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# UNIT 8 STRENGTH OF MATERIALS

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## Structure

- 8.1 Introduction
  - Objectives
- 8.2 Experimental Determination of Strength
  - 8.2.1 Tensile Test
  - 8.2.2 Tensile Properties : Yielding and Yield Strength
  - 8.2.3 Tensile Strength
- 8.3 Technological Properties of Materials
  - 8.3.1 Ductility
  - 8.3.2 Resilience
  - 8.3.3 Toughness
- 8.4 Effects of Variables on Mechanical Properties
  - 8.4.1 Grain Size
  - 8.4.2 Temperature
  - 8.4.3 Strain Rate
  - 8.4.4 Fatigue
  - 8.4.5 Chemical Effects
  - 8.4.6 Elastic Constants under Pressure
- 8.5 Fracture
- 8.6 Summary
- 8.7 Key Words
- 8.8 Answers to SAQs

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## 8.1 INTRODUCTION

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Many **materials**, when in service, are subjected to forces and loads; examples include the aluminium alloy **from** which an airplane wing is constructed **and** the steel **in an** automobile axle. In such situations, it is necessary to know the characteristics of the material and to design the member from which it is made such that any resulting deformation will not be excessive and fracture will not occur. The mechanical **behaviour** of a **material** reflects the relationship between its response or **deformation** to an applied load or force. Important mechanical **properties** are strength, hardness, ductility and stiffness.

The mechanical properties of materials are ascertained by performing carefully designed **laboratory** experiments that replicate as nearly as possible the service conditions. **Factors** to be considered include the nature of the applied load and its **duration**, as well as environmental conditions. It is possible for the load to be tensile, compressive or shear, **and** its magnitude **may** be constant with time, or may fluctuate continuously. Application time **may** be for only a fraction of a second or it may extend over a period of many **years**. **Service** temperature may be **an** important factor. The role of structural engineer is to determine stresses and stress distributions within members that are subjected to well-defined loads. This may be accomplished by experimental testing techniques **and/or** by theoretical and mathematical stress analyses. Materials and metallurgical engineers, on the other **hand**, are concerned with **producing** and fabricating materials to meet **service requirements** as predicted by these stress analyzer. This necessarily involves an understanding of the relationships between the microstructure of materials and their **mechanical** properties' (see Unit 4 on Microstructure of Materials). Materials **are** frequently chosen for **structural** applications because they have desirable combinations mechanical characteristics. **The** present discussion is confined primarily to the mechanical **behaviour** of metals.

## Objectives

At the end of this unit, you will be able to

- understand principal mechanical properties of materials,
- define various terms from stress-strain curve,
- conduct tension test on materials, and
- understand theory of fracture of materials.

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## 8.2 EXPERIMENTAL DETERMINATION OF STRENGTH

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### 8.2.1 Tensile Test

These tests are used to apply a stress to a **material** and record the materials response to this stress. The mathematical **definition** of stress ( $\sigma$ ) is the load ( $P$ ) on a body distributed over the cross-sectional area of the body ( $A$ ).

$$\sigma = \frac{P}{A}$$

A tensile stress tends to pull **member** apart; a compressive stress tends to crush or collapse a body, a shear stress tends to clear a structural member, a torsion stress tends to a twist a member and a bending stress tends to deflect a member. Handbooks on material properties invariably list the properties of materials subjected to tensile loading. The allowable torsion stress a material can tolerate is measured by shear strength and the allowable bending stress. This is because **bending** puts the outer fibres of member in tension.

A materials response to the **three** major forms of stress – tension, compression and shear can be measured on a **universal** testing machine, more commonly referred to **as** a tensile tester. These machines can pull axially on a test sample (tensile load) or push on a test sample to measure response to compression loading shear tests are **run** by loading a pin in special fixture.

These machines apply a tensile load when end of the test sample is attached to a movable cross head with the other end fixed to a stationary member as **depicted** in Figure 8.1.

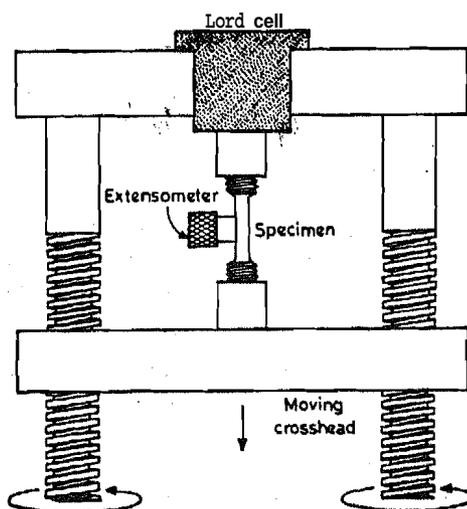


Figure 8.1 : Schematic of the Apparatus Used to Conduct Tensile Stress-strain Test

The cross head is then driven in such a manner or to pull the sample apart. Compressive loading is achieved by driving the cross head against short cylinders placed on a stationary machine pattern. Attachment are used to hold various shaped specimen, but tensile specimens are usually made in a "dog-bone shape" (Figure 8.2). The dog-bone shape ensures that the sample will break in the centre and not in the grip area. A tensile test is performed by extensometer **and** then stretching the sample until it fails.

The output of the test is a stress-strain curve. As seen in the previous unit; deformation in which stress (ordinate) versus **strain** (abscissa) results in a linear relationship, as shown in

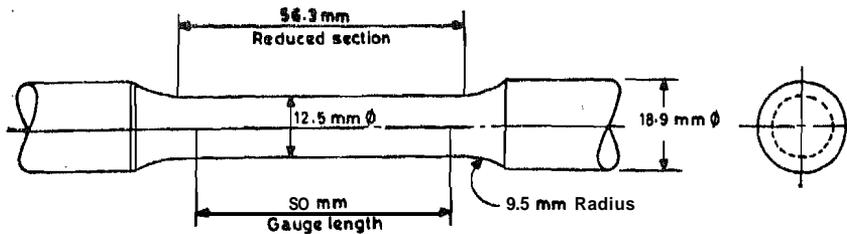


Figure 8.2: A Standard Tensile Test Specimen with Circular Cross-section. Also Known as Dog-bone Specimen

Figure 8.3. The slope of this linear segment corresponds to the modulus of elasticity  $E$ . This modulus may be thought of as stiffness or a material's resistance to elastic deformation.

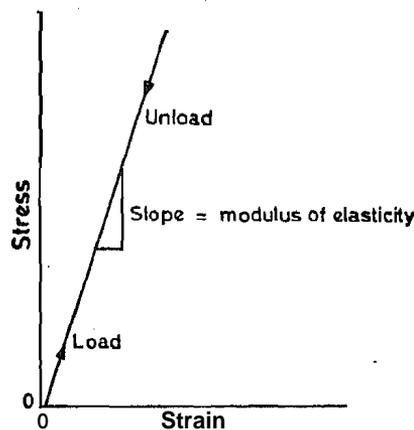


Figure 8.3: Stress-strain Diagram Showing Linear Elastic Deformation for Loading-unloading Cycles

The greater the modulus, the stiffer the material, or the smaller the elastic strain that results from the application of a given stress, The modulus is an important design parameter for computing elastic deformations.

## 8.2.2 Tensile Properties: Yielding and Yield Strength

Most structures are designed to ensure that only elastic deformation will result when a stress is applied. It is therefore desirable to know the stress level at which plastic deformation begins or where the phenomenon of yielding occurs. For metals that experience this gradual elastic-plastic transitions, the point of yielding may be determined as the initial departure from linearity of the stress-strain curve; this is sometimes called the proportional limit as shown by point P in Figure 8.4 (a). In such cases the position of this point may not be determined precisely. As a consequence, a convention has been established wherein a straight line is constructed parallel to the elastic portion of the stress-strain curve at some specified strain offset, usually 0.002. The stress corresponding to the intersection of this line and the stress-strain curve as it bends over in the plastic region is defined as the yield strength  $\sigma_y$ . This is shown in Figure 8.4 (a).

For those materials having a non-linear elastic region (Figure 8.5), use of the strain offset method is not possible, and the usual practice is to define the yield strength as the stress required to produce some amount of strain (for example  $\epsilon = 0.005$ ). For this non-linear behaviour, either tangent or secant modulus is normally used. Tangent modulus is taken as the slope of the stress-strain curve at some specified level of stress, while secant modulus represents the slope of a secant drawn from the origin to some given point of the  $\sigma$ - $\epsilon$  curve. This is shown in Figure 8.5.

Some steels and other materials exhibit the tensile stress-strain behaviour shown in Figure 8.4 (b). The elastic plastic-transition is very well defined and occurs abruptly in what is termed a yield-point phenomenon. At the upper yield point, plastic deformation is initiated with an actual decrease in stress. Continued deformation fluctuates slightly about some constant stress value, termed lower yield point; stress subsequently rises with

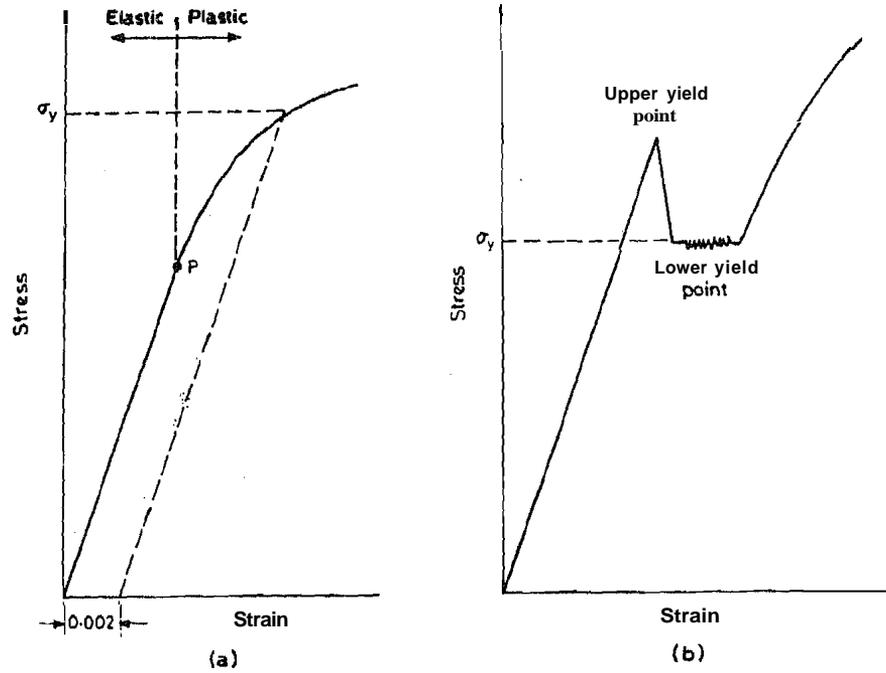


Figure 84 : a) Typical Stress-strain Behaviour for a Metal Showing Elastic and Plastic Deformations, the Proportional Limit P, and the Yield Strength  $\sigma_y$ , as Obtained from 0.002 Strain Offset Method  
b) Stress-strain Behaviour Found for Some Steels Showing the Yield Point Phenomenon

increasing strain. For metals, that display this effect, the yield point strength is taken as the average stress that is associated with the lower yield point, since it is well defined and relatively insensitive to the testing procedures. Thus it is necessary to employ the strain offset method for these materials.

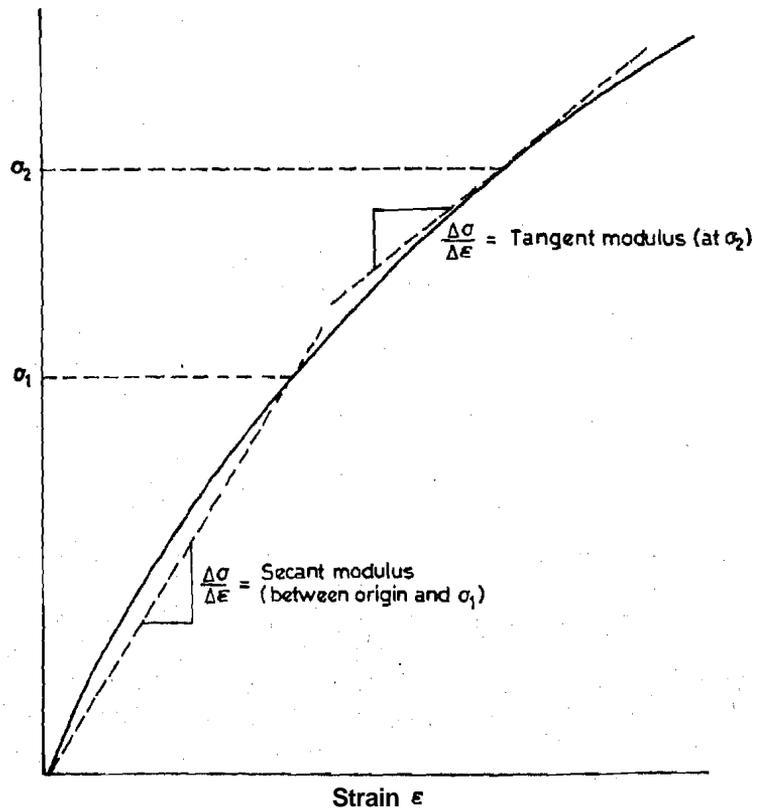


Figure 8.5 : Stress-strain Diagram Showing Non-linear Elastic Behaviour, and how Secant and Tangent Moduli are Determined

### 8.2.3 Tensile Strength

After yielding, the stress necessary to **continue plastic** deformation increases to a **maximum**, point M in Figure 8.6, and then decreases to the eventual fracture, point F. The **tensile strength TS** (unit psi or **MPa**) is the stress at the maximum on the **engineering stress strain curve**. This corresponds to the maximum stress that can be sustained by a structure **in tension**, if this stress is applied the fracture will result. All deformation up to this point is uniform throughout the narrow region of the tensile specimen. However, at this maximum **stress**, a small constriction or neck begins to form at **some** point, and all subsequent deformation is confined at this neck, as shown in Figure 8.5. This phenomenon is termed **necking** and fracture **ultimately** occurs at the neck. The fracture or rupture strength corresponds to the stress and fracture.

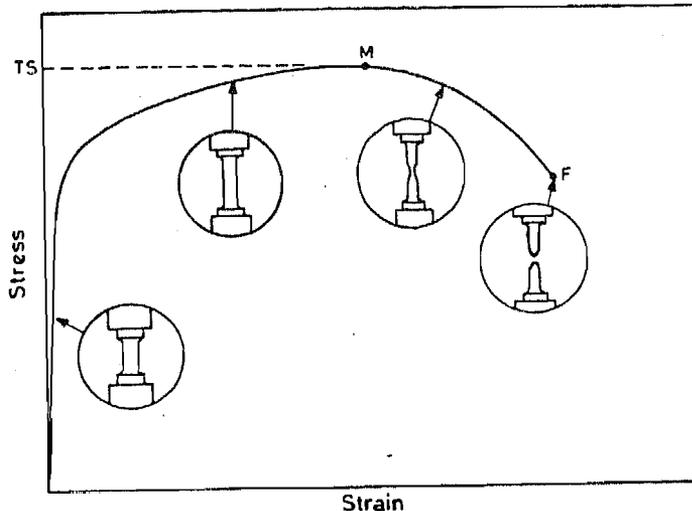


Figure 8.6 : Typical Engineering Stress-strain Behaviour to Fracture, Point F. The Tensile Strength TS is Indicated at Point M. The Circular Insets Represent the Geometry of the Deformed Specimen at Various Points along the Curve

Tensile strength may vary anywhere from 50 MPa (7000 Psi) for an Aluminium to as high as 3000 **MPa** (450,000) for the high-strength steels. Ordinarily, when the strength of a metal is cited for design purposes, the yield strength is used. This is because by the time a stress corresponding to the tensile strength has been applied, often a structure has experienced so much plastic deformation that it is useless. Furthermore, fracture strengths are not normally specified for engineering purposes.

## 8.3 TECHNOLOGICAL PROPERTIES OF METALS

In the previous section we have seen how material behaves when pulled in tension. In addition to the tensile strength **there** are many other properties of materials which can be deduced **from** the stress-strain curve which is generated as an output of the tensile test. Let us now discuss some of these properties which are extremely **useful** while deciding technological application for metals.

### 8.3.1 Ductility

Ductility is another **important** mechanical property. It is a measure of the degree of plastic deformation **that** has been **sustained** at fracture. A **material** that experiences very little or no plastic deformation upon **fracture** is termed brittle. The tensile stress-strain **behaviours** for both ductile **and** brittle materials are shown in Figure 8.7.

Ductility may be expressed quantitatively as either percent elongation or percent area reduction. The percent elongation, % EL, is **the** presence of plastic **strain** at fracture, or

$$\% \text{ EL} = \left( \frac{l_f - l_o}{l_o} \right) \times 100$$

where,  $l_f$  is the fracture length and  $l_o$  is the original gauge length as above. In as much as a significant **proportion** of the plastic deformation at a fracture is confined to the neck region, **the magnitude** of % EL will depend on specimen gauge length. The shorter the  $l_o$ ,

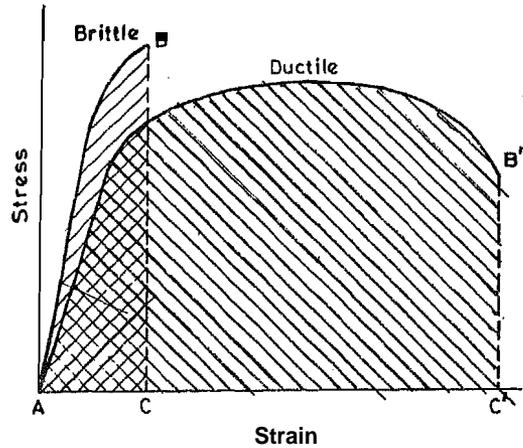


Figure 8.7 : Representation of Tensile Stress-strain Behaviour for Brittle and Ductile Materials

the greater is the fraction of total elongation from the neck end, and consequently, the higher the value of % EL. Therefore,  $l_0$  should be specified when percent elongation values are cited. It is commonly 50 mm.

Percent area reduction, % AR, is defined as

$$\% \text{ AR} = \left( \frac{A_0 - A_f}{A_0} \right) \times 100$$

where,  $A_0$  is the original cross-sectional area and  $A_f$  is the cross-sectional area at the point of fracture. Percent area reduction values are **independent** of both  $l_0$  and  $A_0$ . Furthermore, for a **given material** the magnitudes of % EL and % AR will, in general, be different. Most metals possess at least a moderate degree of ductility at room **temperature**; however, some become brittle as the temperature is lowered.

A knowledge of the ductility of materials is important for at least two reasons. First, it indicates to a designer the degree to which a structure will deform plastically before fracture. Second, it specifies the degree of allowable **deformation** during fabrication operations. We sometimes refer to relatively ductile **materials** as being "forgiving", in the sense that they may experience local deformation without fracture should there be an error in the magnitude of the design stress calculations. Brittle materials are approximately considered to be those having a fracture strain of less **than** about 5 %.

### SAQ 1

A piece of copper originally 305 mm long is pulled in tension with a stress of 276 MPa. If the deformation is entirely elastic, what will be the resultant elongation? Take  $E$  for copper =  $11.0 \times 10^4$  MPa

### SAQ/2

A cylindrical specimen of steel having an original diameter of 12.8 mm is tensile tested to fracture and found to have an engineering fracture strength of 460 MPa. If its cross-sectional diameter at fracture is 10.7 mm; determine

- the ductility in terms of percent area reduction, and
- the true stress at fracture.

### 8.3.2 Resilience

Resilience is the **capacity** of a **material** to absorb energy when it is deformed elastically and then upon **unloading**, to have this **energy** recovered. The associated property is the Modulus of Resilience,  $U_r$ , which is strain energy per unit volume **required** to stress a **material** from an unloaded state up to a point of yielding.

**Computationally**, the modulus of resilience for a specimen subjected to a uniaxial tension test is just the area under the engineering stress-strain curve taken to yielding or

$$U_r = \int_0^{\epsilon_y} \sigma \, d\epsilon$$

Assuming a linear elastic region

$$U_r = \frac{1}{2} \sigma_y \epsilon_y$$

in which  $\epsilon_y$  is the strain at yielding.

**The** units of resilience are the product of the units from each of the two axes of the stress-strain plot. **The** SI units is joules per cubic meter ( $\text{J/m}^3$ , equivalent Pa), Joules is an unit of energy, and thus this area under the stress-strain curve represents energy **absorption** per unit volume of material.

### 8.3.3 Toughness

Toughness is a mechanical term that is used in several contexts; **loosely** speaking, it is a measure of the ability of a material to absorb energy up to fracture. Specimen geometry as well as the manner in which load is applied are important in toughness determination. For dynamic (high strain rate) loading conditions and when a notch (or point of stress concentration) is present, notch toughness is assessed by **using** an impact test.

For static (low **strain** rate) situations, toughness may **be ascertained** from the **results** of a **tensile** stress-strain test. It is the area under the  $\sigma$ - $\epsilon$  curve **upto** the point of fracture. The units for toughness **are** the same **as** for resilience (**i.e.**, energy per unit **volume** of the **material**). For a material to be tough, it must display both strength and ductility; and often, ductile **materials** are tougher than brittle ones. This is demonstrated in Figure 8.7, in which the stress-strain **curves** are plotted for both material types. Hence, even though the brittle material has higher yield and tensile strengths, by virtue of lack of ductility, it has a lower toughness than the ductile one; this can be deduced by comparing the areas ABC and A B' C' in Figure 8.7.

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## 8.4 EFFECTS OF VARIABLES ON MECHANICAL PROPERTIES

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The **mechanical** properties of a material can be affected in **innumerable** ways. Recently **found examples**, such as radiation effects on the behaviour of materials, justify the idea that there are still **some** effects to be discovered. Thus, the variables discovered here should be considered only as a beginning for those dealing with the **strength** of materials. **The** effects of the variables are shown only qualitatively because the magnitude of the change in the mechanical properties may depend considerably on the **material**. The aim here is to **alert** the reader to seek further **information** beyond what is available in standard tables (this **may** involve much effort and expense) about any chosen materials in these two general situations,

- a) There are unusual conditions during fabrication **or** service, and
- b) The design requires the **saving** of material as much as possible.

In some cases even the indicated qualitative effects are not true. A different material or one under different conditions may show **an** opposite tendency in mechanical behaviour.

The following variables with respect to mechanical properties of metals are **often** important to **engineers**.

### 8.4.1 Grain Size

In general, resistance to deformation increases as the **microstructure** (size of the grains) is **made finer**. Cold **working** reduces the average grain size, **increase** the **yields** strength, reduces the **fracture** ductility, but leaves the fracture strength essentially unchanged (Figure 8.8). A cold worked metal can be softened by reannealing it any number of times, For a simple demonstration of this, bend a heavy copper wire repeatedly until it becomes stiff. Throw it in open fire for a few minutes, let it cool, and then bend it again.

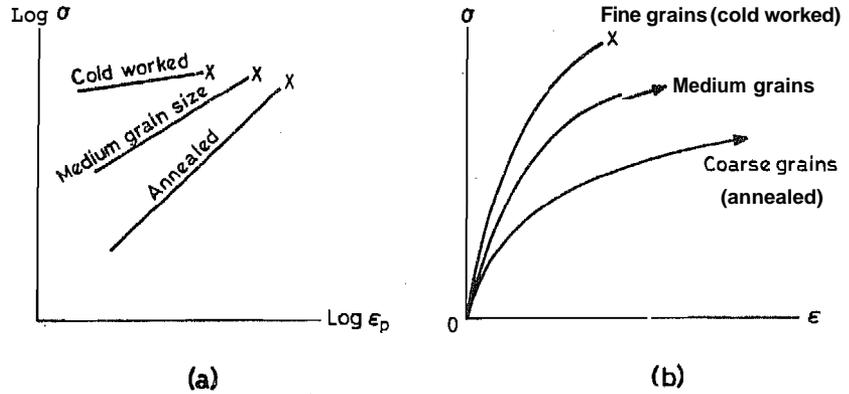


Figure 8.8 : Effect of Microstructure on Strength and Ductility

### 8.4.2 Temperature

The common effect of temperature is that both the yield strength and the fracture strength decrease while the ductility increases **as** the temperature increases (Figure 8.9). These are two important exceptions to this that must be noted here.

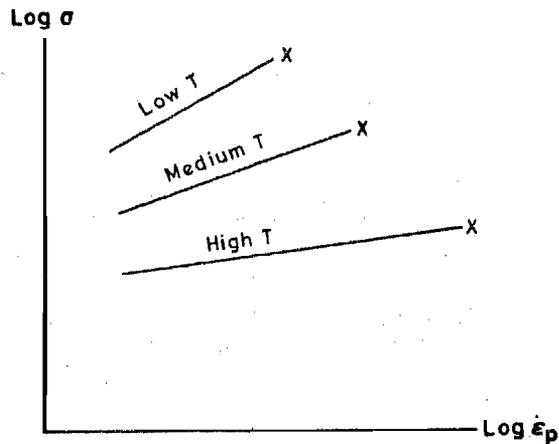


Figure 8.9 : Effect of Temperature on Strength and Ductility

At mildly elevated temperatures the yield strength of low carbon steel **increases** as shown in Figure 8.10. The phenomenon responsible **for** this behaviour is called strain aging. This strengthening mechanism depends on plastic deformation and on the diffusion rate of carbon atoms in the steel. Strain aging appears **as** a factor in several areas of strength of materials.

**The** other exception to the tendencies shown in figure is the ductile-brittle transition found mainly in low **carbon** steel. This is shown schematically in Figure 8.11; which applies to a metal containing flaws or machine notches. The slope and the **horizontal** position of the transition part of the curve depend on the metal, In most cases, the transition occurs somewhere below **+50 °F**. The ductile-brittle transition has enormous practical significance because the metals affected by it are the most common and because the transition temperature (or those below them) occur in many places in the **world**. Designing all members to have the average stresses in them below the lower plateau is a **safe** but uneconomical approach.

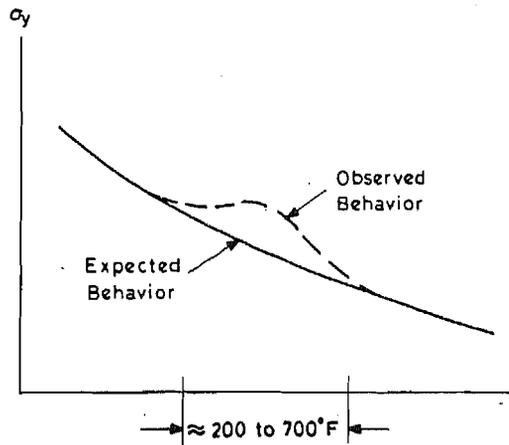


Figure 8.10 : Increase in Yield Strength by Strain Aging in Mild Steel

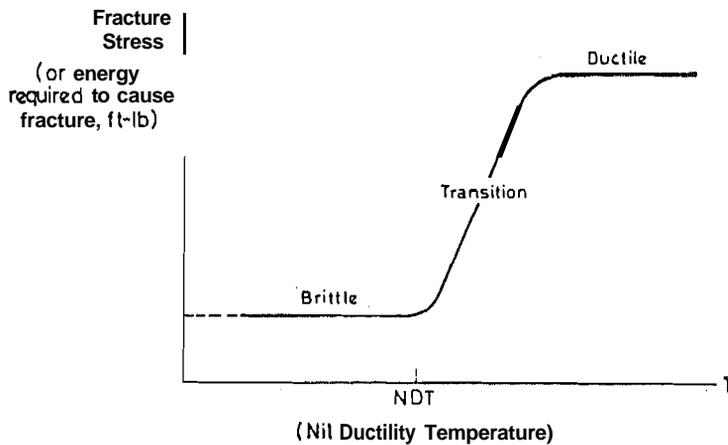


Figure 8.11 : Ductile-brittle Transition in Low Carbon-steel

### 8.4.3 Strain Rate

The rate at which a material is deformed affects its strength as shown in Figure 8.12. Both the yield strength and the fracture strength increases and the ductility decreases at high strain rates. These effects are less pronounced in normal practice than those caused by various temperatures.

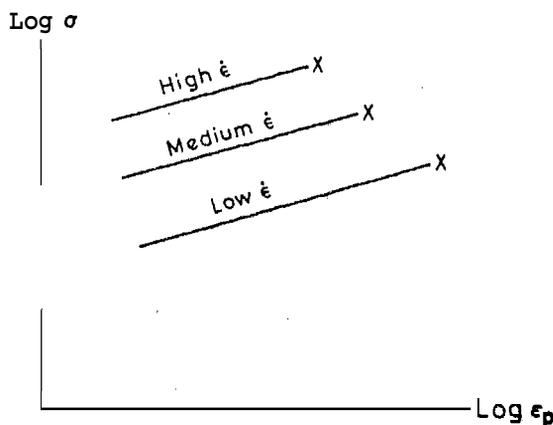


Figure 8.12: Effect of Strain Rate on Strength and Ductility

### 8.4.4 Fatigue

Repeatedly applied loads may cause large changes in the stress strain curve of a metal. A given metal may cyclically soften or harden compared to its tension stress-strain curve as shown in Figure 8.13. Which of these happens depends on the initial condition of the

material. In most cases, if it was soft (annealed) initially, it will cyclically harden and vice-versa. The strain hardening exponent  $n$ , which can be determined in a monotonic test, can give an indication of what will happen in fatigue loading. The material will change little if  $n=0.15$ . There is hardening for  $n$  greater than the stable value and softening for  $n$  below the stable value.

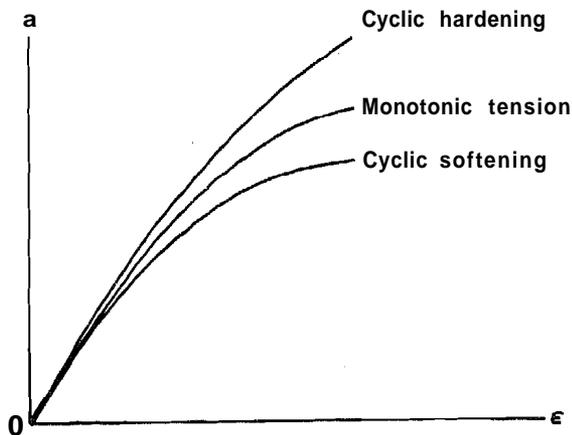


Figure 8.13: Effect of Cyclic Loading on Stress-strain Response

The sharp yielding of many steels can be observed during a single tensile loading can be completely bypassed as demonstrated in the following. A single loading to stress  $\sigma_A$  on the tension stress-strain curve in Figure 8.14 would involve sharp yielding. Assume that tensile and compressive stresses of magnitude  $\sigma_B$  are applied many times to a new specimen of the same metal, after which the stress is raised in a single step. A new stress-strain curve is followed after the cyclic loading as shown in Figure 8.14.

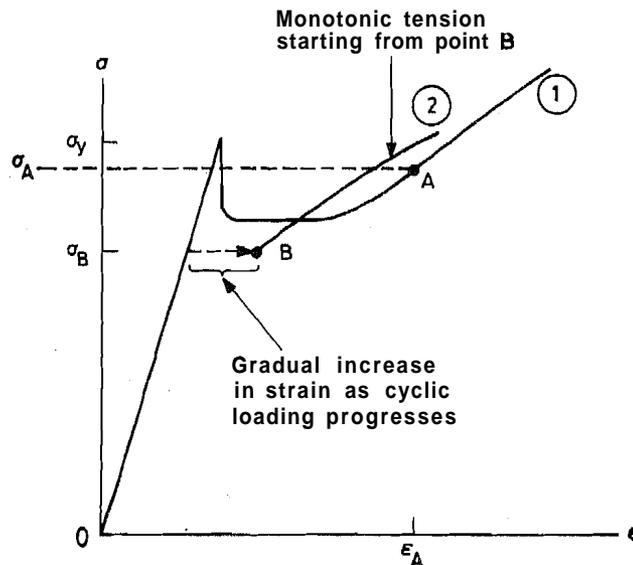


Figure 8.14: Change in Stress-strain Curve after some Cyclic Loading. Note - Curves 1 and 2 may or may not cross

A curve that shows the deformation resistance of a material after it has been deformed cyclically is called a cyclic stress-strain curve. Unfortunately, there is no unique cyclic stress-strain curve for a given temperature and other conditions. Such curves also depend on the extent of prior deformations; they may, in fact be in constant change throughout the life of the member. The practical solution to this problem is to use single cyclic stress-strain curve for each material that represents all possibilities fairly well. The important thing to remember is that in some cases the stress-strain curve obtained in a unidirectional test may not be adequate to describe the stress-strain response of a material throughout its life. The cyclic stress-strain curve is generally more suitable for this purpose.

### 8.4.5 Chemical Effects

Many chemicals are detrimental to the mechanical behaviour of materials. Two broad categories of chemical effects can be established :

- a) **Continuous corrosion attack** (details described in Unit 10) that causes an increase in stress because of the loss of material. The mechanical properties of the remaining material are not changed by this.
- b) **Embrittlement** of a ductile metal without a significant loss of material. Grain boundary attack and stress-corrosion cracking are important technological problems in this category. The macroscopically measurable ductility and strength are reduced by these. Alloys of Aluminium, copper, iron, magnesium, titanium and others are susceptible to these problems.

### 8.4.6 Elastic Constants under Pressure

We are concerned here with the mechanical response of the system which is initially subjected to a large hydrostatic pressure when a further, generally small stress system is superimposed on the initial system. In practical applications there are two important situations to be considered. First is the static response, when the additional stress system and the resultant stress strain are independent of time. This may be very important in engineering applications in which it is essential to know the additional deformations that are produced.

Second situation is dynamic one where the additional stress system varies with time. This includes particular solid. When a solid is subjected to a stress system its internal configuration is changed. This concept of internal configuration is most readily visualized in terms of relative positions of the constituents atom but can also be extended to continuous media with the assumption that each infinitesimal material region of the system can be identified throughout all possible changes. In either case, it is assumed that the position of an individual material point can, in any state of the solid, be specified by a set of co-ordinates with respect to some fixed reference system. For simplicity, this is taken as a cartesian system.

In the solid as a whole, the configuration is described by a collection of such co-ordinates (three for each point). This collection will change with the state of the system, such changes being brought about, for example, by the application of external stresses. For our purposes it is convenient to distinguish three type of states.

## 8.5 FRACTURE

When the stress normal to a crystallographic plane exceeds a critical value, the planes part and the crystal breaks. In single crystals, this is called cleavage when it occurs along well defined planes so that the two halves of the fractured crystal are bounded by flat faces. The cleavage planes are usually planes separated by large interplanar spacings such as the (100) planes in NaCl or  $\alpha$ -Fe. On the other hand, the nature of the interatomic forces in crystals can modify this generalization. For example, crystals having layered cleave along planes parallel to the layers because the forces between layers are frequently weaker than those within a layer. Because crystals tend to cleave along certain planes only the critical stress necessary to produce cleavage depends on the crystals orientation relative to the stress condition.

Another way that a crystal can deform is by the formation of one or more twins in which the two parts of the crystal are so displaced that they appear to be mirror images of each other. It can be shown that there is a critical stress value for twinning similar to the critical stress value for a slip or for fracture. Unlike the normal fracture stress, the critical stresses for slip or twinning depends not only on their orientations relative to the slip or twin.

There are two kinds of fracture in polycrystalline material. The fracture is called brittle fracture if no plastic deformation preceded it or ductile fracture if it follows pronounced plastic deformation. Although many theories those attempted the explanation fracture in polycrystalline aggregates but they are moderately successful. The reasons for this is the difficulty in taking account of all the imperfections that are present in the material and tend to weaken it. For example fracture of glasses is explained by postulating that fracture occurs because of the presence of small cracks normal to the tensile stress. Thus the very high tensile strength of freshly drawn glass fibres is explained by assuming that such fibres

do not contain cracks favourably oriented for fracture since cracks that are parallel to the fibre axis do not affect the tensile strength appreciably. When same theories applied to the metals, however the problem is further complicated by the fact that cracks can be produced during the plastic deformation prior to ductile fracture. Fracture can also occur by a shearing along a slip band shear fracture can be distinguished from cleavage because it leaves a nonplanar, usually **concoidal** surface. Both type of **fracture** can occur in **polycrystalline** materials and is easily distinguished the appearance of the fractured surfaces. If a material is subjected to a series of consecutive stresses even of relatively low magnitude, particularly if the stresses any cylindrically repeated fatigue fracture can occur. This type of fracture is believed to be produced as a consequence of the formation of cracks at the points of stress concentration. A common example of fatigue fracture is the breaking of a wire or a metal strip by repeated bending in opposite directions.

### 8.5.1 The Griffith Theory

Griffith was the first to offer an explanation for the low fracture strength of brittle materials. He postulated that in a brittle material there are **small** cracks which act to concentrate the stress at their tips. We now deal with the theory which he developed from this idea and which now bears his name.

It can be shown that, for a crack of elliptical section, with stress,  $\sigma$ , applied perpendicular to its long axis of length  $2a$ , the stress  $\sigma_{tip}$  at its tip is given by

$$\sigma_{tip} = 2 \sigma \left( \frac{a}{r} \right)^{1/2}$$

where  $r$  is the radius of curvature at the tip. This stress exists within a distance of approximately  $r$  of the tip and if it exceeds the ideal fracture stress (i.e., strength of the bonds) it may be assumed that the crack will propagate **through the material**.

However, it is better to consider a thermodynamic **criterion** for the growth of a crack. This was the approach taken by Griffith who argued that there were two energies to be taken into account when a crack is propagated – a release of elastic energy strain energy **and** an increase in surface energy. Thus as the material separates along a crack, new surfaces are being created and therefore a certain amount of energy must be provided to create these (i.e., some work must be done). Now before the crack propagates, elastic strain energy is stored in the material. This is released when the material relaxes as the crack spreads. Griffith supposed that the crack propagates when the released strain energy is just sufficient to provide the surface energy necessary for the creation of the new surfaces.

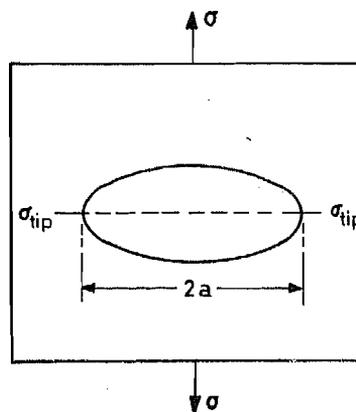


Figure 8.15 : Model for Griffith Crack Theory

The elastic strain energy for unit volume is  $\frac{1}{2} \frac{\sigma^2}{E}$ . For purpose of **calculation** we take the crack to have unit width perpendicular to the plane of the paper as shown in Figure 8.15. Very near to the crack faces the stress falls to **zero** and vary far from crack it is unchanged. So we assume that roughly a region of radius  $a$  around the crack is relieved of its elastic energy. This would, for unit width, give total elastic energy of

$$\frac{\sigma^2}{2E} \times \text{area} \times \text{width} = \frac{\sigma^2}{2E} \times \pi a^2$$

Properly, the strain field should be integrated from **infinity** to the surface of the crack, which makes the elastic energy,  $U_E$ , available per **unit** width twice **the above** value

$$U_E = \frac{\sigma^2 \pi a^2}{E}$$

If the surface energy per unit area is  $\gamma$  joules per square meter then the **surface** energy for a **crack** of length  $2a$  and unit width will be

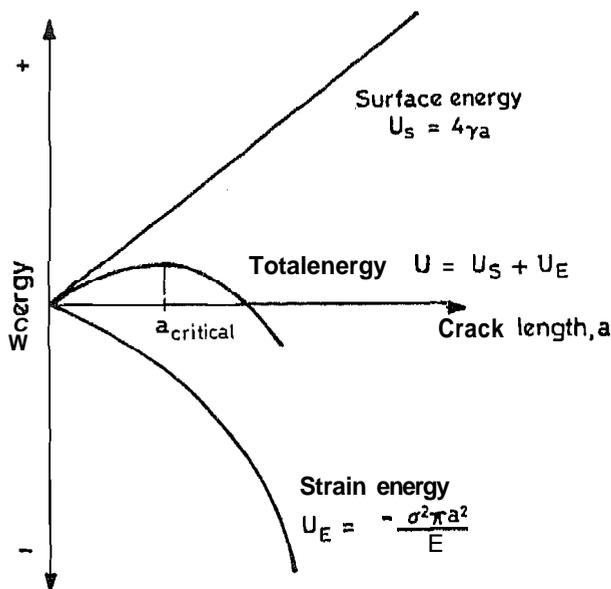
$$U_s = 4\gamma a$$

where we multiply by two because there are two faces. The total energy change,  $U$ , is therefore given by

$$U = 4\gamma a - \frac{\sigma^2 \pi a^2}{E}$$

and these terms are plotted as function of **crack** length  $a$  in **Figure 8.16**. The total energy is **maximum** at length  $a$  critical and consequently a **crack** of that length is unstable and reduces the total energy by growing. The **maximum** in  $U$  occurs when

$$\frac{dU}{da} = \frac{dU_s}{da} = \frac{dU_E}{da} = 0$$



**Figure 8.16 : Variation In the Surface Energy, Strain Energy and Total Energy with Crack Length**

From this equation we can see that the **crack** will grow when the strain energy release rate is equal to or greater than the rate of change of the surface energy with crack length.

Substituting for  $U_s$  and  $U_E$  we obtain

$$\frac{dU}{da} = \frac{d}{da} (4\gamma a) - \frac{d}{da} \left( \frac{\sigma^2 \pi a^2}{E} \right) = 0$$

Therefore,

$$4\gamma - \frac{2\sigma^2 \pi a}{E} = 0$$

and so

$$\sigma = \left( \frac{2\gamma E}{\pi a} \right)^{\frac{1}{2}}$$

This is Griffith equation and shows that the stress necessary to cause a crack to propagate varies inversely as the square root of the crack length. Hence the fracture stress of a brittle material is determined by the length of the largest crack existing before loading. Furthermore, once a crack begins to spread, the stress required to propagation falls as it is increasing and the crack accelerates rapidly.

The observed strength of hard brittle materials such as germanium, glass, silicon, quartz etc. require Griffith crack of only micron dimension. Even if homogeneous specimen having a crack-free surface is prepared it is almost impossible to keep it that way. The slightest contact of such surface with any other solid object can cause minute hard particles to be pushed into the surface with enough force to exceed the theoretical strength locally. Chemical attack at points of high local strain can also produce micron-size cracks. Therefore it seems highly probable that a few such cracks will be present in any specimen of a brittle material of ordinary size. Experiments are consistent with the idea that Griffith cracks are located primarily at the external surfaces. It is also found that the number of such defects need not be large. The highly pure and structurally perfect silicon crystals grown were found have exceptional fracture strength only after very careful chemical polishing of the surface followed by extreme care not to allow the surface to be touched.

The Griffith theory still appears to be a reasonable explanation for the fracture of hard brittle materials. The fracture of materials having some ductility, however, is of greater interest. For a material in which shear deformation occur at any stress below the theoretical strength, the Griffith theory requires modification. Irreversible rearrangements of atoms may take place in the region of high concentration at the tip of the advancing crack. The energy required for crack growth will then be greater than the surface energy by the amount of plastic work. In ductile materials, the formation of crack nuclei and the propagation of a crack become intimately connected with the mechanism of plastic deformation.

For soft ductile materials the mechanism suggested for formation of Griffith cracks at surface unsatisfactory. In many cases the ratio between theoretical and observed fracture strength is so large that cracks several millimeters long would be required. Such cracks, if present hardly escape detection. It must be concluded that crack of sufficient size to grow are not present initially but are formed as a result of the plastic-deformation process. Several different mechanisms for formation of crack nuclei have been proposed. All of them involve the pile-up or running together of dislocations resulting in stress concentrations.

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## 8.6 SUMMARY

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In this chapter we have discussed a number of important properties of materials, mainly metals. Concepts of engineering stress and strain were introduced. Strain represents the amount of deformation induced by a stress; both engineering and true strains are used.

Some mechanical characteristics of metals can be ascertained by simple stress-strain tests. There are four test types: tension, compression, shear and torsion. Tensile are most common. A material that is stressed first undergoes elastic or nonpermanent deformation wherein the stress and strain are proportional. The constant of proportionality is the modulus of elasticity for tension and compression, and is the shear modulus when the stress is shear.

The phenomenon of yielding occurs at the onset of plastic deformation; yield strength is determined by a strain offset method from the stress-strain behaviour; which is indicative of the stress at which plastic deformation begins. Tensile strength corresponds to the maximum tensile stress that may be sustained by a specimen, whereas percent elongation and area reduction are measures of ductility—the amount of plastic deformation that has occurred at fracture. Mechanical strength depends on many factors and there are many variables which drastically alter the mechanical properties for example temperature, strain rate, chemical environment, hardness etc. affect the strength of materials.

## 8.7 KEY WORDS

- Annealing : A generic **term** used to denote a heat treatment wherein the microstructure and consequently the properties of a **material** are altered. Annealing is frequently referred to a heat treatment whereby a previously cold-worked metal is softened by **allowing** it to **recrystallize**.
- Ductile **Fracture** : A mode of Fracture that is attended by extensive gross plastic deformation.
- Ductility : A measure of a material's ability to undergo appreciable plastic deformation before fracture; it may be expressed as percent elongation (% EL) or percent area reduction (% AR) from **tensile** test.
- Elastic **Deformation** : Deformation that is non-permanent, that is totally recovered upon release of **an** applied stress.
- Resilience** : The capacity of a material to absorb energy when it is elastically deformed.
- Strain - Engineering : The change in the gauge length of a specimen divided by its original gauge length.
- Stress - Engineering** : The instantaneous load applied to a specimen divided by its **cross-sectional** area before any deformation.
- Tensile Strength : The **maximum** engineering stress, in tension, that may be sustained **without** fracture. Often **termed** ultimate strength.
- Yielding : The onset of plastic deformation.
- Yield Strength, : The stress required to produce a very slight yield yet specified amount of plastic strain; a strain offset of 0.002 is **commonly** used.

## 8.8 ANSWERS TO SAQs

### SAQ 1

- Since the deformation is elastic, strain is dependent on stress equation and is related by

$$\sigma = \epsilon E.$$

Furthermore, the elongation  $\Delta l$  is related to the original length  $l_0$  through equation

$$\epsilon = \frac{l_1 - l_0}{l_0} = \frac{\Delta l}{l_0}$$

Combining these two expressions and solving  $\Delta l$ , yields

$$\sigma = \epsilon E = \left( \frac{\Delta l}{l_0} \right) E$$

$$\Delta l = \frac{\sigma l_0}{E}$$

The values of  $\sigma$  and  $l_0$  are **given** and the magnitude of E for copper is  $11.0 \times 10^4$  MPa. Elongation is obtained by substitution into the expression above as

$$\Delta l = \frac{(276 \text{ MPa}) (305 \text{ mm})}{11.0 \times 10^4 (\text{MPa})} = 0.76 \text{ mm}$$

SAQ 2

a) Ductility is computed from equation

$$\%AR = \left( \frac{A_o - A_f}{A_o} \right) \times 100 = \frac{\left( \frac{12.8 \text{ mm}}{2} \right)^2 \pi - \left( \frac{10.7 \text{ mm}}{2} \right)^2 \pi}{\left( \frac{12.8 \text{ mm}}{2} \right)^2 \pi} = 30\%$$

b) True stress is defined by equation  $\sigma_T = \frac{F}{A_f}$ , where in this case the area is taken as the fracture area  $A_f$ . However, the load at fracture must first be computed from the fracture strength as

$$F = \sigma_f A_o = (460 \text{ MPa}) (5.0)^2$$

Thus the true stress is calculated as

$$\sigma_T = \frac{F}{A_f} = \frac{(460 \text{ MPa}) (5.0)^2}{(3.25)^2} = 660 \text{ MPa.}$$