# **UNIT 2 SEQUENCES AND SERIES**

#### Structure

- 2.1 Introduction
  Objectives
- 2.2 Sequences
- 2.3 Arithmetic Progression
- 2.4 Geometric Progression
- 2.5 Summary
- 2.6 Answers to SAQs

#### 2.1 INTRODUCTION

In this unit, we shall be introducing the notion of sequences and series of various types. In particular we shall study arithmetic sequences (series) and geometric sequences (series).

#### **Objectives**

After studying this unit, you should be able to

- introduce the concept of sequences, and series,
- define Arithmetic Progression and Geometric Progression,
- compute the sum of a finite (or infinite) AP or GP,
- define Arithmetic Mean (or Means) and Geometric Mean (or Means) between two numbers, and
- insert and compute the AMs and GMs between two numbers.

## 2.2 SEQUENCES

#### **Definition 1**

Let *N* be the set of natural numbers and  $N_n$  be the set of first numbers, i.e.  $N_n = \{1, 2, 3, ..., n\}$  and *X* be a non-empty set.

Then a map  $f: N \to X$  is called a sequence. If X = R or a subset of R, R the set of real numbers then f is called a **real sequence** and if X = C or a subset of C, C the set of complex numbers then f is called a **complex sequence**.

If domain f = N, we call it an **infinite sequence**. And if domain  $f = N_n$  a subset of N, then it is called a **finite sequence**.

Thus, if f is a sequence, then for any  $k \in N_n$  or N, we have  $f(k) = a_k \in X$ .

Hence  $a_1, a_2, a_3, \ldots, a_n$  or  $a_1, a_2, a_3, \ldots, a_n, \ldots$  determine the sequence f.

These sequences are usually denoted by  $\{a_k\}_{k=1}^n$  and  $\{a_k\}_{k=1}^8$ .

The different elements in a sequence are called its terms which are usually denoted by  $T_1, T_2, T_3, \ldots, T_n$  or  $T_1, T_2, \ldots, T_n$ , .... accordingly as the sequence is finite or infinite.  $T_n$ , the  $n^{th}$  term is called the **general term**.

THE PEOPLE'S UNIVERSITY

THE PEOPLE'S UNIVERSITY









#### **Definition 2**

A sequence following some definite rule (or rules) is called a **progression**.

For example, consider the following sequences,

(i) 
$$3, 5, 7, 9, \ldots, 21$$

(ii) 
$$8, 5, 2, -1, -4, \ldots$$

We observe that each term (except first) in (i) is formed by adding 2 to the preceding term; each term in (ii) is formed by subtracting 3 from the preceding term; each term in (iii) is formed by multiplying the preceding term by 3, each term in (iv) is formed by squaring the next natural number. Thus all are sequences, (i) is a finite sequence whereas others are infinite sequences.

If the terms of a sequence are connected by plus (or minus) signs, a series is formed for example

$$3 + 5 + 7 + \ldots + 21$$

$$8+5+2+(-1)+\dots$$

### 2.3 ARITHMETIC PROGRESSION

A sequence is called an **Arithmetic Progression** (abbreviated AP) if any only if the difference of any term from its preceding term is constant.

This constant is denoted by d and is called the **common difference** (CD).

Thus  $a_1, a_2, \ldots, a_n$  or  $a_1, a_2, \ldots, a_n$ , ... is an AP iff  $a_{k+1} - a_k = d$ , a constant independent of  $k, k = 1, 2, \ldots, n$  or  $k = 1, 2, \ldots, n$ , ... as the case may be.

#### **Theorem 1**

Find the general term of an A. P.

#### **Proof**

Let a be the first term and d be the CD of an AP. Let  $T_1, T_2, \ldots, T_n$  denote  $1^{st}, 2^{nd}, \ldots n^{th}$  terms respectively, then we have

$$T_2 - T_1 = d$$

$$T_3 - T_2 = d$$

$$T_4 - T_3 = d$$

$$T_n - T_{n-1} = d$$

By adding we have  $T_n - T_1 = (n-1) d$ 

i.e. 
$$T_n = T_1 + (n-1) d$$

$$= a + (n-1) d$$

If the last terms of an AP consisting of *n* terms is *l*, then l = a + (n - 1) d

THE PEOPLE'S

#### Example 2.1

A sequence  $\{a_n\}$  is given by the formula  $a_n = 10 - 3n$ . Prove that it is an AP.

#### **Solution**

$$a_n = 10 - 3n$$

$$a_{n+1} = 10 - 3 (n+1) = 10 - 3n - 3 = 7 - 3n$$

$$\therefore a_{n+1} - a_n = (7 - 3n) - (10 - 3n)$$

$$= 7 - 3n - 10 + 3n = -3$$

which is independent of n. Hence the sequence is an A. P.

THE PEOPLE'S

#### **Theorem 2**

If the terms of an arithmetic sequence be increased, decreased, multiplied or divided by the same non-zero constant, they remain in arithmetic sequence.

#### **Proof**

Consider an AP

$$a, a + d, a + 2d, a + 3d, \dots$$
 (2.1)

- (i) When each term of (2.1) is increased by a non-zero constant k, we obtain the sequence a + k, a + d + k, a + 2d + k, a + 3d + k, . . . i.e. (a + k), (a + k) + d, (a + k) + 2d, (a + k) + 3d, . . . which is clearly an AP whose first term is a + k, and CD is d.
- (ii) Can be similarly proved.
- (iii) When each term is multiplied by k, we obtain the sequence ak, ak + kd, ak + 2kd, ak + 3kd, . . . which is an AP.
- (iv) Can be similarly proved.

#### **Theorem 3**

Find the sum of *n* terms of an A. P.

#### **Proof**

Let a be the first term, d the common difference and l the last term i.e. the n<sup>th</sup> term. If  $S_n$  denotes the sum of n terms then

$$S_n = a + (a+d) + (a+2d) + \ldots + (l-2d) + (l-d) + l \qquad \ldots (2.2)$$

Rewriting  $S_n$  in the reverse order, we have

$$S_n = l + (l - d) + (l - 2d) + \ldots + (a + 2d) + (a + d) + a \qquad \ldots (2.3)$$

Adding Eqs. (2.2) and (2.3), we have

$$2S_n = (a+l) + (a+l) + \dots + (a+l)$$
 {n times}  
= n (a+l)

$$\therefore S_n = \frac{n(a+l)}{2} = \frac{n}{2} [a+a+(n-1)d]; \{\because l = a+(n-1)d\}$$

Sequences and Series

THE PEOPLE'S UNIVERSITY









$$=\frac{n}{2}\left[2a+(n-1)d\right]$$

#### 2.3.1 Arithmetic Mean

When three numbers are in AP, the middle one is said to be the Arithmetic Mear (written as AM) between the other two.

Similarly if  $a, A_1, A_2, \ldots, A_n, b$  are in AP then  $A_1, A_2, \ldots, A_n$  are called the **n-Arithmetic Means** between a and b.

#### Theorem 4

Find an Arithmetic Mean between two numbers a and b.

Let *A* be the AM between *a* and *b*.

Then a, A, b are in AP

$$\Rightarrow A - a = b - A \Rightarrow 2A = a + b \Rightarrow A = \frac{a + b}{2}$$
Thus the AM between two numbers is half their sum

Thus the AM between two numbers is half their sum.

#### **Theorem 5**

Find *n* arithmetic means between two numbers.

Let  $A_1, A_2, \ldots, A_n$  be the n-arithmetic means between a and b.

Then  $a, A_1, A_2, \ldots, A_n, b$  are in AP.

Let d be the CD. Then b is the  $(n + 2)^{th}$  term.

$$\therefore b = a + (n+2-1) d = a + (n+1) d$$

i.e. 
$$b - a = (n + 1) d$$

i.e. 
$$d = \frac{b-a}{n+1}$$

$$A_1 = a + d = a + \frac{b-a}{n+1} = \frac{na+b}{n+1}$$

$$A_2 = a + 2d = a + 2\left(\frac{b-a}{n+1}\right)$$

$$A_n = a + nd = a + n\left(\frac{b-a}{n+1}\right) = \frac{a+nb}{n+1}$$

The sum of these n-arithmetic means is  $A_1 + A_2 + ... + A_n$ 

$$= a + A_1 + A_2 + \ldots + A_n + b - (a + b)$$

= Sum of (n + 2) term of an AP – (a + b)

$$= \frac{(n+2)}{2}(a+b) - (a+b)$$

$$= (a+b) \left\lceil \frac{n+2}{2} - 1 \right\rceil = n \left( \frac{a+b}{2} \right)$$

Thus we find that the sum of n AM's between two numbers is n times the single AM between the two numbers.

Let a be the first term and d be the common difference of the given AP.

$$T_3 = 18 \implies a + 2d = 18$$

$$T_7 = 30 \implies a + 6d = 30$$
  $\Rightarrow 4d = 12 \implies d = 3$ 

$$a + 2d = 18, d = 3 \implies a = 12$$

THE PEOPLE'S

$$T_n = a + (n-1) d = 12 + (n-1) 3 = 12 + 3n - 3 = 9 + 3n$$

:. The series is

$$12 + 15 + 18 + \ldots + (3n + 9) + \ldots$$

Example 2.3

If 
$$a^2$$
,  $b^2$ ,  $c^2$  are in AP, prove that  $\frac{a}{b+c}$ ,  $\frac{b}{c+a}$ ,  $\frac{c}{a+b}$  are in AP.

**Solution** 

$$\frac{a}{b+c}$$
,  $\frac{b}{c+a}$ ,  $\frac{c}{a+b}$  are in AP.

iff 
$$\frac{a}{b+c}+1$$
,  $\frac{b}{c+a}+1$ ,  $\frac{c}{a+b}+1$  are in AP (adding 1 to each term)

iff 
$$\frac{a+b+c}{b+c}$$
,  $\frac{a+b+c}{c+a}$ ,  $\frac{a+b+c}{a+b}$  are in AP.

iff 
$$\frac{1}{b+c}$$
,  $\frac{1}{c+a}$ ,  $\frac{1}{a+b}$  are in AP (dividing each term by  $(a+b+c)$ )

iff 
$$(c+a)(a+b)$$
,  $(b+c)(a+b)$ ,  $(b+c)(c+a)$  are in AP (multiplying each term by  $(c+a)(a+b)(b+c)$ )

iff 
$$ac + a^2 + bc + ab$$
,  $ab + ac + b^2 + bc$ ,  $bc + c^2 + ab + ac$  are in AP

iff 
$$a^2$$
,  $b^2$ ,  $c^2$  are in AP (subtracting  $ab + bc + ca$  from each term)

$$\therefore$$
 if  $a^2, b^2, c^2$  are in AP

then 
$$\frac{a}{b+c}$$
,  $\frac{b}{c+a}$ ,  $\frac{c}{a+b}$  are in AP

Example 2.4

Find the sum of 19 terms of an AP whose  $n^{th}$  term is 2n + 1.

**Solution** 

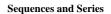
Let a be the first term and d be the common difference.

THE PEOPLE'S

Then 
$$T_1 = a = 2.1 + 1 = 3$$
 (as  $n = 1$ )

$$T_2 = 2.2 + 1 = 5$$
 (as  $n = 2$ )

$$\therefore d = T_2 - T_1 = 5 - 3 = 2$$













Mathematics-I

$$S_n = \frac{n}{2} [2a + (n-1) d]$$

$$= \frac{n}{2} [6 + 2 (n-1)] = \frac{n}{2} [6 + 2n - 2]$$

$$= \frac{n}{2} (4 + 2n) = n (2 + n)$$

# THE PEOPLE'S UNIVERSITY

# $\therefore S_{19} = 19(2+19) = 19 \times 21 = 399$

#### Example 2.5

The sum of three consecutive numbers in an AP is 18 and their product is 192. Find the numbers.

#### **Solution**

Let the three numbers be a - d, a, a + d

Then 
$$a - d + a + a + d = 18 \implies 3 \ a = 18 \implies a = 6$$

and 
$$(a-d) \ a \ (a+d) = 192$$
  
 $a \ (a^2 - d^2) = 192$   
 $6 \ (36 - d^2) = 192 \implies 36 - d^2 = 32$   
 $\Rightarrow d^2 = 36 - 32 = 4$ 

$$\Rightarrow d = \pm 2$$

Taking d = 2, we have the numbers 4, 6, 8.

Taking d = -2, we have the numbers 8, 6, 4.

Hence the required numbers are 4, 6, 8 or 8, 6, 4.

#### Example 2.6

Insert six arithmetic means between 2 and 16. Also prove that their sum is 6 times the arithmetic mean between 2 and 16.

#### **Solution**

Let A be the arithmetic mean between 2 and 16 then

$$A = \frac{2+16}{2} = 9 \tag{2.4}$$

Let  $A_1, A_2, ..., A_6$  be the six arithmetic means between 2 and 16, then 2,  $A_1, A_2, ..., A_6$ , 16 are in AP. Let d be the common difference. Then 16 is the  $8^{th}$  term

$$\therefore 16 = a + 7d \implies 16 = 2 + 7d \implies d = 2$$

$$\therefore$$
  $A_1 = 2 + 2 = 4$ ,  $(A_1 = a + d)$ ,  $A_6 = a + 6d = 2 + 12 = 14$ 

Now,

$$A_1 + A_2 + \ldots + A_6 = \frac{6}{2} (4 + 14)$$
 {:  $S_n = \frac{n}{2} (a + b)$ }  
=  $3 \times 18 = 54$  ...(2.5)



(Proved)

Sequences and Series

#### SAQ1

- (a) Which term of the sequence -3, -7, -11, -15, ... is 403? Also find which term if any of the given sequence is -500?
- (b) The fourth term of an AP is equal to 3 times the first term and the seventh term exceeds twice the third term by 1. Find the first term and the common difference.
- (c) Prove that a, b, c are in AP iff  $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$  are in AP.
- (d) If p times the  $p^{th}$  term of an AP is equal to q times the  $q^{th}$  term, show that the  $(p+q)^{th}$  term is zero.

THE PEOPLE'S UNIVERSITY

### SAQ 2

- (a) The sum of n terms of two arithmetic series are in the ratio of (7n + 1): (4n + 27). Find the ratio of their  $11^{th}$  terms.
- (b) If a, b, c, x are real numbers and  $(x a + b)^2 + (x b + c)^2 = 0$ , prove that a, b, c are in AP.
- (c) The ratio of the  $2^{nd}$  to  $7^{th}$  of n AM between -7 and 65 is 1:7. Find n.
- (d) The sum of two numbers is  $\frac{13}{6}$ . An even number of arithmetic means are inserted between them and their sum exceeds their number by 1. Find the number of means inserted.

[**Hint**: Take 2n to be the number of means inserted.]





# 2.4 GEOMETRIC PROGRESSION

A sequence (finite or infinite) of non-zero terms is called a geometric progression (GP) iff the ratio of any term to its preceding term is constant.

We assume that none of the terms of the sequence is zero. This constant ratio (non-zero) is denoted by r and is called common ratio (CR).



Mathematics-I

Thus 
$$a_1, a_2, \ldots, a_n$$
 or  $a_1, a_2, a_3, \ldots, a_n, \ldots$  is a GP iff  $\frac{a_{k+1}}{a_k} = r$  (constant independent of  $k$ ) for  $k = 1, 2, 3, \ldots, n-1$  or  $k = 1, 2, 3, \ldots$ 

$$\therefore \quad \text{In a GP } T_n = T_{n-1} \cdot r$$

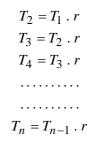
THE PEOPLE'S UNIVERSITY

For example, the sequence  $\frac{1}{2}$ , 1, 2, 4, 8, ..., 512 is a finite GP with r = 2 and the sequence 1,  $-\frac{1}{3}$ ,  $\frac{1}{9}$ ,  $-\frac{1}{27}$ , ... is an infinite GP with  $r = -\frac{1}{3}$ .

#### Theorem 6

#### General term of a GP

Let a be the first term and  $r \neq 0$  be the CR.



Multiplying all these we have

$$T_2 T_3 T_4 \dots T_n = T_1 T_2 T_3 \dots T_{n-1} \cdot r^{n-1}$$

i.e. 
$$T_n = T_1 r^{n-1} = ar^{n-1}$$

If last term of a GP with *n* terms is denoted by *l*, then  $l = ar^{n-1}$ .

#### Theorem 7

#### Sum of n Terms of a GP

Let a be the first term and r be the common ratio of a GP and  $S_n$  the sum of n terms.

Then 
$$S_n = a + ar + ar^2 + ... + ar^{n-1}$$
 ... (2.6)

$$rS_n = ar + ar^2 + \dots + ar^{n-1} + ar^n$$
 ... (2.7)

Subtracting Eq. (2.7) from Eq. (2.6), we have

$$(r-1) S_n = ar^n - a = a (r^n - 1)$$

i.e. 
$$S_n = a \left( \frac{r^n - 1}{r - 1} \right)$$
 or  $a \left( \frac{1 - r^n}{1 - r} \right)$ 

This formula fails when r - 1 = 0, i.e. r = 1. When r = 1, then

$$S_n = a + a + a + \dots n \text{ times} = na$$

#### Sum of an Infinite GP

Consider a GP a, ar,  $ar^2$ , ... with |r| < 1. We know that

$$S_n = a \left( \frac{1 - r^n}{1 - r} \right) = \frac{a}{1 - r} - \frac{ar^n}{1 - r}$$





Since  $|r| < 1 \Rightarrow r^n$  goes on decreasing as n goes on increasing. Ultimately when  $n \to \infty$ ,  $r^n \to 0$ .

$$S_n \to \frac{a}{1-r}$$

In other words if S denotes the sum of infinite terms then  $S = \frac{a}{1-r}$ .



Sequences and Series

#### 2.4.1 Geometric Mean

When three numbers are in GP, the middle one is said to be the Geometric Mean (GM) between the other two.

Similarly if  $a, a_1, a_2, \ldots, a_n$ , b are in GP then  $a_1, a_2, \ldots, a_n$  are called the n geometric means between a and b.



# To find the GM between two members a and b

Let G be the GM. Then a, G, b are in GP.

i.e. 
$$\frac{G}{a} = \frac{b}{G}$$
, i.e.  $G^2 = ab$ , i.e.  $G = \sqrt{ab}$ 

(Conventionally G is taken to be the +ve square root of ab).

#### To find n GMs between the numbers a and b

Let  $a_1, a_2, \ldots, a_n$  be the GMs between a and b.

Then  $a, a_1, a_2, \ldots, a_n, b$  are in GP.

Let r be the common ratio.

Then 
$$b = ar^{n+2-1} = ar^{n+1} \implies r^{n+1} = \frac{b}{a} \implies r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$\therefore G_1 = ar = a \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

$$G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

Cor.

$$a_1 \times a_2 \times \ldots \times a_n = ar \cdot ar^2 \cdot ar^3 \cdot \ldots ar^n$$
  
=  $a^n r^{1+2+3+\ldots+n}$   
=  $a^n r^{\frac{n(n+1)}{2}} (\because 1+2+3+\ldots+n \text{ is AP of } n \text{ terms})$ 











$$= a^n \left[ \left( \frac{b}{a} \right)^{\frac{1}{n+1}} \right]$$

$$=a^{n}\left[\left(\frac{b}{a}\right)^{\frac{1}{n+1}}\right]^{\frac{n(n+1)}{2}}$$



$$=a^n\left(\frac{b}{a}\right)^{\frac{n}{2}}=a^{\frac{n}{2}}b^{\frac{n}{2}}=(\sqrt{ab})^n$$

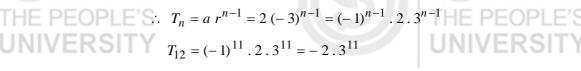
 $= n^{\text{th}}$  power of the single GM between a and b.

#### Example 2.7

Find the  $n^{th}$  term and the  $12^{th}$  term of the sequence  $2, -6, 18, -54, \ldots$ 

#### Solution

The given sequence is a G. P with a = 2 and r = -3.



#### Example 2.8

Find the sum of the series  $\frac{2}{9} + \frac{1}{3} + \frac{1}{2} + ... + \frac{81}{32}$ .

#### **Solution**

The given series is a GP where a, the first term is  $\frac{2}{9}$  and r the common ratio

is 
$$\frac{3}{2}$$
.

Last terms is  $\frac{81}{32} = T_n$  say.

Then 
$$\frac{81}{32} = ar^{n-1} = \frac{2}{9} \left(\frac{3}{2}\right)^{n-1}$$

i.e. 
$$\left(\frac{3}{2}\right)^{n-1} = \frac{81}{32} \times \frac{9}{2} = \left(\frac{3}{2}\right)^6$$
 :  $n-1=6 \Rightarrow n=7$ 

Hence the sum of the 7 term of the series



$$= \frac{\frac{2}{9} \left[ \left( \frac{3}{2} \right)^7 - 1 \right]}{\frac{3}{2} - 1} = \frac{2}{9} \times \frac{2}{1} \left( \frac{2187}{128} - 1 \right)$$
2059

# IGNOU THE PEOPLE'S UNIVERSITY

#### Example 2.9

Find the sum of 50 terms of the sequence  $7, 7.7, 7.77, 7.777, \ldots$ 

It is not a GP but we can relate it to a GP. Let  $S_{50}$  denote the sum of 50 term of this sequence.

$$S_{50} = 7 + 7.7 + 7.77 + 7.777 + \dots$$
 to 50 terms





$$= \frac{7}{9} (9 + 9.9 + 9.99 + 9.999 + \dots) \text{ to 50 terms}$$

$$= \frac{7}{9} [(10 - 1) + (10 - .1) + (10 - .01) + \dots] \text{ to 50 terms}$$

$$= \frac{7}{9} (10 + 10 + \dots \text{ to 50 terms}) - \frac{7}{9} (1 + .1 + .01 + \dots \text{ to 50 terms})$$
This is a GP

This is a GP
$$= \frac{7}{9} (10 \times 50) - \frac{7}{9} 1. \left[ \frac{1 - (.1)^{50}}{1 - .1} \right]$$

$$= \frac{7}{9} \times 500 - \frac{7}{9} \quad \frac{1 - \frac{1}{10^{50}}}{\frac{9}{10}} = \frac{7}{9} \times 500 - \frac{7}{9} \times \frac{10}{9} \left(1 - \frac{1}{10^{50}}\right)$$
$$= \frac{7}{81} \left[ 4500 - 10 + \frac{10}{10^{50}} \right]$$

$$=\frac{7}{81}\left[4490+\frac{1}{10^{49}}\right]$$

#### Example 2.10

If the first term of a GP. exceeds the second term by 2 and the sum of infinite terms is 50, find the GP.

#### **Solution**

Let a be the first term and r with  $|\mathbf{r}| < 1$  be the common ratio.

$$T_1 = T_2 + 2 \implies a = a r + 2 \implies a = \frac{2}{1 - r}$$
 ... (2.8)

$$S_{\infty} = 50 = \frac{a}{1 - r}$$
 ... (2.9)

From Eqs. (2.8) and (2.9), we have

$$50 (1-r) = a = \frac{2}{1-r}$$

$$\Rightarrow (1-r)^2 = \frac{2}{50} = \frac{1}{25} \Rightarrow 1-r = \pm \frac{1}{5} \Rightarrow r = 1 - \frac{1}{5} = \frac{4}{5} \text{ or } \frac{6}{5}$$

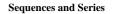
$$r \neq \frac{6}{5} \text{ as } |r| < 1; r = \frac{4}{5}$$

$$\therefore a = \frac{2}{1 - \frac{4}{5}} = \frac{2}{1} = 10$$

The GP is 10, 8, 
$$\frac{32}{5}$$
,  $\frac{128}{25}$ , ...

#### Example 2.11

Insert 4 GMs between 3 and 96. Show that their product is the 4<sup>th</sup> power of the GM between them.



THE PEOPLE'S UNIVERSITY

THE PEOPLE'S UNIVERSITY







#### **Solution**

Let a be the single GM and  $a_1$ ,  $a_2$ ,  $a_3$ ,  $a_4$  be the four geometric means inserted between the two numbers. Let r be the common ratio, then 96 is the  $6^{th}$  term

$$\therefore 96 = ar^5 \implies r^5 = \frac{96}{a} = \frac{96}{3} = 32 \implies r = 2$$

$$\therefore G_1 = ar = 6$$

$$G_2 = ar^2 = 12$$
  $(G = \sqrt{3 \times 96} = \sqrt{288} = 12\sqrt{2})$ 

$$G_3 = ar^3 = 24$$

$$G_4 = ar^4 = 48$$

$$G_1 \ G_2 \ G_3 \ G_4 = 6 \times 12 \times 24 \times 48 = 12^4 \times 2^2 = (12 \ \sqrt{2})^4 = G^4$$

SAQ3

- (a) The second term of a GP is  $\frac{25}{4}$  and the 8<sup>th</sup> term is  $\frac{16}{625}$ , find the GP.
- (b) If the  $p^{th}$ ,  $q^{th}$  and  $r^{th}$  terms of a GP are a, b, c respectively, prove that  $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$ .
- (c) If a, b, c are in GP, show that  $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$  are also in GP.
- (d) Determine the number of terms of a GP  $\{a_n\}$  if  $a_1 = 3$ ,  $a_n = 96$  and  $S_n = 189$ .

## SAQ 4

- (a) Sum to *n* terms the series  $5 + 55 + 555 + \dots$
- (b) Sum the series  $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \dots$  upto n terms.

# SAQ 5

- (a) The sum of four numbers in GP is 60 and the AM of the first and the last is 18. Find the numbers.
- (b) Find the sum of an infinite sequence  $7, -1, \frac{1}{7}, -\frac{1}{49}, \dots$

- (c) Show that AM between two distinct numbers is always greater than their GM.
- (d) If p, q, r are in AP, a is GM between p and q and b is GM between q and r', prove that  $a^2$ ,  $q^2$ ,  $b^2$  are in AP.



## 2.5 SUMMARY

#### AP and GP

If a is the first term and d the common difference of an AP then

(i) 
$$n^{\text{th}}$$
 term of an AP is  $T_n = a + (n-1)d$ ,

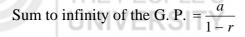
(ii) 
$$S_n = \text{Sum of the } n \text{ terms } = \frac{n}{2} [2a + (n-1)d]$$

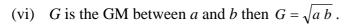
(iii) AMs between a and b is 
$$\frac{a+b}{2}$$
.

(iv) 
$$A_1, A_2, ..., A_n$$
 are  $n$  AMs iff  $a, A_1, ..., A_n, b$  are in AP.

(v) If 
$$1^{st}$$
 term is  $a$  and  $r$  is the common ratio of a GP, then  $n^{th}$  term  $T_n = ar^{n-1}$ 

Sum of first *n* terms 
$$S_n = \frac{a(1-r^n)}{1-r}$$
 or  $\frac{a(r^n-1)}{(r-1)}$ ,  $r \ne 1$ 





# IGNOU THE PEOPLE'S UNIVERSITY

# 2.6 ANSWERS TO SAQs

#### SAQ 1

- (a) -403 is the 101<sup>st</sup> term, and there is no term which is -500.
- (b)  $1^{st}$  term = 3 and common difference = 2.

#### SAQ 2

- (a) Required ratio is  $\frac{4}{3}$
- (c) n = 11
- (d) 12

#### SAQ3

(a) 
$$\frac{125}{8}, \frac{25}{4}, \frac{5}{2}, 1, \dots$$

(d) Number of terms = 6

SAQ4



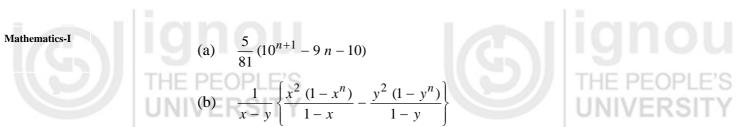






(a) 
$$\frac{5}{81} (10^{n+1} - 9 n - 10)$$

(b) 
$$\frac{1}{x-y} \left\{ \frac{x^2 (1-x^n)}{1-x} - \frac{y^2 (1-y^n)}{1-y} \right\}$$



SAQ5

(b) 
$$\frac{49}{8}$$















