UNIT 7 STAIRCASES

Structure

7.1 INTRODUCTION

Staircase is a space where steps for going (rising) from one floor (height) to another are arranged systematically in stairs (flights).

A staircase may be of different types depending upon the available space, convenience of users, architectural considerations, etc. A few simple types of staircases are shown in Figure 7.1.

Figure 7.1 : A Few Simple Types of Staircase

Rise and tread of a step (Figure 7.2) are planned in such a way that the user may not feel any inconvenience while going. Based on experience the rise and tread are fixed as given below :

Rise = 125 to 200 mm

Tread = 225 to 320 mm

Rise × Tread $\approx$ 4300

2 Rise + Tread $\approx$ 590

Figure 7.2 : Typical Section Showing Components of a Stair

Steps in one flight must have the same tread and rise. Neither more than 15 steps nor less than 3 steps may be provided in one flight.

The head room, i.e. the height between parallel flights or the height between a floor and landing, must be sufficient so that user may not have any difficulty either in going or carrying useful furnitures, luggages, etc. A minimum headroom of 2.1 m is essential.

The width of a flight shall not be less than 0.9 m so that users crossing each other on a step may not collide. The maximum width may be fixed depending on traffic and other considerations.

Objectives

After studying this unit, you should be able to

- define staircase and its functions, and
- design and detail simple types of rectangular staircases.

7.2 EFFECTIVE SPAN OF A FLIGHT FOR DIFFERENT SUPPORT CONDITIONS AND TYPES OF STAIRCASE

Stair Slab Spanning Longitudinally without Stringer Beams

(a) Where a flight is supported on risers by beams/walls, the effective span is the horizontal distance between c/c of the beams/walls (Figure 7.3).

Figure 7.3 : Effective Span for Flight Supported on Risers by Beam or Wall

(b) Where landing slab falls between the goings and spans in the same direction as the stairs, the effective span is the horizontal distance between supports (walls/beams) considering that the goings and landing form a single slab (Figure 7.4).

Figure 7.4 : Effective Span where Landing Falls between the Goings

(c) Where the flight spans on to the edge of a landing slab, which spans parallel to risers, the effective span is equal to the going plus at each end either half the width of the landing or one metre, whichever is smaller (Figure 7.5).

Figure 7.5 : Effective Span for Stairs Supported at Each End by Landing Spanning Parallel with the Risers

Stair Slab spanning Transversely with Stringer Beams

(a) If waist slab is fixed to a wall or a beam along one of its longitudinal edges the effective span is its cantilevering width (Figure 7.6(a)).

> *Though the slab is designed as a simply supported one, the reinforcement at top shall be provided to take care of negative bending moment due to partial fixidity with beams.

(b) If the waist slab is supported longitudinally along the centre line of its width, the effective span of the cantilevering slab will be as shown in Figure 7.6(b).

(c) If the waist slab is supported longitudinally along both of its longitudinal edges, the effective span of the slab is the same as that of a simply supported slab*.

Figure 7.6 : Effective Span for Slab Supported Longitudinally

Effective Span of a Stringer Beam

Where the waist slab is supported on a beam (or beams) the effective span of the beam is the horizontal distance between centres of supports as shown in Figure 7.7.

Figure 7.7 : Effective Span of a Stringer Beam

7.3 EVALUATION OF DESIGN LOADS

7.3.1 Design Load on Unit Horizontal Area for Longitudinal Span

The design loads are calculated on *horizontal area* if the effective span is considered in horizontal direction. Unit horizontal area is generally taken in square meter (m^2).

If rise, tread, depth and inclination of a slab with horizontal be R, T, D, and θ, respectively, load per metre run for unit width of slab (i.e. load per m^2) may be calculated as follows (Figure 7.8).

Load of steps/m = $\frac{1}{2} \times R \times T \times$ no. of steps in 1 m horizontal length × Density

$$= \frac{1}{2} \times R \times T \times \frac{1}{T} \times \text{Density}$$

$$= \frac{1}{2} \times R \times \text{Density}$$

Load due to slab/m = $1 \times 1 \times \sec\theta \times D \times$ Density

The imposed load may be taken directly as they are valid for horizontal area only.

Figure 7.8 : Evaluation of Design Load on Unit Horizontal Area

7.3.2 Design Load on Unit Inclined Area for Transverse Span

When waist slab spans transversely the slab bends perpendicular to the inclined surface due to load component W_n as shown in Figure 7.9. If the inclined length of one step is $l = \sqrt{R^2 + T^2}$ then the no. of steps covered in 1 m inclined length is equal to $\frac{1}{l}$.

Figure 7.9 : Evaluation of Design Load on Unit Inclined Area

Load per meter run for unit width of slab (i.e. load per m^2) may be calculated as follows :

Load of steps/m = $\frac{1}{2} \times R \times T \times$ no. of steps in 1m inclined length × Density

$$= \frac{1}{2} \times R \times T \times \frac{1}{\sqrt{R^2 + T^2}} \times \text{Density}$$

Load due to slab/m = $1 \times 1 \times D \times \cos\theta \times$ Density

Imposed Load (IL)/m = $IL \times \cos\theta$

7.3.3 Design Load Distribution on Common Area

In cases of quarter turn stairs and open well staircase where the spans are at right angles, have common landing area, the loads in such cases shall be one half in each direction (Figure 7.10).

Figure 7.10 : Loading on Stairs with Open Walls

7.3.4 Design Load when the Stair Slab is Engraved in Side Wall

If the waist slab is engraved in side wall for more than 110 mm width, a 150 mm strip may be deducted from the loaded area and the effective breadth of the section increased by 75 mm for the purpose of design (Figure 7.11).

Figure 7.11 : Loading on Stairs Built into Walls

Example 7.1

Design the first flight of the dog-legged stair case (Figure 7.12) for the following data :

Staircase Size = 4.50 m × 2.45 m

Supporting wall thickness = 250 mm

Figure 7.12 : Plan of Staircase

Height between floors = 3.6 m

Rise (R) × Tread (T) =150 × 300

Nominal cover = 20 mm

$f_{ck} = 25$ N/mm^2

$f_y = 415$ N/mm^2

Solution

Fixing Size

Providing two flights

Height at one flight = $\frac{3.6}{2}$ = 1.8 m.

No. of rise in one flight = $\frac{1.8}{0.15}$ = 12 Nos.

No. of treads = (12 – 1) = 11

Going = 11 × 0.3 = 3.3 m

Providing Landing Width = 1.2 m.

Width of flight = 1.2 m.

Effective Length (l_{ef})

$$\text{Effective span } l_{ef} = \text{Going} + \frac{\text{width of landing}}{2} + \frac{\text{width of support}}{2}$$

$$= 3.3 + \frac{1.2}{2} + \frac{0.25}{2} = 4.025 \text{ m}$$

Depth of Waist Slab (D)

Depth from Deflection Control

For simply supported slab

$$\frac{l_{ef}}{d} \leq 20\ k_1\ k_2\ k_3\ k_4$$

$k_1 = 1$ as $l_{ef} < 10$ m

k_2 (for p_b % = 0.36%) = 1.39

for $\quad f_s = 0.58\ f_y \dfrac{\text{Area of cross section of steel required}}{\text{Area of cross section of steel provided}}$

(Assuming A_{st} required = A_{st} provided)

$k_3 = k_4 = 1$

or, $\quad \dfrac{l_{ef}}{d} \leq 20 \times 1 \times 1.39 \times 1 \times 1$

or, $\quad \dfrac{4025}{d} \leq 27.8$

or, $\quad d \geq \dfrac{4025}{27.8}$

or, $\quad d \geq 144.78$ mm

$\therefore \quad D = 144.78 + 20 + \frac{10}{2} = 169.78$ (assuming $\phi = 10$ for main bars and nominal cover = 20).

Hence, provided *D* = 200 mm.

$$\therefore \quad d = 200 - 20 - \frac{10}{2} = 175 \text{ mm}$$

From BM Consideration

Loads

Load on Going

Self weight of waist slab/m = 0.20 × 1 × 25 = 5.0 kN/m.

Self weight of slab/hor. m run = 5.0 sec θ

$$= 5.0 \frac{\sqrt{0.15^2 + 0.3^2}}{0.3} = 5.59 \text{ kN/m}$$

Self weight of steps/m run = $\frac{1}{2} \times 0.15 \times 1 \times 0.3 \times 24 \times \frac{1000}{300}$

= 1.8 kN/m

Floor finish including underside plaster

= 0.05 × 1 × 1 × 20 = 1 kN/m

1L = 4 kN/m

∴ Total load for going = 5.00 + 5.59 + 1.8 + 1 + 4
= 17.39 kN/m

Factored load for going = 1.5 × 17.39 = 26.085 kN/m.

Load on Landing

Self weight of Slab = 0.2 × 1 × 25 = 5 kN/m

Floor finish including underside plaster

= 0.05 × 1 × 1 × 20 = 1 kN/m

Total *DL* = 6 kN/m

IL = 4 kN/m

DL + *IL* = 10 kN/m.

As the area of landing is common between the flight and the landing, only 50% of the above load will be carried by the flight

$$\therefore \quad (DL + IL) \text{ for the flight design} = \frac{1}{2} \times 10 = 5 \text{ kN/m}$$

Factored Load on landing = 1.5 × 5 = 7.5 kN/m

The loaded beam has been shown in Figure 7.13

$$R_1 = \frac{26.085 \times 3.425 \times \left(\frac{3.425}{2} + 0.6\right) + 7.5 \times \frac{0.60^2}{2}}{4.025} = 51.665 \text{ kN}$$

$$R_2 = \frac{\left(26.085 \times \frac{3.425^2}{2} + 7.5 \times 0.6 \times \left(\frac{0.6}{2} + 3.425\right)\right)}{4.025} = 42.176 \text{ kN}$$

Let SF be zero at *x* from LHS.

$$51.665 - 26.085\,x = 0 \quad \text{or} \quad x = \frac{51.665}{26.085} = 1.98 \text{ m}$$

Figure 7.13 : Section with Design Span and Loading

$$\therefore \quad M_{u\,,\max} = 51.165 \times 1.98 - 26.085 \times \frac{1.98^2}{2} = 51.165 \text{ kN/m}$$

$$M_u = 0.36\,\frac{x_{u,\max}}{d}\left(1 - 0.4\,\frac{x_{u,\max}}{d}\right) f_{ck}\, bd^2$$

or, $51.165 \times 10^6 = 0.36 \times 0.48 \times (1 - 0.42 \times 0.48)$
$\times 25 \times 1000 \times d^2$

or, $d = 121.79$ mm; $D = 103.1 + 20 + \dfrac{10}{2}$

$= 146.79$ mm < 200

Tensile Reinforcement (A_{st})

$$M_u = 0.87\, f_y\, A_{st}\, d\left(1 - \frac{A_{st}\, f_y}{bd\, f_{ck}}\right)$$

or, $51.165 \times 10^6 = 0.87 \times 415 \times A_{st} \times 175 \times \left(1 - \dfrac{A_{st} \times 415}{1000 \times 175 \times 25}\right)$

or, $51.165 \times 10^6 = 63183.75\, A_{st} - 5.993\, A_{st}^2$

or, $A_{st}^2 - 10542.925\, A_{st} + 8537460.37 = 0$

or, $A_{st} = \dfrac{(10542.925 \pm \sqrt{(10542.925^2 - 4 \times 8537460.37)})}{2}$

or, $A_{st} = 883.38$ mm^2/m

Hence, provided ϕ 10 @ 125 mm c/c.*

$\therefore$ A_{st} provided = 884 mm^2 and corresponding

$p_t\% = \dfrac{884 \times 100}{1000 \times 175} = 0.51\%$

*All the main reinforcement bars are taken to the top beyond the junction of going and landing slab for the following two reasons :

(a) Negative bending moment of the same magnitude as positive bending moment also develops at the junction of the two slabs.

(b) If main bars are bent at junction it will chip away the concrete due to straightening of these bars due to bending.

Distribution Reinforcement

$A_{st,}$ required $= \frac{0.12}{100} \times bd = \frac{0.12}{100} \times 1000 \times 175 = 210$ mm^2/m

Hence, provided ϕ 8 @ 235 mm c/c.

Check for SF

SF at d from face of support = 51.665 – 26.085 × (0.125 + 0.175)

= 43.839 kN

$$\tau_v = \frac{V_u}{bd} = \frac{43.839 \times 10^3}{1000 \times 175} = 0.251 \text{ N/mm}^2$$

$$\tau_c = 0.36 + \frac{(0.49 - 0.36)}{(0.5 - 0.25)} \times (0.36 - 0.25) = 0.42 \text{ N/mm}^2$$

$k = 1.2$

∴ Design shear strength = $k\tau_c$ = 1.2 × 0.42

= 0.504 N/mm^2 > 0.179 N/mm^2

Hence, O.K.

The reinforcement detailing has been shown in Figure 7.14.

Figure 7.14 : Detailing of Reinforcement

Example 7.2

Design a staircase having cantilevering steps for a residential house where floor to floor height is 3.6 m and the staircase size is 2.0 m × 4.05 m. Use M 20 concrete and Fe 415 steel.

Solution

Fixing Size

Let Rise × Tread = 0.18 m × 0.25 m

∴ No. of riser $= \frac{3.6}{0.18} = 20$

and let width of steps = 0.9 m.

The plan and section at *AA* of the stair case have been shown in Figure 7.15. Since the projection of each steps from the wall is only 0.9 m, it is assumed that if each step has sufficient fixity in the wall, it may be designed as a cantilever from the wall.

Figure 7.15 : Staircase with Cantilever Steps

Effective Span

The effective span,

$l_{ef} = 0.9$ + half of effective depth

$d = D - \frac{\phi}{2} - \phi_{\text{stirrups}}$ – Nominal cover

$= 180 - \frac{8}{2} - 6 - 20 = 150$ mm

$\therefore \quad l_{ef} = 0.9 + \frac{0.15}{2} = 0.975$ m.

Determination of Depth (*D*)

From Deflection Control

$$\frac{l_{ef}}{d} \leq k_B\, k_1\, k_2\, k_3\, k_4$$

$$d \geq \frac{l_{ef}}{k_B\, k_1\, k_2\, k_3\, k_4}$$

$k_B = 7$;

$k_1 = 1$ as $l_{ef} < 10$ m

For M 20 concrete and Fe 415 p_B % = 0.96%

$$f_s = 0.58\, f_y\, \frac{A_{st} \text{ reqd}}{A_{st} \text{ provided}}$$

Assuming A_{st} reqd = A_{st} provided

$f_s = 0.58 \times 415 = 240$ and correspondingly $k_2 = 1$, $k_3 = k_4 = 1$ as the slab is singly reinforced rectangular section.

Substituting all the above values,

$$d \geq \frac{975}{7 \times 1 \times 1 \times 1 \times 1} = 139.3 \text{ mm}$$

$D = 139.3 + \frac{8}{2} + 6 + 20 = 169.3$ mm (assuming ϕ 8 for main bars and ϕ 6 for stirrups and nominal cover = 20 mm)

Hence, adopted $D = 180$ mm

$$\therefore \qquad d = 180 \frac{8}{2} - 6 - 20 = 150 \text{ mm}$$

From BM Consideration

Loads

Self weight = 0.25 × 0.18 × 1 × 25 = 1.125 kN/m

IL = 0.75 kN/m

Total Load = 1.875 kN/m

Factored Load/m = w_u =1.5 × 1.875 = 2.81 kN/m

Assuming wt. of railing = 1 kN at the free end

Factored load due to railing $W'_u = 1.5 \times 1 = 1.5$ kN

Figure 7.16 : Design of Span and Load

From Figure 7.16

$$M_u = \frac{w_u l_{ef}^2}{2} + W'_u \, l_{ef} = \frac{2.81 \times 0.975^2}{2} + 1.5 \times 0.975 = 2.8 \text{ kN-m}$$

$$\therefore \quad M_{u,\text{lim}} = 0.36 \, \frac{x_{u,\max}}{d} \left(1 - 0.42 \frac{x_{u,\max}}{d}\right) bd^2 \, f_{ck}$$

$$2.8 \times 10^6 = 0.36 \times 0.48 \times (1 - 0.42 \times 0.48) \times 250 \times d^2 \times 20$$

or, $\quad d = 63.7 < 150$ mm

Hence, provided $D = 180$ mm and $d = 150$ mm

Tensile Reinforcement

$$M_u = 0.87 \, f_y \, A_{st} \, d \left(1 - \frac{A_{st} \, f_y}{bd \, f_{ck}}\right)$$

$$2.8 \times 10^6 = 0.87 \times 415 \times A_{st} \times 150 \times \left(1 - \frac{A_{st} \times 415}{250 \times 150 \times 20}\right)$$

or, $\quad 29.967 \, A_{st}^{\,2} - 54157.5 \, A_{st} + 2.8 \times 10^6 = 0$

or, $\quad A_{st}^{\,2} - 1807.24 \, A_{st} + 93436.11 = 0$

After solving the above equation, we get

$A_{st} = 53.27$ mm^2

$$A_{st,\,\min} = \frac{0.12 \times 250 \times 180}{100} = 54 \text{ mm}^2\text{/m}$$

Hence, provided 3 ϕ 8 at the top to keep clear distance between two bars less than 180 mm.

Check for Shear

$$V_u = w_u\, l + W' = 2.81 \times (0.9 - 0.15) + 1.5 = 3.61 \text{ kN}$$

$$\tau_v = \frac{V_u}{bd} = \frac{3.61 \times 10^3}{250 \times 150} = 0.096 \text{ N/mm}^2 \leq 0.28 \text{ N/mm}^2 \ (= \tau_{c,\, min})$$

Hence, nominal 2 legged ϕ 6 stirrups @ 100 c/c are provided.

The reinforcement detailing has been shown in Figure 7.17.

Figure 7.17 : Reinforcement Detailing

Example 7.3

Design the slab and the beam of the third flight of staircase (Figure 7.18) for the following parameters :

Rise × Tread =150 × 275

Width of flight = 1.5 m

Supporting wall thickness = 375 m

Imposed Load = 4 kN/m^2

f_{ck} = 20

f_y = 415

Nominal cover = 30 mm

Figure 7.18 : Plan of Staircase

Solution

Design of Slab

Effective Length (l_{ef})

The slab is cantilevering (Figure 7.19) on both sides of beam; therefore,

$$l_{ef} = \frac{1.5}{2} = 0.75 \text{ m}$$

Figure 7.19 : Cantilever Slab with Design Load

Depth of Slab (D)

From Deflection Control

$$d \geq \frac{l_{ef}}{k_B\ k_1\ k_2\ k_3\ k_4}$$

$k_B = 7$ and

$k_1 = k_2 = k_3 = k_4 = 1$ as calculated in Example 7.2.

Substituting these values in the above equation.

$$d \geq \frac{750}{7 \times 1 \times 1 \times 1 \times 1} = 107.14 \text{ mm}$$

$$D = d + \frac{\phi}{2} + \text{Nominal cover} = 107.14 + \frac{8}{2} + 30 = 141.14 \text{ mm}$$

Keeping $D = 160$ mm; $d = 160 - \frac{8}{2} - 30 = 126$ mm.

From BM Consideration

In this case the normal component of load (w_n) causes the waist slab to bend in transverse plane normal to the sloping surface of the slab.

Load acting vertically over each inclined width of a tread

Self wt of slab = 0.16 × 0.313 × 1 × 25 = 1.252 kN/m width

Wt of step = $\frac{1}{2}$ × 0.15 × 0.275 × 24 = 0.495 kN/m width

Total DL = 1.747 kN/m width

$IL = 4 \times 0.275 = 1.100$ kN/m width

Total $(DL + IL) = 2.847$ kN/m width

Factored load causing flexure in the *transverse* direction

$1.5 \times 2.847 \times \cos 28.61^o = 3.749$ kN/m width

$\therefore$ Distributed Factored Load per meter length along inclined slab,

$$w_n = 3.749 \times \frac{1}{0.313} = 11.978 \text{ kN/m width}$$

Assuming wt of railing = 1 kN/m (horizontally)

$\therefore$ Factored load on 1m inclined length = $1.5 \times 1 \times \cos 28.61^o$

= 1.317 kN/m

$$M_u = \frac{w_u\, l_{ef}^2}{2} + W_u' l_{ef}$$

$$= \frac{11.978 \times 0.75^2}{2} + 1.317 \times 0.75 = 4.356 \text{ kN-m/m long length}$$

$$M_{u,\lim} = 0.36\ \frac{x_{u,\max}}{d}\left(1 - 0.42 \frac{x_{u,\max}}{d}\right) bd^2 f_{ck}$$

or, $4.356 \times 10^6 = 0.36 \times 0.48 \times (1 - 0.42 \times 0.48) \times 1000 \times d^2 \times 20$

or, $d = 39.73 < 126$ mm

Hence, provided $D = 160$ mm and $d = 126$ mm.

Tensile Reinforcement (A_{st})

$$M_u = 0.87\, f_y\, A_{st}\, d \left(1 - \frac{A_{st}\, f_y}{bd\, f_{ck}}\right)$$

or, $4.356 \times 10^6 = 0.87 \times 415 \times A_{st} \times 126 \times \left(1 - \frac{A_{st} \times 415}{1000 \times 126 \times 20}\right)$

or, $7.492\, A_{st}^2 - 45492.3\, A_{st} + 4.356 \times 10^6 = 0$

or, $A_{st}^2 - 6072.117\, A_{st} - 581420.18 = 0$

or, $A_{st} = \frac{(6072.117 \pm \sqrt{(6072.117^2 - 4 \times 581420.18)})}{2}$

$= 97.312$ mm^2/m

$$A_{st,\ \min} = \frac{0.12 \times 1000 \times 160}{100} = 192 \text{ mm}^2\text{/m} > 97.312 \text{ mm}^2\text{/m c/c}$$

Spacing of bars for $A_{st} = 192$ mm^2/m = 260 mm c/c

Maximum Spacing

(a) $3\,d = 3 \times 126 = 378$ mm

(b) 300 mm

Hence, provided ϕ 8 @ 260 mm c/c ($A_{st} = 192.3$ mm^2)

Distribution Reinforcement

$$A_{st} = \frac{0.12 \times bD}{100} = \frac{0.12 \times 1000 \times 160}{100} = 192 \text{ mm}^2$$

i.e. ϕ 8 @ 260 mm c/c < 450 < 5 d (5 × 126 = 630)

Hence, provided ϕ 8 @ 260 mm c/c.

The reinforcement detailing has been shown in Figure 7.20.

Figure 7.20 : Detailing Slab Reinforcement

Design of 3rd flight Beam

Effective Span (l_{ef})

As d is not known at the outset

l_{ef} = c/c distance between supports (Figure 7.21)

= (1.5 + 0.275 × 9 + 1.5 + 0.375) = 5.85 m

Figure 7.21 : Third Flight Beam

Depth of Beam (D)

From Deflection Control

$$d \geq \frac{l_{ef}}{k_B\, k_1\, k_2\, k_3\, k_4}$$

For simply supported beam $k_B = 20$

$k_1 = 1$ as $l_{ef} < 10$ m

$k_2 = 1$ as in Example 7.2 for M 20 and Fe 415 grade of concrete and steel respectively.

$k_3 = 1$.

For k_4; $\frac{\text{Web } (b_w)}{\text{Flange width } (b_f)} = \frac{250}{1500} = 0.167$

Correspondly, $k_4 = 0.8$.

Substituting all values in the above equation

$$d \geq \frac{5.85 \times 10^3}{20 \times 1 \times 1 \times 1 \times 0.8} = 365.625 \text{ mm}$$

$$\therefore \qquad D = d + \frac{\phi}{2} + \text{dia of stirrups} + \text{Nominal Cover}$$

$$= 365.625 + \frac{25}{2} + 8 + 30 = 416.125 \text{ mm}$$

(assuming ϕ 25 for main bars and ϕ 8 for stirrups)

Keeping $D = 420$ mm; $d = 420 - \frac{25}{2} - 8 - 30 = 369.5$ mm

Loads

Loads in going

Wt. of 2 railings/m = 2 × 1 = 2.0 kN/m

wt. of steps = $1.5 \times \frac{1}{2} \times 0.15 \times 0.275 \times 1 \times 24 \times \frac{1000}{275}$

= 2.7 kN/m

wt. of slab = 1.5 × 0.16 × 1 × 1 × sec 28.61 × 25 = 6.834 kN/m

wt. of web = (0.42 – 0.16) × 0.25 sec 28.61 × 25 = 1.851 kN/m

Total *DL* = 13.385 kN/m

1L = 1.5 × 4 = 6.000 kN/m

Total (*DL* + *IL*) = 19.385 kN/m

$w_u = 1.5 \times 19.385 = 29.078$ kN/m.

Load on Upper Landing

Wt. of web = (0.42 – 0.16) × 0.25 × 25 = 1.625 kN/m

DL of slab = 0.16 × 1 × 1 × 1.5 × 25 = 6.0 kN/m

Wt. of railing = 2 × 1 = 2.0 kN/m

Total *DL* = 9.625 kN/m

1L = 4 × 1.5 = 6.000 kN/m

Total (*DL* + *IL*) = 15.625 kN/m

$\therefore \quad w_u = 1.5 \times 15.625 = 23.438$ kN/m

Load on Lower Common Landing

Wt. of web = (0.42 – 0.16) × 0.25 × 25 = 1.625 kN/m

$\frac{1}{2}$ (*DL* of slab) = $\frac{1}{2} \times 0.16 \times 1 \times 1 \times 1.5 \times 25$ = 3.000 kN/m

Total *DL* = 4.625 kN/m

$\frac{1}{2}$ (IL) = $\frac{1}{2}$ × 4 × 1.5 = 3.000 kN/m

Total (DL + IL) = 7.625 kN/m

$\therefore \quad w_u = 1.5 \times 7.625 = 11.438$ kN/m.

The loading beam is shown in Figure 7.22.

Figure 7.22 : Third Flight with Design Load

Reactions

$$R_A = \frac{\left(11.438 \times \frac{1.688^2}{2} + 29.078 \times 2.475 \times \left(\frac{2.475}{2} + 1.688\right) + 23.438 \times 1.688 \times \left(\frac{1.688}{2} + 2.475 + 1.688\right)\right.}{(1.688 + 2.475 + 1.688)}$$

or, $$R_A = \frac{(16.295 + 210.543 + 198.094)}{5.851} = 72.626 \text{ kN}$$

$$R_B = \frac{\left(23.438 \times \frac{1.688^2}{2} + 29.078 \times 2.475 \times \left(\frac{2.475}{2} + 1.688\right)\right) + 11.438 \times 1.688 \times \left(\frac{1.688}{2} + 2.475 + 1.688\right)}{(1.688 + 2.475 + 1.688)}$$

or, $$R_B = \frac{33.391 + 210.543 + 96.672}{5.851} = 58.213 \text{ kN}$$

Let SF be zero at x from LHS.

$$72.626 - 23.438 \times 1.688 - 29.078\ (x - 1.688^2) = 0$$

or, $$x = \frac{(72.626 - 23.438 \times 1.688 + 29.078 \times 1.688)}{29.078} = 2.825 \text{ m}$$

$$M_{u,\,\max} = 72.626 \times 2.825 - 23.438 \times 1.688 \times \left(2.825 - \frac{1.688}{2}\right)$$

$$- \frac{29.078}{2} \times (2.825 - 1.688)^2 = 108 \text{ kN-m}$$

For isolated T-beam,

$$b_f = \frac{l_0}{\left(\frac{l_0}{b}\right) + 4} + b_w$$

$$= \frac{5851}{\left(\frac{5851}{1500}+4\right)}+250$$

$$= 740.57 + 250 = 990.57$$

Assuming x_u = thickness of slab = 160

$$M_u = 0.36\, f_{ck}\, x_u\, b\, (d - 0.42\, x_u)$$

$$= 0.36 \times 20 \times 160 \times 990.57 \times (369.5 - 0.42 \times 160)$$

$$= 344.966 > 108 \text{ kN-m}$$

Hence n.a. will fall in flange and the beam will designed as rectangular beam.

Tensile Reinforcement (A_{st})

$$M_u = 0.87\, f_y\, A_{st}\, d \left(1 - \frac{A_{st}\, f_y}{bd\, f_{ck}}\right)$$

or, $108 \times 10^6 = 0.87 \times 415 \times A_{st} \times 369.5 \times \left(1 - \frac{A_{st} \times 415}{990.57 \times 369.5 \times 20}\right)$

or, $7.563\; A_{st}^2 - 133407.98\, A_{st} + 108 \times 10^6 = 0$

or, $A_{st}{}^2 - 17639.56\, A_{st} + 14280047.6 = 0$

or, $A_{st} = \frac{(17639.56 \pm \sqrt{(17639.56^2 - 4 \times 14280047.6)})}{2} = 850.56 \text{ mm}^2$

$$A_{st,\ \min\%} = \frac{0.85}{f_y} \times 100 = \frac{0.85}{415} \times 100 = 0.205\%$$

$$A_{st,\ \min} = \frac{0.205}{100} \times 250 \times 369.5 = 89.37 \text{mm}^2 < 850.56 \text{ mm}^2$$

$$A_{st,\ \max} = 0.04\, bD = 0.04 \times 250 \times 420$$

$$= 4200 \text{ mm}^2 > 850.56 \text{ mm}^2$$

Hence, provided 2 ϕ 25.

Provision for Shear Reinforcement

V_u at d from support = 76.626 – 23.438 × 0.3695 = 67.966 kN

$$\tau_v = \frac{V_u}{bd} = \frac{67.966 \times 10^3}{250 \times 369.5} = 0.736 \text{ N/mm}^2$$

$$\frac{100 A_s}{bd} = \frac{100 \times 2 \times 491}{250 \times 369.5} \% = 1.063\%$$

$$\tau_c = 0.62 + \frac{(0.67 - 0.62)}{(1.25 - 1.0)} \times (1.063 - 1) = 0.633 \text{ N/mm}^2$$

$$V_{us} = V_u - \tau_c\, bd = 67.966 - 0.633 \times 250 \times 369.5 \times 10^{-3}$$

$$= 9.493 \text{ kN}$$

$$V_{us} = \frac{0.87\, f_y\, A_{su}\, d}{s_v}$$

$$s_v = \frac{0.87 \times 415 \times 2 \times 50 \times 369.5}{9.493 \times 10^3} = 1405.33 \text{ kN}$$

Minimun shear reinforcement is given by formula

$$\frac{A_{sv}}{b\, s_v} \geq \frac{0.4}{0.87\, f_y}$$

or, $$s_v \leq \frac{2 \times 50 \times 0.87 \times 415}{250 \times 0.4} = 361.05$$

$$s_v \leq 0.75 \times d = (0.75 \times 369.5 = 277.13)$$

$$s_v \leq 300$$

Hence, provided 2-legged ϕ 8 stirrups @ 275 mm c/c.

As only two bars as tensile reinforcement are provided hence, they will be extended into the support without curtailment.

Positive Moment Reinforcement at Simple Support

$$L_d \leq \frac{M_1}{V} + L_o$$

$$M_1 = 0.87\, f_y\, A_{st}\, d \left(1 - \frac{A_{st}\, f_y}{bd\, f_{ck}}\right)$$

$$= 0.87 \times 415 \times 2 \times 491 \times 369.5 \times \left(1 - \frac{2 \times 491 \times 415}{990.57 \times 369.5 \times 20}\right)$$

$$= 123.713 \times 10^6$$

$$V = 72.626 \times 10^3 \text{ N}$$

Substituting these values in the above equation,

$$L_o \geq \left(\frac{\phi \sigma_s}{4 \tau_{bd}} - \frac{M_1}{V}\right)$$

$$L_o \geq \frac{25 \times 0.87 \times 415}{4 \times 1.6 \times 1.2} - \frac{1.3 \times 123.713 \times 10^6}{72.626 \times 10^3}$$

$$L_o < (-\,1039.16)$$

Hence no extension of tensile reinforcement beyond the centre of supports. The detailing of reinforcement is shown in Figure 7.23.

Figure 7.23 : Detailing Beam Reinforcement

SAQ 1

(a) Sketch simple types of rectangular staircases.

(b) What are the criteria for fixing tread and rise?

(c) How effective span of a waist slab without stringer beam is determined?

(d) Write short notes on evaluation of design load on a slab span.

(e) Sketch showing reinforcement detailing at the junction of going and landing.

7.4 SUMMARY

Planning as well as structural design and detailing of flights with or without stringer beams of rectangular staircases have been described in this Unit.

The waist slab of a flight without stringer beam is designed as a slab spanning horizontally between the supports and loaded with gravity loads ($DL + IL$). Where a waist slab spans on to the edge of a landing slab, which spans parallel with the riser, the horizontal span of such slab is taken as going plus at each end either half the width of landing or one meter whichever is smaller.

A waist slab supported at one of its longitudinal edges by a beam or fixed in a wall is designed as a cantilever inclined slab spanning transversely loaded perpendicular to the plane of the slab.

Sometimes each tread of a flight is separate from the other and cantilevering from the edge beam is designed as a cantilever slab for gravity loads.

A waist slab supported on both edges is designed as a simply supported inclined transverse slab.

A supporting beam is designed as a beam spanning horizontally.

The principles of planning, design and detailing have been explained with illustrations.

The loads at the junction of two perpendicular flights is shared equally by both flights.

7.5 ANSWERS TO SAQs

SAQ 1

(a) Refer Section 7.1.

(b) Refer Section 7.1.

(c) Refer Section 7.2.

(d) Refer Section 7.3.

(e) Refer Example 7.1.