UNIT 3 MOMENT DISTRIBUTION

Structure

3.1 INTRODUCTION

The **moment distribution method** is a very convenient and useful method for finding the bending moment in a rigid jointed structure, like *portal fames* and *continuous beams*.

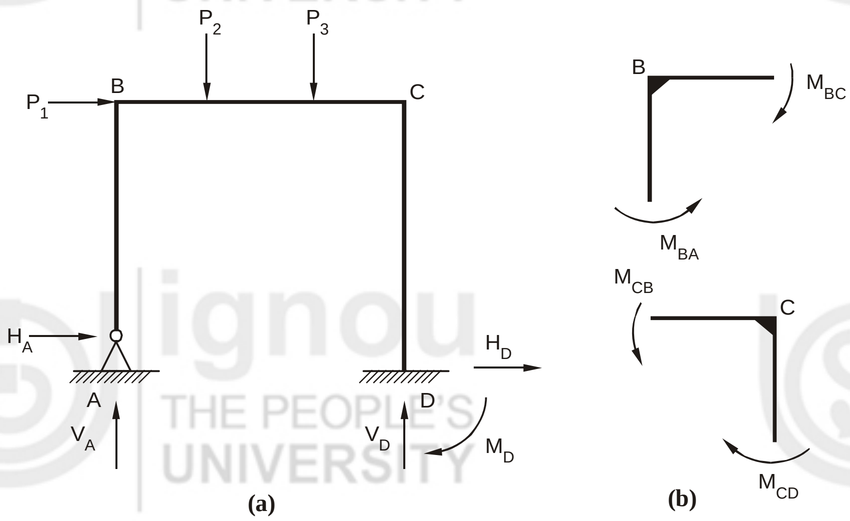


Figure 3.1 : A Portal Frame

Figure 3.1 shows a portal frame with vertical members (column members) *AB* and *CD* and the horizontal member (beam member) *BC* which are rigidly joined at *B* and *C* so that the angles between the members remain fixed (in Figure 3.1 these are a right angles). As the joints *B* and *C* are not hinged, but fixed therefore, bending moments will appear both in members *BA* and *BC* at joint *B* and similarly in *CB* and *CD* at joint *C*. In Figure 3.1(b) we have the conditions of static equilibrium at the joints stating that the sum of all the moments acting at the joint must be zero.

i.e. $M_{BA} + M_{BC} = 0$ . . . (3.1)

and $M_{CB} + M_{CD} = 0$

However, the moments themselves are unknowns and can be termed as *internal redundants*. Also the portal may be supported in any manner. Here, in Figure 3.1, the support *A* is a hinge where two reactions (V_A and H_A) are present, while the support *D* is fixed causing three reactions V_D, H_D and M_D there. Thus, the portal has five unknown support reactions and we have only three equations of static equilibrium to determine them. Hence, *externally also it is indeterminate to the second degree* (5 – 3 = 2). To solve them by compatibility method will involve a solution of so many simultaneous equations which is a laborious process.

In the moment distribution method, however, we are not required to solve any equations to determine the reaction etc. But we directly start finding the fixed end moments at all the joints and support points. After determining the moments, the reactions can be found out by equations of statics. Here, the joints are assumed first to be all rigidly fixed; also without any translation (i.e. without horizontal movement or sway of any joint). *The fixed end moments* are first determined for each member and then the joints are released for the sake of *compatibility* to attain *equilibrium*, which is called *balancing*. This act of balancing results in introduction of *carry over moments* which cause loss of equilibrium again. Hence, a second balancing is required, and then a second carry over and so on.

This process of repeated adjustments or balancing is carried out till we reach a point where sufficient accuracy of result is achieved. In fact we may stop at any point, as the process is a convergent one.

As you will very soon see that the method is quite simple and does not need solution of a number of simultaneous equations as is usually required for frames having a large number of redundancies. Figure 3.2 is a portal frame with 4 bays and 3 storeyes, having 12 beam members and 15 column members.

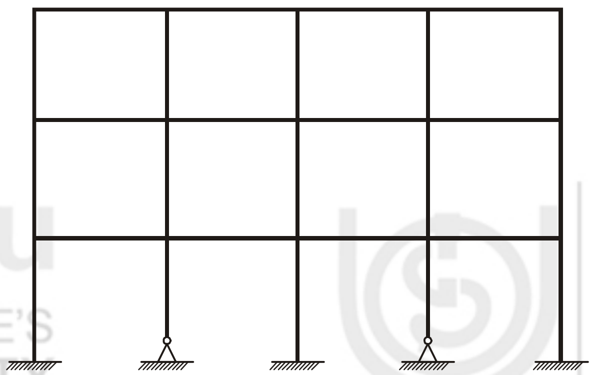


Figure 3.2

The number of joints are 20. As you can easily see that it has a large number of external unknown reactions, and internal unknown joint moments which is difficult to solve without the use of fast digital computers. But it can be solved mechanically by the moment distribution method. The method was first proposed by Prof. Handy Cross in 1932 when digital computers and hand calculators were not available. However, by simple manual calculations fairly accurate results could be obtained.

Objectives

After studying this unit, you should be able to

- find the moments and reactions at the supports of a continuous beam by the moment distribution method, and draw the BM and SF diagram,
- find out the joint and support moments of a single storey, single bay portal frame *without sway* by moment distribution, and to draw its bending moment and shearing force diagram, and
- solve a similar portal frame when *side sway* is also present.

3.2 SIGN CONVENTIONS AND OTHER PRELIMINARIES

The moment distribution method requires a sign convention which must be clearly defined. As equilibrium of joint moments are to be considered, we assume all clockwise moments as positive (and therefore anticlockwise moments as negative). Now with reference to bending of a member it will have a different meaning depending upon the end at which it is applied. For example, in Figure 3.3(a), a clockwise moment applied to the left hand end *A* of the member *AB* produces a sagging (or positive) bending moment. The same clockwise moment when applied to the right hand end *B* produces a hogging (or negative) bending moment. Opposite effects will be produced by anticlockwise moments.

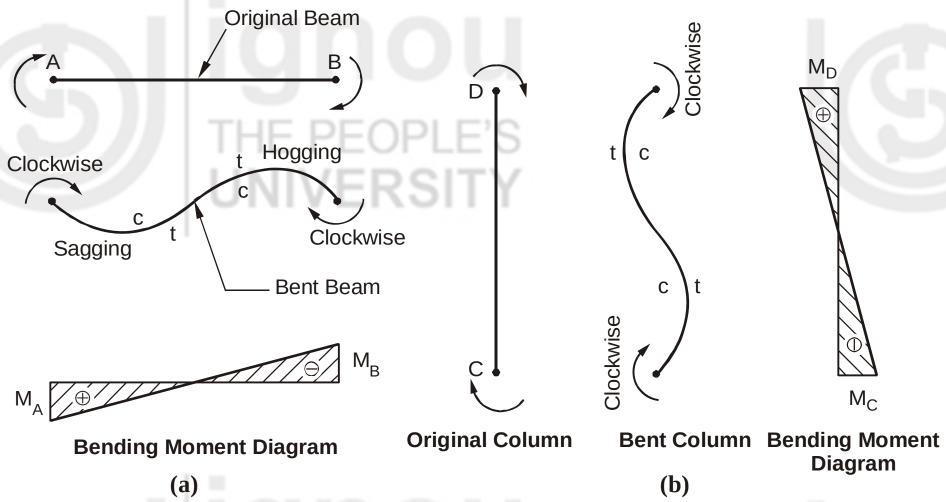
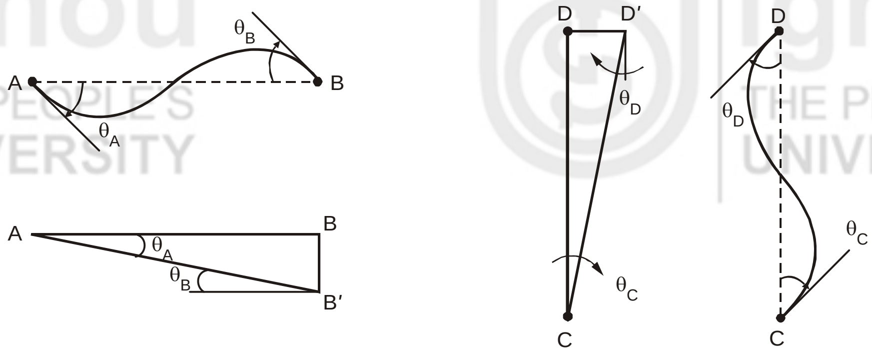


Figure 3.3

Similarly, on a vertical member *CD* (Figure 3.3(b)) a clockwise moment at the lower end *C* produces tension on the right hand face (compression on left hand face) marked '*t*' and '*c*' respectively. A similar clockwise moment at the upper end will produce tension on the left hand face (and compression on the right hand face). So the sign of the bending moments at the ends *C* and *D* will be opposite although the moments are both clockwise in direction. They may not be called hogging or sagging now. The best way to show their signs are to draw the positive bending moment on the tension side of the original axis line as shown above at Figures 3.3(a) and (b). Similarly, in moment distribution notation all joint rotations or chord rotation is *positive* if *clockwise* and negative if anticlockwise. Hence, the downward deflection of right hand end *B* with respect to left hand *A* will cause a clockwise (positive) rotation as shown in Figure 3.4(a).



(a) (b)

Figure 3.4

So will be the case with vertical member where upper end *D* moves to right with respect to the lower end *C* as shown in Figure 3.4(b). The opposite movements will result in anticlockwise rotations, and shall be given negative sign.

3.3 SLOPE DEFLECTION EQUATIONS FOR A MEMBER

In this section, we will develop some basic equations needed for further discussion in the moment distribution context.

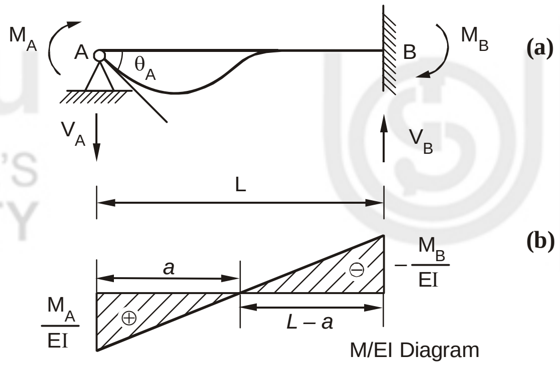


Figure 3.5

Figure 3.5(a) shows a beam hinged at the end *A* and fixed at *B*. A clockwise moment M_A at *A* produces a clockwise rotation θ_A at end *A*, the moment at end *B* is assumed M_B while the slope at the fixed end *B* is zero. The vertical reactions at V_A and V_B are equal in magnitude but their directions will be opposite as shown in the diagram producing an anticlockwise couple to balance the couple due to the end moments M_A and M_B.

i.e. $$-V_A = V_B = \frac{M_A + M_B}{L} \quad \ldots (3.1)$$

To find out M_A and M_B we shall apply the moment area method. The bending moment (÷ *EI*) diagram is shown in Figure 3.5(b). At the left hand end clockwise M_A is positive (sagging) BM while at the right hand end clockwise M_B is a negative (sagging) BM.

As the change in slope from end *B* to *A* is θ_A, it is equal to the area of $\frac{M}{EI}$ diagram between these points i.e. $$\theta_A = \frac{M_A}{EI} \cdot \frac{a}{2} - \frac{M_B}{EI} \frac{(L - a)}{2} \qquad \ldots (3.2)$$

Also the intercept on the tangent at *B* made by the vertical through *A* is equal to the moment of $\frac{M}{EI}$ diagram about *A*. As the tangent at *B* passes through *A*, so this intercept is zero, hence,

$$\left(\frac{M_A}{EI} \cdot \frac{a}{2}\right)\left(\frac{a}{3}\right) - \left(\frac{M_B}{EI} \frac{L-a}{2}\right)\left(a + \frac{2(L-a)}{3}\right) = 0 \qquad \ldots (3.3)$$

Also we know from similar triangles that $\frac{a}{L-a} = \frac{M_A}{M_B}$ . . . (3.4)

From Eq. (3.3),

$$M_A \cdot a^2 = M_B (L - a) [3a + a^2 - 2 (L - a)]$$

Dividing by $(L - a)^2$ we have

$$M_A \left(\frac{a}{L-a}\right)^2 = M_B \left[3 \frac{a}{L-a} + 2\right]$$

On Substituting Eq. (3.4), we get

$$\frac{M_A}{M_B}\left(\frac{M_A}{M_B}\right)^2 = 3\frac{M_A}{M_B} + 2$$

If $\frac{M_A}{M_B} = r$ this gives $r^3 = 3r + 2$ or $r^3 - 3r - 2 = 0$

Giving $(r + 1)^2 (r - 2) = 0$ hence $r + 1 = 0$ or $r - 2 = 0$

As the ratio $r = \frac{M_A}{M_B} = -1$ is not acceptable since $M_A = -M_B$ gives the case of a uniform sagging moment all over the beam, hence,

$$\therefore \quad \frac{M_A}{M_B} = 2 \text{ or } M_A = 2M_B \qquad \ldots (3.5)$$

Also $$\frac{a}{L-a} = \frac{M_A}{M_B} = 2 \text{ or a} = \frac{2L}{3} \qquad \ldots (3.6)$$

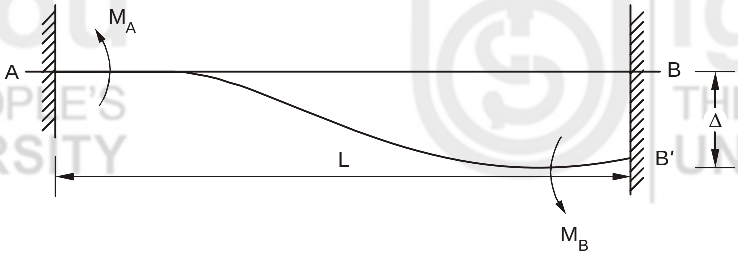
Substituting Eqs. (3.5) and (3.6) in Eq. (3.2), we get

$$M_A = \frac{4EI\theta_A}{L} \qquad \ldots (3.7)$$

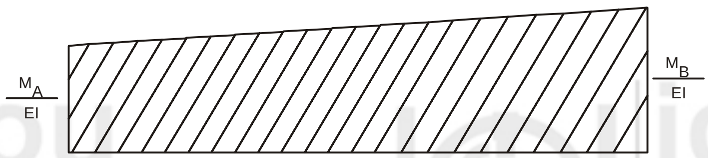
and $$\therefore \quad M_B = \frac{M_A}{2} = \frac{2EI\theta_A}{L} \qquad \ldots (3.8)$$

Thus, a rotation of θ_A at hinged end *A* is produced by a moment $\frac{4EI\theta_A}{L}$ at the end *A*, while at the fixed end *B* the moment induced is half of this, i.e. $\frac{2EI\theta_A}{L}$ and both are clockwise if θ_A is clockwise.

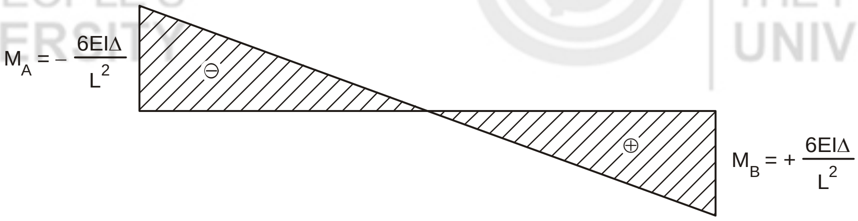
Next, let us calculate the fixed end moments due to a support settlement or end deflection as shown in Figure 3.6(a).



(a) Support Settlement



(b) BM Diagram



(c) Actual BM Diagram

Figure 3.6

In the beam *AB*, the end *B* moves through a distance $BB' = \Delta$, such that chord rotation AB' is positive (clockwise).

Hence, M_A and M_B, the fixed end moments, *both are anticlockwise* as shown. As there is no change in slope between *A* and *B* (both tangents remain horizontal).

From the *first moment area theorem* the area of the $\dfrac{M}{EI}$ diagram between *A* and *B* is zero.

$$\therefore \qquad \frac{(M_A + M_B)\,L}{2EI} = 0 \quad \Rightarrow \quad M_A + M_B = 0 \qquad \ldots (3.9)$$

Also since the intercept on tangent at *A*, on a vertical through *B* is Δ, we have, by taking moment of the $\dfrac{M}{EI}$ diagram about *B* (*Second moment area theorem*)

$$\left(\frac{M_A + M_B}{2EI}\right) L \left(\frac{2M_A + M_B}{M_A + M_B} \cdot \frac{L}{3}\right) = -\,\Delta$$

Here Δ is taken negative as B' is below *B*.

$$\text{Or} \qquad 2\,M_A + M_B = -\,\frac{6E\,I\,\Delta}{L^2} \qquad \ldots (3.10)$$

But $\qquad M_A + M_B = 0$

$$\text{and} \quad \therefore \quad \left.\begin{aligned} M_A &= \frac{-\,6E\,I\,\Delta}{L^2} \\ M_B &= \frac{6E\,I\,\Delta}{L^2} \end{aligned}\right\} \qquad \ldots (3.11)$$

Thus, M_A is negative (hogging) bending moment and M_B is a positive (sagging) bending moment and is equal to $\frac{6EI\Delta}{L^2}$ in magnitude (Figure 3.6(b)).

Thus, a support settlement Δ at the right hand end *B* of the beam *AB* induces a negative bending moment

$$M_A = \frac{6EI\Delta}{L^2} \qquad \text{. . . (3.12(a))}$$

on the left end *A* and a positive *bending moment*

$$M_B = \frac{6EI\Delta}{L^2} \qquad \text{. . . (3.12(b))}$$

at the end *B*.

Both M_A and M_B are anticlockwise movements and have, therefore, negative sign. However, the cases with support settlement are beyond the scope of this course.

3.4 DISTRIBUTION FACTOR

The process of moment distribution can be explained by the following example of a redundant joint :

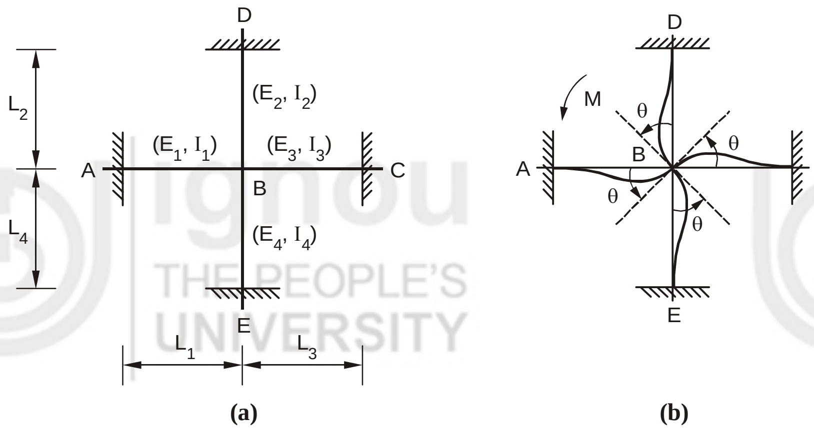


Figure 3.7

In Figure 3.7(a), four bars *AB*, *DB*, *CB* and *EB* are meeting at joint *B*, which is a rigid joint (not hinged).

Also the ends *A*, *D*, *C*, *E* are fixed-ended supports. The modulus of elasticity, moment of inertia and lengths of the bars are (E_1, I_1, L_1), (E_2, I_2, L_2) etc as shown.

Now if the bars *AB*, *BC*, etc. are loaded by transverse loads fixed ends moments (FEM) develop at both ends of each bar. Thus, bar *AB* will have M_{FAB} and M_{FBA} as fixed end moments at ends *A* and *B* respectively. Finally, at the joint *B* we will have four fixed end moments M_{FBA}, M_{FBD}, M_{FBC} and M_{FBE} which may have their own magnitudes and sign [clockwise (+) or anticlockwise (–)].

For statical equilibrium of the joint *B*, the algebraic sum of all the moments must be zero. If it is not zero a balancing moment *M* has to be applied such that

$$[M_{FBA} + M_{FBD} + M_{FBC} + M_{FBE}] + M = 0 \qquad \text{. . . (3.13)}$$

Now the balancing moment *M* will cause a rotation at the joint so that each member will rotate through an angle θ. This rotation will be the same for all the members as the joint '*B*' is a rigid one and angles between the bars cannot change. This is shown in Figure 3.7(b). However, this rotation θ will introduce a moment at each of the member ends at *B*.

Now as bars BA is fixed at A, and its elastic modulus; moment of inertia and length is E_1, I_1, L_1 then the moment M'_{BA} which can cause a rotation θ at end B will be equal to $\frac{4 E_1 I_1}{L_1} . \theta$ from Eq. (3.7).

Similarly, for member BD, $M'_{BD} = \frac{4 E_2 I_2}{L_2} . \theta$ and so on.

But the sum of all these central moments should be equal to the balancing moment M. In other words the balancing moment M will be distributed in certain ratios to each of these members. Algebraically it can be expressed as

$$M'_{BA} + M'_{BD} + M'_{BC} + M'_{BE} = M$$

or $$\frac{4 E_1 I_1}{L_1} \theta + \frac{4 E_2 I_2}{L_2} \theta + \frac{4 E_3 I_3}{L_3} \theta + \frac{4 E_4 I_4}{L_4} \theta = M$$

Now, if all the bars are of the same material, E is same for all of them and can be taken as E.

$$\therefore \quad (4E\theta)\left(\frac{I_1}{l_1} + \frac{I_2}{l_2} + \frac{I_3}{l_3} + \frac{I_4}{l_4}\right) = M$$

or $$4E\theta = \frac{M}{\sum \frac{I}{l}}$$

Here, the ratio, $\frac{\text{Moment of inertia}}{\text{Length}} = \frac{I}{l}$ of any member is called its **stiffness ratio** and is denoted by K.

$$\therefore \quad 4E\theta = \frac{M}{\sum K} = \frac{M}{K_1 + K_2 + K_3 + K_4} \qquad \ldots (3.14)$$

Substituting back in M'_{BA} etc., we get

$$\left.\begin{aligned} M'_{BA} &= (4E\theta)\ \frac{I_1}{L_1} = \frac{M}{\sum K}\left(\frac{I_1}{L_1}\right) = M\ \frac{K_1}{\sum K} \\ M'_{DA} &= \frac{M}{\sum K}\left(\frac{I_2}{L_2}\right) = M\ \frac{K_2}{\sum K} \\ M'_{CA} &= \frac{M}{\sum K}\left(\frac{I_3}{L_3}\right) = M\ \frac{K_3}{\sum K} \\ M'_{EA} &= \frac{M}{\sum K}\left(\frac{I_4}{L_4}\right) = M\ \frac{K_4}{\sum K} \end{aligned}\right\} \qquad \ldots (3.15)$$

Thus, the balancing moment M has to be allotted to each member in ratio of their respective stiffnesses, and the multiplying factors. $\frac{K_1}{\sum K}$ etc. are called the *moment distribution factors* or simply **distribution factors** (DF).

$$DF = \frac{K_i}{\sum K} \qquad \ldots (3.16)$$

It is interesting to note that the *sum of the distribution factors of all the members meeting at a joint is equal to 1*.

3.5 CARRY OVER MOMENTS

As soon as we distribute the balancing moment *M* to each member as in Eq. (3.15), a fixing moment equal to half its value appears at the other end of the member. For example, if we distribute a moment $M'_{BA} = M \left(\frac{K_1}{\sum K} \right)$ to end *B* of member *BA*, a moment $\left(= \frac{M'_{BA}}{2} \right)$ appears at the other end *A* of the member which is fixed. Refer Eq. (3.8) in Section 3.3. This happens to all the other members *DB*, *CB* and *EB* which meet at *B*. These moments are called **carry over moments**.

Now, if the structure is not a simple joint as shown in Figure 3.7 but forms part of an extended structure which may extend beyond the joints *A*, *D*, *C*, *E*, etc. then these carry over moments produce an unbalanced condition at all these remote ends. This needs a second cycle of adjustments or moment distribution, with the next round of carry over and so on. The cycle is repeated.

But as stated above, the distribution moments get smaller in value each time and the moment distribution process converges fast enough as we shall soon see.

3.6 MOMENT DISTRIBUTION PROCEDURE

With the knowledge gained in previous sections, we see that the moment distribution method involves the following steps :

(a) Determination of **fixed end moments** due to externally applied loads for all the members (this is given in **Appendix A** for some common cases).

(b) Determination of **distribution factors** for members meeting at each joint.

(c) The *sum* of the fixed end moments at each joint is calculated. Next calculate the **balancing moments** which is equal in magnitude but opposite in sign to this sum.

(d) At each joint distribution of **balancing moments** to each member in the ratios of distribution factors.

(e) **Carry over** half the distributed moments to the other end of each member.

(f) Repeat the cycle (d) and (e) once it is completed for all the joints of the structure.

This process continues till sufficient accuracy of results is achieved.

However, it must be remembered that the process **should stop only after a distribution** and **never** after a *carry over*.

Finally, the sum of all the moments at each joint is added to get the final resulting bending moment. Thus, we see that the moment distribution is an approximate method of analysis and the degree of accuracy depends upon the number of cycles you perform. However, it can be easily seen that after certain number of cycles sufficient accuracy has been achieved.

3.7 CONTINUOUS BEAMS

Now, this will be explained by an example of a *continuous beam* given below.

3.7.1 Moment Distribution Method Applied to Continuous Beam

Example 3.1

A continuous beam fixed at ends *A* and *D* is loaded as shown in Figure 3.8. The end spans *AB* and *CD* have moment of inertia *I* and for the middle span *BC* the moment of inertia is 1.5 *I*. Determine the moments and reactions at the support and draw the BM and SF diagrams.

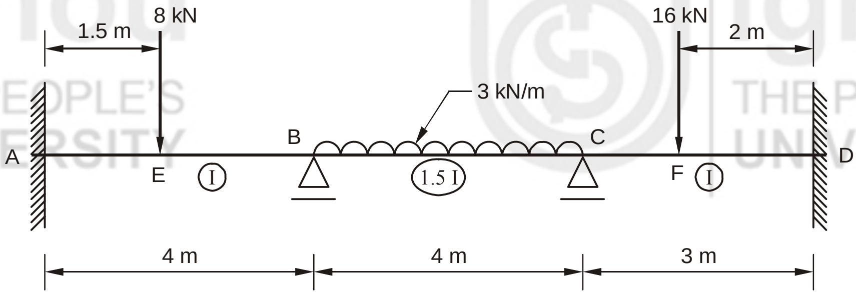


Figure 3.8

Solution

Step 1

The fixed end moments for each member is determined below :

Member *AB*

$$M_{FAB} = \frac{8 \times 1.5 \times (2.5)^2}{4^2} = -\ 4.6875 \text{ kNm}$$

$$M_{FBA} = \frac{8 \times (1.5)^2 \times 2.5}{4^2} = +\ 2.8125 \text{ kNm}$$

Member *BC*

$$M_{FBC} = \frac{3 \times 4^2}{12} = -\ 4.0 \text{ kNm}$$

$$M_{FCB} = \frac{3 \times 4^2}{12} = +\ 4.0 \text{ kNm}$$

Member *CD*

$$M_{CD} = \frac{16 \times 1 \times 2^2}{3^2} = -\ 7.1111 \text{ kNm}$$

$$M_{DC} = \frac{16 \times 1^2 \times 2}{3^2} = +\ 1.7778 \text{ kNm}$$

Step (2)

The *distribution factors* for members meeting at each joint is determined below :

At joint A

The joint *A* is fixed and there is only one member *AB*, hence, *no distribution* is required here.

At joint B

The joint *B* is simply supported and two members *BA* and *BC* meet at this joint

$$K_{BA} = \frac{I}{l} = \frac{I}{4}$$

$$K_{BC} = \frac{1.5\,I}{4}$$

$\therefore$ By Eq. (3.16), distribution factor for *BA*,

$$= \frac{K_{BA}}{\sum K} = \frac{\frac{I}{4}}{\frac{I}{4} + \frac{1.5\,I}{4}} = 0.4$$

Distribution factor for *BC*, $= \dfrac{K_{BA}}{\sum K} = \dfrac{\frac{1.5\,I}{4}}{\frac{I}{4} + \frac{1.5\,I}{4}} = 0.6$

At joint C

There are two members *CB* and *CD* and we have $K_{CB} = \dfrac{1.5\,I}{4}$ and $K_{CD} = \dfrac{I}{3}$.

and $$DF_{CB} = \frac{\frac{1.5\,I}{4}}{\frac{1.5\,I}{4} + \frac{I}{3}} = 0.53$$

and $$DF_{CD} = \frac{\frac{I}{3}}{\frac{1.5\,I}{4} + \frac{I}{3}} = 0.47$$

At joint D

As the joint is fixed there will *not be any distribution* of moment at the only member *DC* meeting at this joint (*DF* = 0).

The actual process of distribution of moments begins. This is usually done in a tabular form. Various authors have suggested various forms. For facility of calculation for a continuous beam the simple scheme in Table 3.1 appears to be good enough.

Table 3.1

Joint	**A**	**B**		**C**		**D**
Distribution Factor	0	0.4	0.6	0.53	0.47	0
Members	AB	BA	BC	CB	CD	DC
Fixed End Moments	– 4.6875	+ 2.8125	– 4.0	+ 4.0	– 7.1111	+ 1.7778
1st Distribution	0	+ 0.4750	+ 0.7125	+ 1.6489	+ 1.4622	0
Carry over	+ 0.2375	0	+ 0.8244	+ 0.3563	0	+ 0.7311
2nd Distribution	0	– 0.3298	– 0.4946	– 0.1888	– 0.1675	0
Carry over	– 0.1649	0	– 0.0944	– 0.2473	0	– 0.0837
3rd Distribution	0	+ 0.0378	+ 0.0566	+ 0.1311	+ 0.1162	0
Carry over	+ 0.0189	0	+ 0.0655	+ 0.0283	0	+ 0.0581
4th Distribution	0	– 0.0262	– 0.0393	– 0.0150	– 0.0133	0
TOTAL	**– 4.5960**	**+ 2.9693**	**– 2.9693**	**+ 5.7134**	**– 5.7134**	**+ 2.4833**

In Table 3.1, the first row denotes the '*joints'* which in this case are the four support points *A*, *B*, *C*, *D*. Next two rows denote the '*distribution factors'* and names of the *members meeting* at the joint. At end supports *A* and *D*, there are one member each (e.g. *AB* and *DC*); at the intermediate supports *B* and *C* two members one from either side meet (e.g. at *B* we have *BA*, *BC* and at *C*, we have *CB*, *CD*). As the joints *A* and *D* are fixed no distribution is to be done at this point and the *DF*s are zero.

At joint *B*, the distribution factors for members *BA* and *BC* which are 0.4 and 0.6 as calculated above are noted. Similarly, at joint *C*, the distribution factors for the members *CB* and *CD* which are 0.53 and 0.47 are noted. A double line at this point is given showing the beginning of the process.

In the next row, the fixed end moments for each member as calculated previously in step (1) are noted. The first moment distribution for the unbalanced joint moments is then done in the next row. Below member *AB* at joint *A*, we shall write 0 as there is no distribution done here. Next at the joint *B*, we have fixed end moment + 2.8125 in member *BA* and – 4.0 kNm in member *BC*. Thus, there is an unbalanced moment of + 2.8125 – 4.0 = – 1.1875 kNm. The balancing moment (*M*) will be equal in magnitude but opposite in sign, i.e. + 1.1875 kNm. Now this moment has to be distributed in the ratio of 0.4 : 0.6 ratio in the two members *BA* and *BC*.

∴ Balancing moment for *BA* = + 1.1875 × 0.4 = + 0.475 kNm

and Balancing moment for *BC* = + 1.1875 × 0.6 = + 0.7125 kNm.

These two moments are noted in their respective columns in the row '*1st Distribution'*. The same process is done at joints *C* and *D* which can be verified. After completing the first distribution, half the moments at each end are to be carried over to the other end as explained above. Hence in the first carry over operation for member *AB* as the end *A* had 0 moment corrections, therefore, carry over to end *B* will be 0 and end *B* has +0.4750 kNm correction so

$+\frac{0.4750}{2} = +0.2375$ is carried over to end *A*. This is shown by a pair of arrows.

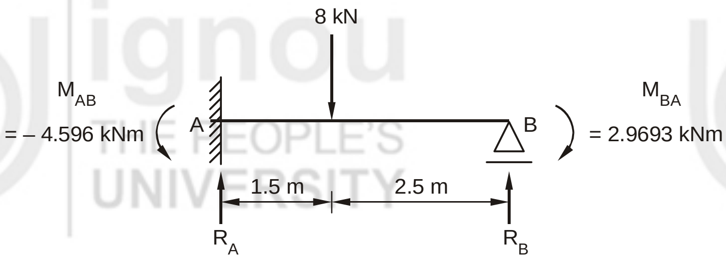
Similar carry overs are done for members *BC* (*CB*) and *CD* (*DC*).

As you cannot stop after a carry over a second distribution must be done this is shown in the next column. Thus, two cycles of distributions are completed. In the table shown two more cycles (i.e. four distributions) of operations are performed. After the fourth distribution we see that the correction moments have become less the 0.05 kNm. Hence the moment distribution process is stopped and all the moments in the same vertical line are algebraically added. This gives the final bending moments at each joint (member – end).

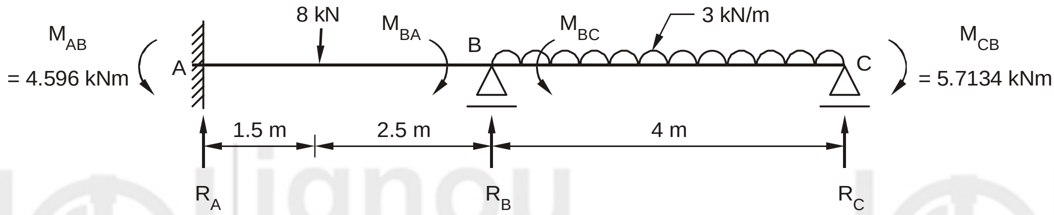
In member *AB* : the left hand end *A* is acted upon by a negative moment – 4.5960 kNm which is anticlockwise and, therefore, causes a hogging (or negative) bending moment. At end *B* we have a positive moment + 2.9693 kNm which is clockwise and since it is acting at right hand end it will be again hogging (or negative) BM. By extending the above argument we see that all the support bending moments are hogging (i.e. negative) BM.

Support Reactions

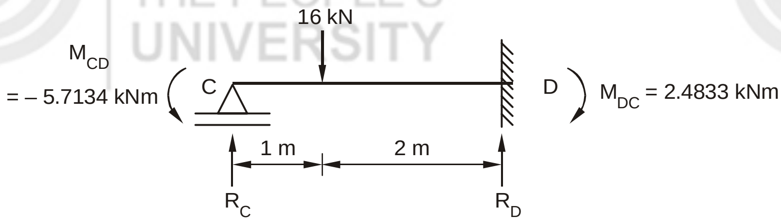
To find the support reaction we consider the free body diagrams of each beam span as shown below :



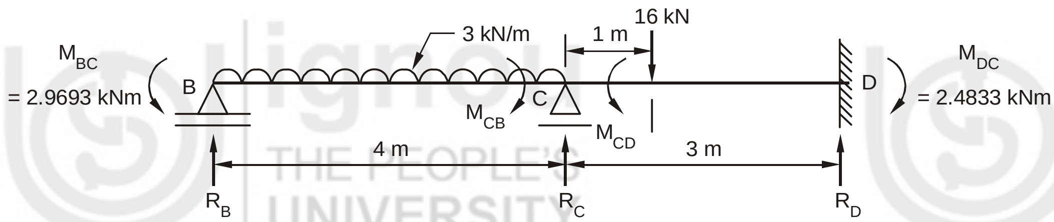
(a) Free Body Diagram of AB



(b) Free Body Diagram of ABC



(c) Free Body Diagram of CD



(d) Free Body Diagram of BCD

Figure 3.9 : Free Body Diagrams

(a) Considering free body diagram of *AB*, shown in Figure 3.9(a), it is acted upon by a anticlockwise couple of – 4.5960 at end *A*, and + 2.9693 kNm at end *B*; while a load of 8 kN is acting downwords at point 1.5 m to right of *A*. Let the support reactions at *A* and *B* be R_A and R_B.

Taking moments about end *B*, we have,

$$-4.596 - 8 \times 2.5 + R_A \times 4 + 2.9693 = 0$$

giving $\boldsymbol{R_A = 5.4066}$ **kN** . . . (i)

(b) Next considering the free body diagram of the two spans *ABC* shown in Figure 3.9 (b) the end couples are $M_{AB} = -4.596$ kNm and $M_{BC} = +5.7134$ kNm. The moment M_{BA} and M_{BC} at the intermediate support *B* cancel out each other as they are equal in magnitude but opposite in sign. The concentrate load of 8 kN is acting in span *AB* and a uniformly distributed load of 3 kN/m over span *BC*. The support reaction at *A*, *B*, *C* are R_A, R_B, R_C of which R_A is known to be 5.4066 kN.

∴ Taking moment of all forces about point C, we have

$$-4.5960 + 5.7134 - 8 \times 6.5 - (3 \times 4) \times 2 + 5.4066 \times 8 + R_B \times 4 = 0$$

giving $\boldsymbol{R_B = 7.9076}$ **kN** . . . (ii)

(c) Similarly, from free body of CD, shown in Figure 3.9(c), we get the equation (taking moment about point C)

$$-5.7134 + 2.4833 + 16 \times 1 - R_D \times 3 = 0$$

giving $\boldsymbol{R_D = 4.2564}$ **kN** . . . (iii)

Again from free body of BCD shown in Figure 3.9(d), taking moments about B, we get,

$$-2.9693 + 2.4833 + (3 \times 4) \times 2 + 16 \times 5$$
$$-R_C \times 4 - 4.2566 \times 7 = 0$$

giving $\boldsymbol{R_C = 18.4294}$ . . . (iv)

Thus, the four support reaction are as follows :

$R_A = 5.4066$ kN

$R_B = 7.9076$ kN

$R_C = 18.4294$ kN

$R_D = 4.2564$ kN

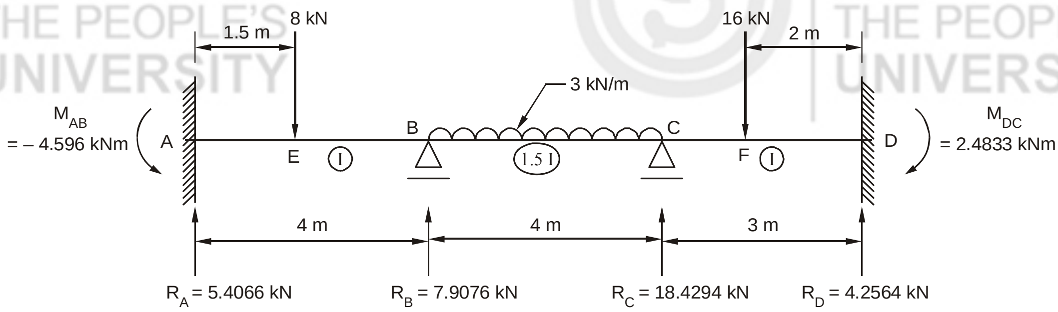
The sum of all these reaction is 36 kN which is the same as the total downward load, i.e. 8 + 3 × 4 + 16 = 36 kN.

The shear force diagram can be now easily drawn (Figure 3.10(b)).

For bending moment two diagrams are drawn in Figure 3.10(c) the free positive bending moments due to the vertical loads in each span is first drawn, over which the negative BM diagrams are superimposed to cut off the negative values from the positive ones.

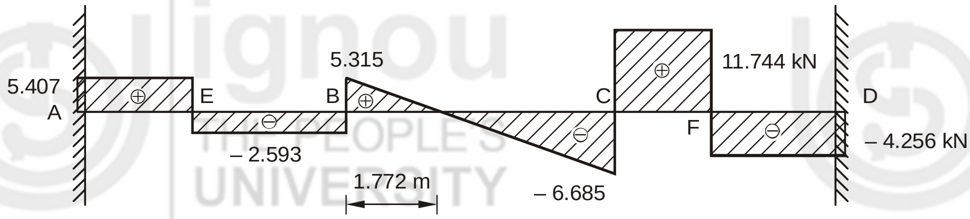
However, a better depiction of bending moments is obtained when the some diagram is drawn on a common horizontal base as shown in Figure 3.10(d). The BMs and the points of contraflexture may be verified.

The maximum positive BM in central span BC occurs, where the SF = 0, which is obtained in Figure 3.10(b) at 1.772 m from support B. The value of the maximum positive BM may then be calculated and found to be + 1.659 kNm.

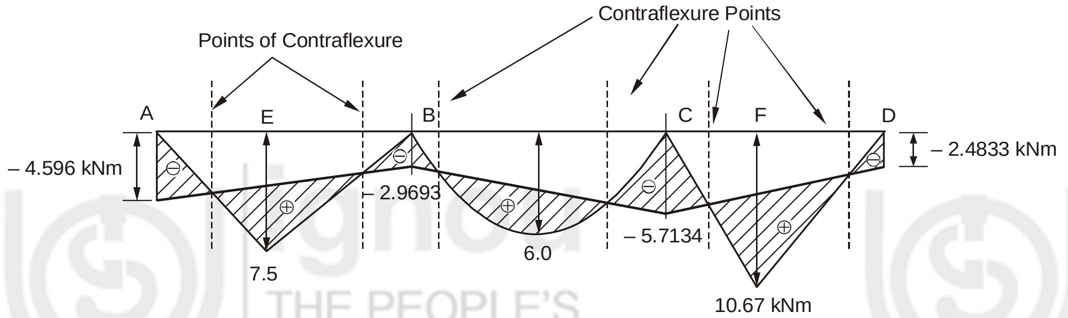


(a) Reactions

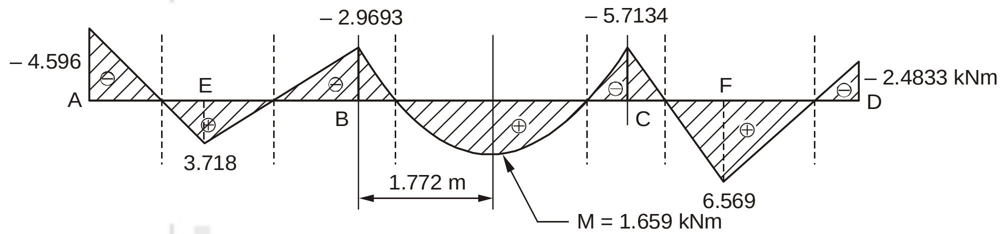
Figure 3.10



(b) SF Diagram



(c) BM Diagram (Superimposed)



(d) BM Diagram (Common Base)

Figure 3.10

3.7.2 Continuous Beam with Hinged End Supports

In the above example, the end supports of the continuous beam are fixed, hence no distribution is required there. However, in the case of end supports being simply supported (hinges) the bending moments will be zero and at this point the fixed end moments will have to be fully redistributed. This is explained in the next example.

Example 3.2

Draw the BM and SF diagrams for the continuous beam *ABC* shown in Figure 3.11, using moment distribution method. The moment of inertia in span *AB* is double that of *BC*.

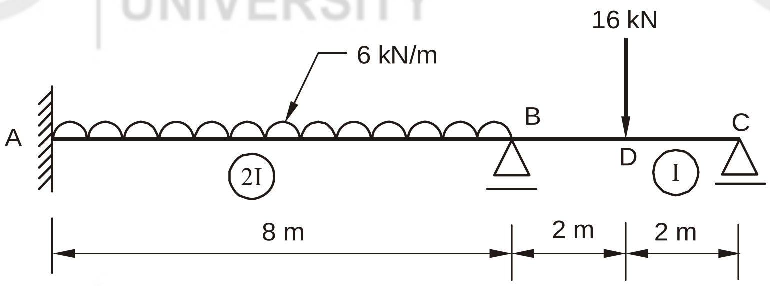


Figure 3.11

Solution

Fixed End Moments

$$M_{FAB} = -\frac{6 \times 8^2}{12} = -32 \text{ kNm}, \quad M_{FBA} = +32$$

$$M_{FBC} = -\frac{16 \times 4}{8} = -8 \text{ kNm}, \quad M_{FCB} = +8 \text{ kNm}.$$

Distribution Factors

At joint *A*, *DF* = 0, since it is fixed.

At joint *B*, there are two member $= K_{BA} = \frac{2I}{8}$ and $K_{BC} = \frac{I}{4}$

$$\therefore \quad (DF)_{BA} = \frac{\frac{2I}{8}}{\frac{2I}{8} + \frac{I}{4}} = 0.5; \quad (DF)_{BC} = \frac{\frac{I}{4}}{\frac{2I}{8} + \frac{I}{4}} = 0.5$$

At joint *C*, Distribution Factor = 1, since it is hinged.

The process of moment distribution is next carried out in the table below and does not need any further explanation.

Table 3.2

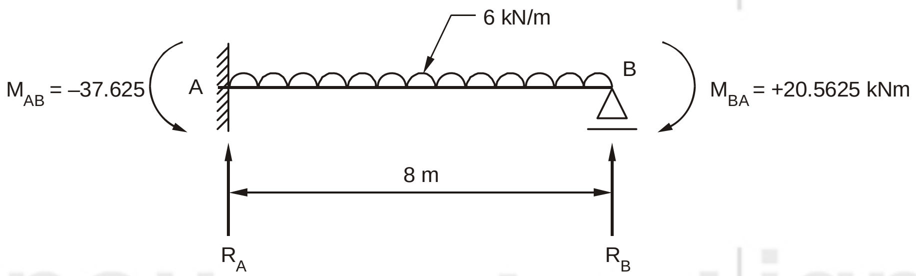
Joint	A	B		C
Distribution Factors	0	0.5	0.5	1
Members	AB	BA	BC	CB
Fixed end moments	– 32.0	+ 32.0	– 8.0	+ 8.0
1st Distribution	0	– 12.0	– 12.0	– 8.0
Carryover	– 6.0	0	– 4.0	– 6.0
2nd Distribution	0	+ 2.0	+ 2.0	+ 6.0
Carryover	+ 1.0	0	+ 3.0	+ 1.0
3rd Distribution	0	– 1.5	– 1.5	– 1.0
Carryover	– 0.75	0	– 0.5	– 0.75
4th Distribution	0	+ 0.25	+ 0.25	+ 0.75
Carryover	+ 0.125	0	+ 0.375	+ 0.125
5th Distribution	0	– 0.1875	– 0.1875	– 0.125
TOTAL	**– 37.6250**	**+ 20.5625**	**– 20.5625**	**0**

Support Reactions

(i) From Free Body Diagram of *AB* shown in Figure 3.12(a), taking moments about *B*,

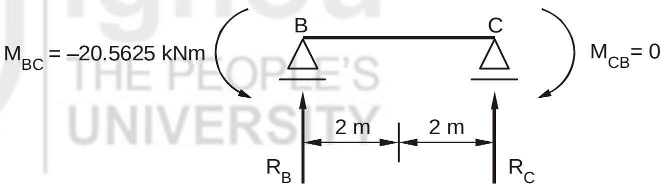
$$-37.625 + 20.5625 - (6 \times 8) \times 4 + R_A \times 8 = 0$$

giving $\boldsymbol{R_A = 26.133}$ **kN**



(a) Free Body Diagram of AB

Figure 3.12



(b) Free Body Diagram of BC

Figure 3.12

(ii) From Free Body Diagram of *BC* shown in Figure 3.12(b).

Taking moments about *B*

$$-20.5625 + 16 \times 2 - R_C \times 4 = 0$$

giving $\boldsymbol{R_C}$ **= 2.859 kN**.

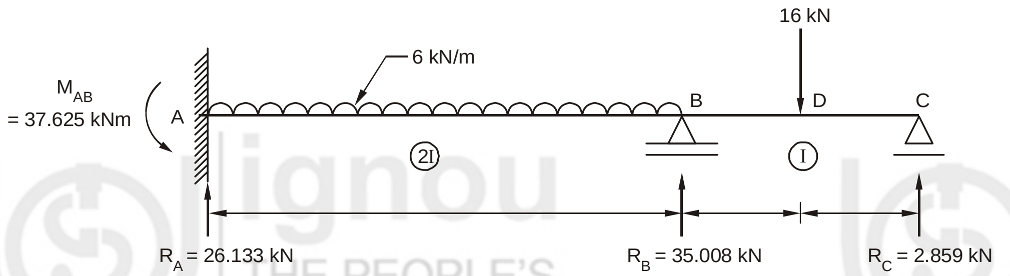
Next considering vertical equilibrium of the structure

$$R_A + R_B + R_C = 6 \times 8 + 16 = 64$$

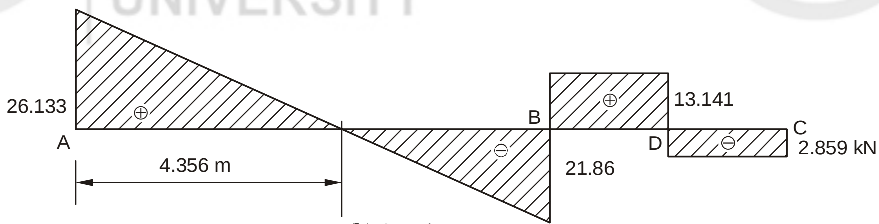
Substituting the values $26.133 + R_B + 2.859 = 64$

giving $\boldsymbol{R_B}$ **= 35.008 kN**.

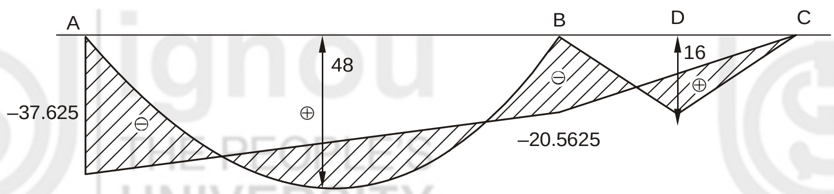
Now, the Bending Moment Diagram (BMD) and Shear Force Diagram (SFD) can be drawn as shown in Figure 3.13.



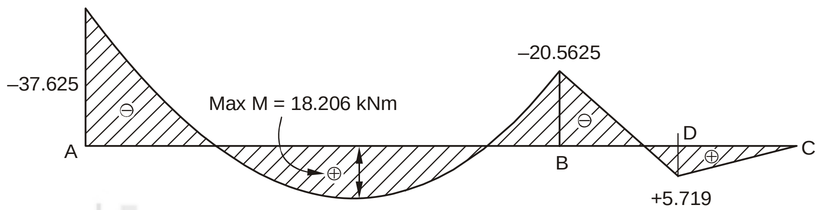
(a) Loading and Reactions



(b) SF Diagram



(c) BM Diagram (Superimposed)



(d) BM Diagram (Common Base)

Figure 3.13

Note : Show that the maximum positive BM in the span *AB* occurs at 4.356 m from *A* and its value is **18.906** kNm. Also find the position of points of contraflexure (BM = 0).

SAQ 1

(a) Analyze the continuous beam shown in Figure 3.14. The beam has constant *EI* throughout. Draw the BM and SF diagrams.

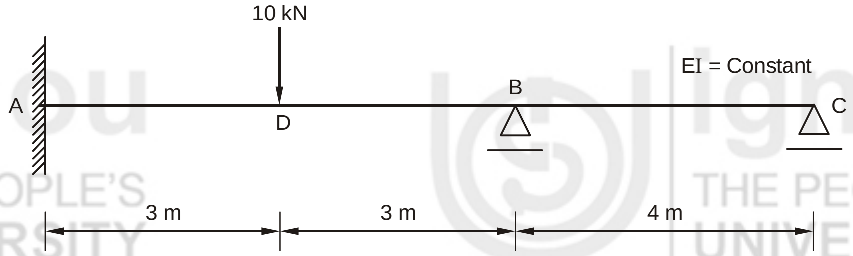


Figure 3.14

(b) Using moment distribution method, find out the support reactions and draw the BM and SF diagrams for the continuous beam shown in Figure 3.15 (*EI* is constant throughout).

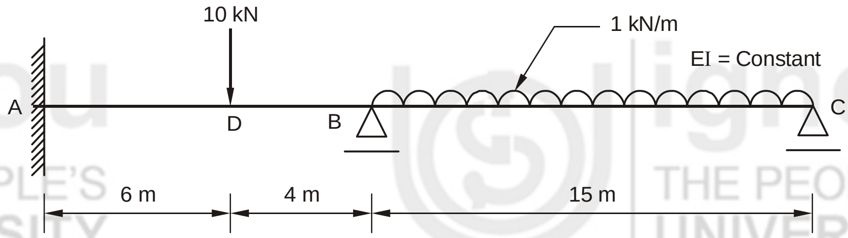


Figure 3.15

(c) Determine the support moments and reactions of the continuous beam shown in Figure 3.16. Draw the bending moment and shear force diagrams. Use moment distribution method. The relative moments of inertia of the beam is shown within circles in each span.

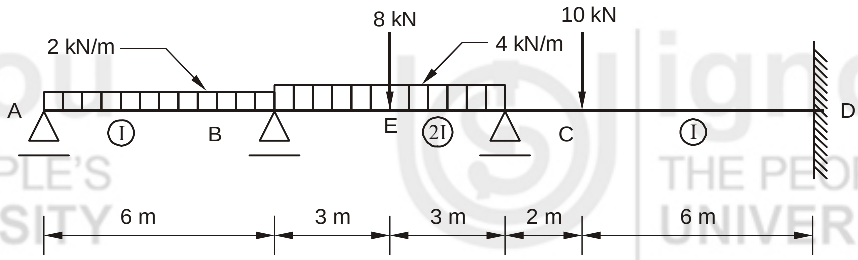


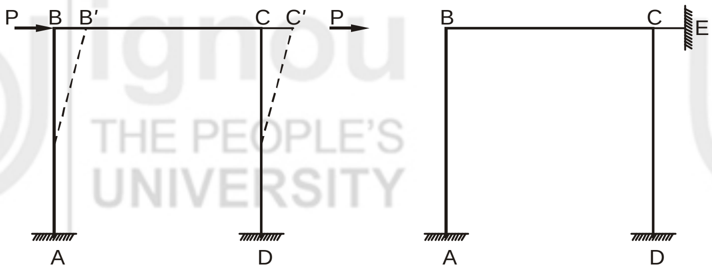
Figure 3.16

3.8 PORTAL FRAMES

As explained in the introductory section of this unit, portal frames are rigid jointed structures which may have one or more spans, or one or more storey. Generally, they are statically indeterminate and the moment distribution method is specially suited to solve them. Here, we shall consider portals of single span and single storey only. The supports may be either fixed, hinged or roller.

3.8.1 Portal Frames and Sway

We have seen that the supports of a continuous or fixed beam may move vertically due to settlement of foundations, etc. As they are statically indeterminate structures these support movements introduce internal stress resultants in them. In portal frames, in addition to vertical support movements, another type of joint movement is possible which is in the horizontal direction and is known as sway. This sway movement is also capable of affecting the internal stress resultants in the portal members, and hence cannot be ignored. However, not all portals are subjected to sway. Look at Figure 3.17(a).



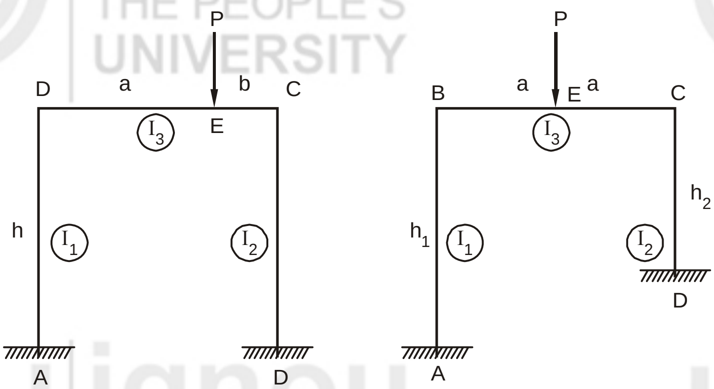
(a) Portal Frame with Sway **(b) Portal Frame without Sway**

Figure 3.17

The portal *ABCD* is subjected to a horizontal load *B*, hence, the joint *B* and *C* having freedom to move towards the right may occupy the position, *B*′ and *C*′ as shown by dotted lines. The movements *BB*′ (which is same as *CC*′) will be called the amount of sway at this level *BC*. However, in Figure 3.17(b), the portal is held at joint *C* to the fixed support *E* by another member *CE*. This will not allow free sway of *C* (or *B*). Hence, this portal will be without any sway. In a multistoreyed portal, independent sways are possible at each ‘floor’ or ‘storey’ level. However, we are not discussing here multi-storeyed portals.

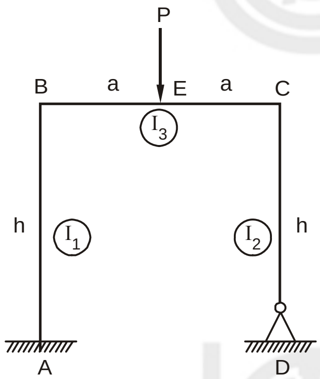
Sway is not only possible due to horizontal loads, but due to lack of symmetry either in *loading* or *support conditions* or *geometrical properties* of the portal.

In Figure 3.18(a), the loading is unsymmetric (*a* and *b*), being unequal even though the portal is symmetric. Hence, there will be a sway. In Figure 3.18(b), the load is symmetrically placed on the beam but the columns being of unequal height causes unsymmetry in the structure and hence, sway will take place. Sway will take place even in Figure 3.18(c) where the lack of symmetry may be due to different type of supports on the two sides (fixed at *A*, hinged at *D*) or due to unequal moments of inertia ($I_1 \neq I_2$).



(a) Sway Due to Unsymmetrical Loading (b) Sway Due to Unsymmetrical Geometry of Frame

Figure 3.18



(c) Sway Due to Unsymmetrical Support

Figure 3.18

The sway *BB′* causes a chord rotation of member *AB* (similarly, *CC′* for member *DC*). Due to this chord rotation, fixed end moments $-\dfrac{6EI\,\Delta}{l^2}$ will be introduced at either end of the member where $\Delta = BB'$.

Now, we shall take the case of symmetrical portals loaded with symmetrical loads only, in the example below. The moment distribution is carried out in a similar tabular form as used for continuous beams in the previous section.

Example 3.3

Analyze the portal frame shown in Figure 3.19 and draw the BM and SF diagrams. The moments of inertia are shown within circles.

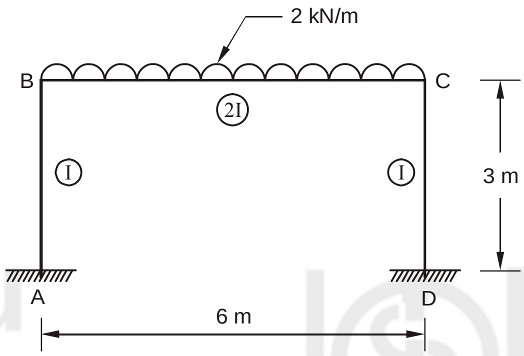


Figure 3.19

Fixed End Moment

As there are no loads on columns *AB* and *CD*, there will not be any fixed moments at their ends. However, the beam member *BC* will have fixed end moments.

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{2 \times 6^2}{12} = -\,0.6 \text{ kNm}; \; M_{FCB} = +\,6.0 \text{ kNm}.$$

There are no sway moments as the frame and loading is symmetrical.

Distribution Factors

Joints A and D are fixed hence, there will not be any distribution of moments there and $(DF)_{AB} = 0$ and $(DF)_{DC} = 0$.

At joint B, there are two members BA and BC meeting. Their relative stiffnesses are $K_{BA} = \frac{I}{3}$, and $K_{BC} = \frac{2I}{6} = \frac{I}{3}$

$$\therefore \qquad (DF)_{BA} = \frac{\frac{I}{3}}{\frac{I}{3} + \frac{I}{3}} = 0.5 \text{ and } (DF)_{BC} = 0.5$$

Similarly, at joint C we have $(DF)_{CB} = 0.5$ and $(DF)_{CD} = 0.5$.

The moment distribution is made joint-wise and member-wise as shown below :

Joint	**A**	**B**		**C**		**D**
Distribution Factor	0	0.5	0.5	0.5	0.5	0
Members	AB	BA	BC	CB	CD	DC
Fixed End Moments	0	0	– 6.0	+ 6.0	0	0
1st Distribution	0	+ 3.0	+ 3.0	– 3.0	– 3.0	0
Carryover	+ 1.5	0	– 1.5	+ 1.5	0	– 1.5
2nd Distribution	0	+ 0.75	+ 0.75	– 0.75	– 0.75	0
Carryover	+ 0.375	0	– 0.375	+ 0.375	0	– 0.375
3rd Distribution	0	+ 0.187	+ 0.188	– 0.188	– 0.187	0
Carryover	+ 0.093	0	– 0.094	+ 0.094	0	– 0.093
4th Distribution	0	+ 0.047	+ 0.047	– 0.047	– 0.047	0
Carryover	+ 0.024	0	– 0.024	+ 0.024	0	– 0.024
5th Distribution	0	+ 0.012	+ 0.012	– 0.012	– 0.012	0
TOTAL	**+ 1.992**	**+ 3.996**	**– 3.996**	**+ 3.996**	**– 3.996**	**– 1.992**

Calculation of Support Reactions

The support is fixed hence, it will be subjected to three reactions, sign conventions being horizontal reaction H_A ($\rightarrow$ positive), V_A ($\uparrow$ positive) and M_A (positive). From the above result $M_A = + 1.992$ kNm.

Considering the free body diagram of vertical member AB (Figure 3.20(a)), we find that it is acted upon by clockwise moments of $M_A = + 1.992$ and $M_B = + 3.996$ kNm at ends A and B respectively; in addition to the unknown reactions H_A, V_A at end A and H_B, V_B at end B. Taking moments about the point B, we have $1.992 + 3.996 - H_A \times 3 = 0$, giving $H_A = 1.9968$ kN and is acting toward right as shown.

By consideration of symmetry (or calculation if you so like), the horizontal reaction at D will be of same magnitudes, but opposite in direction. For finding V_A, taking moments of all the forces acting on the frame as shown in Figure 3.20(b) about the point D, we have

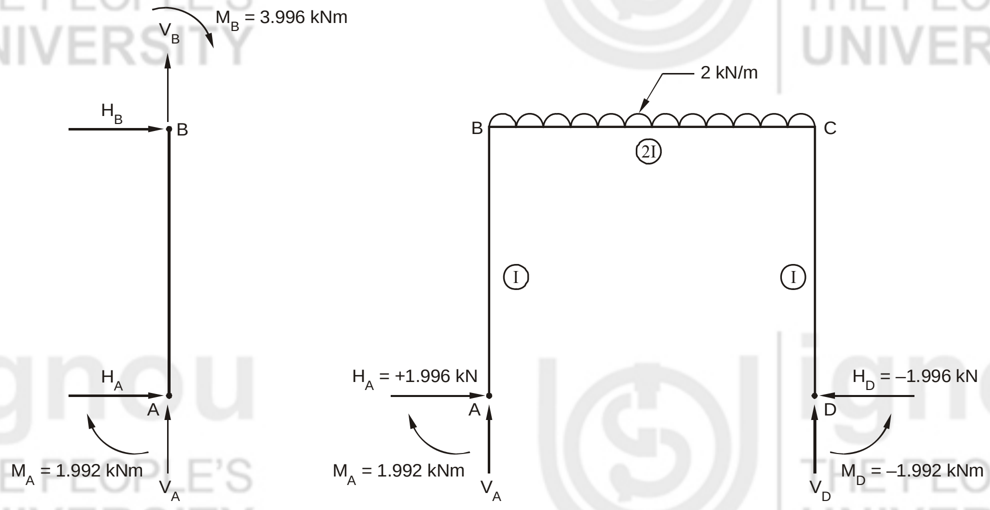
$$+ 1.992 - 1.992 + V_A \times 6 - (2 \times 6) \times 3 = 0$$

giving $\qquad V_A = 6$ kN.

The frame with the reactions and loading is shown in Figure 3.21(a). By vertical equilibrium of whole frame

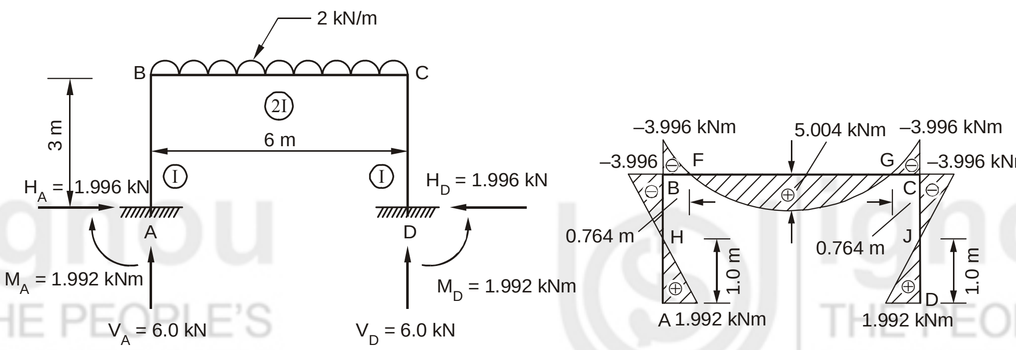
$$V_A + V_D = 2 \times 6$$

$$\therefore \qquad V_D = 6 \text{ kN}.$$

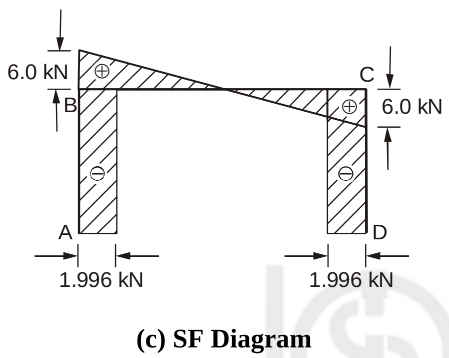


(a) Free Body Diagram of AB **(b) Free Body for Portal**

Figure 3.20



(a) Reactions and Loads **(b) BM Diagram**



(c) SF Diagram

Figure 3.21

(a) While drawing the **BM diagram,** for the beam *BC*, the same convention is used for sagging and hogging bending moments; namely the positive sagging moment is drawn below (– ve) the reference line, i.e. on that side of the beam where fibres are in tension, and negative (hogging) moments are drawn above the line where fibres are in tension in this case.

As we cannot have sagging or hogging concept for the columns *AB* and *CD* we shall continue with the convention of drawing the BM on the tension side as in the beam *BC*.

For member *AB*, the positive end moment of + 1.992 kNm causes a bending moment producing tension on the right hand face; while at end *B* the positive end moment of + 3.996 kNm produce a tension on the opposite, i.e. left hand face. Thus, they are plotted on opposite *sides of line AB* and joined by a straight line. Similarly, for column *CD* also the complete BM diagram is shown at Figure 3.21(b). The compression side is marked '*C*' the opposite side of the member will be in tension. The points of contraflexure (where the bending moments change the sign) are the points of zero bending moments and can be easily calculated.

For vertical members *AB* :

$$AH = \frac{1.992}{1.992 + 3.996} \times AC = \frac{1.992}{5.998} \times 3 = \mathbf{1.0\ m}$$

For horizontal member *BC*, *BF* is obtained by putting the moment equation to zero.

$$M_x = -3.996 + 6 \cdot x - 2 \cdot x \frac{x}{2}$$

For point of contraflexure $M_x = 0$, when x **= 0.764 m** and **5.236 m** (from the above quadratic equation).

The maximum positive BM in the beam is at the point where,

$\frac{dM_x}{dx} = 0$, or $6 - 2x = 0$ or x = **3 m** (i.e. at the centre of the BM).

The maximum positive BM

$$\frac{wl^2}{8} - FEM = \frac{2 \times 6^2}{8} - 3.996 = + \mathbf{5.004\ kN\,m}$$

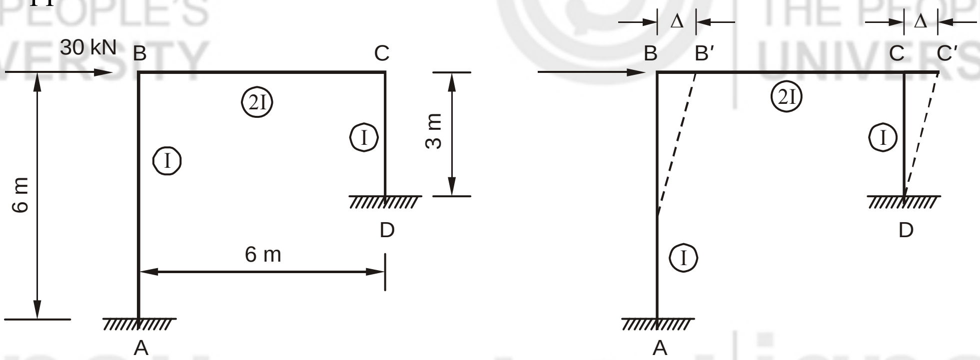
(b) The **shear force diagram** is shown in Figure 3.21(c). For the beam member *BC*, you are already familiar with the method. For the vertical members *AB*, *CD* the shear force is equal to the horizontal reaction ± any lateral force acting on them. As there are no lateral forces, the members are subjected to constant SF of 1.996 kN as shown in Figure 3.21(c).

3.8.2 Portal Frames with Sway Due to Horizontal Load

It has been shown above that a portal frame having a lack of symmetry or subjected to unsymmetrical or horizontal loads will exhibit 'sway'. This means that the joints free to move in the horizontal direction will move in that direction causing the concerned members to rotate in their plane. If this 'member rotation' or 'cord rotation' is clockwise, it has been shown in Section 3.3 (Eq. (3.10)) that it will induce fixed end moments equal to $-\frac{6EI\,\Delta}{l^2}$ (anticlockwise moments) where Δ is the amount of 'sway' or relative lateral movement of the member. In the moment distribution method, first it is assumed that there is no sway and the moment distribution is carried out for the fixed end moments caused by external loading only. Next a second distribution known as '*sway distribution*' is carried out for the frame for *sway moments* only. These 'sway moments' have to be adjusted by multiplying them by *a* '*factor*' so as to satisfy the conditions of horizontal equilibrium. As Δ is generally not known the sway moments which are in the ratio of $\left(\frac{E_i \cdot I_i}{L_i^2}\right)$ of the concerned members are assumed as convenient integral numbers in that ratio. Finally, the correction factor and correct moments are determined by writing down the conditions of horizontal equilibrium. These '*sway moments*' are then algebraically added to the fixed end moments due to external loads (already determined), giving the final joint moments. In the example here a case of *pure sway* (without any external load moments) is given and the method of solving it is shown.

Example 3.4

Analyse the rigid jointed portal frame shown in the Figure 3.22(a) subjected to a horizontal load of 30 kN at the joint *B*, acting to the right. The relative moments of inertia of the members are shown within the circles. The supports *A* and *D* are fixed.



(a) **(b)**

Figure 3.22

Solution

Fixed End Moments

There are no *fixed end moments* in any as the load in the members due to external loading of 30 kN is acting at one of the joints. However, there will be fixed end moments due to chord rotations of the members caused by sway. Figure 3.22(b) shows the deflected shape of the structure. Here, the joint *B* moves horizontally to *B′* and *C* moves to *C′*. Since *B* and *C* are connected by the rigid member *BC* the movement *BB′* = *CC′*= Δ (say). The vertical member *AB* then takes the new position *AB′* which is a clockwise rotation and will cause fixed end moment $-\frac{6EI\,\Delta}{6^2} = -\frac{EI\,\Delta}{6}$ at both ends

$$\therefore \qquad M_{FAB} = M_{FBA} = -\frac{EI\,\Delta}{6}$$

Similarly, the vertical member *DC* rotates clockwise to *DC′* and the fixed end moments will be

$$\therefore \qquad M_{FCD} = M_{FDC} = -\frac{6\,EI\,\Delta}{3^2} = -\frac{2\,EI\,\Delta}{3}.$$

As there is no vertical movement of the joints *B* and *C*, there will not be any chord rotation and, therefore, no fixed end moments in the horizontal member *BC*.

Now, as we do not know the actual valued of *E*, *I*, or *Δ* so we cannot calculate the fixed end moments in absolute terms.

However, we can say that $\dfrac{M_{FAB}}{M_{FCD}} = \dfrac{-EI\,\dfrac{\Delta}{6}}{-2EI\,\dfrac{\Delta}{3}} = \dfrac{1}{4}$ this shows that the *fixed end moment in member CD is four times that in member AB.*

Here, we will assume some arbitrary values of fixed end moments in these members in this ratio and perform the moment distribution operation and later on we will calculate the actual values by considerations of horizontal equilibrium.

So we begin with fixed end moments of – 10 kNm in member *AB* and – 40 kNm in member *CD*. In member *BC*, the fixed end moments are 0 at each end.

Distribution Factors

As the supports *A* and *D* are fixed, the distribution factor in the members *AB* and *DC* are both zero, $(DF)_{AB} = (DF)_{DC} = 0$.

At joint *B*, the relative stiffnesses of the members are

$$K_{BA} = \frac{I}{6} \quad \text{and} \quad K_{BC} = \frac{2I}{6} = \frac{I}{3}$$

$$\therefore \qquad (DF)_{BA} = \frac{\frac{I}{6}}{\frac{I}{6} + \frac{I}{3}} = \frac{1}{3} \quad \text{and} \quad (DF)_{BC} = \frac{2}{3}$$

At joint *C*, the relative stiffnesses of members are

$$K_{CB} = \frac{2I}{6} = \frac{I}{3} \quad \text{and} \quad K_{CD} = \frac{I}{3}$$

$$(DF)_{CB} = \frac{\frac{I}{3}}{\frac{I}{3} + \frac{I}{3}} = \frac{1}{2} \quad \text{and} \quad (DF)_{CD} = \frac{1}{2}$$

The moment distribution with the above assumed moments are given in the table below :

Joint	A	B		C		D
Distribution Factor	0	1/3	2/3	1/2	1/2	0
Members	AB	BA	BC	CB	CD	DC
Fixed end moments	– 10	– 10	0	0	– 40	– 40
1st Distribution	0	+ 3.333	+ 6.667	20	+ 20	0
Carryover	+ 1.667	0	+ 10	+ 3.333	0	+ 10
2nd Distribution	0	– 3.333	– 6.667	– 1.667	– 1.666	0
Carryover	– 1.667	0	– 0.833	– 3.333	0	– 0.833
3rd Distribution	0	+ 0.278	+ 0.555	+ 1.667	+ 1.666	0
Carryover	+ 0.139	0	+ 0.833	+ 0.278	0	+ 0.833
4th Distribution	0	– 0.278	– 0.555	– 0.139	– 0.139	0
Carryover	– 0.139	0	– 0.070	– 0.278	0	– 0.070
5th Distribution	0	+.023	+ 0.067	+ 0.139	+ 0.139	0

Total	**– 10**	**– 9.977**	**+ 9.977**	**+ 20**	**– 20**	**– 30.070**
After further distribution the values of the BMs will converge to						
	(– 10K)	**(– 10K)**	**(+ 10K)**	**(+ 20K)**	**(– 20K)**	**(– 30K)**

where K is an unknown multiplying factor.

After the above distribution it is clear that the fixed end moments are in the ratios as indicated in the last row, where K is an unknown multiplying factor. To find K we find the support reactions and use the equations of equilibrium.

Considering the free body diagram of AB as shown in Figure 3.23(a) and taking moments about B, we have

$$- H_A \times 6 + M_{AB} + M_{BA} = 0$$

$$- \quad H_A \times 6 - 10\,K - 10\,K = 0 \text{ giving} \qquad \boldsymbol{H_A = \frac{-10\ K}{3}}$$

Similarly, considering the free body diagram of CD at Figure 3.23(b); taking moments about D

or $\qquad - H_D \cdot 3 + M_{CD} + M_{DC} = 0$

or, $\qquad - H_D \cdot 3 - 20\,K - 30\,K = 0$ giving $\quad \boldsymbol{H_D = \frac{-50\ K}{3}}$

Now considering the horizontal equilibrium of the whole frame ($\sum H \equiv 0$)

We have, $\qquad H_A + H_D + 30 = 0;$

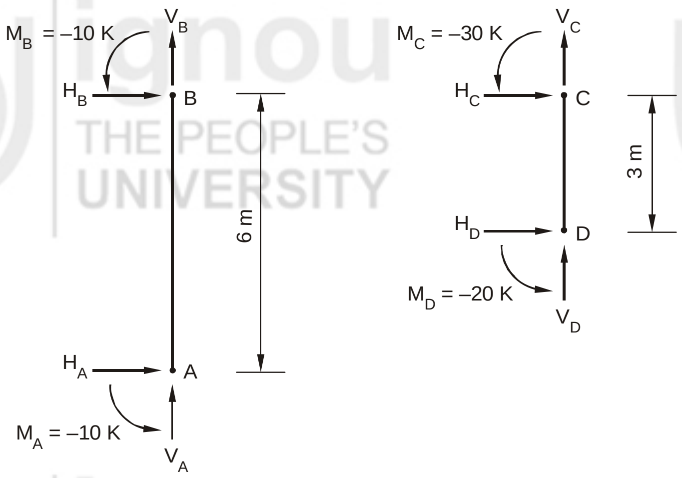
On substitution $\quad -\frac{10\,K}{3} - \frac{50\,K}{3} + 30 = 0$ giving $\boldsymbol{K = 1.5}$.

Hence, $H_A = -\frac{10}{3} \times 1.5 = -\ 5\ \text{kN},$ towards left and

$H_D = -\frac{50}{3} \times 1.5 = -\ 25\ \text{kN}$ towards right. The negative sign shows that they will be acting in a direction opposite to that shown (i.e. toward left).

The moment for the various members will be also multiplied by this factor, $K = 1.5$, and the final values (in kNm) are shown in kNm is given below :

	AB	**BA**	**BC**	**CB**	**CD**	**DC**
Relative Values	– 10 K	– 10 K	+10 K	+ 20 K	– 20 K	– 30 K
Actual Values (in kNm)	– 15	– 15	+15	+ 30	– 30	– 30



(a) **(b)**

Figure 3.23

The vertical reaction V_A is obtained by taking moment of all forces about D in the free body diagram of Figure 3.23(a).

$$V_A \times 6 + 30 \times 3 - 15 - 45 + 5 \times 3$$

giving $\quad \boldsymbol{V_A = -\ 7.5 \text{ kN}}$

showing that V_A acts vertically downward.

Similarly, considering the vertical equilibrium of the frame $V_A + V_D = 0$.

$\therefore \quad V_D = 7.5$ kN acting upwards (↑)

The reactions are shown in Figure 3.24.

The BM and SF diagrams are shown in Figures 3.25(a) and (b), respectively.

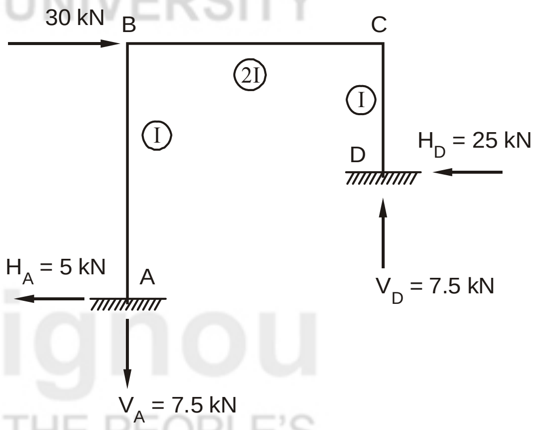
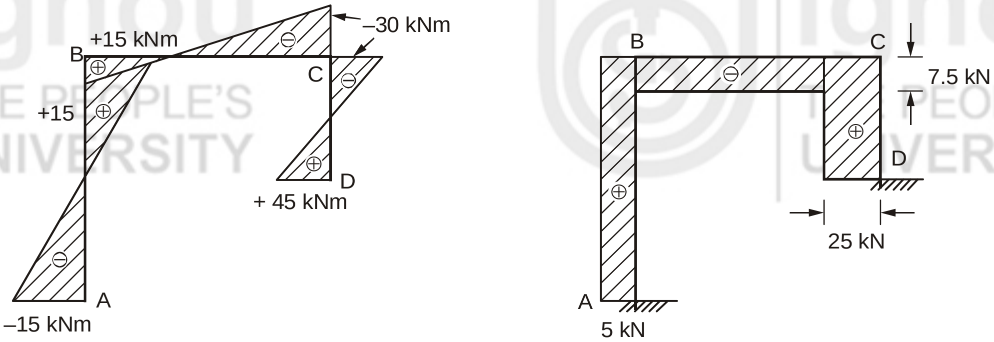


Figure 3.24

Explanation

As shown in Figure 3.25(a), for the beam BC, the clockwise moment 15 kNm at left hand end B produces a sagging (positive) bending moment. At end C, the clockwise moment of 30 kNm produces a hogging (negative) bending moment.



(a) BM Diagram **(b) SF Diagram**

Figure 3.25

For the column *AB*, the anticlockwise moment of 15 kNm at end *A* produces tension on left hand or outer face (as plotted). The anticlockwise moment of 15 kN at end *B* produces tension on right hand (inner) face. They will be plotted on the opposite sides of the *AB*.

Similarly, for column *CD*, the BM diagram may be drawn.

The shear force diagram given in Figure 3.25(b) is self-explanatory.

SAQ 2

(a) Analyze the rigid-jointed frame shown in Figure 3.26. Find the reactions and draw the BM and SF diagrams. The moment of inertia of the beam is 1.5 times that of the column.

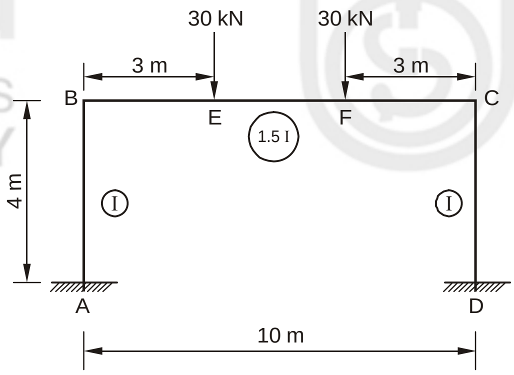


Figure 3.26

(b) Portal frame *ABCD* shown in Figure 3.27 is fixed at support *A* and hinged at *D*. It carries horizontal load of 15 kN at the joint *B*. Find the support reactions and draw the BM and SF diagram.

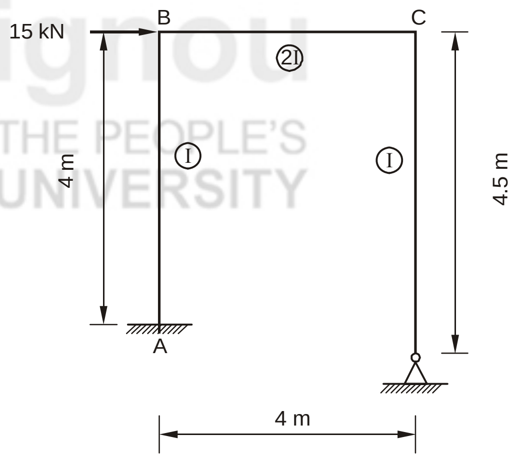


Figure 3.27

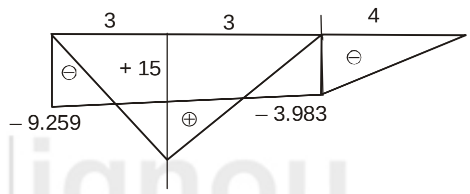
3.9 SUMMARY

In this unit, you have learnt about moment distribution procedure after deriving the slope deflection equation for the members. The concepts of distribution factor and carryover moment along with basic rules have been explained. The illustrative examples of continuous beams and portal frames with and without sway have been presented in this unit.

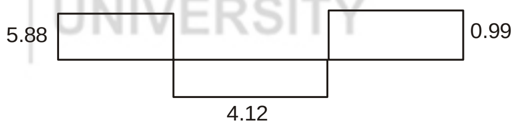
3.10 ANSWERS TO SAQs

SAQ 1

(a)



BM Diagram



SF Diagram

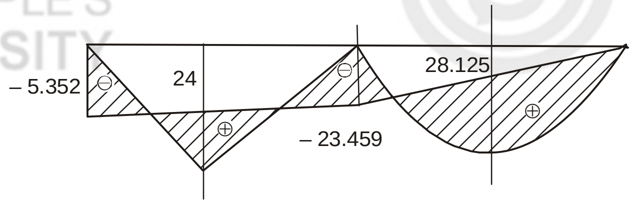
$$M_{AB} = -\ 9.259 \text{ kNm}$$

$$M_{BA} = -\ M_{BC} = -\ 3.983 \text{ kNm}$$

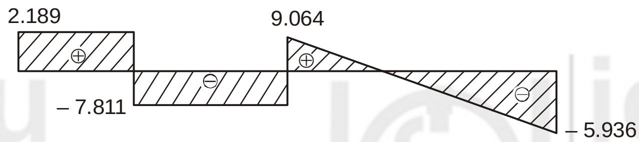
$$M_{CB} = 0$$

$V_A = 5.88 \uparrow$; $V_B = 5.12 \uparrow$; $V_C \downarrow$ kN

(b)



BM Diagram



SF Diagram

$M_{AB} = -5.352$ kNm

$M_{BA} = -M_{CA} = -23.459$ kNm

$M_{CB} = 0$

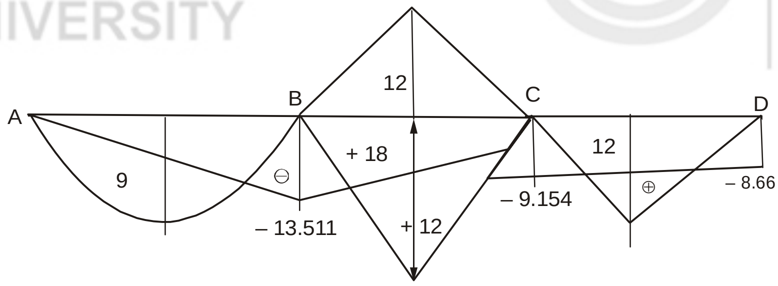
$V_A = 2.189$ kN $\uparrow$; $V_B = 16.875$ kN $\uparrow$; $V_C = 5.936$ kN $\uparrow$

(c) $M_{AB} = 0$

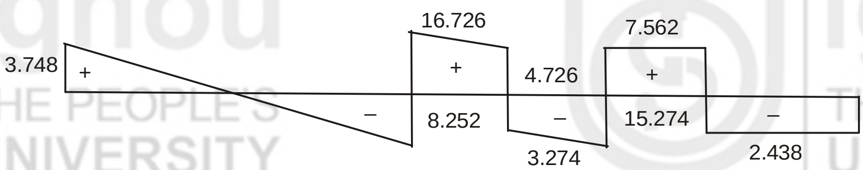
$M_{BA} = -M_{BC} = -13.511$ kNm

$M_{CB} = -M_{CD} = -9.154$ kNm

$M_{DC} = -8.66$ kNm



BM Diagram



SF Diagram

$V_A = 3.748$ kN $\uparrow$

$V_B = 24.978$ kN $\uparrow$

$V_C = 22.836$ kN $\uparrow$

$V_D = 2.438$ kN $\uparrow$

SAQ 2

(b) $M_{AB} = -27.695$ kN

$M_{BA} = -M_{BC} = -21.768$

$M_{CB} = -M_{CD} = +11.855$

$M_{DC} = 0$

$H_A = 12.348$ leftwards

$H_D = 2.652$

$V_B = 8.408$

$V_A = -8.408$.