UNIT 3 FLOW THROUGH ORIFICES

Structure

3.1 INTRODUCTION

An orifice is an opening with a closed perimeter made in the wall or the base of a vessel through which the fluid flows. Orifices are used for the measurement of flow. The top edge of the orifice is always below the free surface. If the free surface is below the top edge of the opening, the orifice becomes a weir.

Orifices are of various shapes; commonly used shapes are circular, square, triangular and rectangular.

The upstream edge of the orifice may be rounded or sharp. The orifice with a sharp upstream level edge causes a line contact between the fluid and the edge. The rounded orifice has a surface contact. The orifice with a sharp upstream edge is commonly used, which is also known as a **standard orifice.**

An orifice is termed small when its dimensions are small compared to the head causing flow. The orifice is termed large if its dimensions are comparable with the head causing flow. In a small orifice, the velocity does not vary appreciably from the top to the bottom edge of the orifice and is assumed to be uniform. In large orifices, the variation in the velocity from the top to the bottom edge is considerable, and is, therefore, accounted for.

An orifice is said to be discharging free when it discharges into atmosphere. It is said to be submerged when it discharges into another liquid. The jet of the fluid coming out of the submerged orifice is not falling freely, as it is buoyed up by the surrounding liquid.

Objectives

After studying this unit, you should be able to

- conceptualise the phenomena of discharge through a sharp-edged circular orifice,
- explain the use and applications of different types of orifice, and
- handle problems pertaining to time of emptying of tanks through an orifice.

3.2 DISCHARGE THROUGH A SHARP-EDGED CIRCULAR ORIFICE

Let us consider a small circular orifice with sharp edges in the side wall of a tank (Figure 3.1). Let the centre of the orifice be at a depth *H* below the free surface. Let us assume that the orifice is discharging free into atmosphere. As the fluid flows through the orifice, it contracts and attains a parallel form (i.e. streamlines become parallel) at a distance of about *d/*2 from the plane of orifice. This is due to the fact that the fluid particles in the tank cannot change their directions abruptly. The section at which the streamlines first become parallel is termed the *venacontracta*. The area of cross-section is minimum at the venacontracta.

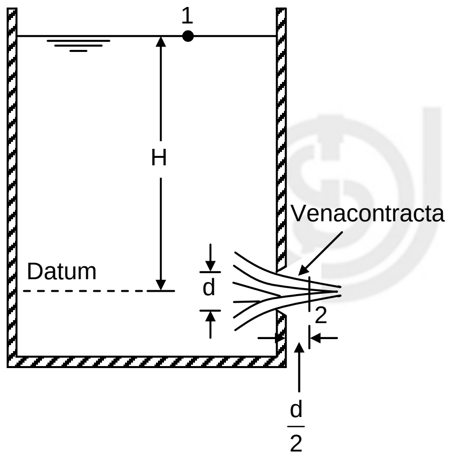

Figure 3.1

The cross-sectional area of the jet at the venacontracta is less than the area of cross-section of the orifice. The fluid particles before they reach the orifice have the velocity components parallel to the plane of the orifice and because of inertia, they cannot make abrupt changes in the direction, and they take curvilinear paths. This causes the contraction of the jet at the venacontracta. The ratio of the cross-sectional area of the jet at the venacontracta (a_c) to the cross-sectional area of the orifice (a) is called the *coefficient of contraction* and is usually designated by C_c.

Thus $$C_c = \frac{a_c}{a}$$

Taking datum through the axis of the orifice and applying Bernoulli's equation to points 1 and 2,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + H = \frac{p_2}{\gamma} + \frac{V_2^2}{2g}$$

(**Note :** In the problems of orifices, it is convenient to work in terms of gauge pressures, i.e. the pressures are measured above the atmospheric pressure.)

As the pressure at points 1 and 2 is atmospheric, $p_1 = p_2 = 0$. Further, if the cross-sectional area of the tank is very large, the liquid at point 1 is practically standstill and hence $V_1 = 0$.

Thus $$\frac{V_2^2}{2g} = H$$

or $$V_2 = \sqrt{2gH} \qquad \text{. . . (a)}$$

Eq. (a) is known as *Torricelli's theorem.* This equation gives the theoretical (or ideal) velocity of efflux at the venacontracta. It may be noted that this velocity is also equal to the velocity of a particle falling under gravity through a height H. Owing to friction between the jet and the walls of the orifice, the actual velocity (V) is slightly less than the theoretical, and is given by

$$V = C_v \sqrt{2gH} \qquad \text{. . . (b)}$$

in which C_v is a coefficient called the coefficient of velocity. The coefficient of velocity is defined as the ratio of the actual velocity to the theoretical velocity.

Thus $$C_v = \frac{V}{\sqrt{2gH}}$$

At the venacontracta, the streamlines are straight and parallel and are perpendicular to the cross-section of the jet. The discharge (Q) may be obtained from the continuity equation

$$Q = a_c V = (C_c\ a)\ C_v \sqrt{2gH}$$

$$Q = C_d\ a \sqrt{2gH} \qquad \text{. . . (3.1)}$$

in which C_d is a coefficient called the coefficient of discharge. The coefficient of discharge is equal to the product of the coefficient of velocity and the coefficient of contraction.

Thus $$C_d = C_c \times C_v \qquad \text{. . . (3.2)}$$

Theoretical discharge through the orifice is given by

$$Q \text{ (theoretical)} = a \sqrt{2gH} \qquad \text{. . . (c)}$$

From Eqs. (3.1) and (c), it would be evident that the coefficient of discharge is also equal to the ratio of the actual discharge to the theoretical discharge.

For a sharp-edged orifice, the value of C_c varies from 0.61 to 0.69. Its theoretical value, obtained from classical hydrodynamics, is given by

$$C_c = \frac{\pi}{\pi + 2} = 0.611$$

For very small orifices under low heads, the effects of surface tension and capillary action increases the coefficient of contraction. On the other hand, the coefficient of contraction decreases with an increase in the diameter and the head.

For a sharp-edged orifice, the value of the coefficient of velocity ranges from 0.95 to 0.99; the smaller values are for small orifices under low heads.

The value of the coefficient of discharge (C_d) for a sharp-edged orifice ranges from 0.59 to 0.68. Its value depends upon the coefficients of velocity and contraction. It also depends on the nominal Reynolds number (N_R) and is sometimes determined by the relation

$$C_d = 0.592 + \frac{4.50}{\sqrt{N_R}}$$

where, $$N_R = \frac{Vd}{\nu} = \frac{\sqrt{2gH} \times \text{Diameter of the orifice}}{\text{Kinematic viscosity of fluid}}$$

Example 3.1

A sharp-edged orifice of 5 cm diameter discharges water under a head of 5 m. Find the values of the coefficients of velocity, contraction and discharge if the measured rate of flow is 0.012 cumecs. The diameter of the jet at the venacontracta is 4 cm.

Solution

From Eq. (3.1),

$$Q = C_d\, a \sqrt{2gH}$$

$$0.012 = C_d \times \left(\frac{\pi}{4}\right) \times (0.05)^2 \times \sqrt{2 \times 9.81 \times 5}$$

$$C_d = 0.617$$

Now, $$C_c = \frac{a_c}{a} = \frac{\frac{\pi}{4} \times 4^2}{\frac{\pi}{4} \times 5^2} = 0.64$$

From Eq. (3.2),

$$C_v = \frac{C_d}{C_c} = \frac{0.617}{0.64} = 0.964$$

SAQ 1

A sharp-edged orifice of 4 cm diameter discharges water under a head of 3 m. Determine the discharge if the coefficients C_v = 0.98 and C_c = 0.63.

3.3 EXPERIMENTAL DETERMINATION OF THE COEFFICIENTS OF CONTRACTION, VELOCITY AND DISCHARGE

Coefficient of Contraction

The coefficient of contraction may be determined experimentally by measuring the area of the jet at the venacontracta by an instrument called the micrometer contraction gauge (Figure 3.2). The instrument consists of a small ring having four radial screws which are equally spaced along the periphery. The ring is held at the venacontracta and adjusted so that the jet just passes through its centre. Then the screws are adjusted such that their points just touch the periphery of the jet. The instrument is then removed and the space between the screw points is measured by means of a micrometer screw gauge. The coefficient of contraction is calculated from the measured diameter (d_c) of the cross-section of the jet and the diameter (d) of the orifice. Thus

$$C_c = \frac{a_c}{a} = \frac{d_c^2}{d^2}$$

The above method for the determination of coefficient of contraction is not very satisfactory, as the section of the jet is not absolutely regular. Moreover, it is difficult to adjust the screws accurately. A better method of finding the coefficient of contraction is to find the values of C_d and C_v experimentally and then calculate C_c from Eq. (3.2),

$$C_c = \frac{C_d}{C_v}$$

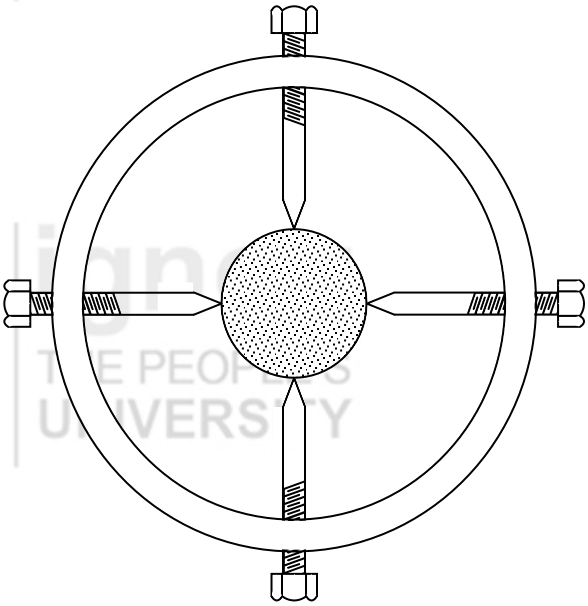

Figure 3.2

Coefficient of Velocity

There are three methods as described below for the determination of the coefficient of velocity which are commonly used.

(a) By measurement of coordinates (Trajectory Method).

(b) By momentum method.

(c) By pitot tube

By Measurement of Coordinates (*Trajectory Method*)

The coefficient of velocity for a vertical orifice may be determined experimentally by measuring the horizontal and vertical coordinates of the jet as it falls under gravity (Figure 3.3).

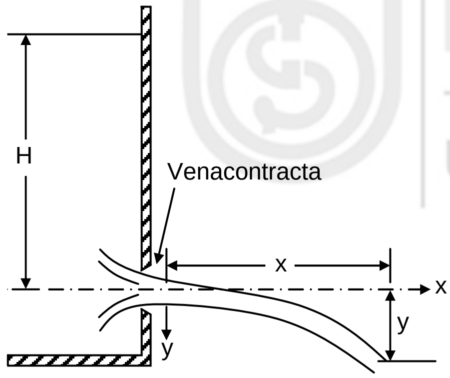

Figure 3.3

As mentioned before, the jet contracts and forms the venacontracta at a distance of *d/2* from the plane of the orifice. Beyond the venacontracta, the jet falls under gravity. This case is similar to motion of a particle with initial horizontal velocity and falling under gravity.

Let V be the velocity of the jet at the venacontracta. This velocity is horizontal, as the streamlines are parallel. Let us take the origin at the venacontracta and let x and y be the coordinates of a point on the jet after time 't'. Now applying the equation of motion,

$$x = V\,t \qquad \text{. . . (a)}$$

and

$$y = 0 + \frac{1}{2}\,g t^2 \qquad \text{. . . (b)}$$

Eliminating 't' between Eqs. (a) and (b),

$$x = \sqrt{\frac{2y\,V^2}{g}}$$

But $V = C_v\,\sqrt{2gH}$ (See Section 3.2)

Therefore,

$$x = \sqrt{\frac{2y\,C_v^2\,(2gH)}{g}} = C_v\,\sqrt{4yH}$$

or

$$C_v = \sqrt{\frac{x^2}{4yH}} \qquad \text{. . . (3.3)}$$

By measuring the coordinates x and y, and the head H, the coefficient of velocity (C_v) may be obtained from Eq. (3.3).

Momentum Method

A triangular beam is passed through the tank and is supported on the knife-edges on either side of the tank. A lever arm carrying a load P at its one end is attached to the triangular beam (Figure 3.4).

The jet issuing from the orifice has a rate of change of momentum in horizontal direction given by $(W/g)\,V$, where W is the weight of liquid issuing from the orifice per second, and V is the velocity of the jet.

According to the impulse-momentum equation, the force is equal to the rate of change of momentum. Therefore,

$$F = \left(\frac{W}{g}\right) V$$

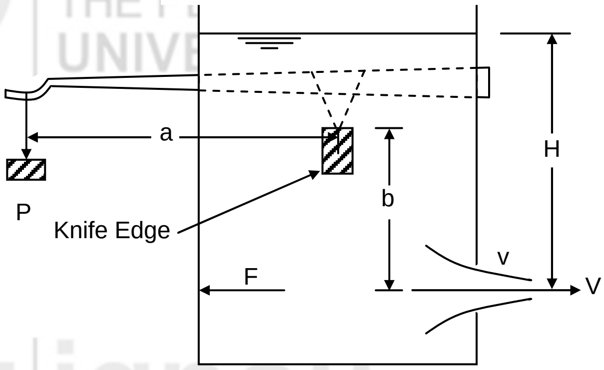

Figure 3.4

The reaction will act backward and try to tilt the tank as soon as the liquid starts discharging through the orifice. The weight *P* will exert a balancing moment and the tank will remain in the vertical position. For equilibrium, the tilting moment of *F* about the knife edge will just be equal to the balancing moment of the weight *P*.

Thus $$F \times b = P \times a$$

or $$\frac{W}{g} \times V \times b = P \times a$$

or $$V = \frac{P}{W} \times g \times \frac{a}{b} \qquad \ldots \text{(a)}$$

But velocity, $V = C_v \sqrt{2gH}$ (See Section 3.2)

Therefore, $$C_v = \frac{\frac{P}{W} \times g \times \frac{a}{b}}{\sqrt{2gH}} \qquad \ldots (3.4)$$

By Pitot Tube

The coefficient of velocity (C_v) may also be determined by measuring the actual velocity (V) at the venacontracta by means of a pitot tube.

In this case $$C_v = \frac{V}{\sqrt{2gH}}$$

However, this method is not convenient because of practical difficulties in the measurement of actual velocity.

Coefficient of Discharge

The coefficient of discharge of an orifice is determined by measuring the volume of liquid which has actually come out of the orifice in a known interval of time. The actual discharge *Q* is equal to the measured volume of the liquid divided by time. The coefficient of discharge C_d is obtained from the relation :

$$C_d = \frac{Q}{a\sqrt{2gH}} \qquad \ldots (3.5)$$

where *Q* is the actual measured discharge.

Example 3.2

A circular orifice, 3.5 cm diameter, is made in the vertical wall of a tank. The jet falls vertically through 0.5 m while moving horizontally through a distance of 1.5 m. Calculate the coefficient of velocity if the head causing flow is 1.2 metres. If the discharge is 2.80×10^{-3} cumecs, calculate C_c and C_d.

Solution

From Eq. (3.3),

$$C_v = \sqrt{\frac{x^2}{4yH}} = \sqrt{\frac{1.5^2}{4 \times 0.5 \times 1.20}} = 0.97$$

From Eq. (3.5), $$C_d = \frac{Q}{a\sqrt{2gH}}$$

$$= \frac{2.80 \times 10^{-3}}{\left(\frac{\pi}{4}\right) \times (0.035)^2 \sqrt{2 \times 9.81 \times 1.20}} = 0.60$$

From Eq. (3.2), $$C_c = \frac{C_d}{C_v} = \frac{0.60}{0.97} = 0.62$$

Example 3.3

A circular sharp-edged orifice, 6 cm^2 in area, is made in the vertical side of a tank, which is suspended, from knife-edges 1.65 m above the level of the orifice. If, when the head of water is 1.215 m, the discharge is 1079.1 N/m, while the weight of 48.17 N at a leverage of 30 cm is required to keep the tank vertical, determine the coefficients of velocity, contraction, and discharge of the jet.

Solution

From Eq. (3.4),

$$C_v = \frac{\frac{P}{W} \times g \times \frac{a}{b}}{\sqrt{2gH}}$$

Substituting the values,

$$C_v = \frac{60 \times \left(\frac{48.17}{1079.1}\right) \times 9.81 \times \left(\frac{0.30}{1.65}\right)}{\sqrt{2 \times 9.81 \times 1.215}} = 0.978$$

$$\text{Actual discharge} = \frac{1079.1 \times 1}{9810 \times 60} = 1.8333 \times 10^{-3} \text{ cumecs}$$

From Eq. (3.5), $$C_d = \frac{Q}{a\sqrt{2gH}} = \frac{1.833 \times 10^{-3} \times 10^4}{6 \times \sqrt{2 \times 9.81 \times 1.215}} = 0.626$$

From Eq. (3.2), $$C_c = \frac{C_d}{C_v} = \frac{0.626}{0.978} = 0.64$$

SAQ 2

(a) A sharp-edged circular orifice of 4 cm diameter projects a jet horizontally under a head of 2 m. If the jet shrikes at a point 1.36 m horizontally and 0.24 m vertically from the venacontracta, calculate the coefficient of velocity C_v.

(b) If the diameter of the jet at the venacontracta is 3.2 cm, calculate the coefficients of contraction and discharge.

3.4 COEFFICIENT OF RESISTANCE

The walls of the orifice cause resistance to the fluid as it comes out. It is because of this resistance that the actual velocity is less than the theoretical value. The loss of head may be obtained from the head causing flow and the velocity head at the exit.

Since $$C_v = \frac{V}{\sqrt{2gH}}$$

or $$H = \frac{1}{(C_v)^2}\left(\frac{V^2}{2g}\right) \qquad \text{. . . (a)}$$

Since the velocity of the jet at the venacontracta is V, the velocity head at that point is $V^2/2g$.

Therefore, loss of head, $$h_L = H - \frac{V^2}{2g} = \frac{1}{(C_v)^2}\left(\frac{V^2}{2g}\right) - \left(\frac{V^2}{2g}\right)$$

or $$h_L = \frac{V^2}{2g}\left[\frac{1}{(C_v)^2} - 1\right] \qquad \text{. . . (3.6a)}$$

This loss of head can also be written in terms of H

$$h_L = H - H\,(C_v)^2$$

or $$h_L = H\,[1 - C_v{}^2] \qquad \text{. . . (3.6b)}$$

Loss of power, $$P = \gamma\, Q h_L$$

$$= \gamma\, QH\,(1 - C_v^2)\ \text{kW} \qquad \text{. . . (3.6c)}$$

where γ is in kN/m^3, Q in m^3/s and H in metres.

The coefficient of resistance (C_r) is defined as the ratio of the loss of head in the orifice to the head available at the exit of the orifice. Thus

$$C_r = \frac{h_L}{\left(\dfrac{V^2}{2g}\right)}$$

or $$C_r = \frac{\left(\dfrac{V^2}{2g}\right)\left[\dfrac{1}{(C_v)^2} - 1\right]}{\dfrac{V^2}{2g}}$$

Also $$C_r = \frac{1 - C_v^2}{C_v^2} = \left(\frac{1}{C_v^2} - 1\right) \quad \ldots (3.7)$$

3.5 SUBMERGED ORIFICE

A submerged orifice discharges into another liquid instead of discharging into atmosphere. Figure 3.5 shows a submerged orifice. The orifice discharges into another tank containing a liquid with its free surface above the orifice. For the orifice to act as a submerged orifice, the free surface of the liquid on the downstream must be above the orifice; otherwise the orifice will act as a free or a partially submerged orifice.

Taking datum through the axis of the orifice and applying Bernoulli's equation to points 1 and 2,

$$0 + 0 + H_1 = \frac{V_2^{\,2}}{2g} + H_2 + 0$$

or $$\frac{V_2^2}{2g} = H_1 - H_2 = H$$

where H is the *difference* of levels on the two sides.

[It may be noted that the pressure head (gauge pressure) at point 2 is H_2 and not zero.]

Thus $$V_2 = \sqrt{2gH}$$

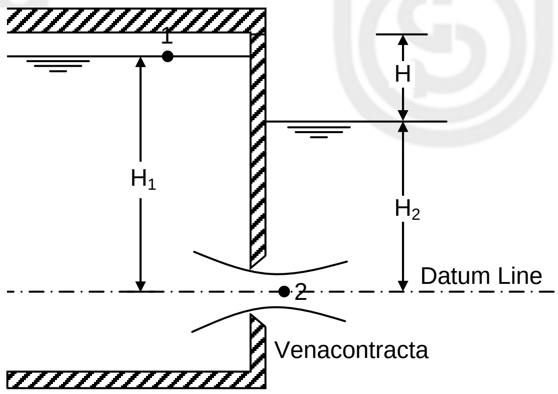

Figure 3.5

The velocity through a submerged orifice depends upon the difference of liquid levels on the two sides of the orifice, irrespective of the position of the orifice. This equation is identical to the equation for the ideal velocity through a free orifice. The only difference is that in a submerged orifice, the head H is the difference of liquid levels on two sides, whereas it is the head on the orifice in the case of a free orifice.

The actual velocity will be slightly less than the theoretical value because of frictional resistance at the walls of the orifice.

Therefore, $$V = C_v \sqrt{2gH}$$

where C_v is the coefficient of velocity.

The area of the jet as the venacontracta (a_c) is given by,

$$a_c = C_c\ a$$

Thus, discharge

$$Q = a_c\ V = C_c\ C_v\ a\ \sqrt{2gH}$$

or $$Q = C_d\ a\ \sqrt{2gH} \qquad \text{(same as Eq. (3.1));}$$

where C_d is the coefficient of discharge.

The coefficient of discharge C_d for a submerged orifice is slightly less than that for a similar orifice discharging free. This is due to the interference of the liquid on the downstream side. It may be remembered that H is the difference of levels on the two sides.

Example 3.4

A 2.5 cm diameter orifice connects two tanks. Water levels on the two sides are 2 m and 1 m above the axis of the orifice. Calculate the discharge. $C_d = 0.60$

Solution

From Eq. (3.1) $Q = C_d\ a\ \sqrt{2gH}$

$$= 0.60 \times \left(\frac{\pi}{4}\right)(0.025)^2\ \sqrt{2 \times 9.81 \times (2-1)}$$

$$= 0.0013 \text{ cumecs.}$$

3.6 VELOCITY OF APPROACH

So far the discussions were limited to the case when the velocity of the liquid in the tank was either zero or negligibly small. When the velocity in the tank is considerable, it must be taken into account for more accurate results. Let us consider a small orifice discharging free from the walls of a small vessel. As the surface area of the vessel is small, the velocity at the free surface is not negligible. Let the velocity of the liquid at point 1 in the vessel be V_1. The head due to this velocity is $V_1^2/2g$. This head is known as the head due to the velocity of approach and is represented by H_a (Figure 3.6).

Applying Bernoulli's equation to points 1 and 2, with datum at the axis of the orifice,

$$\frac{V_1^2}{2g} + H + 0 = \frac{V_2^2}{2g} + 0 + 0 \qquad \text{. . . (a)}$$

or, $$H_a + H = \frac{V_2^2}{2g}$$

or, $$V_2 = \sqrt{2g\ (H + H_a)}$$

Thus the effect of the velocity of approach is to increase the effective head and hence to increase the discharge.

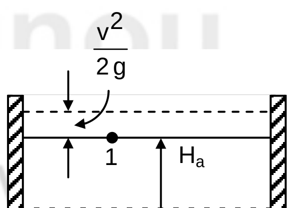

Figure 3.6

Let A_1 be the surface area of the tank, and a_c be the cross-sectional area at the venacontracta. For the continuity of flow,

$$A_1 V_1 = a_c V_2 \text{ or } V_1 = \left(\frac{a_c}{A_1}\right) V_2$$

Substituting this value of V_1 in Eq. (a),

$$\left(\frac{a_c}{A_1} V_2\right)^2 \frac{1}{2g} + H = \frac{V_2^2}{2g}$$

or
$$\frac{V_2^2}{2g}\left[1 - \left(\frac{a_c}{A_1}\right)^2\right] = H$$

or
$$V_2 = \sqrt{2gH}\sqrt{\frac{A_1^2}{A_1^2 - a_c^2}} \qquad \ldots (3.8)$$

where V_2 is the theoretical velocity at the venacontracta. This expression indicates that the velocity is increased in the ratio $\sqrt{\frac{A_1^2}{A_1^2 - a_c^2}} = \frac{A_1}{\sqrt{A_1^2 - a_c^2}}$ due to velocity of approach.

Evidently, the discharge is also increased in the same ratio. It may be noted that if the surface area A_1 of the tank is very large compared with a_c, the above ratio becomes approximately unity and the effect of the velocity of approach may be neglected. *Unless otherwise mentioned, the velocity of approach will be neglected.*

3.7 LARGE VERTICAL RECTANGULAR ORIFICE

When the dimensions of the orifice, as compared to the head causing flow, are large, the velocity through the orifice can no longer be regarded as uniform since the effective head on various laminae in the orifice is different. Therefore, the variation of head in the orifice must be taken into account.

Figure 3.7 shows a large rectangular orifice of width b in the sidewall of a tank. The side view of the orifice is also shown. Let the height of the liquid above the top edge and the bottom edge be respectively H_1 and H_2. Assuming that all points at the same depth have the same velocity, the expression for discharge can be obtained. Let us consider a horizontal strip of the orifice of thickness dH at a depth H below the free surface.

Area of flow of the strip at the venacontracta = $C_c\ b\ dH$.

where C_c is the coefficient of contraction.

Since the area of strip is small, the small orifice formula is applicable.

Thus, velocity of flow through the strip $= C_v \sqrt{2gH}$

Discharge through the strip, $dQ = (C_c\ b\ dH)\ (C_v \sqrt{2gH})$

or, $$dQ = C_d\ b \sqrt{2gH}\ dH$$

where C_d is the coefficient of discharge which is equal to the product of C_c and C_v.

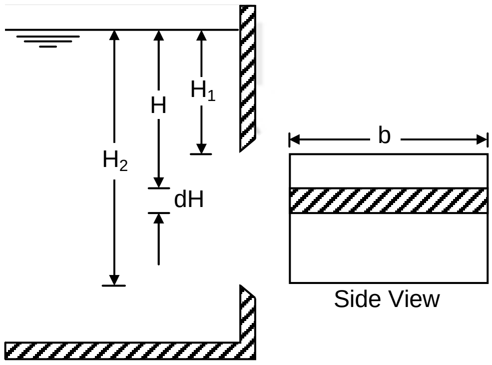

Figure 3.7 : Large Vertical Orifice

Total discharge through the orifice, $Q = \int dQ$

$$Q = \int_{H_1}^{H_2} C_d\, b \sqrt{2gH}\ dH$$

$$Q = C_d\ b \sqrt{2g} \left[\frac{2}{3} H^{3/2} \right]_{H_1}^{H_2}$$

$$Q = \frac{2}{3} C_d\ b \sqrt{2g}\ (H_2^{3/2} - H_1^{3/2}) \qquad \ldots (3.9)$$

Submerged Large Orifice

If a large orifice is completely submerged at the exit, discharge may be obtained, as in the case of a small submerged orifice by considering the difference of liquid levels on the two sides.

Thus, $$Q = C_d\ a \sqrt{2gH} \qquad \ldots (3.10)$$

where a = area of orifice, and

H = difference of liquid levels on the two sides.

Partially Submerged Large Orifice

Let us consider the case when the liquid surface on the exit side is below the top edge of the orifice but above the bottom edge (Figure 3.8). This liquid level divides the orifice in two portions. The lower portion of the orifice acts as a submerged orifice and the upper portion as an orifice discharging free. Total discharge may be obtained by adding the discharges in two pc

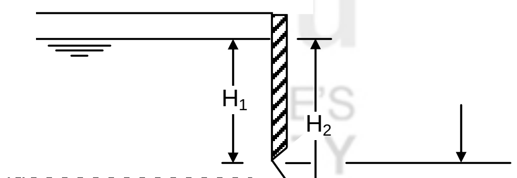

Figure 3.8 : Partially Submerged Orifice

From Eq. (3.9), discharge through the portion treated as free,

$$Q_1 = \frac{2}{3} C_{d_1}\ b \sqrt{2g}\ [H_2^{3/2} - H_1^{3/2}]$$

From Eq. (3.10), discharge through the submerged portion

$$Q_2 = C_{d_2}\ bd \sqrt{2gH_2}$$

where C_{d_1} is coefficient of discharge for free portion and C_{d_2}, for submerged portion.

Total discharge, $Q = Q_1 + Q_2 = \frac{2}{3} C_{d_1}\ b \sqrt{2g}\ [H_2^{3/2} - H_1^{3/2}] + C_{d_2}\ bd \sqrt{2gH_2}$

If $\quad C_{d_1} = C_{d_2} = C_d$,

$$Q = \frac{2}{3} C_d\ b \sqrt{2g}\ [H_2^{3/2} - H_1^{3/2}] + C_d\ bd \sqrt{2gH_2} \qquad \text{. . . (3.11)}$$

Example 3.5

A reservoir discharges water through a large orifice 1 m wide and 1.5 m deep. The top of the orifice is 0.80 m below the water level in the reservoir. Assuming that the downstream water level is below the bottom of the orifice, calculate

(a) The discharge through the orifice if $C_d = 0.60$, and

(b) The percentage error if the orifice is treated as small.

Solution

(a) In this case (refer Figure 3.7), $H_1 = 0.80$, $H_2 = 2.30$,

From Eq. (3.9),

$$Q = \frac{2}{3} C_d\ b \sqrt{2g}\ [H_2^{3/2} - H_1^{3/2}]$$

or $$Q = \frac{2}{3} \times 0.60 \times 1 \sqrt{2 \times 9.81}\ (2.30^{3/2} - 0.80^{3/2})$$

or $\quad Q = 4.91$ cumecs

(b) Considering the orifice as small, head at the centre of the orifice,

$$H = \frac{0.8 + 2.30}{2} = 1.55 \text{ m}$$

Therefore from Eq. (3.10),

$$Q = C_d\, a \sqrt{2gH}$$

$$Q = 0.60 \times (1.50 \times 1) \times \sqrt{2 \times 9.81 \times 1.55} = 4.96 \text{ m}^3/\text{s}$$

Percentage error, $= \dfrac{4.96 - 4.91}{4.91} \times 100 = 1.02\%$

It may be noted that the percentage error is not large even if the orifice is considered as small.

SAQ 3

Water is discharged from a tank through a rectangular orifice 1m wide and 2 m deep. The top edge of the orifice is 1.0 m below the water level in the tank. If the coefficient of discharge is 0.60, determine the discharge when the downstream water level is

(a) below the bottom edge of the orifice,

(b) 1 m above the bottom edge of the orifice, and

(c) 2.5 m above the bottom edge of the orifice.

3.8 BELL-MOUTHED ORIFICE

A bell-mouthed orifice is of the shape of a horizontal bell. In this type of orifice, the curvature of its walls conforms to the curvature of the streamlines of the jet coming out of the orifice (Figure 3.9).

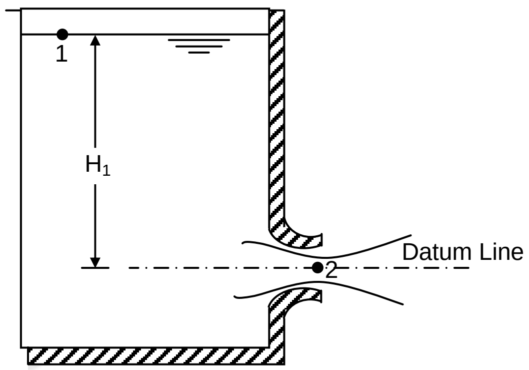

Figure 3.9 : Bell-Mouthed Orifice

Since no further contraction of the jet is possible, the coefficient of contraction of the orifice is unity. The coefficient of velocity, as well as the coefficient of discharge, ranges from 0.95 to 0.995, i.e. $C_v = C_d = 0.95$ to 0.995.

Because of practical difficulties in its construction, a bell-mouthed orifice is seldom used in practice.

3.9 TIME OF EMPTYING A TANK THROUGH AN ORIFICE

The discussions so far were limited to the case when the head causing flow was constant. Occasionally, we come across problems in which the head causing flow does not remain constant. In this case, the orifice discharges under a varying head.

(**Note :** Strictly speaking, the flow is unsteady. However, if the head changes gradually, the flow is termed *quasi-steady*, and the equations of steady flow are applied to obtain approximate results.)

Let us consider the vessel of an arbitrary shape with an orifice at the base as shown in Figure 3.10. At any instant when the head over the orifice is H, let A be the surface area of the liquid. If 'a' is the cross-sectional area of the orifice, discharge through the orifice at that instant is given by

$$Q = C_d\, a\, \sqrt{2gH}$$

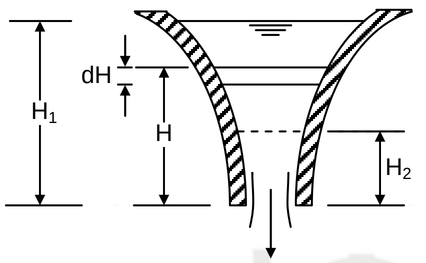

Figure 3.10

In a small interval of time dt, the volume of liquid which flows out of the orifice is given by

$$\text{Volume} = Q\, dt = (C_d\ a\ \sqrt{2gH})\ dt \qquad \text{. . . (a)}$$

If the head over the orifice falls by a height dH during this time, then

$$\text{Volume of liquid withdrawn} = -A\, dH \qquad \text{. . . (b)}$$

The minus sign has been taken as H decreases with time. Equating these two volumes,

$$C_d\, a\, \sqrt{2gH}\ dt = -\, A dH$$

or $$dt = \frac{-\, A dH}{C_d\, a\, \sqrt{2gH}} \qquad \text{. . . (3.12)}$$

If the area of the vessel (A) can be expressed in terms of H, the time required to lower the liquid level from the head H_1 to H_2 can be determined by integrating Eq. (3.12). The integration is much simplified if the area of the vessel is constant.

Time required to lower the liquid from H_1 to H_2 if the area of the vessel is constant is given by

$$t = \frac{-\, A}{C_d\ a\ \sqrt{2g}} \int_{H_1}^{H_2} H^{-\frac{1}{2}}\, dH$$

or $$T = \frac{+\, 2A}{C_d\, a\, \sqrt{2g}} [H_1^{\frac{1}{2}} - H_2^{\frac{1}{2}}] \qquad \text{. . . (3.12a)}$$

For a rectangular vessel of the length L and width B, Eq. (3.12a) becomes

$$T = \frac{2BL}{C_d\, a\, \sqrt{2g}}\,[H_1^{\frac{1}{2}} - H_2^{\frac{1}{2}}] \quad \text{. . . (3.12b)}$$

3.10 TIME OF EMPTYING A CIRCULAR CYLINDRICAL HORIZONTAL TANK THROUGH AN ORIFICE

Figure 3.11 shows a cylindrical tank with its axis horizontal. In this case, the horizontal cross-sectional area A of the tank varies with head H over the orifice.

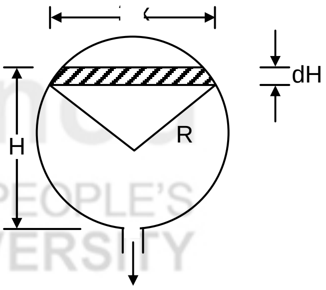

Figure 3.11

From geometry,

$$x^2 = R^2 - (H - R)^2 = R^2 - (H^2 - 2RH + R^2)$$

or $$x^2 = 2RH - H^2$$

where R is the radius of the tank.

The width of the liquid surface is equal to $2x$.

Therefore, surface area of liquid,

$A = 2xL$ where L is the length of the tank

or $$A = (2\sqrt{2RH - H^2})\,L \quad \text{. . . (a)}$$

From the continuity of flow,

$$-A\,dH = Q\,dt = \text{volume of liquid discharged}$$

where Q is the instantaneous discharge when the head causing flow is H and there is a drop of dH in time dt.

Substituting $Q = C_d\, a\, \sqrt{2gH}$ from Eq. (3.10) and the value of A from Eq. (a),

$$-(2\sqrt{2RH - H^2})\,L\,dH = (C_a\, a\, \sqrt{2gH})\,dt$$

or $$dt = -\left[\frac{\left(\sqrt{2RH - H^2}\right)2L}{C_d\, a\, \sqrt{2gH}}\right] dH$$

or $$dt = \frac{-2L}{C_d\, a\, \sqrt{2g}}\,(\sqrt{2R - H})\,dH$$

The time of emptying when the liquid level falls from H_1 to H_2 is given by

$$t = \frac{4L}{3C_d\, a\, \sqrt{2g}}\,[(2R - H_2)^{3/2} - (2R - H_1)^{3/2}] \quad \text{. . . (3.13)}$$

If the tank is initially full of liquid, $H_1 = 2R$ and if the tank is to be completely emptied, $H_2 = 0$.

Therefore, $$t = \frac{4L}{3C_d\, a\, \sqrt{2g}} [(2R)^{3/2}] = \frac{8\sqrt{2}\, LR^{3/2}}{3C_d\, a\, \sqrt{2g}} \qquad \ldots (3.14)$$

SAQ 4

A cylindrical boiler of diameter 2 m and length 10 m is lying in the horizontal position with its axis horizontal. The boiler is half full of water and is to be emptied through an orifice at the bottom by an orifice of 44.2 cm^2 and coefficient of discharge of 0.62. How long will it take to empty the boiler?

3.11 TIME OF EMPTYING A HEMISPHERICAL TANK THROUGH AN ORIFICE

Figure 3.12 shows a hemispherical tank with an orifice at its bottom. Let the radius of the tank be R. At any instant, let H be the head over the orifice.

From geometry,

$$x^2 = R^2 - (R - H)^2$$

or $$x^2 = 2RH - H^2$$

Surface area of the vessel at that instant $A = \pi\, x^2$

or $$A = \pi\, [2RH - H^2]$$

From Eq. (3.12),

$$dt = \frac{-A\, dH}{C_d\, a\, \sqrt{2gH}}$$

$$= \frac{-\pi\, [2RH - H^2]}{C_d\, a\, \sqrt{2g}}\, H^{-1/2}\, dH$$

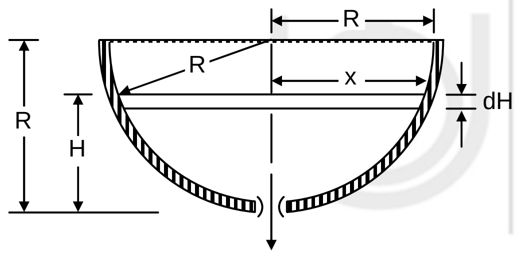

Figure 3.12

Time required for the water level to drop from H_1 to H_2,

$$t = \int dt = \frac{-\pi}{C_d\, a\, \sqrt{2g}} \int_{H_1}^{H_2} (2RH^{1/2} - H^{3/2})\, dH$$

$$= \frac{-\pi}{C_d\, a\sqrt{2g}} \left[\frac{4}{3} R H^{3/2} - \frac{2}{5} H^{5/2}\right]_{H_1}^{H_2}$$

$$= \frac{\pi}{C_d\, a\sqrt{2g}} \left[\frac{4}{3} R\,(H_1^{3/2} - H_2^{3/2}) - \frac{2}{5}\,(H_1^{5/2} - H_2^{5/2})\right] \qquad \ldots (3.15)$$

If the vessel is full at the beginning, $H_1 = R$; and if it is to be completely emptied, $H_2 = 0$. Substituting these values in above equation,

$$t = \frac{\pi}{C_d\, a\sqrt{2g}} \left[\frac{4}{3} R\,(R)^{3/2} - \frac{2}{5} R^{5/2}\right] = \frac{14}{15}\,\frac{\pi}{C_d\, a\sqrt{2g}}\, R^{5/2} \qquad \ldots (3.16)$$

SAQ 5

Calculate the time required to empty a hemispherical vessel of radius 1 m full of water by an orifice at the bottom with a cross-sectional area of 50 cm^2 and coefficient of discharge of 0.60.

3.12 TIME OF FLOW FROM ONE TANK TO ANOTHER TANK

Figure 3.13 shows two tanks connected by an orifice. Let H_1 be the initial difference of head and H_2 be the final difference of head. At any instant, let the difference of head on the two sides of the orifice be H. In a small interval of time dt, let us assume that the level of the liquid in the vessel 1 drops by x. The level of the liquid in vessel 2 rises by y.

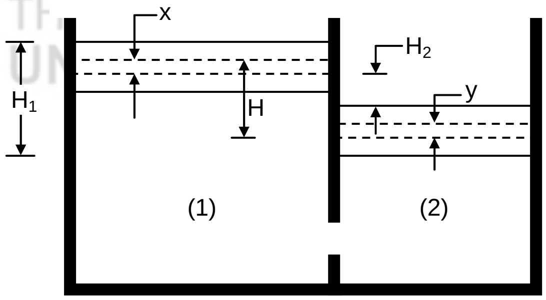

Figure 3.13

From continuity, $A_1\, x = A_2\, y$ or $y = \left(\frac{A_1}{A_2}\right) x$

where A_1 is the area of the tank with higher liquid level and A_2 is the surface area of the tank with lower liquid level.

Net difference of head after time dt

$$= H - x - y = H - x - \frac{A_1}{A_2}\, x = H - x\left(1 + \frac{A_1}{A_2}\right)$$

Therefore, change of difference of head,

$$dH = H - \left[H - x\left(\frac{A_1}{A_2}\right)\right] = x\left(1 + \frac{A_1}{A_2}\right) \qquad \ldots (a)$$

Equating the volume of liquid discharged from vessel 1 to vessel 2 to the volume of liquid discharged through the orifice,

$$-A_1\, x = (C_d\, a\, \sqrt{2gH})\, dt$$

or
$$dt = \frac{-A_1\, x}{C_d\, a\, \sqrt{2g}}\, H^{-1/2}$$

Substituting the value of x from Eq. (a)

$$dt = \frac{-A_1}{C_d\, a\, \sqrt{2g}} \left[\frac{dH}{1 + \frac{A_1}{A_2}} \right] H^{-1/2}$$

Time required to bring the difference of levels from H_1 to H_2,

$$t = \int dt = \frac{-A_1}{C_d\, a\, \sqrt{2g} \left(1 + \frac{A_1}{A_2}\right)} \int_{H_1}^{H_2} H^{-1/2}\, dH$$

Interchanging,

$$T = \frac{A_1}{C_d\, a\, \sqrt{2g} \left(1 + \frac{A_1}{A_2}\right)} \int_{H_2}^{H_1} H^{-1/2}\, dH$$

or
$$T = \frac{A_1}{C_d\, a\, \sqrt{2g} \left(1 + \frac{A_1}{A_2}\right)} \left[\frac{2}{1} H^{1/2} \right]_{H_2}^{H_1}$$

$$= \frac{2A_1}{C_d\, a\, \sqrt{2g} \left(1 + \frac{A_1}{A_2}\right)} [H_1^{1/2} - H_2^{1/2}] \qquad \ldots (3.17)$$

If the surface areas of the vessels are not constant, the integration procedure becomes difficult. In such cases, the numerical integration methods may be used.

It must be noted that H_1 and H_2 represent the difference of heads, and not the heads. Also note that A_1 is the surface area of the vessel with higher liquid level.

Example 3.6

A tank 4 m long and 2 m wide is divided into two parts by a partition so that the area of one portion is twice the area of the other. The partition contains an orifice of 50 cm^2 area through which the water may flow from one part to the other. If the initial difference of level is 4 m, find the time required to reduce the difference to 1 m. C_d = 0.60. Water level in the larger portion is higher.

Solution

$$A_1 = \frac{2}{3}(4 \times 2) = \frac{16}{3}\ \text{m}^2; \quad A_2 = \frac{1}{3}(4 \times 2) = \frac{8}{3}\ \text{m}^2$$

where A_1 and A_2 are areas of the larger and smaller portions respectively.

Substituting the values in Eq. (3.17),

$$T = \frac{2A_1}{C_d\, a\, \sqrt{2g}\left(1 + \frac{A_1}{A_2}\right)} [H_1^{1/2} - H_2^{1/2}]$$

$$= \frac{2 \times \frac{16}{3}}{0.60 \times (50 \times 10^{-4}) \times 4.43 \times (1 + 2)} [4^{1/2} - 1^{1/2}]$$

$= 267.5$ seconds $= 4$ minutes 27.5 secs.

3.13 SUMMARY

- An orifice is a small opening in a vessel through which the liquid can flow. Different types of orifices are explained. In practice, generally a sharp-edged small orifice is used. It is also known as a standard orifice.
- The basic equation for discharge through a sharp-edged circular orifice is derived. The coefficients C_c, C_v, C_d and C_r are explained. The discharge coefficient (C_d) generally varies between 0.59 to 0.68 for most orifices.
- Experimental methods for the determination of C_c, C_v and C_d are discussed.
- In a submerged orifice, there is a liquid on the downstream of the orifice which affects discharge. The effective head is equal to the difference of liquid levels.
- The effect of the variation of head on a large orifice is discussed and the discharge equation of a rectangular large orifice is derived.
- The time of emptying a tank through an orifice can be determined considering the continuity of flow and the discharge equation.
- The equations for time of emptying a tank of a constant cross-sectional area, circular cylindrical horizontal tank, and hemispherical tank have been derived.
- The time of flow from one tank to another connected tank is explained.

3.14 ANSWERS TO SAQs

SAQ 1

Coefficient of discharge, $C_d = C_v \times C_c$

$= 0.98 \times 0.63 = 0.62$

Now $Q = C_d\, a\, \sqrt{2gH}$

$$= 0.62 \times \frac{\pi}{4} \times (0.04)^2 \times \sqrt{2 \times 9.81 \times 3} = 5.98 \times 10^{-3}\ \text{m}^3/\text{s}$$

SAQ 2

(a) $C_v = \sqrt{\frac{x^2}{4yH}} = \sqrt{\frac{(1.36)^2}{4 \times 0.24 \times 2}}$

or $C_v = 0.981$

(b) $C_c = \frac{a_c}{a} = \frac{(d_c)^2}{(d)^2} = \frac{(3.2)^2}{(4)^2} = 0.641$

$C_d = C_c \times C_v = 0.641 \times 0.981$

or $C_d = 0.629$

SAQ 3

(a) The orifice is discharging free.

$$Q = \frac{2}{3} C_d\, b \sqrt{2g}\ (H_2^{3/2} - H_1^{3/2})$$

$$= \frac{2}{3} \times 0.69 \times 1.0 \times \sqrt{2 \times 9.81}\ (3^{3/2} - 1^{3/2}) = 7.436 \text{ m}^2/\text{s}$$

(b) The orifice is partially submerged.

$$Q = \frac{2}{3} C_d\, b \sqrt{2g}\ (H_2^{3/2} - H_1^{3/2}) + C_d\, bd \sqrt{2g}\ H_2$$

$$= \frac{2}{3} \times 0.69 \times 1.0 \times \sqrt{2 \times 9.81}\ (2^{3/2} - 1^{3/2}) + 0.60 \times 1.0 \times 1.0 \sqrt{2 \times 9.81 \times 2}$$

$$= 3.240 + 3.759 = 6.999 \text{ m}^3/\text{s}$$

(c) The orifice is fully submerged

$$Q = C_d\, a \sqrt{2gH}$$

$$= 0.60 \times (1.0 \times 2.0) \sqrt{2 \times 9.81 \times 0.50} = 3.759 \text{ m}^3/\text{s}$$

SAQ 4

For a cylindrical tank, the time of emptying is given by

$$t = \frac{4L}{3C_d\, a \sqrt{2g}} [(2R - H_2)^{3/2} - (2R - H_1)^{3/2}]$$

$$= \frac{4 \times 10.0}{3 \times 0.62 \times 44.2 \times 10^{-4} \times \sqrt{2 \times 9.81}} [(2 - 0)^{3/2} - (2 - 1)^{3/2}]$$

$$= 1098.3\ (2.828 - 1) = 2008 \text{ s (33 min 28 s).}$$

SAQ 5

For a hemispherical tank, the time of emptying is given by

$$t = \frac{14\pi R^{5/2}}{15C_d\, a \sqrt{2g}}$$

$$= \frac{14 \times \pi \times (1)^{5/2}}{15 \times 0.60 \times 50 \times 10^{-4} \times \sqrt{2 \times 9.81}} = 220.6 \text{ s (3 min 40.6 s)}.$$