UNIT 3 BEAMS

Structure

3.1 INTRODUCTION

The principles of design and detailing of reinforced concrete section have been explained in the previous two Units. Here those principles have been applied for design and detailing of actual beams. Three types of beam, namely, simply supported rectangular T-beam and cantilever rectangular beam, are dealt with in a systematic manner.

Objectives

After studying this unit, you should be able to

- describe the sequence of design and detailing,
- analyse the forces for applied loads,
- understand the design of critical sections, and
- explain the details of reinforcements along a beam.

3.2 DESIGN AND DETAILING OF A SIMPLY SUPPORTED RECTANGULAR BEAM

Simply supported rectangular beams may be designed in the following steps :

Step I : Determine Effective Span (l_{ef})

The effective span shall be taken as clear span plus effective depth of beam or centre to centre of supports or whichever is less.

Step II : Determine Depth of Beam (D)

Design of a section means fixing dimension, $b \times D$, of the concrete section and area of reinforcement, A_{st}. As all the above three quantities may not be found out with the number of available equations, design process becomes a method of iteration. For the first trial, D and b are chosen based on Thumb Rules, Serviceability and Lateral Stability Criteria.

Total depth, D, is chosen as explained below :

Thumb Rule

Depth (D) is generally assumed between $\frac{l_{ef}}{10}$ and $\frac{l_{ef}}{20}$ depending on span and loading on the beam. In other words, for lesser span and lesser load, the depth will be lesser.

Control of Deflection

As explained in Section 1.5.1 for a simply supported beam

$$d \not< \frac{l_{ef}}{k_B\, k_1\, k_2\, k_3\, k_4}$$

where $k_B = 20$

Moment of Resistance Consideration

$$M_{u,\text{lim}} = 0.36\, \frac{x_{u,\max}}{d} \left(1 - 0.42\, \frac{x_{u,\max}}{d}\right) b\, d^2\, f_{ck} \qquad \text{. . . (3.1)}$$

Substituting all other values except d, the value of d can be determined.

Step III : Determine Breadth of Beam (*b*)

The breadth may be chosen between $\frac{D}{3}$ and $\frac{2D}{3}$. The breadth shall also comply with the limits for slenderness ratio for beam to ensure lateral stability which states that the clear distance between the lateral restraints shall not exceed 60 b or $\frac{250\, b^2}{d}$ or whichever is less.

Step IV : Determine Area of Tensile Reinforcement (A_{st})

A_{st} is determined by the equation (Section 1.6.2)

$$M_u = 0.87\, f_y\, A_{st}\, d \left(1 - \frac{f_y\, A_{st}}{f_{ck}\, b\, d}\right)$$

A_{st} must lie in the range of $A_{st,\ \min}$ and $A_{st,\ \max}$,

i.e. $A_{st,\ \min} < A_{st} < A_{st,\ \max}$.

where $A_{st,\min} = \frac{0.85}{f_y}\, bd$

and $A_{st,\ \max}$ may be calculated as follows :

$$\frac{x_u}{d} = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b\, d} \qquad \text{. . . (3.2)}$$

Substituting $x_u = x_{u,\ \max}$ and $A_{st} = A_{st,\ \max}$ in the above equation

$$\frac{x_{u,\max}}{d} = \frac{0.87\, f_y\, A_{st,\max}}{0.36\, f_{ck}\, bd}$$

The values of $\frac{x_{u,\max}}{d}$ are 0.53, 0.48 and 0.46 for grades of reinforcements Fe 250, Fe 415 and Fe 500, respectively. Substituting $\frac{x_{u,\max}}{d}$ for a particular grade of reinforcement $A_{st,\ \max}$ may be determined. However, $A_{st,\ \max}$ shall not exceed 0.04 bD (4% of the gross sectional area of the beam).

Step V : Curtailment and Detailing of Reinforcement

As explained in Section 2.4.

Step VI : Provision of Shear Reinforcements

As explained in Section 2.2 and 2.4.

Step VII : Detailing Near Supports

As explained in Section 2.4.

Step VIII : Side Face Reinforcement

As explained in Section 2.4.4.

Example 3.1

Design a simply supported beam of 5 m clear span loaded with a uniformly distributed load of 15 kN/m including its self weight. The design parameters are : Support width = 300 mm; f_{ck} = 20 N/mm^2; f_y = 415 N/mm^2 for main reinforcement and f_y = 250 N/mm^2 for shear reinforcement.

Solution

Effective Span (l_{ef})

$$l_{ef} = 5 + 0.3 = 5.3 \text{ m}$$

or l_{ef} = 5 + effective depth of beam

Assuming effective depth to be *more than* 0.3 m to start with

$$l_{ef} = 5.3 \text{ m.}$$

Depth of Beam (D)

From Thumb Rule

$$\frac{l_{ef}}{10} \text{ to } \frac{l_{ef}}{20} \text{ or } \frac{5.3 \times 1000}{10} = 530 \text{ to } \frac{5.3 \times 1000}{20} = 265 \text{ mm}$$

Hence, adopted D = 470 mm

From Control of Defection

$$d \geq \frac{l_{ef}}{k_B\, k_1\, k_2\, k_3\, k_4}; \qquad k_B = 20$$

For simply supported beam of l_{ef} < 10 m, k_1 = 1. Assuming balanced section for M 20 concrete and Fe 415 steel, p_t% = 0.96% and correspondingly k_2 = 1. (Figure 1.2), $k_3 = k_4 = 1$ as the beam is singly reinforced rectangular one.

From Moment of Resistance Consideration

$$M_u = \frac{1.5\, w l_{ef}^2}{8} = \frac{1.5 \times 15 \times 5.3^2}{8} = 79 \text{ kN-m} \qquad \text{(Figure 3.1)}$$

Figure 3.1 : Beam with Loading

From Eq. (1.8)

$$M_{u,\lim} = 0.36 \frac{x_{u,\max}}{d}\left(1 - 0.42 \frac{x_{u,\max}}{d}\right) b\, d^2 f_{ck}$$

or, $\quad 79 \times 10^6 = 0.36 \times 0.48 \times (1 - 0.42 \times 0.48) \times 250 \times d^2 \times 20$

or $\quad d = 338.41$ mm

Hence, provided $D = 470$ mm

Assuming ϕ 16 for main reinforcement and ϕ 6 for shear reinforcement

$$d = 470 - (20 + \frac{16}{2} + 6) = 436 \text{ mm}$$

Breadth of beam between $\frac{D}{3}$ and $\frac{2D}{3}$

Let $\quad b = 250$ mm

Lateral Stability consideration

$$5 \times 10^3 \not> 60 \times b \text{ or } b \not< \frac{5 \times 1000}{60} = 83.33 \text{ mm}$$

or $$5 \times 10^3 > \frac{250\, b^2}{d} \text{ or } b \geq \sqrt{\frac{5 \times 10^3 \times 436}{250}} = 93.38 \text{ mm}$$

Hence, provided $b = 250$ mm.

Tensile Reinforcement (A_{st})

$$M_u = 0.87 f_y A_{st}\, d\left(1 - \frac{A_{st} f_y}{b\, d\, f_{ck}}\right)$$

$$79 \times 10^6 = 0.87 \times 415 \times A_{st} \times 436 \times \left(1 - \frac{A_{st} \times 415}{250 \times 436 \times 20}\right)$$

$$79 \times 10^6 = 157417.8\, A_{st} - 29.97\, A_{st}^2$$

or, $$A_{st}^2 - 5252.51\, A_{st} + 2635969.30 = 0$$

$$A_{st} = \frac{\left(5252.51 \pm \sqrt{(5252.51^2 - 4 \times 2635969.30)}\right)}{2} = 561.98 \text{ mm}^2$$

Figure 3.2 : Evaluation of Effective Depth

Provided 6 ϕ 12 bar in two layers as shown in Figure 3.2.

Curtailment and Detailing of Reinforcement

3 ϕ 12 bars in upper layer is to be curtailed as shown in Figure 3.3. Hence, continuing bars to the supports has area 50% of total reinforcement provided Moment of Resistance of section with continuing bars.

Figure 3.3 : Designed Section

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

$$= 0.87 \times 415 \times 3 \times 113 \times 436 \times \left(1 - \frac{3 \times 113 \times 415}{250 \times 436 \times 20}\right)$$

$$= 49.921 \text{ kN-m}$$

Let theoretical cut off section from support = x

$$\therefore \quad M_u = \frac{w_u l_{ef}}{2} x - \frac{w_u x^2}{2}$$

or $$49.921 = \frac{1.5 \times 15 \times 5.3}{2} x - \frac{1.5 \times 15 \times x^2}{2}$$

or $$49.921 = 59.63 x - 11.25 x^2$$

or $$x^2 - 5.3 x + 4.437 = 0$$

or $$x = \frac{\left(5.3 \pm \sqrt{(5.3^2 - 4 \times 4.437)}\right)}{2}$$

or $x = 1.04$ m. In other words, section will lie at 1.61 m $\left(= \frac{5.3}{2} - 1.04\right)$ from centre of the beam. Actually, upper bars will be curtailed at 1.61 + (12 ϕ or d or whichever is greater) from centre of the beam

i.e. 1.61 + 0.410 = 2.02 m

$\therefore$ The curtailment of upper bars will be done at 2.02 m from centre of the beam.

$$L_d = \frac{\phi \, \sigma_s}{4 \tau_{bd}} = \frac{12 \times 0.87 \times 415}{4 \times 1.6 \times 1.2} = 0.564 \text{ m} < 2.02 \text{ m}$$

Length available beyond the actual cut off point from centre of supports

$$= \frac{l_{ef}}{2} - 2.02 = \frac{5.3}{2} - 2.02 = 0.63 \text{ m}$$

Provision of Shear Reinforcement

Shear at support

$$V'_u = \frac{w_u\, l_{ef}}{2} = \frac{1.5 \times 15 \times 5.3}{2} = 59.625 \text{ kN}$$

(a) **Design Shear Force** at distance d = 410 from face of support, i.e. at distance $\left(\frac{l_{ef}}{2} - \frac{\text{Support width}}{2} - d\right)$ from centre to beam

$\therefore$ DC (Figure 3.4) = $\frac{5.3}{2} - \frac{0.3}{2} - 0.41 = 2.09$ m

Figure 3.4 : SFD

$\therefore$ From similarity of triangles,

$$V_u = \frac{59.625}{\left(\frac{5.3}{2}\right)} \times 2.09 = 47.025 \text{ kN}$$

$$\therefore \quad \tau_v = \frac{V_u}{bd} = \frac{47.025 \times 10^3}{250 \times 410} = 0.459 \text{ N/mm}^2$$

For $$p_t\% = \frac{100 \times 3 \times 113}{250 \times 410} = 0.33\%$$

$$\tau_c = 0.36 + \frac{0.48 - 0.36}{0.5 - 0.25} \times (0.33 - 0.25) = 0.398 \text{ N/mm}^2$$

$$\therefore \quad V_{us} = 47.025 - 0.398 \times 250 \times 436 \times 10^{-3} = 3.643 \text{ kN}$$

Here, d = 436 mm (= 410 + 20 + 6) as the upper layer of bars does not continue up to the critical section for shear force.

$$V_{us} = \frac{0.87\, f_y\, A_{sv}\, d}{s_v}$$

or $$s_v = \frac{0.87 \times 250 \times 2 \times 28 \times 436}{3.643 \times 10^3} = 1458 \text{ mm c/c}$$

$$s_{v,\min} = 0.75\, d = 0.75 \times 436 = 327 \text{ mm}$$

or $$s_{v,\min} = 300 \text{ mm}$$

or $s_{v,\min}$ is given by the following formula

$$\frac{A_{sv}}{b\, s_{v,\min}} \geq \frac{0.4}{0.87 \times f_y}$$

or $\dfrac{2 \times 28}{250 \times s_{v,\min}} \geq \dfrac{0.4}{0.87 \times 250}$

or $s_{v,\min} \leq \dfrac{2 \times 28 \times 0.87 \times 250}{250 \times 0.4} = 121.8$ mm

Hence, provided 2-legged stirrups ϕ 6 @ 120 mm c/c.

(b) **Check Shear Strength at Cut Off Point as per Cl 26.2.3.2(a)**

Shear at cut off point $= \dfrac{w_u \, l_{ef}}{2} - w_u \left(\dfrac{l_{ef}}{2} - 2.02 \right)$

$$= \frac{1.5 \times 15 \times 5.3}{2} - 1.5 \times 15 \times \left(\frac{5.3}{2} - 2.02 \right) = 45.45 \text{ kN}$$

Shear strength of the section with ϕ 6 @120 c/c stirrups

$$= \frac{2}{3} (\tau_c bd + V_{us}) = \frac{2}{3} \left(0.398 \times 250 \times 436 + \frac{0.87 f_y A_{sv} d}{s_v} \right)$$

$$= \frac{2}{3} \left(0.398 \times 250 \times 436 + \frac{0.87 \times 250 \times 2 \times 28 \times 436}{120} \right) \times 10^{-3} \text{ kN}$$

$$= 58.424 \text{ kN} > 39.375 \text{ kN}$$

As 2 legged stirrups ϕ 6 @ 120 mm c/c is the minimum shear reinforcement, hence the same shall be continued for full span of the beam.

Detailing Near Support

L_d = 564 mm

(a) As per L_d requirement the continuing bars shall continue into the support = 564 – 630 = – 66 mm from centre of support.

(b) $\dfrac{L_d}{3} = \dfrac{564}{3} = 188$, i.e. $188 - \dfrac{300}{2} = 38$ from centre of support

(c) $L_d > \dfrac{1.3 M_1}{V} + L_0$

or $L_0 \not< L_d - \dfrac{1.3 M_1}{V} = 564 - \dfrac{1.3 \times 49.921 \times 10^3}{59.625} = -524.42$ mm

Figure 3.5 : Detailing of Reinforcements (All Dimensions in mm)

Hence bars at support shall extend 38 mm beyond centre of support. The detailing of reinforcement is shown in Figure 3.5.

SAQ 1

(a) There are three unknowns *b*, *d* and A_{st} in the design of a rectangular section. Why all these three cannot be solved simultaneously?

(b) Explain the meaning and application of modification factors k_1, k_2, k_3, and k_4.

(c) How breadth *b* of a rectangular section is determined?

(d) How area of tensile steel, A_{st}, is calculated? What are the minimum and maximum limits of A_{st} and how they are calculated?

(e) Explain curtailment and detailing of reinforcement along a beam and also detailing of reinforcement at supports.

(f) How shear reinforcement is provided? Also why extra shear reinforcement is provided at section of curtailment?

(g) Explain provision of side face reinforcement with sketch.

3.3 DESIGN AND DETAILING OF A CANTILEVER BEAM

The steps for design and detailing are almost the same as those for simply supported beams. The difference in contents of various steps is being mentioned here sequentially.

Step I : Determine Effective Span (l_{ef})

The effective span is equal to the length from the face of support plus half the effective depth except where it forms the end of a continuous beam where the length to the centre of the support shall be taken.

Step II : Determine Depth of Beam (*D*)

Depth (*D*) of beam is determined by deflection control formula, i.e.

$$d \nless \frac{l_{ef}}{k_B\, k_1\, k_2\, k_3\, k_4}$$

where $k_B = 7$ for beams up to 10 m span.

If the span of the beam is greater than 10 m, $k_1 = \frac{10}{\text{span}}$. Deflection may be calculated as given in Annexure C of IS : 456 - 2000.

Step III : Determine Breadth of Beam (*b*)

The breadth of the beam is chosen from lateral stability consideration as follows :

The clear distance from the free end of the cantilever to the lateral restraint shall not exceed 25 *b* or $\frac{100\, b^2}{d}$ or whichever is less.

Step IV : Determine Area of Tensile Reinforcement (A_{st})

Same as that for simply supported beam section (Section 1.6.2).

Step V : Curtailment and Detailing of Reinforcement

Curtailment rules for simple support or end of a cantilever are the same as for single supported beam. At fixed end, the tensile bars shall extend into the support for full development length (Figure 3.6). If necessary hooks or bends to be provided.

Figure 3.6 : Curtailment of Main Reinforcement of a Cantilever Beam

Step VI : Provision of Shear Reinforcement

Same as that for simply supported beams (Section 2.2 and 2.4).

Step VII : Detailing Near Support

See Figure 3.6.

Step VIII : Side Face Reinforcement

Same as that for simply supported beam (Section 2.4.4).

Example 3.2

Design a cantilever beam of clear span 3 m loaded with a uniformly distributed load 28 kN/m including self weight (Figure 3.7). Design parameters are as follows : f_{ck} = 25 N/mm^2 and f_y = 415 N/mm^2.

Solution

Effective Span (l_{ef})

$$l_{ef} = \text{clear span} + \frac{d}{2} = 3 + \frac{0.5}{2} = 3.25 \text{ m}$$

(Assuming d = 500 mm to start with)

Figure 3.7 : Beam with Loading

Depth of Beam (*D*)

From Control of Deflection

$$\frac{l_{ef}}{d} \not> k_B \, k_1 \, k_2 \, k_3 \, k_4$$

or $$d \nless \frac{l_{ef}}{k_B\, k_1\, k_2\, k_3\, k_4} = \frac{3.25}{7 \times 1 \times k_2\, k_3\, k_4}$$

$k_1 = 1$ as $l_{ef} < 10$ m

Assuming the section to be doubly reinforced, percentage of tensile reinforcement, $p_t\% = 1.5 > 1.19$ for balanced section $k_2 = 0.9$, $k_3 = 1.15$ assuming percentage of compression steel $p_c\% = 0.5\%$ and $k_4 = 1$ as it is a rectangular section. Substituting the above values

$$d \nless \frac{3.25}{7 \times 1 \times 0.9 \times 1.15 \times 1} = 0.449 \text{ m}$$

Hence, provided $d = 450$ mm.

Breadth of Beam (*b*)

Assuming total depth ≈ 500

According to thumb rule, b will fall in the range of

$$\frac{D}{3} = \frac{500}{3} = 167 \quad \text{to} \quad \frac{2D}{3} = 333 \text{ mm}$$

From lateral stability consideration

$$b > \frac{l_c}{25} \left(= \frac{3 \times 10^3}{25} = 120 \text{ mm} \right)$$

$$b > \sqrt{\frac{l_c \times d}{100}} = \sqrt{\frac{3 \times 10^3 \times 450}{100}} = 116 \text{ mm}$$

Hence, provided $b = 300$ mm.

Area of Reinforcements (A_{st} and A_{sc})

$$l_{ef} = \text{clear span} + \frac{d}{2}$$

$$= 3 + \frac{0.45}{2} = 3.225 \text{ m}$$

$$M_u = \frac{w_u\, l_{ef}^2}{2} = \frac{1.5 \times 28 \times 3.225^2}{2} = 218.413 \text{ kN-m}$$

To Determine A_{st1}

For balanced section.

$$M_{u,\text{lim}} = 0.36\, \frac{x_{u,\max}}{d} \left(1 - 0.42\, \frac{x_{u,\max}}{d} \right) f_{ck}\, bd^2$$

$$= 0.36 \times 0.48 \times (1 - 10.42 \times 0.48) \times 2\,5 \times 300 \times 450^2$$

$$= 209.532 \text{ kN-m} < 218.413 \text{ kN-m}$$

Hence, doubly reinforced section shall be provided

$$M_u = 0.87\, f_y\, A_{st}\, d \left(1 - \frac{A_{st}\, f_y}{bd\, f_{ck}} \right)$$

$$= 0.87 \times 415 \times A_{st1} \times 450 \times \left(1 - \frac{A_{st} \times 415}{300 \times 450 \times 25} \right)$$

or $$209.532 \times 10^6 = 162472.5\ A_{st1} - 19.978\ A_{st1}^2$$

or $$A_{st1}^2 - 8132.571\ A_{st1} + 10488136.95 = 0$$

or, $$A_{st1} = 1607.314 \text{ mm}^2$$

$$M_2 = 218.413 - 209.532 = 8.881 \text{ kN-m}$$

To Determine A_{sc}

$$M_2 = f_{sc}\, A_{sc}\, (d - d')$$

Assuming $d' = 50$

$$\frac{x_{u,\max}}{d} = 0.48$$

or $$x_{u,\max} = 0.48 \times 450 = 216$$

$$\varepsilon_c = \frac{0.0035}{216} \times (216 - 50) = 0.0027 \quad \text{(Figure 3.8)}$$

or $$f_{sc} = 342.8 + \frac{(351.8 - 342.8)}{(0.00276 - 0.00241)} \times (0.0027 - 0.00241)$$

$$= 350.26 \text{ N/mm}^2.$$

Substituting value of f_{sc} in the above equation

$$8.881 \times 10^6 = 350.26 \times A_{sc}\,(450 - 50)$$

or $$A_{sc} = 63.69 \text{ mm}^2$$

Hence, provided 2 ϕ 10 bars as compression reinforcement.

Figure 3.8 : Section with ε-Diagram

To Determine A_{st2}

$$M_2 = 0.87\, f_y\, A_{st2}\,(d - d')$$

or $$8.881 \times 10^6 = 0.87 \times 415 \times A_{st2} \times (450 - 50)$$

or $$A_{st2} = \frac{8.881 \times 10^6}{0.87 \times 415 \times 400} = 61.494 \text{ mm}^2$$

∴ Total $A_{st} = A_{st1} + A_{st2} = 1607.314 + 61.49 = 1668.804 \text{ mm}^2$

Hence provided 6 ϕ 20 bars in two layers

($A_{st} = 1884 \text{ mm}^2 > 1668.804 \text{ mm}^2$).

Figure 3.9 : Designed Section

Curtailment and Detailing of Reinforcement

Assuming 50% of Curtailment

Let theoretical cut off section be at x from free end for lower 3 ϕ 20 bars; therefore, BM at x from free end,

$$M_u = \frac{w_u x^2}{2} = \frac{1.5 \times 28 x^2}{2} = 21x^2 \text{ kN-m}$$

Effective cover = 30 + 8 + $\frac{20}{2}$ = 48 mm

Providing D = 520 mm, d = 520 – 48 = 472 mm

M_u with 3 ϕ 20 as tensile reinforcement at that section

$$M_u = 0.87 \; f_y \; A_{st} \; d \times \left(1 - \frac{A_{st} \; f_y}{bd \; f_{ck}}\right)$$

$$= 0.87 \times 415 \times 3 \times 314 \times (520 - 48) \times \left(1 - \frac{3 \times 314 \times 415}{300 \times 472 \times 25}\right)$$

= 142.804 kN-m

Substituting value of M_u in the above equation

$142.804 = 21x^2$ or x = 2.608 m,

i.e. from face of support cutoff section is at 3 – 2.608 = 0.392 m.

Actual cut off section will be at greater of

(a) 0.392 +12 ϕ = 0.392 + 12 × 0.02 = 0.632, or

(b) 0.392 + 0.472 = 0.864 m

from face of support.

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{20 \times 0.87 \times 415}{4 \times 1.6 \times 1.4} = 0.806 \text{ m}$$

Hence, actual curtailment will be at 0.864 m from face of support.

Provision of Shear Reinforcement

(a) **At Support**

$$V_u = w_u l = 1.5 \times 28 \times 3 = 126 \text{ kN}$$

$$\tau_v = \frac{V_u}{bd} = \frac{126 \times 10^3}{300 \times 450} = 0.933 \text{ N/mm}^2$$

$$p_t\% = \frac{6 \times 314 \times 100}{300 \times 450} = 1.396\%$$

$$\tau_c = 0.7 + \frac{(0.74 - 0.7)}{(1.5 - 1.25)} \times (1.396 - 1.25)$$

$$= 0.723 \text{ N/mm}^2 < 0.933 \text{ N/mm}^2 \ (= \tau_v)$$

$\therefore \quad V_{us} = V_u - \tau_c\, bd = (126 - 0.723 \times 300 \times 450 \times 10^{-3}) = 28.395$ kN

Providing 2-legged vertical stirrups ϕ 8,

$$V_{us} = 0.87 f_y A_{sv} \frac{d}{s_v} = 0.87 \times 415 \times 2 \times 50 \times \frac{450}{s_v}$$

(i) $s_v = \dfrac{0.87 \times 415 \times 2 \times 50 \times 450}{28.395 \times 10^3} = 572.187$ mm c/c

(ii) $0.75\, d = 0.75 \times 450 = 337.5$ mm c/c

(iii) 300 mm

(iv) $\dfrac{A_{sv}}{b\, s_v} \geq \dfrac{0.4}{0.87 f_y}$

(v) $s_v \leq \dfrac{A_{sv} \times 0.87 f_y}{0.4\, b} = \dfrac{2 \times 50 \times 0.87 \times 415}{0.4 \times 300} = 300.9$ mm c/c

Hence provided 2-legged stirrups ϕ 8 @ 300 mm c/c.

(b) **At Cut Off Section $V_u < \frac{2}{3}(V_c + V_{us})$**

V_u at cut off section = $w_u (l_c - 0.806) = 1.5 \times 28 \times (3 - 0.806)$

= 92.148 kN

For $\quad V_c \quad p_t\,\% = \dfrac{3 \times 314 \times 100}{300 \times 472}\% = 0.665\,\%$

$$\tau_c = 0.49 + \frac{(0.57 - 0.49)}{(0.75 - 0.5)} (0.665 - 0.5) = 0.543 \text{ N/mm}^2$$

$\therefore \quad V_c = \tau_c bd = 0.543 \times 300 \times 472 \times 10^{-3} = 76.89$ kN

$$V_{us} = 0.87 f_y A_{sv} \frac{d}{s_v} = 0.87 \ \ 415 \times 2 \times 50 \times \frac{472}{300} \times 10^{-3} = 56.81 \text{ kN}$$

$$\frac{2}{3} (V_c + V_{us}) = \frac{2}{3} (76.89 + 56.81) = 89.11 < V_u$$

Let $\quad s_v = 250$ mm

$$\therefore V_{us} = 0.87\, f_y\, A_{sv}\, \frac{d}{s_v} = 0.87 \times 415 \times 2 \times 50 \times \frac{472}{250} \times 10^{-3} = 68.17 \text{ kN}$$

$$\frac{2}{3}(V_c + V_{us}) = \frac{2}{3}\,(76.89 + 68.17) = 96.71 > 92.148 \text{ kN}$$

Hence, O.K.

Anchorages

At Support

The detailing of reinforcement as per calculation above is shown in Figure 3.10.

Figure 3.10 : Detailing of Reinforcement

At Free End

The continuing 3 ϕ 20 bars has a length (3 – 0.806) m = 2.194 m > L_d up to the free end. Hence, no anchorages is required at the free end. However, these bars will be anchored nominally at the free end as shown in Figures 3.10 and 3.11.

Figure 3.11 : Section after Curtailment of Inner Layer of Bars

SAQ 2

(a) How effective span of a cantilever beam is determined?

(b) What are the basic values of $\frac{L_{ef}}{d}$ ratio, k_B for different types of beams?

(c) How deflection is evaluated if the effective span of the beams is more than 10 m?

(d) For a cantilever beam of 4 m clear span and effective depth (d) 625, what should be the minimum width, b, for fire resistance of 2 hrs and lateral stability criteria.

3.4 DESIGN AND DETAILING OF A SIMPLY SUPPORTED FLANGED BEAM

Design and detailing of a simply supported flanged beam is similar to that for a simply supported rectangular beam. The difference in design is due to cross sectional shapes of the two. These differences have been explained under different steps.

Step I : Determine Effective Span (l_{ef})

Same as that for a simply supported rectangular beam.

Step II : Determine Depth of Beam (D)

Same as those for a simply supported rectangular beam. From 'Thumb Rule' procedure, D may be somewhat smaller due to the fact that a large area of concrete in the form of flange is available for resisting flexural compressive force.

Step III : Determine Breadth of Beam (b_w and b_f)

Breadth of web (b_w) is guided by architectural considerations as well as accommodation of tensile reinforcement. Hence, b_w may be taken between $\frac{D}{3}$ and $\frac{2D}{3}$. Lateral stability is no problem in flanged beam as the part of the web of the beam in compression is throughout supported laterally by floor/roof slab forming flange.

The effective width of flange (b_f) may be taken as the following but in no case greater than the breadth of the web plus half the sum of the clear distances to the adjacent beams on either side.

(a) For T-beam $b_f = \frac{l_0}{6} + b_w + 6D_f$

(b) For L-beam $b_f = \frac{l_0}{12} + b_w + 3D_f$

For isolated beams, the effective width shall be obtained as below but in no case it shall be greater than the actual width.

(a) For T-beam $b_f = \dfrac{l_0}{\left(\dfrac{l_0}{b}\right) + 4} + b_w$

(b) For L-beam $b_f = \dfrac{0.5 l_0}{\left(\dfrac{l_0}{b}\right) + 4} + b_w$

where, b_f = Effective width of flange,

l_o = Distance between points of zero moments in the beam*,

b_w = Breadth of web,

D_f = Thickness of flange, and

b = Actual width of flange.

* For continuous beams and frames, 'l_o' may be assumed as 0.7 times the effective span.

Step IV : Determine Area of Tensile Reinforcement (A_{st})

Bending moment (M_u) for the external loads is calculated and then A_{st} is calculated as follows :

Calculate M'_u assuming $x_u = D_f$

In other words, for a flanged beam having $x_u \le D_f$

$$M_u = 0.36\, f_{ck}\, b_f\, x_u\, (d - 0.42\, x_u)$$

as that for a rectangular beam of breath b_f.

Substituting $x_u = D_f$

$$M'_u = 0.36\, f_{ck}\, b_f\, D_f\, (d - 0.42\, D_f)$$

If $M'_u \ge M_u$, n.a. will lie in the flange and the section will be analyzed as a rectangular beam of cross section $b_f \times D$ and A_{st} may be obtained from formula.

$$M_u = 0.87\, f_y\, A_{st}\, d \left(1 - \frac{A_{st}\, f_y}{b_f\, d\, f_{ck}}\right)$$

But if $M'_u < M_u$, n.a. will be in the web for which two cases may arise :

If $\frac{D_f}{d} \le 0.2$, n.a. may be determined from the equation

$$M_u = 0.36\, f_{ck}\, b_w\, x_u\, (d - 0.42\, x_u) + 0.45\, f_{ck}\, (b_f - b_w) \left(d - \frac{D_f}{2}\right)$$

A_{st} may be calculated by equating

$$C = T$$

or, $\quad 0.36\, f_{ck}\, b_w\, x_u + 0.45\, f_{ck}\, (b_f - b_w)\, D_f = 0.87\, f_y\, A_{st}$

If $\frac{D_f}{d} > \mathbf{0.2}$, n.a. may be calculated from the equation

$$M_u = 0.36\, f_{ck}\, b_w\, x_u\, (d - 0.42\, x_u) + 0.45\, f_{ck}\, (b_f - b_w)\; y_f \left(d - \frac{y_f}{2}\right)$$

where $\quad y_f = (0.15\, x_u + 0.65\, D_f) \le D_f$

A_{st}, then, may be obtained from force equilibrium equation, i.e. $C = T$

or, $\quad 0.36\, f_{ck}\, b_w\, x_u + 0.45\, f_{ck}\, b_f\, y_f = 0.87\, f_y\, A_{st}$

Step V : Curtailment and Detailing of Reinforcement

Same as for simply supported rectangular beam (Section 2.4).

Step VI : Provision of Shear Reinforcement

Same as that for a simply supported rectangular beam (Sections 2.2 and 2.4) except that b_w is substituted for b.

Step VII : Detailing Near Supports

Same as that for simply supported rectangular beam (Section 2.4).

Step VIII : Side Face Reinforcement

Same as that for simply supported beam (Section 2.4.4).

Example 3.3

Design an interior beam as T-beam of the floor shown in Figure 3.12 for the following specifications :

Slab = 120 mm thick; Floor finish load = 2 kN/m^2;
Imposed loads = 3 kN/m^2; f_{ck} = 20 N/mm^2; f_y = 415 N/mm^2; and
Nominal cover = 20 mm.

Figure 3.12 : Plan of Floor

Solution

Span (l_{ef})

Clear span + Effective depth or

c/c of support = 5 + 0.3 = 5.3 m (Cl. 22.2)

Effective Width of Flange

$$\frac{l_o}{6} + b_w + 6\,D_f = \frac{5.3}{6} + 0.25 + 6 \times 0.12 = 1.853 \text{ m}$$

or c/c distance between adjacent spans = 3.5 m, or whichever is less.

Hence, b_f = 1.853 m.

Depth of Beam (D)

From Thumb Rule

$$D = \frac{l_{ef}}{10} \text{ to } \frac{l_{ef}}{20} \text{ i.e. } \frac{5.3}{10} \text{ to } \frac{5.3}{20} = 530 \text{ to } 265 \text{ mm}$$

From Deflection control

$$\frac{L_{ef}}{d} \ngtr k_B\, k_1\, k_2\, k_3\, k_4$$

k_B = 20

$k_1 = 1$ as $l_{ef} < 10$ m

k_2 = 1 for p_t% = 0.96% tensile reinforcement (assuming balanced section at the outset for rectangular section)

k_3 = 1 as it is singly reinforced

$$k_4 = 0.8 \text{ for } \frac{b_w}{b_f} = \frac{0.25}{1.853} = 0.135$$

$$\therefore \quad d \nless \frac{5.3}{20 \times 1 \times 1 \times 1 \times 0.8} = 0.331 \text{ m} = 331 \text{ mm}$$

Hence, provided D = 500 mm

$$\therefore \quad d = 500 - 20 - 8 - \frac{20}{2} = 462 \text{ mm}$$

(Assuming ϕ 20 for main bars and ϕ 8 for stirrups.)

Applied Bending Moment Consideration

Loads

Rib	= 0.25 × (0.5 – 0.12) × 1 × 25	= 2.375 kN/m
Slab	= 0.12 ×1 × 3.5 × 25	= 10.5 kN/m
Finishing	= 2 × 1 × 3.5	= 7.0 kN/m
IL	= 3 × 1 × 3.5	= 10.5 kN/m
Total Load		= 30.375 kN/m

$$\therefore \quad M_u = \frac{w_u \times l_{ef}^2}{8} = \frac{1.5 \times 30.375 \times 5.3^2}{8} = 159.981 \text{ kN-m}$$

Assuming $x_u = D_f$

$$M'_u = 0.36 f_{ck}\, b_f\, D_f\, (d - 0.42\, D_f)$$

or $$M'_u = 0.36 \times 20 \times 1853 \times 120 \times (462 - 0.42 \times 120) \times 10^{-6}$$

$$= 658.968 \text{ kNm} > 159.981 \text{ kN-m}$$

Hence, n.a. will lie in the flange and section will be analyzed as a rectangular beam.

Area of Tensile Reinforcement (A_{st})

$$M_u = 0.87 f_y\, A_{st}\, d \left(1 - \frac{A_{st} f_y}{b_f d\, f_{ck}}\right)$$

or $$159.981 \times 10^6 = 0.87 \times 415 \times A_{st} \times 462 \times \left(1 - \frac{A_{st} \times 415}{1853 \times 462 \times 20}\right)$$

or $$159.981 \times 10^6 = 166805.1\, A_{st} - 4.043\, A_{st}^2$$

or $$A_{st}^2 - 41257.754\, A_{st} + 39569873.86 = 0$$

or $$A_{st} = 982 \text{ mm}^2$$

Hence, provided 5 ϕ 16.

$$\therefore \quad d = 500 - 20 - 8 - \frac{16}{2} = 464 \text{ mm}$$

Provision for Shear Reinforcements

Providing 2-legged ϕ 8 stirrups

At support $$V_u = \frac{w_u l_{ef}}{2} = \frac{1.5 \times 30.375 \times 5.3}{2} = 120.74 \text{ kN}$$

∴ V_u at d from face of support (Figure 3.13).

$$= \frac{120.74}{5.3/2} \times \left(\frac{5}{2} - 0.464\right) = 92.764 \text{ kN}$$

Figure 3.13 : SFD

$$\tau_v = \frac{V_u}{b_w d} = \frac{92.764 \times 10^3}{250 \times 464} = 0.8 \text{ N/mm}^2$$

$$p_t \% = \frac{5 \times 201 \times 100}{250 \times 464} \% = 0.87\%$$

$$\therefore \quad \tau_c = 0.56 + \frac{(0.62 - 0.56)}{(1 - 0.75)} (0.87 - 0.75)$$

$$= 0.589 < 0.8 \ (= \tau_v)$$

$$\therefore \quad V_{us} = V_u - \tau_c b_w d = 92.764 - 0.589 \times 250 \times 464 \times 10^{-3} = 24.44 \text{ kN}$$

$$V_{us} = 0.87 f_y \, A_{sv} \frac{d}{s_v}$$

or $$s_v = \frac{0.87 \times 415 \times 2 \times 50 \times 464}{24.44 \times 10^3} = 685.46 \text{ mm}$$

$$0.75\, d = 0.75 \times 464 = 348 \text{ mm} > 300 \text{ mm}$$

$$\frac{A_{sv}}{b_w \, s_v} \geq \frac{0.4}{0.87 f_y}$$

or $$s_v \leq \frac{A_{sv} \times 0.87 f_y}{0.4 b_w} = \frac{2 \times 50 \times 0.87 \times 415}{0.4 \times 250} = 361 \text{ mm}$$

or $$s_v \leq 361 \text{ mm}$$

Hence, provided 2-legged ϕ 8 stirrups @ 300 c/c throughout. Curtailment of bars and anchorages may be done as for rectangular beam (Example 3.1).

SAQ 3

(a) How b_w and b_f of a T-beam are determined?

(b) Why lateral stability considerations are not taken cognizance of while deciding b_w of a T-beam?

3.5 SUMMARY

Basically, design of a beam for transverse loading is performed in four steps as follows :

(a) Determination of depth.

(b) Determination of breadth.

(c) Provision of flexural, shear and side face reinforcements.

(d) Detailing of reinforcement, i.e. curtailment of flexural reinforcement complying development length requirements and other requirements such as minimum and maximum distances between reinforcements, minimum and maximum percentage of reinforcements, etc.

The analytical formulations for the above requirements have been developed in Unit 1 and 2 and applied for design in this Unit.

3.6 ANSWERS TO SAQs

SAQ 1

(a) Refer Section 3.2.

(b) Refer Section 3.2.

(c) Refer Section 3.2.

(d) Refer Section 3.2.

(e) Refer Section 3.2.

(f) Refer Section 3.2.

(g) Refer Section 3.2.

SAQ 2

(a) Refer Section 3.3.

(b) Refer Section 1.5.1.

(c) Refer Section 3.3.

(d) For lateral Stability

$$b > \frac{4000}{25} \; (= 160)$$

or, $$b > \sqrt{\frac{4000 \times 625}{100}} = (158.11)$$

For fire resistance of 2hrs

$$b \geq 200$$

Hence $b = 200$

SAQ 3

(a) Refer Section 3.4.

(b) Refer Section 3.4.