UNIT 6 FLOW THROUGH COMPLEX PIPES

Structure

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6.1 INTRODUCTION

Fundamentals of flow through pipes were discussed in Unit 5. In this unit, you will be introduced the flow through pipes in series, pipes in parallel, branch pipes connecting three reservoirs, syphons, etc.

Objectives

After studying this unit, you should be able to

- conceptualise the situation of flow through complex pipes specially when pipes are in series or parallel,
- explain "three reservoirs" and "four reservoir" problems under flow through pipes, and
- describe use and application of siphons.

6.2 PIPES IN PARALLEL

Figure 6.1 shows three pipes 1, 2 and 3 connected in parallel. From the continuity consideration, the total discharge is given by

$$Q = Q_1 + Q_2 + Q_3 \qquad \ldots \text{(a)}$$

where Q_1, Q_2 and Q_3 are discharges in pipes 1, 2 and 3. Since the pipes are connected in parallel, the loss of head in all pipes will be the same. Thus

$$h_{f1} = h_{f2} = h_{f3} \qquad \ldots \text{(b)}$$

There are generally two types of problems :

(i) The loss of head (H) is given and the discharge is required.

(ii) The discharge Q is given and the head loss and the distribution of discharge in different branches are required.

In the first type of problems, the discharge in each pipe may be obtained from the loss of head.

$$H = h_{f1} = f\left(\frac{L_1}{D_1}\right)\frac{V_1^2}{2g}$$

$$H = h_{f2} = f\left(\frac{L_2}{D_2}\right)\frac{V_2^2}{2g}$$

$$H = h_{f3} = f\left(\frac{L_3}{D_3}\right)\frac{V_3^2}{2g}$$

Knowing H, the values of V_1, V_2 and V_3 may be obtained from the above equations and hence the discharges Q_1, Q_2 and Q_3 obtained.

Then $$Q = Q_1 + Q_2 + Q_3 \quad \ldots (6.1)$$

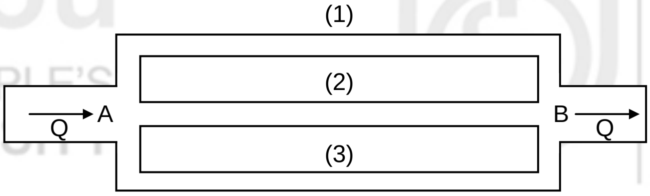

Figure 6.1 : Pipes in Parallel

In the second type of problems, the discharges Q_1, Q_2 and Q_3 can be expressed in terms of H.

We know $$H = f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g}$$

or $$V_1 = \sqrt{\frac{2gH}{f_1\left(\frac{L_1}{D_1}\right)}}$$

$$Q_1 = A_1 V_1 = A_1\sqrt{\frac{2gH}{f_1\left(\frac{L_1}{D_1}\right)}}$$

or $$Q_1 = A_1\sqrt{\frac{2g}{f_1\left(\frac{L_1}{D_1}\right)}}\sqrt{H} = k_1\sqrt{H}$$

where $$k_1 = A_1\sqrt{\frac{2g}{f_1\left(\frac{L_1}{D_1}\right)}}$$ = a constant for a pipe

Similarly, $$Q_2 = k_2\sqrt{H} \quad \text{and} \quad Q_3 = k_3\sqrt{H}$$

Therefore, $$Q = Q_1 + Q_2 + Q_3$$

or $$Q = k_1\sqrt{H} + k_2\sqrt{H} + k_3\sqrt{H}$$

or $$Q = \sqrt{H}\,(k_1 + k_2 + k_3) \quad \ldots (6.2)$$

As the discharge Q is known and k_1, k_2 and k_3 are the constants, the value of H may be obtained from Eq. (6.2). After H has been calculated, the discharge in individual pipes may be obtained.

An approximate solution of the system of pipes in parallel can be obtained by determining the percentage distribution of discharge. A reasonable value of the loss of head *H* is assumed and the discharge in each pipe is found for the assumed value of *H*. The distribution of discharge as the percentage of the total discharge is worked out. It has been found that the percentage distribution of discharge does not change much with the head. This means whatever be the total discharge, the percentage distribution remains more or less constant. Thus from the percentage distribution of discharge for assumed *H*, the discharge in each pipe can be calculated for the given total discharge assuming that the percentage distribution remains the same.

Example 6.1

Two reservoirs are connected by 2 pipes of the same length laid in parallel. The diameters of the pipe are 10 cm and 30 cm respectively. If the discharge through 10 cm diameter pipe is 0.01 cumecs, what will be the discharge through 30 cm pipe? Assume that *f* is the same for both pipes.

Solution

For such problems, it is more convenient to express the Darcy-Weisbach equation in terms of discharge as

$$h_f = f\,\frac{L}{D}\,\frac{V^2}{2g} = f\,\frac{L}{D}\,\frac{Q^2}{\left(\frac{\pi}{4}D^2\right)^2}\times\frac{1}{2g}$$

or $$h_f = \frac{16\,f}{\pi^2}\times\frac{L}{D^5}\times\frac{Q^2}{2g}$$

As the pipes are parallel, $h_{f1} = h_{f2}$

or $$\frac{16\,fL}{2g\,\pi^2}\times\frac{1}{D_1^5}\times Q_1^2 = \frac{16\,fL}{2g\,\pi^2}\times\frac{1}{D_2^5}\times Q_2^2$$

or $$\frac{1}{(0.1)^5}\,Q_1^2 = \frac{1}{(0.30)^5}\,Q_2^2$$

$$Q_2^2 = 243\,Q_1^2$$

Substituting $Q_1 = 0.01$, $Q_2{}^2 = 243 \times 0.01 \times 0.01$

or $Q_2 = 0.156$ cumecs

SAQ 1

Two reservoirs are connected by three pipes of diameter *D*, 2*D* and 3*D* in parallel. What will be the discharge of the other two pipes if discharge through the smallest pipe is 1 m^3/s?

Assume that all the pipes are of the same length and have the same friction factor. Neglect minor losses.

6.3 PIPES IN SERIES

Let us now consider the case when the pipes joining the two reservoirs are connected in series. Figure 6.2 shows a system of three pipes in series. Total loss of head (H) is given by

$$H = h_{L1} + h_{f1} + h_{L2} + h_{f2} + h_{L3} + h_{f3} + \frac{V_3^2}{2g} \qquad \text{. . . (6.3)}$$

where h_{L1}, h_{L2} and h_{L3} are losses at entrance, contraction and enlargement, respectively,

h_{f1}, h_{f2} and h_{f3} are losses in three pipes due to friction, and

V_3 is the velocity in pipe 3.

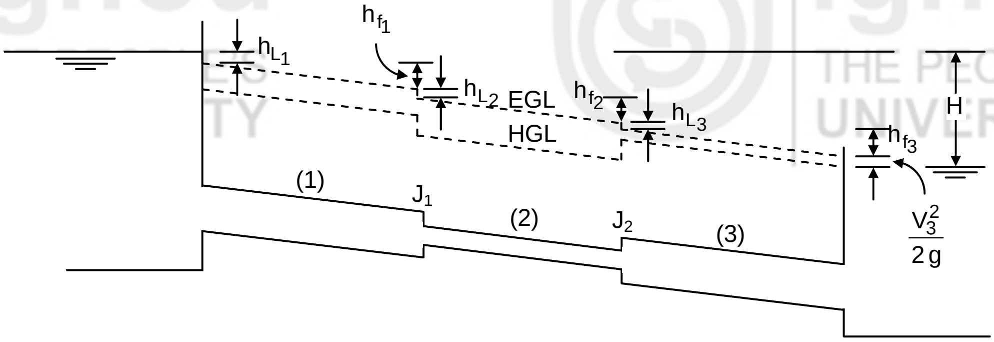

Figure 6.2

The hydraulic gradient line is below the energy gradient line, and the vertical distance between the two lines is equal to the corresponding velocity head. The reader should note a distinguishing characteristic between the two lines. *The energy gradient line always slopes down in the direction of flow, whereas the hydraulic gradient line may rise or fall depending upon the velocity and pressure changes*. As the velocity in pipe 2 is more than the velocity in pipe 1, there is a drop of the hydraulic gradient line at the junction point J_1. On the other hand, since the velocity in pipe 3 is less than that in pipe 2, the hydraulic gradient line rises at the junction point J_2.

For the pipe system shown, from continuity,

$$Q_1 = Q_2 = Q_3 = Q$$

or $$A_1 V_1 = A_2 V_2 = A_3 V_3 = Q \qquad \text{. . . (a)}$$

There are two types of problems in the pipe system :

(a) The discharge Q is known and the loss of head is required.

(b) The loss of head is known and the discharge is required.

In the first type of problems, as the discharge is known, velocities in different pipes can be calculated using Eq. (a). Then the loss of head (H) is obtained using Eq. (6.3). In the second type of problems, all the losses can be expressed in terms of the velocity in any one pipe. The relation between the velocities in different pipes can be obtained using Eq. (a). Substituting the values of all the losses in terms of any one velocity in Eq. (6.3), that velocity can be obtained. The discharge is obtained using Eq. (a).

Example 6.2

Two reservoirs are connected by a pipe line consisting of two pipes, one of 15 cm diameter and length 6 m and the other of diameter 22.5 cm and 16 m length. If the difference of water levels in the two reservoirs is 6 m, calculate the discharge and draw the energy gradient line. Take $f = 0.04$

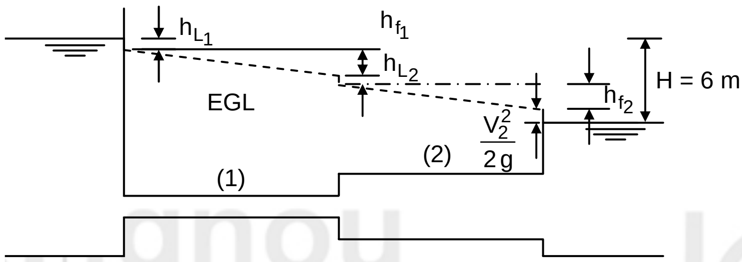

Figure 6.3

Solution

Let the velocities in pipes 1 and 2 be V_1 and V_2, respectively.

From the continuity equation,

$$\left(\frac{\pi}{4}\right) \times 15^2 \times V_1 = \left(\frac{\pi}{4}\right) \times (22.5)^2 \times V_2 \;\text{ or }\; V_1 = 2.25\, V_2$$

Loss of head at entrance

$$h_{L1} = 0.5 \left(\frac{V_1^2}{2g}\right)$$

or
$$h_{L1} = \frac{0.5\,(2.25\,V_2)^2}{2g} = 2.53 \left(\frac{V_2^2}{2g}\right)$$

Loss of head due to friction in the pipe 1,

$$h_{f1} = f\,\frac{L_1}{D_1}\,\frac{V_1^2}{2g}$$

$$h_{f1} = 0.04 \times \frac{6}{0.15}\,\frac{V_1^2}{2g} = 1.6\,\frac{V_1^2}{2g} = 8.1\,\frac{V_2^2}{2g}$$

Loss of head due to sudden enlargement

$$h_{L2} = \frac{(V_1 - V_2)^2}{2g}$$

or
$$h_{L2} = (2.25 - 1)^2\,\frac{V_2^2}{2g} = 1.56\,\frac{V_2^2}{2g}$$

Loss of head due to friction in pipe 2,

$$h_{f2} = f\left(\frac{L_2}{D_2}\right)\frac{V_2^2}{2\text{g}}$$

or
$$h_{f2} = \frac{0.04 \times 16}{0.225} \times \frac{V_2^2}{2g} = 2.84\,\frac{V_2^2}{2g}$$

Loss of head at exit $= \dfrac{V_2^2}{2g}$

Now $\quad h_{L1} + h_{f1} + h_{L2} + h_{f2} + \dfrac{V_2^2}{2g} = 6$

$$(2.53 + 8.1 + 1.56 + 2.84 + 1)\,\frac{V_2^2}{2g} = 6 \text{ or } V_2 = 2.71 \text{ m/s}$$

Now $\quad Q = A_2 V_2 = \dfrac{\pi}{4} \times (0.225)^2 \times 2.71 = 0.108$ cumecs

Energy gradient line (EGL) is shown in Figure 6.3.

SAQ 2

(a) A pipe consists of 3 pipes in series as follows :

- (i) 300 m long, 15 cm diameter
- (ii) 150 m long, 10 cm diameter
- (iii) 240 m long, 20 cm diameter

The first pipe takes off from a reservoir with water level at an elevation of 500.00. If the elevation of the pipe at the exit is 400.00, find the discharge. Assume $f = 0.04$. Neglect minor losses.

(b) Two pipes of diameters $2D$ and D are connected in parallel, and when a discharge passes through them, the loss of head is H_1. When the same two pipes are connected in series, the loss of head is H_2 for the same discharge. Find the relationship between H_1 and H_2. Assume both the pipes are of the same length and have the same f. Neglect minor losses.

(c) For the distribution main of a town water supply, a 500 mm diameter pipe is required. As pipes of 500 mm diameter are not available, it is decided to lay two smaller pipes of equal diameter in parallel. Find the diameter of these pipes.

6.4 BRANCH PIPES CONNECTING THREE RESERVOIRS

Figure 6.4 shows three reservoirs A, B and C at different water surface elevations connected by three pipes 1, 2 and 3 meeting at the junction J. If a piezometer is inserted at J, the liquid will rise in the piezometer, indicating the pressure at that point. Let p be the pressure at J. Let the frictional losses in three pipes be h_{f1}, h_{f2} and h_{f3}. Minor losses are usually neglected. Alternatively, the minor losses are indirectly incorporated by increasing the lengths of pipes by a suitable amount known as equivalent length due to minor losses.

If the pressure head at J is more than that at B, the liquid flows from J to B. From the continuity equation,

$$Q_1 = Q_2 + Q_3 \qquad \ldots \text{(a)}$$

where Q_1, Q_2 and Q_3 are the discharges in pipes 1, 2 and 3 respectively.

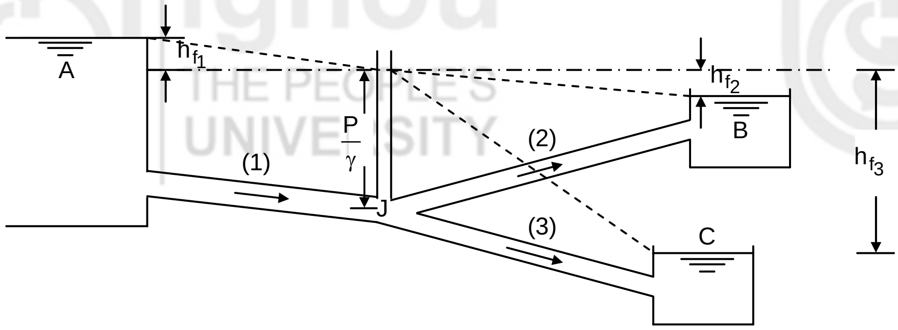

Figure 6.4 : Three Reservoirs System

However, if the pressure head *J* is less than that at *B*, the direction of flow in pipe 2 would be reversed and the continuity equation becomes

$$Q_1 + Q_2 = Q_3 \quad \text{. . . (b)}$$

Three types of problems that commonly occur in practice are given below:

Type 1

Given

Pipe lengths, diameters, the surface elevations of reservoirs *A* and *B*, and the discharge Q_1,

Required

To find the surface elevation of reservoir *C*.

Procedure

(i) Compute the velocity V_1 in pipe from Q_1 and then find the loss of head in pipe 1,

$$h_{f1} = f\left(\frac{L_1}{D_1}\right)\frac{V_1^2}{2g}$$

and determine the pressure head (p/γ) at *J* from

$\frac{p}{\gamma} + z_j = \frac{p_A}{\gamma} + z_A - h_{f1}$ where z_j and z_A are elevations of *J* and *A*, respectively

(ii) From the water elevations at point *J* and at reservoir *B*, find direction of flow and the difference of heads (h_{f2}), and hence calculate the discharge Q_2.

$$h_{f2} = f\left(\frac{L_2}{D_2}\right)\frac{V_2^2}{2g}$$

(iii) Calculate Q_3 from the continuity Eqs. (a) or (b), depending upon the direction of flow.

(iv) From the known value of Q_3, compute the loss of head in pipe 3, and hence determine the elevation of reservoir *C*.

Type 2

Given

Pipe lenghts and diameters, the elevations of liquid surface in reservoirs *A* and *C* and the discharge Q_2 to reservoir *B*.

Required

To find the elevation of liquid surface in reservoir *B*.

Procedure

The problem is solved by trial and error method. From the given data, the sum of losses ($h_{f1} + h_{f3}$) and the difference of discharge $Q_1 - Q_3 = Q_2$ are known.

(a) Assume some suitable distribution of Q_1 and Q_3 satisfying the condition $Q_1 - Q_3 = Q_2$.

(b) Compute the loss of head h_{f1} and h_{f3} and see whether they satisfy the requirement $h_{f1} + h_{f3}$ = level in reservoir *A* – level in reservoir *C*.

Repeat the procedure till a satisfactory solution is obtained.

Alternative

(a) Assume some suitable elevation of the piezometer level at *J* to distribute the sum ($h_{f1} + h_{f3}$) into two parts h_{f1} and h_{f3}.

(b) Compute the discharge Q_1 and Q_3 for the values of h_{f1} and h_{f3}, and see if the continuity equation $Q_1 - Q_3 = Q_2$ is satisfied.

(c) If not, assume different values of h_{f1} and h_{f3} till the continuity equation is satisfied.

(d) Compute h_{f2} for the value of Q_2 obtained in Step (ii).

(e) Obtain the reservoir level at *B* from the value of h_{f2} computed in Step (iv).

Type 3

Given

Pipe lengths and diameters, and the elevations of all reservoirs.

Required

To find the discharge in each pipe.

Procedure

This is the classic *three-reservoir problem*. In this case, it is not known whether flow is in or out of the reservoir *B*.

(a) Assume that no flow occurs in pipe 2, i.e. the piezometric level at *J* is assumed at the water elevation of reservoir *B*.

(b) Compute the discharge Q_1 and Q_3.

(c) If $Q_1 > Q_3$, the liquid flows from *J* to *B*, and the continuity equation $Q_1 = Q_2 + Q_3$.

(d) If $Q_1 < Q_3$, the liquid flows from *B* to *J*, and the continuity equation $Q_1 + Q_2 = Q_3$.

(e) When the direction of flow in pipe 2 is established, the problem becomes rather simple. Let us assume that the direction of flow is from *J* to *B*. Thus $Q_1 = Q_2 + Q_3$.

Method 1

The piezometric level at J is assumed and the discharge in each pipe is determined. Another value of the piezometric level at J is assumed and the procedure is repeated. The procedure is continued till the resulting flow in each pipe satisfies the discharge condition. While making assumptions of the elevation at J, it is helpful to plot the computed values of Q_1 against the error $[Q_1 - (Q_2 + Q_3)]$. The error may be positive or negative depending upon the assumptions. The intersection of the line with Q_1- axis gives the required discharge. This corresponds to zero error (see Example 6.3).

Method 2

The problem can also be solved analytically once the direction of flow in pipe 2 is known. Let us say that the direction of flow is from J to B. We have the equations :

$$H_1 = h_{f1} + h_{f2} = f\left(\frac{L_1}{D_1}\right)\frac{V_1^2}{2g} + f\left(\frac{L_2}{D_2}\right)\frac{V_2^2}{2g} \quad \text{. . . (i)}$$

$$H_2 = h_{f1} + h_{f3} = f\left(\frac{L_1}{D_1}\right)\frac{V_1^2}{2g} + f\left(\frac{L_3}{D_3}\right)\frac{V_3^2}{2g} \quad \text{. . . (ii)}$$

where H_1 = Elevation at A – Elevation at B;

H_2 = Elevation at A – Elevation at C,

and $Q_1 = Q_2 + Q_3$, assuming the flow from J to B

or $$D_1^2 V_1 = D_2^2 V_2 + D_3^2 V_3 \quad \text{. . . (iii)}$$

Eqs. (i), (ii) and (iii) can be solved simultaneously for V_1, V_2 and V_3 and then the discharge Q_1, Q_2 and Q_3 can be obtained.

Example 6.3

Figure 6.5 shows three-reservoirs connected by pipes. Find the discharge in each pipe. Take f = 0.04. All the pipes are 1500 m long and 30 cm in diameter.

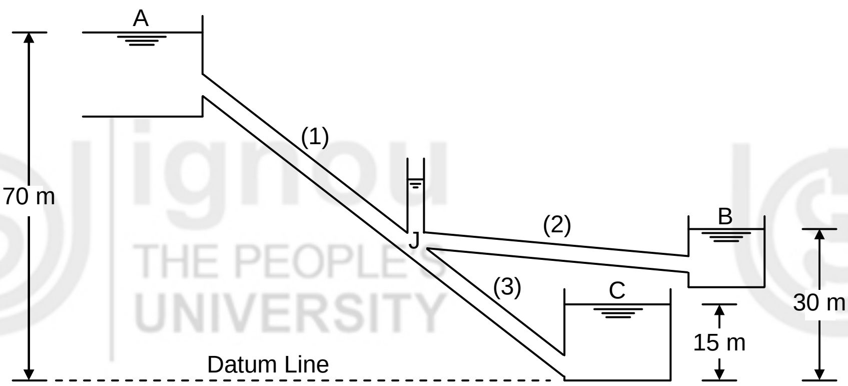

Figure 6.5

Solution

To determine the direction of flow in pipe 2, let us assume that no flow occurs in pipe 2. That is, the piezometric level at J is 30 m above the datum.

$$h_{f1} = 70 - 30 = 40$$

Therefore, $40 = 0.04 \times \left(\frac{1500}{0.3}\right) \times \frac{V_1^2}{19.62}$ or $V_1 = 1.98$ m/sec

$$Q_1 = \frac{\pi}{4} \times 0.30^2 \times 1.98 = 0.140 \text{ cumecs}$$

Now $h_{f3} = 30 - 15 = 15$

Therefore, $15 = 0.04 \times \left(\frac{1500}{0.3}\right) \times \frac{V_3^2}{19.62}$ or $V_3 = 1.21$ m/sec

$$Q_3 = \frac{\pi}{4} \times 0.30^2 \times 1.21 = 0.0855 \text{ cumecs}$$

Since $Q_1 > Q_3$, the direction of flow is from *J* to *B*. The problem will first be solved analytically.

Considering the flow from reservoir *A* to *B*,

$$40 = h_{f1} + h_{f2} = 0.04 \times \left(\frac{1500}{0.3}\right) \times \left(\frac{V_1^2}{19.62} + \frac{V_2^2}{19.62}\right)$$

$$= 10.2\ (V_1^2 + V_3^2)$$

or $V_2 = \sqrt{3.92 - V_1^2}$ . . . (a)

Likewise, considering the flow from reservoir *A* to *C*,

$$55 = \frac{0.04 \times 1500}{0.3 \times 2 \times 9.81}\ (V_1^2 + V_3^2)$$

$$= 10.2\ (V_1^2 + V_3^2)$$ . . . (b)

or $V_3 = \sqrt{5.39 - V_1^2}$

From the continuity of flow, $Q_1 = Q_2 + Q_3$

or $V_1 A_1 = V_2 A_2 + V_3 A_3$

Since $A_1 = A_2 = A_3, \quad V_1 = V_2 + V_3$ . . . (c)

From Eqs. (a), (b) and (c),

$$V_1 = \sqrt{3.92 - V_1^2} + \sqrt{5.39 - V_1^2}$$ . . . (d)

Eq. (d) may be solved by trial and error. The solution gives $V_1 = 1.90$ m/sec.

From Eqs. (a) and (b), $V_2 = 0.557$ m/sec, $V_3 = 1.335$ m/sec.

Thus $Q_1 = 0.134$ cumecs, $Q_2 = 0.039$ cumecs, and $Q_3 = 0.094$ cumecs.

Alternative Method

The problem may also be solved graphically after the direction of flow in pipe 2 has been ascertained. For the first trial, let the piezometric level at $J = 50$ m above the datum.

$$h_{f1} = 70 - 50 = 20$$

$$20 = 0.04 \times \left(\frac{1500}{0.3}\right) \times \frac{V_1^2}{19.62}$$

or $V_1^2 = 1.96$ or $V_1 = 1.40$ m/sec

$$Q_1 = \frac{\pi}{4} \times 0.30^2 \times 1.40 = 0.099 \text{ cumecs}$$

Now $\quad h_{f2} = 50 - 30 = 20$

Therefore, $\quad 20 = 0.04 \times \left(\frac{1500}{0.3}\right) \times \frac{V_2^2}{19.62}$ or $V_2 = 1.40$ m/sec

$$Q_2 = \frac{\pi}{4} \times 0.30^2 \times 1.40 = 0.099 \text{ cumecs}$$

Now $\quad h_{f3} = 50 - 15 = 35$

Therefore, $\quad 35 = 0.04 \times \left(\frac{1500}{0.3}\right) \times \frac{V_3^2}{19.62}$ or $V_3 = 1.85$ m/sec

$$Q_3 = \frac{\pi}{4} \times 0.30^2 \times 1.85 = 0.130 \text{ cumecs}$$

Error $= Q_1 - (Q_2 + Q_3)$

$= 0.099 - (0.099 + 0.130) = - 0.130$ cumecs

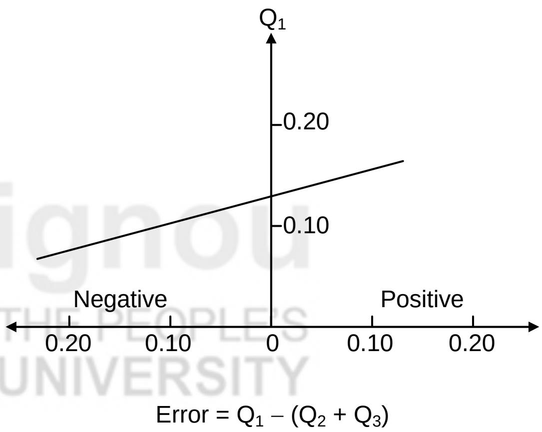

Figure 6.6

Similarly, assuming the level at J = 40 m above the datum, we get

Q_1 = 0.121 cumecs, Q_2 = 0.07 cumecs, and Q_3 = 0.11 cumecs

Error $= Q_1 - (Q_2 + Q_3)$

$= 0.121 - (0.07 + 0.11) = - 0.059$ cumecs

Assuming the level at J = 35 m above the datum, we get

Q_1 = 0.132 cumecs, Q_2 = 0.049 cumecs, and Q_3 = 0.098 cumecs

Error $= Q_1 - (Q_2 + Q_3)$

$= 0.132 - (0.049 + 0.098) = - 0.015$ cumecs

Figure 6.6 shows a plot between Q_1 as ordinate and the error $[Q_1 - (Q_2 + Q_3)]$ as abscissa.

From the plot, the correct value of Q_1, corresponding to zero error, is **0.134 cumecs**. For this value of Q_1, the values of Q_2 and Q_3 may be calculated.

SAQ 3

Three reservoirs *A*, *B* and *C* are connected by branch pipes 1, 2 and 3 meeting at a junction *J*. Pipe 1 connects reservoir *A* to *J*, pipe 2 connects *J* to reservoir *B*, and pipe 3 connects *J* to reservoir *C*. The water level in reservoirs A and B are respectively 50 m and 30 m. If the discharge in pipe 1 is 0.10 cumecs, determine the water level of reservoir C. All the pipes are 1000 m long and have 300 mm diameter and $f = 0.04$. Neglect minor losses.

6.5 BRANCH MAINS CONNECTING FOUR RESERVOIRS

Figure 6.7 represents 4 reservoirs *A*, *B*, *C* and *D* connected by a system of pipes. Reservoir *A* supplies liquid to reservoirs *B*, *C* and *D* through branch mains 2, 3 and 4. If the water elevations of the reservoirs are known, the discharge in each main can be calculated.

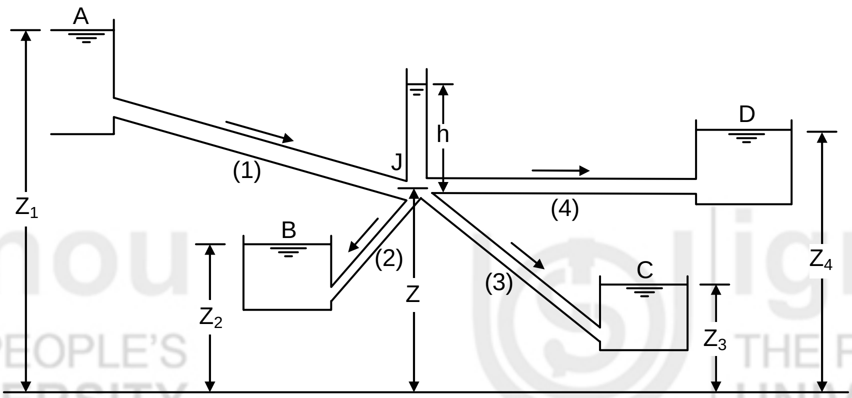

Figure 6.7

Let *h* be the piezometric height at *J*. Let us denote the heights of liquid levels in reservoirs about datum by Z_1, Z_2, Z_3 and Z_4 and the height of the junction *J* by *Z*. Applying Bernoulli's equation,

$$Z_1 = Z + h + h_{f1}$$

or
$$Z_1 = Z + h + f\left(\frac{L_1}{D_1}\right)\left(\frac{V_1^2}{2g}\right) \qquad \ldots \text{(a)}$$

Also
$$Z_2 + h_{f2} = Z + h$$

or
$$Z_2 = Z + h - h_{f2}$$

or
$$Z_2 = Z + h - f\left(\frac{L_2}{D_2}\right)\left(\frac{V_2^2}{2g}\right) \qquad \ldots \text{(b)}$$

Likewise
$$Z_3 = Z + h - f\left(\frac{L_3}{D_3}\right)\left(\frac{V_3^2}{2g}\right) \qquad \ldots \text{(c)}$$

and
$$Z_4 = Z + h - f\left(\frac{L_4}{D_4}\right)\left(\frac{V_4^2}{2g}\right) \qquad \ldots \text{(d)}$$

Also from the continuity of flow,

$$Q_1 = Q_2 + Q_3 + Q_4$$

or $$V_1 A_1 = V_2 A_2 + V_3 A_3 + V_4 A_4 \quad \ldots \text{(e)}$$

Eqs. (a) to (e) may be solved for V_1, V_2, V_3, V_4 and h. After the velocities have been determined, the discharge in each pipe can be calculated.

The problems can also be solved by trial and error. Different values of 'h' are assumed till the flow conditions satisfy the continuity equation, i.e. $Q_1 = Q_2 + Q_3 + Q_4$.

6.6 SYPHONS

When a pipe is laid in such a manner that a part of it is above the hydraulic gradient line, it is called a syphon pipe, or simply a syphon. Figure 6.8 shows a pipe in which the part *CDE* is above the hydraulic gradient line (the vertical scale is exaggerated in the figure). The pressure head at any point along the axis of the pipe is equal to the distance between the hydraulic gradient line and the axis. It follows that the pressure at points *C* and *E* is zero, i.e. the pressure is atmospheric. The pressure in the reach *CDE*, where the pipeline is above the hydraulic gradient line is, negative. The minimum pressure will be at the summit point *D* where the vertical intercept between the point and the hydraulic gradient line is maximum.

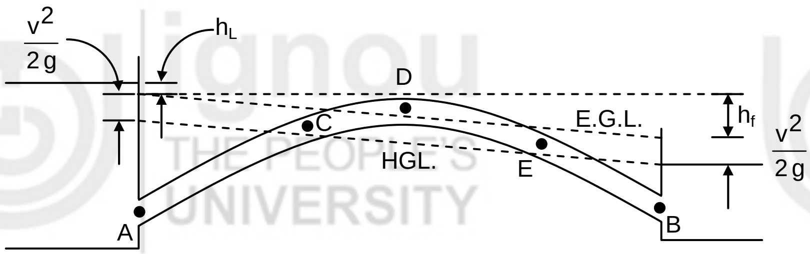

Figure 6.8 : Syphon

If the absolute pressure at the point *D* reaches the vapour pressure, cavitation occurs and the gases are liberated. For water, the maximum height of summit above the hydraulic gradient, for normal temperature and pressure, is about 7.80 m (i.e. vacuum pressure = 7.8 m of water and absolute pressure = 2.5 m of water).

The discharge through a syphon may be obtained as in an ordinary pipe connecting two reservoirs. Thus

$$H = h_L + h_f + \frac{V^2}{2g}$$

where h_L = loss at entrance and h_f = loss due to friction.

Example 6.4

A pipe of 1 m diameter connects two reservoirs having a difference of level of 6 m.

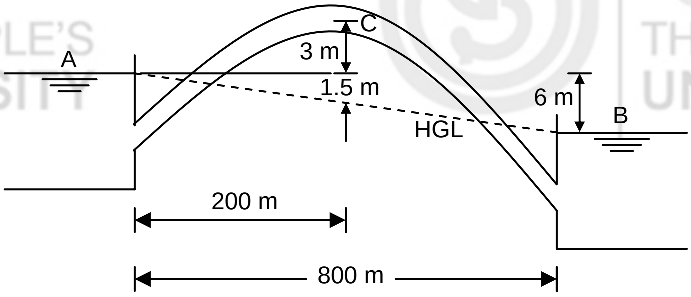

Figure 6.9

The total length of the pipe is 800 m and rises to a maximum height of 3 m above the level of water in the higher reservoir at a distance of 200 m from the entrance. Find the discharge in the pipe and pressure at the highest point. Take $f = 0.04$, and neglect minor losses.

Solution

$$H = h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$

or
$$6 = 0.04 \times \left(\frac{800}{1}\right) \times \frac{V^2}{2g}$$

$$V = 1.92 \text{ m/sec}$$

$$Q = AV = \frac{\pi}{4} \times 1^2 \times 1.92 = 1.51 \text{ cumecs}$$

Loss of head upto point C, assuming uniform loss,

$$h_L = 0.04 \times \left(\frac{200}{1}\right) \times \frac{(1.92)^2}{2 \times 9.81} = 1.50 \text{ m}$$

Negative pressure at $C = 3 + 1.50 = 4.50$ m of water.

$$\frac{p_c}{\gamma} = -\ 4.50 \text{ m of water}$$

A more accurate value of the negative pressure at C may be obtained by applying Bernoullis' equation to points *A* and *C*, taking datum at the reservoir level at *A*,

$$0 = \frac{p_c}{\gamma} + 3 + \frac{V^2}{2g} + h_f$$

or
$$0 = \frac{p_c}{\gamma} + 3 + \frac{(1.92)^2}{2 \times 9.81} + 1.50$$

$$\frac{p_c}{\gamma} = -\ 4.69 \text{ m of water.}$$

It may be noted that the difference between the two values of the pressure at *C* is due to the velocity head.

SAQ 4

A pipe 200 mm diameter and 1200 m long connects two reservoirs, one being 30 m lower than the other. The pipe crosses a ridge whose summit is 2.5 m above the upper reservoir.

Determine the depth of the pipe apex below the ridge in order to ensure that the pressure in the pipe does not fall below 7.80 (vacuum). The length of the pipe from the upper reservoir to the pipe apex is 300 m. Take $f = 0.03$.

6.7 SUMMARY

- Flow through two or more parallel pipes is explained. In parallel pipes, the loss of head is the same in all pipes. Discharge, $Q = Q_1 + Q_2 + Q_3 + \ldots$
- Flow through two or more pipes in series is discussed. In such pipes, the discharge is the same in all pipes. Loss of head, $h_f = h_{f1} + h_{f2} + h_{f3} + \ldots$
- When branch pipes connect three or more reservoirs, the discharge cannot be directly determined.
- In three-reservoir problems, the piezometric head at the junction point (J) of the three pipes is important. This pressure should be determined from the given flow conditions.
- If the piezometric head at the junction point cannot be determined directly, it should be assumed. The assumed piezometric head should be corrected till the continuity equation is satisfied.
- The flow through branch mains connecting four reservoirs can be determined by trial and error. Alternatively, a digital computer can be used.
- A syphon pipe lies above the hydraulic gradient. The pressure in a syphon pipe is negative.
- To avoid cavitation, the absolute pressure in a syphon pipe should be greater than the vapour pressure.

6.8 ANSWERS TO SAQs

SAQ 1

Since the pipes are in parallel, the losses of head is the same in all the pipes. If Q_1, Q_2 and Q_3 are the discharges in the pipes of diameters D_1, D_2 and D_3, respectively,

then $$h_f = \frac{16\, fL}{h^2 \times 2g} \times \frac{Q_1^2}{D^5} = \frac{16\, fL}{\pi^2 \times 2g} \times \frac{Q_2^2}{(2D)^5} = \frac{16\, fL \times Q_3^2}{\pi^2 \times 2g \times (3D)^5}$$

or $$\frac{Q_1^2}{D^5} = \frac{Q_2^2}{32 D^5} = \frac{Q_3^2}{243 D^5}$$

Therefore, $$Q_2 = (32)^{1/2}\ (Q_1) = 5.66 \times 1.0 = 5.66 \text{ m}^3/\text{s}$$

$$Q_3 = (243)^{1/2}\ (Q_1) = 15.59 \times 1.0 = 15.59 \text{ m}^3/\text{s}$$

SAQ 2

(a) For the pipes in series,

$$h_f = f\,\frac{L_1}{D_1}\left(\frac{V_1^2}{2g}\right) + f\,\frac{L_2}{D_2}\left(\frac{V_2^2}{2g}\right) + f\,\frac{L_3}{D_3}\left(\frac{V_3^2}{2g}\right)$$

From continuity equation,

$$\frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4} D_2^2 V_2 = \frac{\pi}{4} D_3^2 V_3$$

or $$\frac{\pi}{4} \times (0.15)^2 \times V_1 = \frac{\pi}{4} \times (0.10)^2 \times V_2 = \frac{\pi}{4} \times (0.20)^2\ V_3$$

Therefore, $$V_2 = \left(\frac{0.150}{0.10}\right)^2 \times V_1 = 2.25\, V_1$$

$$V_3 = \left(\frac{0.150}{0.20}\right)^2 \times V_1 = 0.5625\, V_1$$

Now $$H = 0.04 \times \frac{300}{0.15} \times \frac{V_1^2}{2g} + 0.04 \times \left(\frac{150}{0.10}\right) \times \left(\frac{V_2^2}{2g}\right) + 0.04 \times \frac{240}{0.20} \times \frac{V_3^2}{2g}$$

or $$500 - 400 = 0.04 \times \frac{300}{0.15} \times \frac{V_1^2}{2g} + 0.04 \times \left(\frac{150}{0.10}\right) \times \left(\frac{2.25\, V_1}{2g}\right)^2$$

$$+ 0.04 \times \frac{240}{0.20} \times \left(\frac{0.5625}{2g}\right)^2 V_1^2$$

or $$100 = \frac{0.04\, V_1^2}{2g}\,[2000 + 7593.75 + 379.69]$$

$$V_1 = 2.22 \text{ m/s}$$

$$Q = \frac{\pi}{4} \times (0.15)^2 \times 2.22 = 0.0392 \text{ m}^3/\text{s}$$

(b) Let us express h_f in terms of Q.

$$h_f = f\,.\,\frac{L}{D}\,.\,\frac{V^2}{2g} = f\,.\,\frac{L}{D}\,\frac{Q^2}{\left(\frac{\pi}{4} D^2\right)^2} \times \frac{1}{2g}$$

$$= \frac{16\, fL}{\pi^2 \times 2g}\,.\,\frac{Q^2}{D^5}$$

or $$h_f = k\,\frac{Q^2}{D^5} \text{ where } k = \frac{16\, fL}{\pi^2 \times 2g}$$

When the two pipes are connected in parallel,

$$H_1 = k\,\frac{Q_1^2}{D^5} = k\,\frac{Q_2^2}{(2D)^5}$$

or $$Q_2^2 = Q_1^2 \left(\frac{2D}{D}\right)^5$$

or $$Q_2 = 5.657\, Q_1$$

But $Q_1 + Q_2 =$ Total discharge Q

Therefore, $Q_1 + 5.657\, Q_1 = Q$

or $Q_1 = 0.15\, Q$

$$H_1 = k\,\frac{Q_1^2}{D^5} = k \times \left(\frac{(0.15\,Q)^2}{D^5}\right) = 0.0225\, k\,\frac{Q^2}{D^5} \qquad \text{. . . (a)}$$

When the two pipes are connected in series,

$$H_2 = k\,\frac{Q_1^2}{D^5} + k\,\frac{Q_2^2}{(2D)^5}$$

But $Q_1 = Q_2 = Q$

Therefore, $$H_2 = \frac{k\,Q^2}{D^5}\left(1 + \frac{1}{32}\right) = \frac{33}{32}\,k\,\frac{Q^2}{D^5} \qquad \text{. . . (b)}$$

From Eqs. (a) and (b),

$$\frac{H_2}{H_1} = \frac{33}{32 \times 0.0225} = 45.83$$

(c) Let D be the diameter of the single main and d be the diameter of each smaller pipe in parallel.

$$h_f = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g}$$

or $$V = \sqrt{\frac{2g\,h_f\,D}{fL}}$$

$$Q = \frac{\pi}{4}\,D^2 V = \frac{\pi}{4}\,D^2 \sqrt{\frac{2g\,h_f\,D}{fL}} \qquad \text{. . . (a)}$$

If q is the discharge in each parallel pipe,

$$q = \frac{Q}{2}$$

Also $$q = \frac{\pi}{4}\,d^2 \sqrt{\frac{2g\,h_f\,d}{fL}}$$

or $$Q = \frac{\pi}{4}\,d^2 \sqrt{\frac{2g\,h_f\,d}{fL}} \times 2 \qquad \text{. . . (b)}$$

From Eqs. (a) and (b),

$$2 \times \frac{\pi}{4}\,d^2 \sqrt{\frac{2g\,h_f\,d}{fL}} = \frac{\pi}{4}\,D^2 \sqrt{\frac{2g\,h_f\,D}{fL}}$$

or $$d^{5/2} = 0.50 \times (D)^{5/2}$$

$$d = (0.50)^{2/5} \times D$$

$$= 0.758 \times 500$$

$$= 379 \text{ mm} = 380 \text{ mm (say)}$$

SAQ 3

Velocity in pipe 1,

$$V_1 = \frac{0.10}{\left(\frac{\pi}{4}\right) \times (0.30)^2} = 1.415 \text{ m/s}$$

Loss of head in pipe 1,

$$h_{f1} = 0.04 \times \frac{1000}{0.30} \times \left(\frac{V_1^2}{2g}\right)$$

$$= 0.04 \times \frac{1000}{0.30} \times \frac{(1.415)^2}{19.62} = 13.6 \text{ m}$$

Piezometric head at $J = 50 - 13.6 = 36.4$ m

Therefore, flow in pipe 2 is from J to B.

$$h_{f_2} = 0.04 \times \left(\frac{1000}{0.30}\right) \times \left(\frac{V_2^2}{2g}\right)$$

or $$36.4 - 30 = 0.04 \times \left(\frac{1000}{0.30}\right) \times \frac{V_2^2}{19.62}$$

or $$V_2 = 0.942 \text{ m/s}$$

$$Q_2 = \frac{\pi}{4}(0.30)^2 \times 0.942 = 0.066 \text{ m}^3/\text{s}$$

$$Q_3 = 0.10 - 0.066 = 0.034 \text{ m}^3/\text{s}$$

$$V_3 = \frac{0.034}{\frac{\pi}{4}(0.3)^2} = 0.48 \text{ m/s}$$

$$h_{f_3} = 0.04 \times \frac{1000}{0.30} \times \left(\frac{(0.48)^2}{19.62}\right) = 1.56 \text{ m}$$

Level of water level in $C = 36.40 - 1.56 = 34.84$ m

SAQ 4

$$h_f = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g}$$

or $$30 = 0.03 \times \frac{1200}{0.20} \times \frac{V^2}{2g}$$

$$V = 1.81 \text{ m/s}$$

Applying Bernoulli's theorem to a point on the water surface of the upper reservoir and the pipe apex,

$$0 = \frac{p}{\gamma} + x + \frac{V^2}{2g} + h_f$$

where p is the pressure at the pipe apex and x is the height of apex above the water surface in the upper reservoir.

Substituting the values,

$$0 = -7.80 + x + \frac{(1.81)^2}{19.62} + 0.03 \times \frac{300}{0.20} \times \frac{(1.81)^2}{19.62}$$

$$-x = -7.80 + 0.17 + 7.51$$

or $x = 0.12$ m

Depth of the pipe apex below the ridge

$$= 2.50 - 0.12 = 2.38 \text{ m.}$$