UNIT 1 INFLUENCE LINES

Structure

1.1 INTRODUCTION

Study of Applied Mechanics and Strength of Materials have enabled you to compute the reactions, shear force (SF), bending moments (BM), deflections etc. in a beam and bar forces in a pin-jointed truss, subjected to a given static load system which remains *stationary*. Quite frequently, a beam or a truss is subjected to a load which is not exactly stationary and may be *moving* along a certain path (which may be the length of the beam or top or bottom chord of a truss etc.). Such moving loads are called *live loads* as opposed to stationary loads which may be called *dead loads*. Instances of live loads are quite common, e.g. a railway train moving across a rail bridge or a vehicle moving along a road bridge. Obviously, the value of any of the desired quantities (e.g., shear force, bending moment or bar force) depends upon the position of the load. For the design of the members, it is important to find out the position of the loads for which the stresses caused in the structure is maximum at any point or in any member. For this purpose, a graphical representation (or a curve), depicting the values of the desired quantity for various load positions, is drawn and is used for calculation. Such curves or lines are called **influence lines** for the quantity.

Objectives

After studying this unit, you should be able to

- conceptualise and define influence line,
- calculate the variation of a particular quantity (BM, SF, axial force etc.) due to a unit load moving across a structure,
- depict the variation of the quantity, graphically, through *influence lines*,
- discuss the properties of the *influence line* and to interpret it for direct use in structural analysis, and
- calculate the magnitude of the quantity under a given system of live loads moving across the structure.

1.2 DEFINITION

An influence line is a curve, the ordinate of which at any point is equal to the value of some structural quantity, when a unit load is placed at that point.

The structural quantity could be external support reactions, (e.g. vertical or horizontal reactive forces or bending moments), internal stress resultants (e.g. axial force, SF or BM) or deformations (e.g. slope and deflections). The above definition can be explained by the following simple examples.

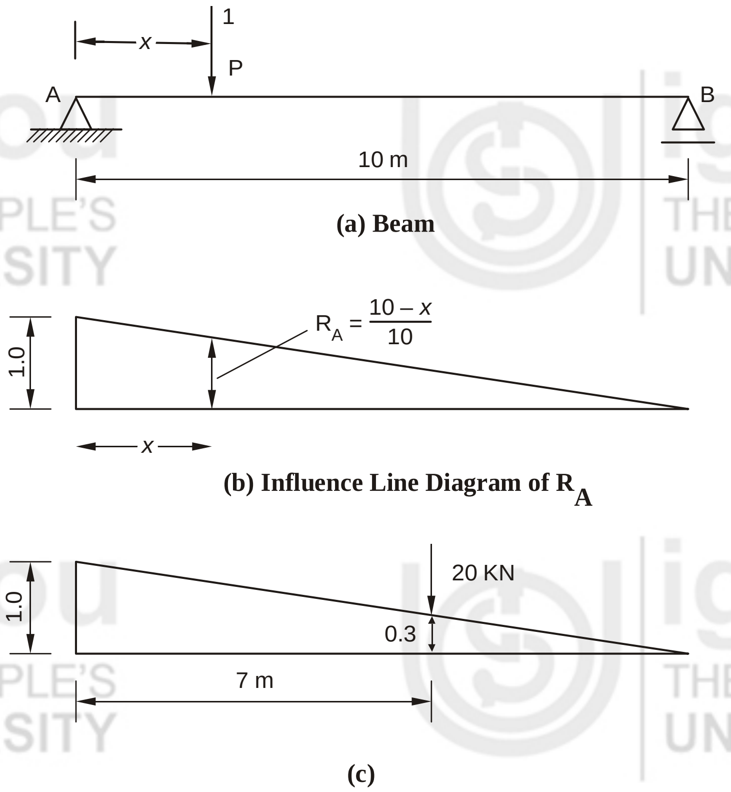


Figure 1.1

Figure 1.1(a) shows a beam which is simply supported at *A* and *B* and has a span of 10 m. If we want to find the influence line for the reaction R_A at the support *A*, we have to place a unit load (e.g. 1 kN) at various points on the beam and to find the corresponding values of R_A. These values are plotted at the points where the load is placed. For example we know that if the unit load is placed at *A*, the reaction $R_A = 1$, and if it is placed at *B*, $R_A = 0$. Now if the moveable unit load is placed at any point *P* on the beam, which is between *A* and *B*, at a distance of *x* from *A*, then taking moments of all forces about *B*, we have

$$R_A \cdot 10 - 1 \times (10 - x) = 0$$

$\Rightarrow$ giving $$R_A = \frac{10 - x}{10} \qquad \ldots (1.1)$$

Eq. (1.1), as can be seen, is the equation of a straight line shown in Figure 1.1(b). Thus, Figure 1.1(b) is the influence line for the reaction R_A of the beam.

Similarly, we may draw influence lines for bending moment or shearing forces at a point on the beam. This will be shown in the next sections.

1.3 PROPERTIES OF INFLUENCE LINES

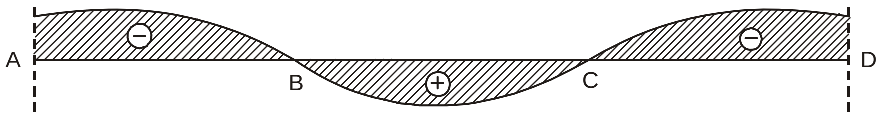
Here, we shall state certain important properties of an influence line, to show how they can be utilized successfully in structural engineering.

(a) To obtain the maximum value of a structural quantity due to a single concentrated moving load, the load should be placed at that point where the ordinate to the influence line is maximum. (This property is obvious from the definition.)

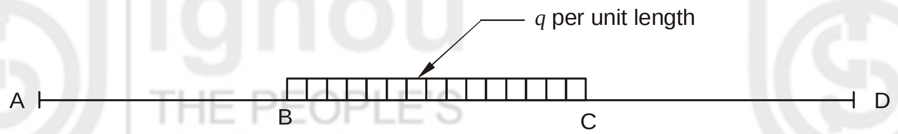
(b) The value of a structural quantity due to a single concentrated moving load is equal to the product of the magnitude of the load and the value of the corresponding ordinate of the influence line. Thus, in Figure 1.1(c), the ordinate of the influence line for R_A at a distance of 7 m from A is $\frac{10 - 7}{10} = 0.3$.

Now if a load of 20 kN is placed at this point the magnitude of reaction R_A will be $20 \times 0.3 = 6$ kN. This can be easily verified.

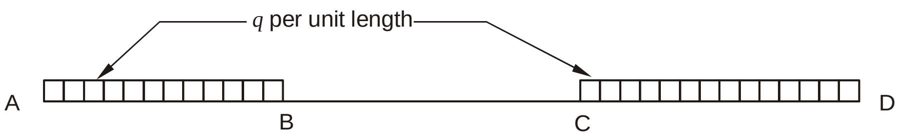
(c) To find the maximum value of a structural quantity due to a uniformly distributed live load, the load should be placed over all those portions of the structure where its influence line ordinates have the same sign.



(a) Influence Line Diargam



(b) Position of *udl* for Maximum Positive Value



(c) Position of *udl* for Maximum Negative Value

Figure 1.2

For example, Figure 1.2(a) shows the influence line diagram of a particular quantity for the beam *ABCD*. The ordinates are positive between *B* and *C*; and are negative between *A* and *B*, and again between *C* and *D*. Hence, for maximum positive value of the quantity the uniformly distributed load should cover the entire portion from *B* to *C* (Figure 1.2(b)). Similarly, for maximum negative value it should cover the portion *AB* and *CD* (Figure 1.2(c)).

(d) The value of a structural quantity due to a uniformly distributed live load is equal to the product of the loading intensity (q) and the net area of the influence line diagram under that portion.

Here, in Figure 1.2, the value of the structural quantity due to the uniformly distributed load covering portion *AB* only is given by

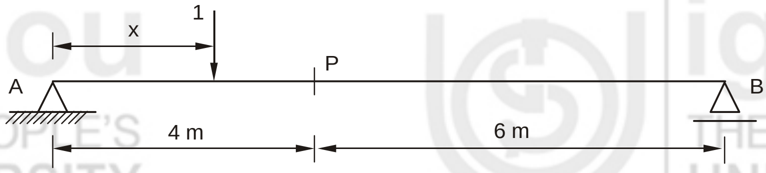
$q \times$ (area of IL diagram over *AB* as shown shaded).

1.4 INFLUENCE LINE DIAGRAM FOR BENDING MOMENT (SIMPLY SUPPORTED BEAM)

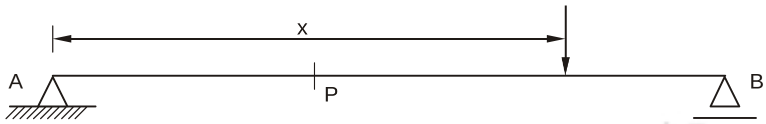
In the following examples, influence line diagram for bending moment and shear force of some common structures are shown, and also how they are used in actual practice.

Example 1.1

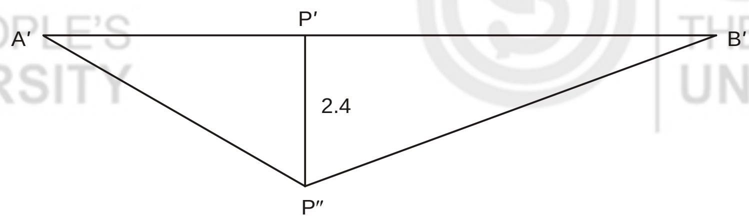
Draw the influence line diagram for bending moment at point *P* of the simply supported beam *AB*, shown in Figure 1.3.



(a) Unit Load between A and P



(b) Unit Load between P and B



(c) Influence Line Diagram for M_P

Figure 1.3

Solution

Case (i) : Load between *A* and *P* (Figure 1.3 (a))

If the unit load lies between *A* and *P* taking moment of forces to left of *P*, then BM at *P* is

$$M_P = R_A \cdot 4 - 1 \times (4 - x) = \frac{10 - x}{10} \times 4 - (4 - x) = \frac{6x}{10}$$

This is a straight line $A'P''$ (Figure 1.3(c)) with IL ordinate = 0 at A (where $x = 0$) and IL ordinate = 2.4 at point *P* (where $x = 4$ m).

Case (ii) : Load between *P* and *B* (Figure 1.3 (b))

When the load crosses the point *P* to the right, bending moment at *P* is

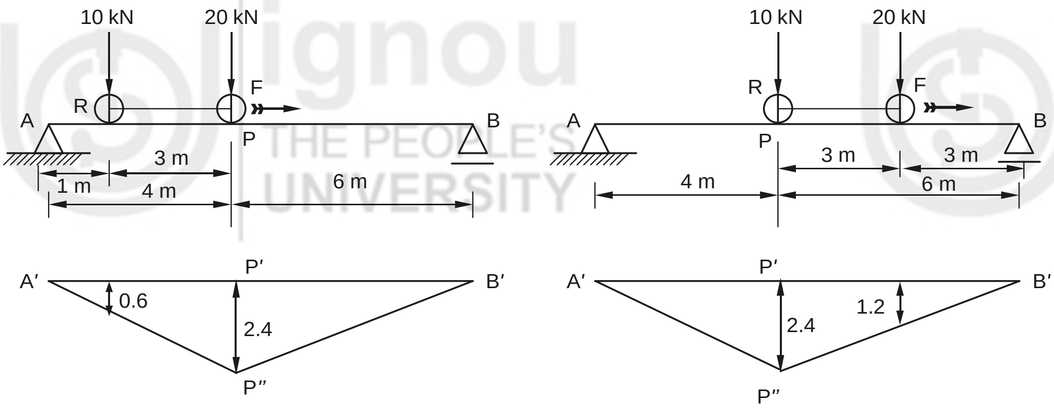
$$M_P = R_A \cdot 4 = \frac{10 - x}{10} \times 4$$

This is again a straight line $P''B'$ and the influence line ordinate is $\frac{10-4}{10} \times 4 = 2.4$ at P; and at B the IL ordinate is $\frac{10-10}{10} \times 4 = 0$.

This is shown in Figure 1.3(c).

Example 1.2

Two connected wheels (wheel base = 3 m) cross the beam in Figure 1.4 from left to right. The front wheel is carrying a load of 20 kN and the rear wheel 10 kN. Find the maximum bending moment at point P due to these wheels.



(a) **(b)**

Figure 1.4

Solution

The influence line diagram is the most convenient method to solve such problems. It is obvious that since the maximum ordinate of the IL diagram is 2.4 (at point P), the maximum value of the bending moment will be obtained when one of the wheels is placed on this point P. There can be two possibilities, *either the* front wheel is at P (Figure 1.4 (a)) *or* the rear wheel is at P (Figure 1.4 (b)). Both are discussed below :

Case (i) : Front Wheel at Point *P* (Figure 1.4(a))

Rear wheel R will be 3 m to left of it (i.e. 4 – 3 = 1 m to right of A).

The ordinate at R (from similar triangles) is equal to $\frac{2.4}{4} \times 1 = 0.6$. Now by Property (b) of the IL diagrams, the total BM at point P due to the two wheels will be $M_P = 20 \times 2.4 + 10 \times 0.6 = 54$ kN m.

Case (ii) : When Rear Wheel is at *P* (Figure 1.4(b))

The front wheel F will be at F, i.e. 3 m to right of it.

The ordinate at F will be $\frac{2.4}{6} \times 3 = 1.2$

Hence, BM at P will be $M_P = 20 \times 1.2 + 10 \times 2.4 = 48$ kN m.

Hence, Case (i) will give the bigger value of bending moment, i.e. as shown in Figure 1.4(a) and it will be 54 kN m.

This will be the maximum bending moment at P due to the wheels crossing across the span.

Example 1.3

For the beam in Figure 1.5 if a uniformly distributed load (udl) of 3 kN/m longer than the span crosses it from left to right, what will be the maximum bending moment at *P*?

Solution

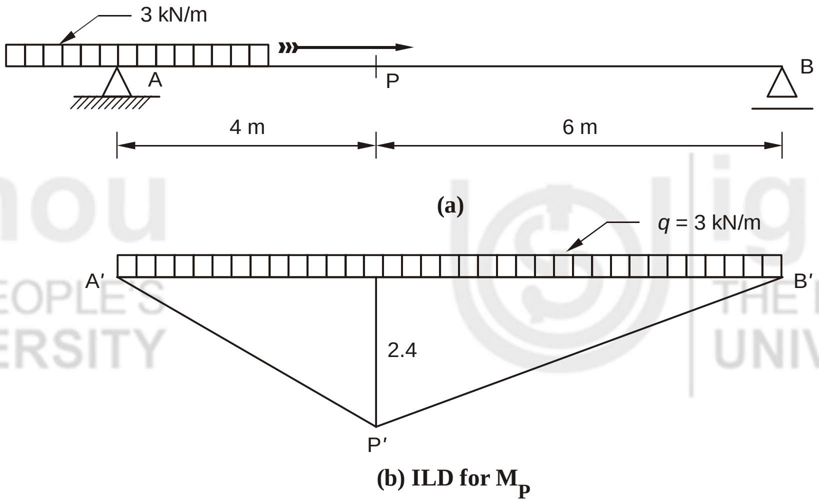


Figure 1.5

In Figure 1.5, it can be seen that as the influence line diagram for bending moment at *P* (M_P) is positive over the whole span *AB*, the moving udl has to cover the entire span for maximum value.

Hence, by Property (d) of the IL Diagram as stated above the maximum M_P = (Intensity of loading) × (Area of IL Diagram)

$$= 3 \times \left(\frac{1}{2} \times 2.4 \times 10\right) = 36 \text{ kNm}.$$

Example 1.4

If the uniformly distributed load crossing the span in Figure 1.5 is smaller than the span, say 4 m long, find the maximum bending moment at *P*.

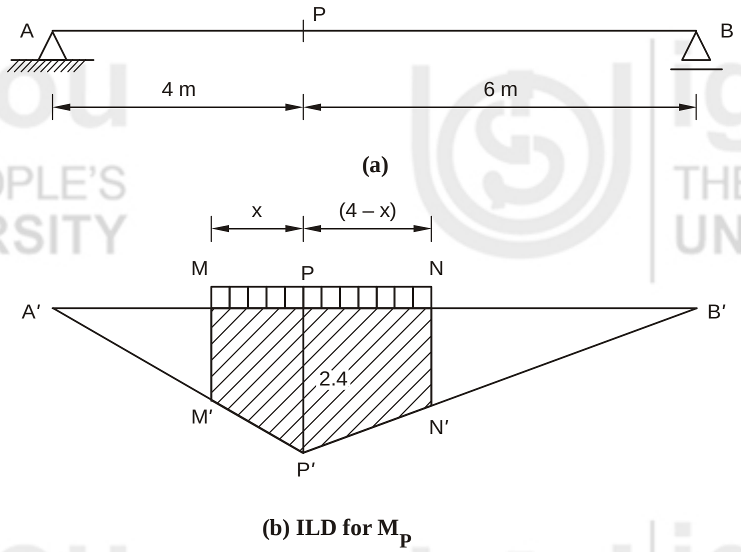


Figure 1.6

Solution

Here, the moving load can cover only a portion of the whole span. Obviously, the maximum value of M_P will occur when the moving load is passing over the region where the ordinates are largest, that is, near the point *P* itself.

Let us assume that the load occupies a position *MN* covering either side of *P*, such that *MP* = *x*, then *PN* = 4 – *x* (Figure 1.6(b)).

By Property (d), the bending moment at *P* is given by,

M_P = (Intensity of load) × (Hatched area of *IL* diagram below the load).

Now, the hatched area of the IL diagram is composed of two trapeziums *MM′P′P* and *NN′P′P*.

Since the ordinate *PP′* = 2.4, by similar triangles

Ordinate $MM' = \frac{2.4}{4} \times (4 - x) = 0.6\,(4 - x)$ and

Ordinate $NN' = \frac{2.4}{6}\,[6 - (4 - x)] = 0.4\,(2 + x)$

$$\therefore \quad \text{Area } MM'N'N = \frac{2.4 + 0.6(4 - x)}{2}\,x + \frac{2.4 + 0.4(2 + x)}{2}\,(4 - x)$$

$$= 6.4 + 1.6x - 0.5x^2$$

$$\therefore \quad M_P = (\text{Load intensity}) \times (\text{Area of IL diagram})$$

$$= 3 \times (6.4 + 1.6\,x - 0.5x^2) \qquad \text{. . . (1.2)}$$

Now M_P will be maximum when $\frac{dM_P}{dx} = 0$

Differentiating Eq. (1.2), we get,

$$3 \times (1.6 - 0.5 \times 2x) = 0$$

giving $x = 1.6$ m.

Hence, when the maximum BM at *P* occurs 1.6 m of the moving load is towards its left and (4 – 1.6) = 2.4 m is towards its right and the value of the maximum blending moment will be given from Eq. (1.2) above.

$$M_P = 3 \times [6.4 + 1.6 \times 1.6 - 0.5\,(1.6)^2] = 45.312 \text{ kNm.}$$

It is interesting to compare the value of the two end ordinates *MM′* and *NN′* of the influence line diagram when the load occupies the maximum *BM* position.

$$MM' = 0.6\,(4 - x) = 0.6\,(4 - 1.6) = 1.44$$

$$NN' = 0.4\,(2 + x) = 0.4\,(2 + 1.6) = 1.44$$

Thus, we see that under the above conditions the value of the influence line ordinates at the two ends of the loads are equal.

This hints to determine the value of *x* easily without going through the process of differentiation etc. For example,

$$MM' = 0.6\,(4 - x) \text{ and } NN' = 0.4\,(2 + x)$$

For maximizing the area, we must have *MM ′* = *NN ′*

From the above equation, $x = 1.6$ m.

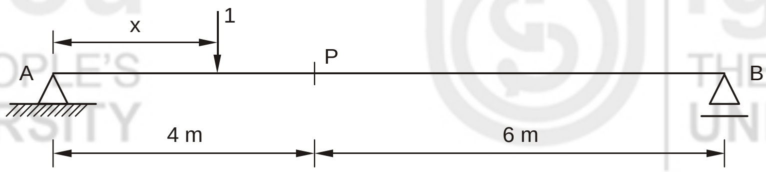
And then we can proceed with finding out the areas etc.

1.5 INFLUENCE LINE DIAGRAM FOR SHEARING FORCE (SIMPLY SUPPORTED BEAM)

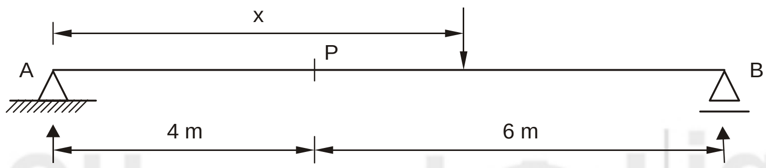
Next, we shall study how to draw the influence line diagram for **shear force** in a simply supported beam. For illustration we take the same beam and the same point *P* as in Example 1.1 (Figure 1.3), for which we now proceed to draw the shear force influence line diagram.

Example 1.5

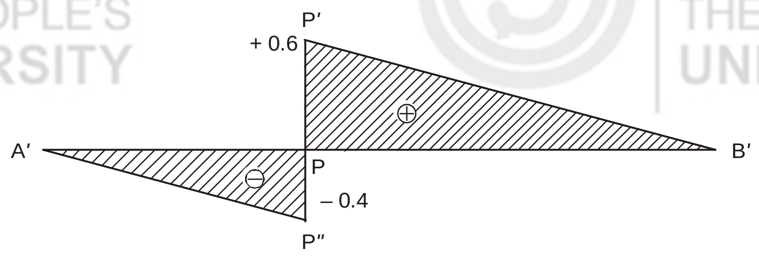
Draw the influence line diagram for shear force at point *P* in the simply supported beam *AB* of span 10 m. *P* is 4 m from the support *A*.



(a)



(b)



(c) ILD for SF at P.

Figure 1.7

Solution

Let us assume that the moving unit load is at a distance *x* from *A*. the reactions $R_A = \frac{10 - x}{10}$, and $R_B = \frac{x}{10}$ as determined earlier.

Case (a) : If the unit load is between the points *A* and *P*, then considering forces to left of *P*,

Shear force at $P = R_A - 1 = \frac{10 - x}{10} - 1 = -\frac{x}{10}$. By our sign convention, a shear force is negative when the left hand portion of the beam tends to move downward. Hence, the shear force in this case will be negative and will depend upon the value of *x* (i.e. its distance from *A*). The ordinate of the diagram will be zero then the load is at *A* ($x = 0$) and it will be

$-\frac{4}{10} = -0.4$ when the rolling load is at P (i.e. $x = 4$). The diagram will be a straight line $A'P'$.

Case (b) : When the unit load crosses to the right of the point P and lies between P and B, then considering forces to left of P the shear force at $P = R_A = \frac{10 - x}{10}$, which is also a straight line $P''B$ such that the ordinate at P (when the unit load has just crossed to right) is equal to $\frac{10 - 4}{10} = +0.6$ and it is zero when the unit load is at B, ordinate $= \frac{10 - 10}{10} = 0$.

Thus, we see that the shear force influence line consists of two parts : the part between A and P has negative ordinates and the part between P and B has positive ordinates, showing that the SF changes sign as the unit rolling load crosses the point P.

Example 1.6

Find the maximum positive and negative shear force at point P in beam of Figure 1.8 which is crossed by two connected wheel loads 3 m aparts moving from right to left. The front wheel carries a load of 20 kN and the rear wheel 10 kN.

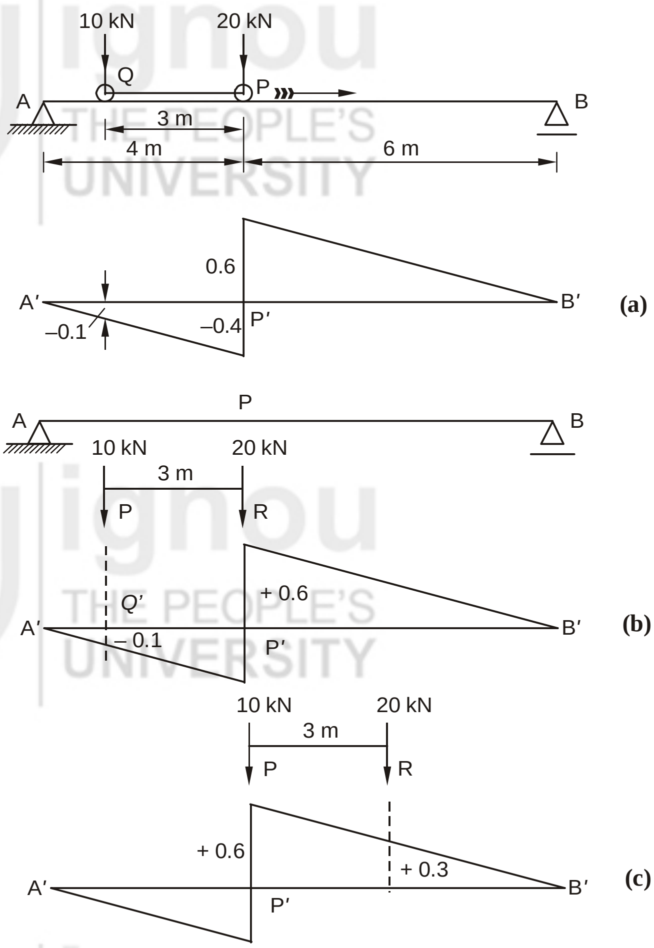


Figure 1.8

Solution

Maximum Negative SF

For maximum negative shear force at *P*, the heavier wheel (20 kN) should be placed just to the left of *P*, the other wheel (10 kN) will then lie at *Q* which is 3 m to left of *P* (Figure 1.8(a)). The ordinate of IL diagram at *P* is – 0.4 and that at *Q* is – 0.1 (by similar triangles).

Hence, maximum negative shear force at *P*,

$$V_P = 20 \times (-0.4) + 10\,(-0.1) = -9 \text{ kN}.$$

Maximum Positive SF

Here, two cases need to be examined

(a) When the heavier wheel 20 kN is just crossed to right of *P*, and the lighter wheel (10 kN) is at *Q* 3 m behind it (Figure 1.8(b)). Hence the shear force

$$V_P = 20 \times (+0.6) + 10 \times (-0.1) = 11 \text{ kN}.$$

(b) When the lighter wheel (10 kN) has just crossed to right of *P* and the front wheel (20 kN) is 3 m to right of it at *R* as in Figure 1.8 (c).

The ordinate of IL diagram at *P* is + 0.6 and at *R* it is + 0.3.

Hence, shear force $V_P = 20 \times (+0.3) + 10 \times (+0.6) = 12$ kN.

The second position gives the higher value, hence, the maximum positive SF will be 12 kN.

SAQ 1

(a) Draw the influence line diagram for the *bending moment* at point *P* of the simply supported beam *AB* in Figure 1.9.

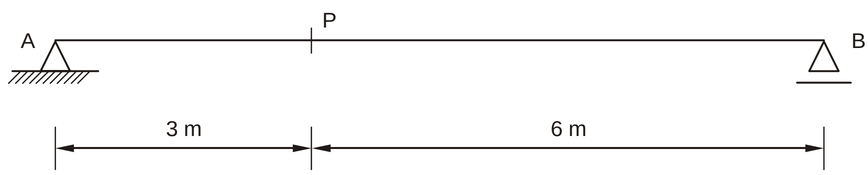


Figure 1.9

Using this diagram find the maximum bending moment at *P*, due to following moving loads

- A uniformly distributed load of 4 kN/m longer than the span.
- A uniformly distributed load of 4 kN/m of 3 m length.
- Two connected wheel loads of 10 kN each, 3 m apart.

(b) Draw the influence line for *shear force at* the same point *P* of the above beam (Figure 1.9).

Determine the maximum shear force at *P* for the same load combinations as given in SAQ 1(a) above.

1.6 INFLUENCE LINE DIAGRAM FOR A CANTILEVER

In this section, we shall explain how to draw the influence line diagrams for the **support reactions** and **BM** and **SF** at any point in a **cantilever beam**. You should verify all the steps yourself and try to solve the numerical examples given in the SAQ below which is based on this diagram.

In Figure 1.10, the cantilever whose free end is *A* and fixed end *B* has a span of *L*.

(a) **Influence Line Diagram for Support Reactions**

If a unit load moves from *A* to *B* along the beam, the vertical reaction R_B at *B* remains constant and is equal to 1.0. However, the fixed end moment $M_B = -1 \times (L - x) = -(L - x)$ and hence the influence line coordinates for M_B varies from $-L$ at A to 0 at *B*. (You should carefully observe that it is just opposite to the BM diagram due to an unit load at *A*. Why?)

The IL Diagrams are shown at Figures 1.10(b) and (c).

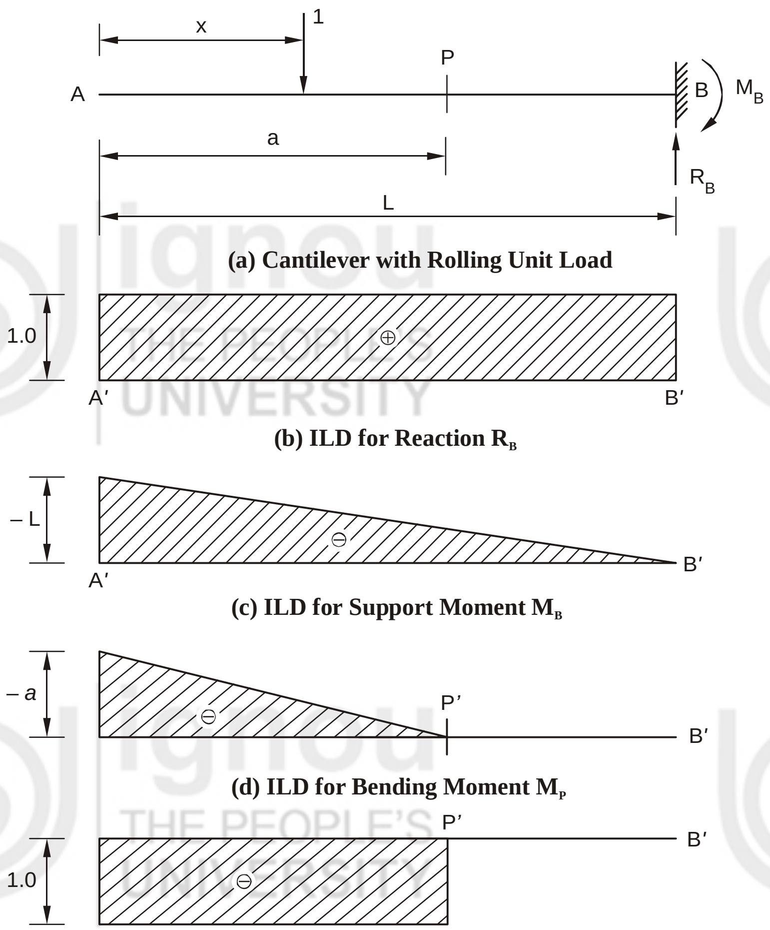


Figure 1.10

(b) **Influence Line Diagram for Bending Moment at *P***

Next, we shall draw the influence line diagram for bending moment (M_P) at a point *P* which is at a distance '*a*' from end *A* :

When the load is between *A* and *P* the BM at *P* is

$$M_P = -1 \times (a - x) = -(a - x)$$

Hence, the ordinate of the IL diagram is $-a$ when $x = 0$ (at point A) and it is 0 when $x = a$ (at point P).

As soon as the load crosses over P to the right hand side the BM at $P = 0$ and remains as such till the end, as shown in Figure 1.10(d).

(c) **Influence Line Diagram for Shear Force at *P***

Next, we draw the influence line for shear force (V_P) at P :

When the load is between A and P, the shear force (considering loads to left of P) is $V_P = -1$ (downwards $\therefore$ negative)

As soon as the load crosses P to the right of P, there is no load to left of it, hence $V_P = 0$. This is shown in Figure 1.10(e).

SAQ 2

A cantilever 6 m span is free at end A and fixed at B. Using influence line diagrams for a point P, 3 m from free end A, find the maximum SF and BM at P due to the following moving loads :

(a) A uniformly distributed load of 3 kN/m longer than the span.

(b) A set of 3 connected wheel loads, shown in Figure 1.11, moving from left to right.

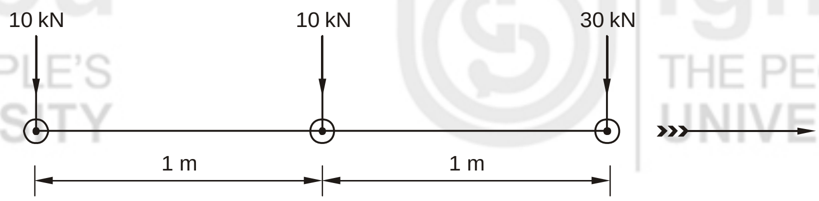


Figure 1.11

1.7 SUMMARY

In this unit, you have learnt how to find the values of a structural quantity (BM, SF, axial force support reactions etc.) for a unit moving load by means of influence line diagrams. You have, thus, learnt the properties of the influence line diagram and its uses.

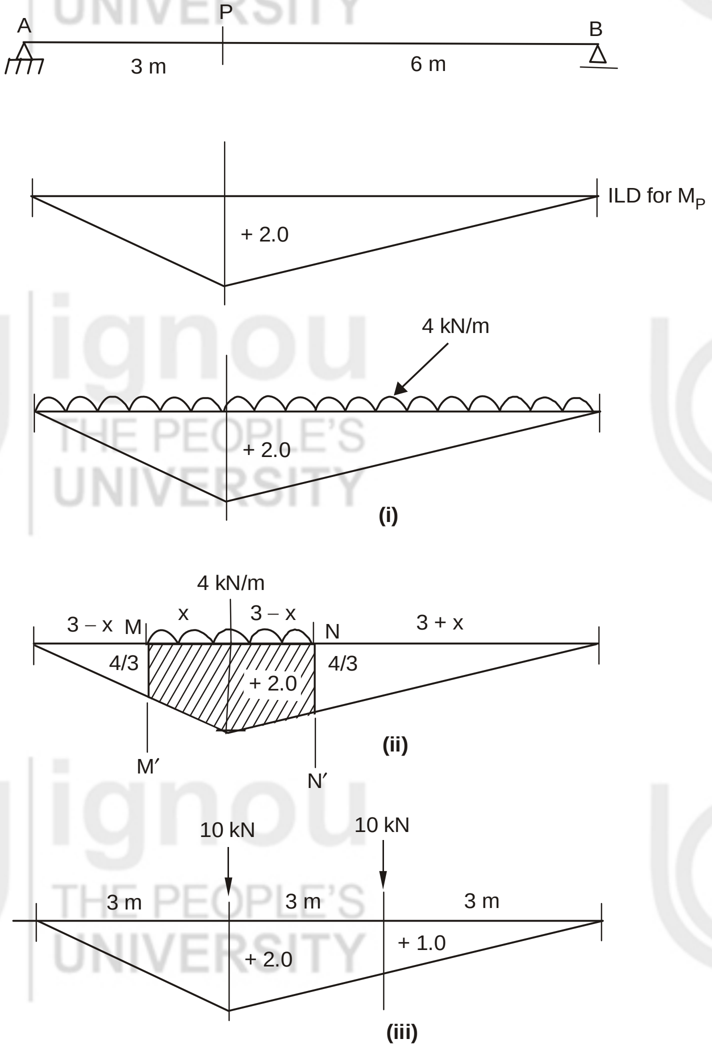
You have also learnt to draw the IL diagrams for support reactions, BM and SF at any point for a simply supported beam; and a cantilever.

In the next unit, we will study theorem of three moments applied to fixed and continuous beams.

1.8 ANSWERS TO SAQs

SAQ 1

(a)



(i) Position of moving load for maximum M_p

$$M_{P(\max)} = (\text{Area of } ILD) \times (\text{Intensity of loading})$$

$$= \left(\frac{1}{2} \times 2.0 \times 9\right) \times 4 = 36 \text{ kNm}$$

(ii) Position of load for maximum M_P

Ordinate MM' = Ordinate NN'

$$\frac{2}{3}(3 - x) = \frac{2}{6}(3 + x)$$

giving $x = 1$ m and ordinates = 4/3

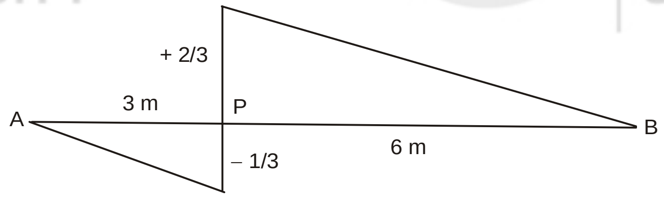
$\therefore$ $$M_{P(\max)} = (\text{Shaded area}) \times (\text{Load intensity})$$

$$= \left(\frac{\frac{4}{3} + 2}{2} \times 3\right) \times 4 = 5 \text{ m}^2 \times 4 \text{ kN/m} = 20 \text{ kNm}$$

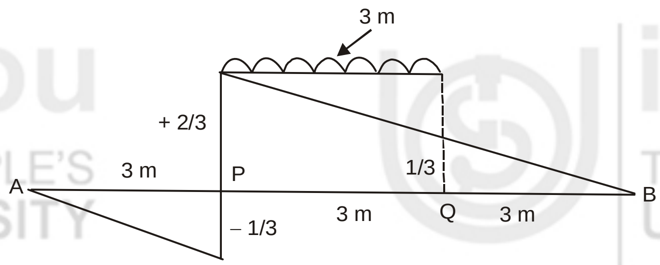
(iii) Position of connected loads for maximum M_P

$$M_{P\,(\max)} = 10 \times 2 + 10 \times 1 = 30 \text{ kNm}$$

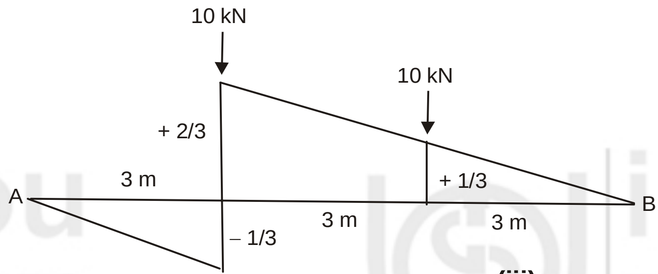
(b)



(i)



(ii)



(iii)

(i) For udl

V_P (+) is max when *PB* is covered

$$V_{p\,(\max)} = \left(\frac{1}{2} \times \frac{2}{3} \times 6\right) \times 4 = +\ 8 \text{ kN}$$

V_P (–) is max when *AP* is covered only

$$V_{p\,(\max)} = -\left(\frac{1}{2} \times \frac{1}{3} \times 3\right) \times 4 = -\ 2 \text{ kN}$$

(ii) For udl of 3 m length *PQ* is covered

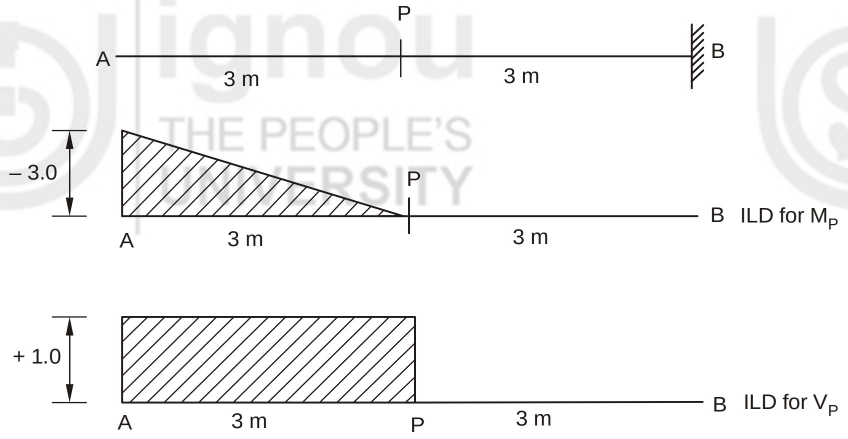
$$V_{p\,(\max)} = \text{Area} \times \text{Intensity}$$

$$= \left(\frac{\frac{1}{2} \times \frac{2}{3}}{2} \times 3\right) \times 4 = 6 \text{ kN}$$

(iii) For two connected conc. loads

$$V_{p\,(\max)} = 10 \times \frac{2}{3} + 10 \times \frac{1}{3} = 10 \text{ kN}$$

SAQ 2



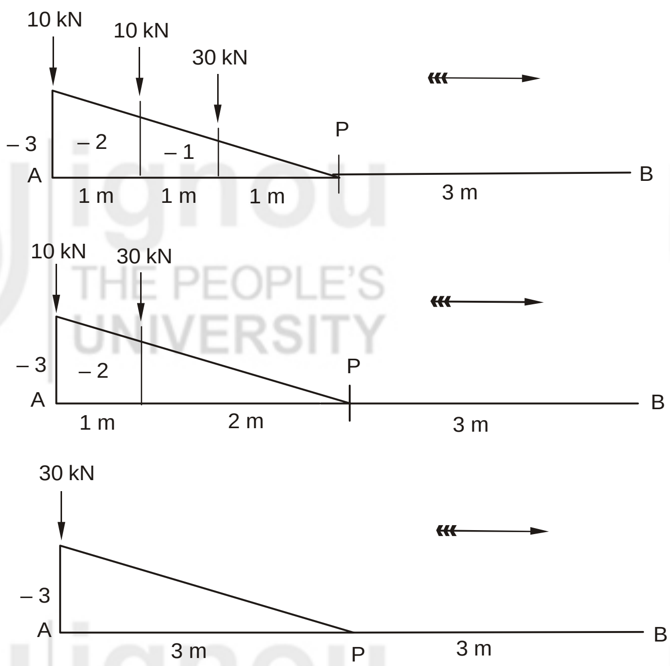
Case (a)

$$M_{P\,(\text{max})} = \left(\frac{1}{2} \times 3 \times 3\right) \times 3 = -\,13.5 \text{ kNm}$$

$$V_{P\,(\text{max})} = (1 \times 3) \times 3 = 9 \text{ kN}$$

Case (b)

Considering the following three cases for M_P



In the first case, $M_P = -\,10 \times 3 - 10 \times 2 - 30 \times 1 = -\,80$ kNm

In the second case, $M_P = -\,10 \times 3 - 30 \times 2 = -\,90$ kNm

In the third case, $M_P = -\,30 \times 3 = -\,90$ kNm

So the second and third cases produce maximum BM of – 90 kNm.

For SF, as the diagram is rectangle, all the loads have the same result placed anywhere between *A* and *P*.

$\therefore \qquad V_{P\,(\text{max})} = 10 \times 1 + 10 \times 1 + 30 \times 1 = 50$ kN