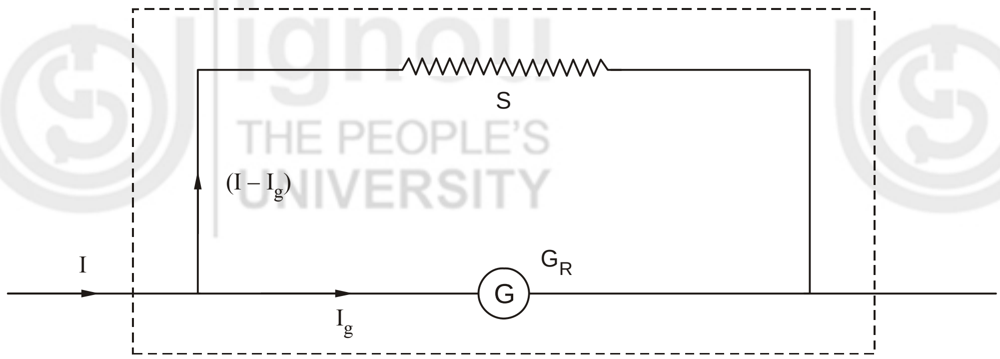

Figure 2.21

Since the galvanometer and S are in parallel, therefore, potential difference across them will be equal. Then,

$$(I - I_g) \times S = I_g \, G_R \qquad \text{. . . (2.70)}$$

$$S = \frac{I_g}{(I - I_g)} G_R \qquad \text{. . . (2.71)}$$

which is the value of shunt resistance required to be connected in parallel with the galvanometer to convert it to an ammeter capable of reading current up to I amperes.

The total resistance R_T of the ammeter will be

$$\frac{1}{R_T} = \frac{I}{G_R} + \frac{1}{S} \qquad \text{. . . (2.72)}$$

$$R_T = \frac{SG_R}{S + G_R} \qquad \text{. . . (2.73)}$$

The resistance of the ammeter is very low as compared to that of galvanometer. Therefore, the ammeter is placed in series in a circuit. The resistance of the circuit practically remains unchanged and consequently current in the circuit remains unaffected.

To measure potential difference, voltmeter is required. We now discuss about the conversion of galvanometer into voltmeter.

2.6.3 Voltmeter

Voltmeter is an instrument used to measure the potential difference across the two points in a circuit. Voltmeter is essentially a galvanometer having a known high resistance (R) in series with it. It is denoted by the symbol (V) in an electrical circuit.

Galvanometer is converted into voltmeter so that it can measure potential difference across a conductor without causing any change in the current flowing through the conductor.

The value of R is adjusted so that if a battery of V volt is connected, the galvanometer gives full-scale deflection. As galvanometer G and high resistance R is connected in series, so that the total resistance of voltmeter is = $G_R + R$, then, using ohm's law

$$I_g = \frac{V}{G_R + R} \qquad \text{. . . (2.74)}$$

$$I_g \, (G_R + R) = V$$

$$R = \frac{V}{I_g} - G_R \qquad \ldots (2.75)$$

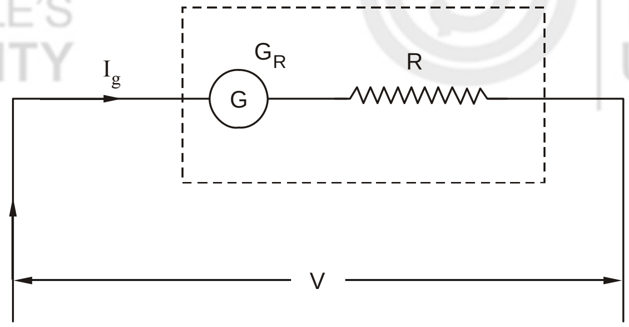

Figure 2.22 : Voltmeter

Voltmeter is a high resistance instrument, therefore it is connected in parallel to conductor. The range of a voltmeter can be both increased and decreased by changing the value of *R*.

2.7 HYSTERESIS

Literally, the meaning of 'hysteresis' is to lag behind or 'coming late'. It is defined as the lagging of the magnetic induction *B* behind the corresponding magnetising field *H*. In detail, this can be explained as follows.

Figure 2.23 : Unmagnetized Steel Ring over which a Wire is Wound

Consider a specimen of unmagnetised steel in the form of ring over which a wire is wound as shown in Figure 2.23. Consider a current is flowing through the wire, such that the magnitude and direction of the current can be varied. It is noted that when the current is gradually increased from zero to maximum in one direction, the magnetising field *H* also increases gradually from zero to maximum in one direction. For this increasing current, the corresponding values of magnetic induction $\vec{B}$ in the ring are noted at different stages. On gradually decreasing the current from maximum to zero, it is found that *H* is also gradually decreased from maximum to zero and the corresponding values of *B* are noted. Similarly, when the current is increased gradually from zero to maximum in the other direction, the magnetising field *H* will also gradually increase from zero to maximum in the opposite direction. The corresponding values of *B* are noted in the steel ring. On reducing the current from the maximum to zero, the magnetising field *H* also decreases from maximum to zero. Hence, the corresponding values of magnetic induction *B* are determined. Finally, the current is increased from zero to maximum in the first direction and different corresponding values of magnetic induction are noted. Therefore, a cycle of magnetisation is said to be completed when magnetising field *H* and the magnetic induction *B* are varied from zero to maximum in one direction, and then back through zero to maximum in the opposite direction and finally back again through zero to the first maximum.

A graph between the values of *H* (along x-axis) and *B* (along y-axis) is plotted as shown in Figure 2.24. It is noted that when the magnetising field *H* is increased from zero to the maximum value *Og*, the induction follows the curve *Oa*.

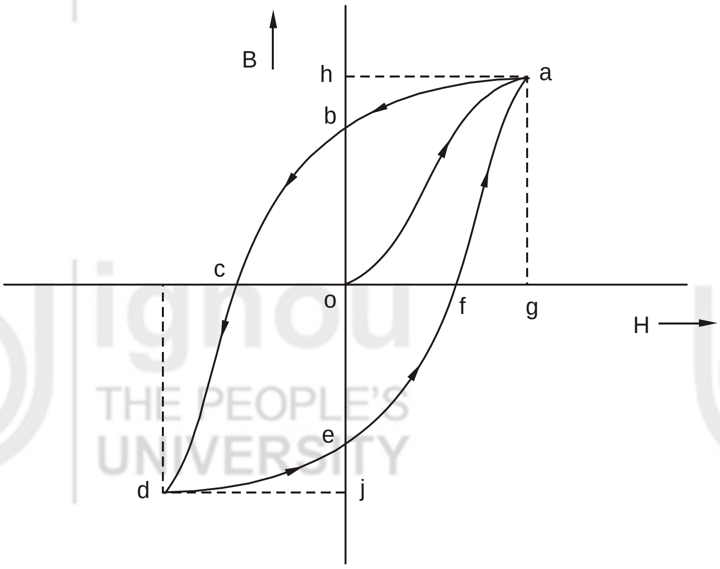

Figure 2.24 : A Plot between *B* and *H*

However, when *H* is brought to zero, the induction does not follow back the curve *Oa* but a different curve *ab*. The same pattern of variations is found for rest of the curve. Hence, the *B-H* curve for decreasing *H* does not coincide with the *B-H* curve for increasing *H*. Even when the *H* is zero, the *B* (*Ob* or *Oe*) is still present in the specimen. It is denoted by *Br* and is called **remanence** or **retentivity residual magnetism.**

After the specimen has been magnetised to saturation (*Oh* or *Oj*), a reversed magnetised field (equal to *Oc* and *Of*) is required to reduce the magnetic induction to zero. This is called the **coercivity** or **coercive force** *Hc*. When the magnetic intensity *H* was varied between equal positive and negative values (as discussed above), the corresponding changes in the flux density lead to symmetrical curve. Other ways and ranges of changing *H* would result in other forms of hysteresis curves.

The hysteresis loop gives a clear indication of the dissipation of energy due to hysteresis. This is because when a ferromagnetic material is taken through a cycle of magnetisation, heat is produced. It has been observed that in each cycle of magnetisation, the heat developed per unit volume of the material is proportional to the area enclosed by the hysteresis loop. A narrow hysteresis loop indicates less dissipation of energy. For the core of a transformer, we select iron or iron alloy having narrow hysteresis loop. Similarly, while making permanent magnets, we select that material which has high retentivity.

2.8 SUMMARY

- The SI unit of magnetic field is Tesla.
- The number of magnetic field lines crossing the surface perpendicularly is known as **magnetic flux**. The SI unit of magnetic flux is Weber (Wb) and its dimensional formula is $ML^2T^{-2}A^{-2}$.

- The force F between two magnetic poles of strength m_1 and m_2 placed distantce r apart is given by Coulomb's law as

$$F = \frac{k \cdot m_1 \; m_2}{r^2}$$

where $k = 10^{-7}$ Wb $A^{-1}m^{-1}$.

- **Ohm's law** for magnetic circuits is

Magnetomotive force = Flux × Reluctance

Mathematically, it can be expressed as

$$mmf = \phi \times S$$

- $\text{Reluctance} = \dfrac{1}{\text{Permeability}} \times \dfrac{\text{Length}}{\text{Area}}$

- The expression for Biot-Savarts law is

$$B = \frac{\mu_0}{4\pi} \int \frac{Idl \sin \alpha}{r^2}$$

- According to Faraday's law and Lenz's law, the emf induced in a coil of N turns is given by

$$e = -N \frac{d\phi}{dt}$$

- Eddy currents have various applications in instruments like deadbeat galvanometer, electromagnetic damping, induction motors, speedometers, electric brake etc.

- Induction is of two types : Self-induction and Mutual induction. Self-inductance of a coil depends upon the number of turns, cross-sectional area and permeability of the core on which the coil is wound. Mutual induction between two coils depends upon the shape and size of the two coils, separation between them and the permeability of the material of the core on which two coils are wound. It also depends upon the manner in which the coils are oriented relative to each other.

- The magnetic field inside a solenoid is given by

$$B = \mu_0 \, n \, I$$

- $L = \mu_0 \, n^2 Al$ is the expression of self-inductance of a long solenoid in terms of length l, area of cross-section A and number of turns per unit length n.

- Energy stored in an inductor carrying current I_0 is

$$w = \frac{1}{2} L I_0^2 .$$

- Capacitance of a parallel plate capacitor is

$$C = \frac{\varepsilon_0 \; \varepsilon_r \; A}{d} .$$

- Energy stored in a capacitor having voltage V across its plates is

$$w = \frac{1}{2} CV^2 .$$

- Equivalent capacitance, C_s, of n series connected capacitors is given by

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}.$$

- Equivalent capacitance, C_p, of n parallel connected capacitors is

$$C_p = C_1 + C_2 + \ldots + C_n.$$

- **Current Sensitivity** is defined as the deflection produced in the galvanometer on passing unit current through its coil.

$$\text{Current Sensitivity} = \frac{\alpha}{I} = \frac{NBA}{K}$$

where α is the deflection produced when a current I is passed.

- **Voltage Sensitivity** is defined as the deflection produced in the galvanometer when a unit voltage is applied across its coil.

$$\text{Voltage sensitivity} = \frac{\alpha}{V} = \frac{NBA}{KR} = \frac{\text{Current sensitivity}}{R}$$

- A galvanometer can be converted into an ammeter by connecting a low resistance known as shunt (S) in parallel and connecting a high resistance in series with it can convert the galvanometer into voltmeter.

- Hysteresis is defined as the lagging of magnetic induction B behind the corresponding magnetising field H.

2.9 ANSWERS TO SAQs

SAQ 1

(a)

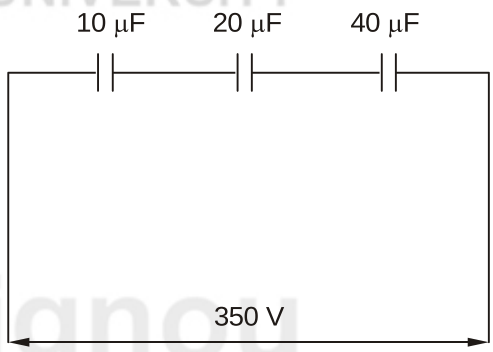

Figure 2.25

(i) Equivalent capacitance $C = \dfrac{C_1 C_2 C_3}{C_1 C_2 + C_2 C_3 + C_3 C_1}$

$$= \frac{10 \times 20 \times 40}{10 \times 20 + 20 \times 40 + 40 \times 10} = \frac{40}{7}\ \mu\text{F} \quad \text{Ans.}$$

(ii) $Q = C\,V = \dfrac{40}{7} \times 10^{-6} \times 350 = 2000 \times 10^{-6}$ Coulombs

Since capacitors are in series, so charge on each capacitor is the same and equals 2000×10^{-6} C.

(iii) $V_1 = \frac{Q}{C_1} = \frac{2000\times10^{-6}}{10\times10^{-6}} = 200$ Volts

$$V_2 = \frac{Q}{C_2} = \frac{2000\times10^{-6}}{20\times10^{-6}} = 100 \text{ Volts}$$

$$V_3 = \frac{Q}{C_3} = \frac{2000\times10^{-6}}{40\times10^{-6}} = 50 \text{ Volts}$$

(iv) Total energy stored $= \frac{1}{2}CV^2$

$$= \frac{1}{2}\times\frac{40}{7}\times10^{-6}\times(350)^2 = 0.35 \text{ Joules}$$

(b) Equivalent capacitance, C_P, of parallel combination, as shown in Figure 2.26, is given by

$$C_P = 4 + 8 = 12\ \mu\text{F}$$

Now, C_P of 12 μ F is connected is series with 24 μF capacitor.

(i) Total Capacitance C is given by

$$\frac{1}{C} = \frac{1}{12} + \frac{1}{24} = \frac{1}{8}$$

or $C = 8$ μF

(ii) Total charge, $Q = CV$

$$= 8\times10^{-6}\times32 = 256\times10^{-6} \text{ Coulomb}$$

(iii) $Q = CV$ or, $V = \frac{Q}{C}$

PD across parallel combinations

$$V_1 = \frac{Q}{C_p} = \frac{256\times10^{-6}}{12\times10^{-6}} = 21.33 \text{ Volt}$$

Charge on capacitor 4 μF = $CV = (4 \times 10^{-6}) \times 21.33$

$= 85.32 \times10^{-6}$ C

Charge on capacitor 8 μF = $(8 \times10^{-6}) \times 21.33$

$= 170.64 \times10^{-6}$ Coulomb

V_2 = PD across 24 μF capacitor = 32 – 21.33

= 10.67 Volts

The charge on 24 μF capacitor = $24 \times10^{-6} \times 10.67$

$= 256.08 \times 10^{-6}$ Coulomb

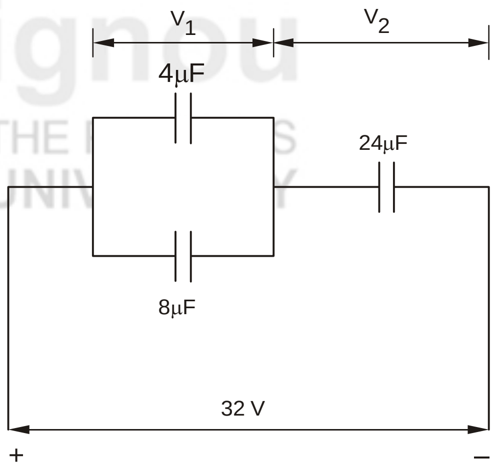

Figure 2.26

(c) (i) Charge on 50 μF capacitor when charged to 200 V

$$Q = CV = 50 \times 10^{-6} \times 200 = 0.01 \text{ Coulomb}$$

When the other uncharged capacitor is connected in parallel, the total capacitance becomes

$C' = 30 + 50 = 80$ μF and the charge of 0.01 Coulomb is distributed between two capacitors. PD across parallel combination is

$$V' = \frac{Q}{C'} = \frac{0.01}{80 \times 10^{-6}} = 125 \text{ volts}$$

(ii) Energy stored before connection

$$= \frac{1}{2} CV^2 = \frac{1}{2} \times (50 \times 10^{-6}) \times (200)^2$$

= 1 Joule

(iii) Energy stored after connection

$$= \frac{1}{2} C' V'^2 = \frac{1}{2} \times (80 \times 10^{-6}) \times (125)^2$$

= 0.625 Joule

(d) Surface area of $A = 0.25 \times 0.25 = 0.0625$ m^2.

Distance between plates $t = 0.5$ cms = 0.005 m

Relative permittivity, $\varepsilon_r = 6$

The capacitance of capacitor $C = \varepsilon_0 \, \varepsilon_r \, \dfrac{A}{t}$

$$= \frac{(8.854 \times 10^{-12}) \times 6 \times 0.0625}{0.005}$$

= 664.05 μ μF = 664.05 pF

SAQ 2

$N = 400$, $I = 5A$, Cross-section area, $A = 2 \times 10^{-4}$ m^2

Length = $2 \pi r = \pi D = 0.25 \pi$ m

(i) $mmf = NI = 400 \times 5 = 2000$ AT

(ii) Reluctance $(S) = \frac{1}{\mu} \times \frac{l}{A}$

But $\mu_r = \frac{\mu}{\mu_0} \Rightarrow \mu = \mu_0 \, \mu_r$

$$\therefore \quad S = \frac{1}{\mu_0 \, \mu_r} \frac{l}{A}$$

$$= \frac{0.25\pi}{4\pi \times 10^{-7} \times 450 \times 2 \times 10^{-4}}$$

$$= \frac{0.785 \times 10^{11}}{11304} = 6.94 \times 10^{6} \ \frac{\text{AT}}{\text{Wb}}$$

(iii) Flux $\phi = \frac{mmf}{S} = \frac{2000\text{AT}}{6.94 \times 10^{6} \ \frac{\text{AT}}{\text{Wb}}}$

$$= 0.288 \times 10^{-3} \text{ Wb}$$

$$= 0.288 \text{ mWb.}$$

SAQ 3

(a) $I = 5\text{A}$

$$a = 10 \text{ cm} = \frac{10}{100} \text{ m} = 0.1 \text{ m}$$

Using the formula,

$$B = \frac{\mu_0}{4\pi} \frac{2I}{a} = \frac{10^{-7} \times 2 \times 5 \times 10}{1}$$

$$B = 10^{-5} \text{ T}$$

(b) $n = 1000; \; a = \frac{0.50}{2} = 0.25 \text{ m}; \; I = 7\text{A}$

$x = 0.15$ m

Using the expression for magnetic field at a point on the axis of the coil, then

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi \, n \, I \, a^2}{(a^2 + x^2)^{\frac{3}{2}}}$$

$$\text{B} = \frac{10^{-7} \times 2 \times 3.14 \times 1000 \times 7 \times (0.25)^2}{[(0.25)^2 + (0.15)^2]^{3/2}}$$

$$B = \frac{2.7475 \times 10^{-4}}{[(0.0645 + 0.0225)^2]^{3/2}} = \frac{2.7475 \times 10^{-4}}{(0.085)^{3/2}}$$

$$B = 110 \times 10^{-4} \text{ T}$$

At the centre of the coil, $x = 0$, therefore the expression is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi \, n \, I}{a}$$

$$= \frac{10^{-7} \times 2 \times 3.14 \times 1000 \times 7}{0.25}$$

$$B = 1.76 \times 10^{-2} \text{ T}$$

SAQ 4

(a) Initial magnetic flux density, $B_1 = 0.10$ Wbm^{-2}

Final magnetic flux density, $B_2 = 0.70$ Wbm^{-2}

Time taken, $t = 0.02$ sec.

Number of turns of the coil = $N = 100$

Area = 0.16m^2

Change in flux = Final flux − Initial flux

$$= \phi_2 - \phi_1 = NA\,(B_2 - B_1)$$

Rate of change of flux $= \dfrac{\phi_2 - \phi_1}{t} = \dfrac{NA\,(B_2 - B_1)}{t}$

$$= \frac{100 \times 0.16 \times (0.70 - 0.10)}{0.02}$$

$$= 16 \times \frac{60}{02}$$

Hence, $|e|$ = Magnitude of induced emf = 480 volt

(b) Area $A = \pi r^2 = \pi \left(\dfrac{D}{2}\right)^2 = \pi \left(\dfrac{1.5 \times 10^{-2}}{2}\right)^2$

$$= 1.77 \times 10^{-4}\ \text{m}^2$$

The inductance L is given by

$$L = \mu_0\, \mu_r\, N^2 \frac{A}{l}$$

$$L = 4\pi \times 10^{-7} \times 1 \times (900)^2 \times \frac{1.77 \times 10^{-4}}{15 \times 10^{-2}}$$

$L = 120 \times 10^{-5}$ H = 1.20 mH.