UNIT 6 DEFORMATION OF MATERIALS

Structure

6.1 INTRODUCTION

The property for which solids are probably valued and utilised in engineering is their "cohesion". The resistance they offer to forces tending to change their shapes or dimensions. In the forms of shelter, building, weapons and tools, solids have been used since the dawn of civilization to sustain or apply force. That this action was in reality a reaction on the part of solid tending to resist deformation had passed largely unnoticed until the second half of the 17th century. This can be understood because the building materials mostly used that time, stone and masonry, deformed so little before fracture that the existence of any deformation could easily have remained undetected by the instruments then available. We know today that this deformation largely disappears when the load is removed.

Experiments show that in almost all solids deformation behaves that way if it is small enough. We say that deformation in solid are elastic when they are recoverable. The word elastic implies springing back – a behaviour readily observable in springs and rubber bands. Springs are not used so much to sustain loads as lo store elastic energy, the form of energy which is associated with elastic deformation. Since the deformation is recoverable elastic energy is recoverable too, and it can be released either gradually, as in watches, or suddenly as in trigger mechanisms.

Objectives

After studying this unit, you should be able to

- understand the effect of force on solids qualitatively as well as quantitatively, and
- understand how defects influence the mechanical properties of the materials.

6.2 DEFINITIONS

6.2.1 Stress

The intensity of the force (force per unit area) developed within the material is called the **unit stress,** or often simply the **stress. The average** unit stress on any plane in the homogeneous member subjected to an axial load may be evaluated by dividing the total force acting upon that plane by the area of the plane section. Expressed in equation form, the relation is

$$S = \frac{P}{A}$$

where,

S = unit stress,

P = total force, and

A = area.'

In metric system stress is usually expressed in kilogram per square centimeter.

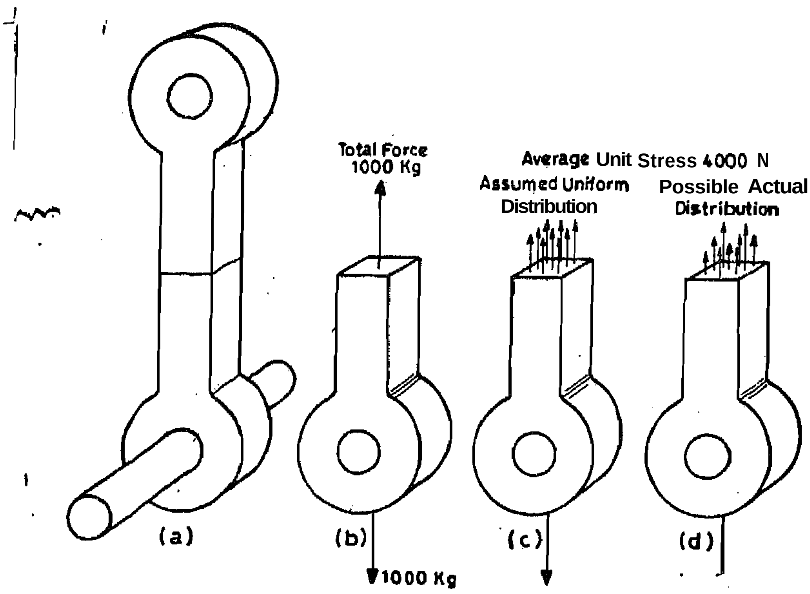

Figure 6.1 : Free-body Diagram and Stress Distribution

In Figure 6.1, 500 kg weight is put on the bar and area of the cross-section 1/2 cm X 1/2 cm. Thus stress will be then 2000 kg/cm^2. In real situation the maximum unit stress at point B will exceed 2000 kg/cm^2 because stress will not be uniformly distributed. Actually not all of the particles in cut section will offer equal resistance to the force, and distribution of stress might be as shown in Figure 6.1 (c). If the distribution of stress is not perfectly uniform then $\frac{P}{A}$ gives only an average value of the unit stress on the plane, which is always less than the maximum intensity of stress. Irregularities, cracks and other points of discontinuity all tend to cause stresses much greater than the average. Similarly, if the load is in not axial or if bending is present because of any other cause, the stress will not be uniformly distributed, even in an ideal material without flaws. The increase in stress caused by the bending action may be determined from flexural formula

$$S = \frac{MC}{I}$$

where,

S = Unit stress,

M = Resultant moment of the forces producing bending,

C = . The distance from centroidal to the point where stress is desired, and

I = The moment of inertia of the area with respect to centroidal axis.

This equation indicates that the stress due to bending varies with the distance from centroidal axis of the section and is greatest at the outside. The definition of stress is general and is independent of nature of the material. The response of the material to a given stress however may be elastic, elastomeric, plastic and viscoelastic as discussed in latter sections.

Similarly, twisting or buckling tendencies will produce a non-uniform stress distribution throughout the member. The stress in the bar in Figure 6.1 is called normal stress because it acts normal (at right angles) to the plane. Normal stress is designated as tension if it tends to stress the object and compression if it tends to shorten the object.

There are tangential or shearing stresses. The stress on the inclined plane indicated in Figure 6.2 may be determined by first drawing a free-body diagram of one portion of the bar. By means of Triangle law or the Parallelogram law, the axial force or the inclined plane may be resolved into two components; a force normal to the plane and a force parallel to the plane. This is indicated in Figure 6.2 (b).

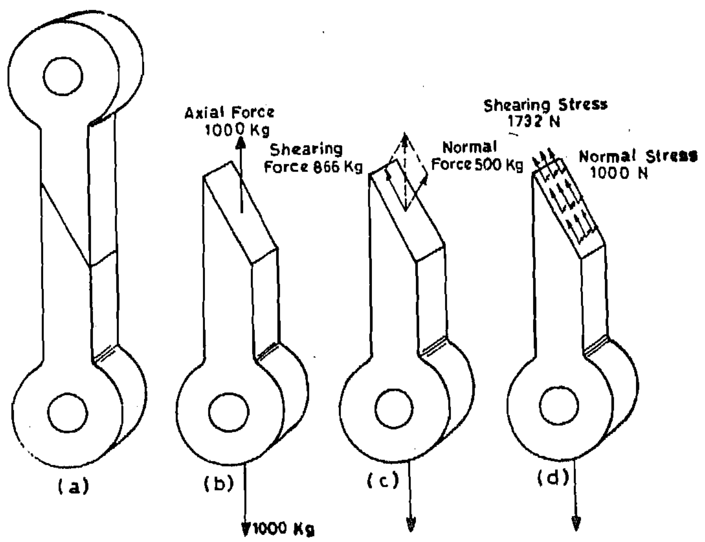

Figure 6.2 : Normal and Tangential Components of Forces

The normal component of the force will produce tensile stresses on the inclined plane, while the parallel component will produce shearing stress. For example, if the plane **makes** an angle of **60°** with horizontal, then normal component of 500 kg force will be 250 kg and tangential will 433 kg. Since the **area** of inclined square is 114 cm^2, the average tensile stresses on the inclined plane is 500 kg/cm^2 and the average shearing stress is 860 kg/cm^2.

In an object subjected to axial tension or compression, the highest average shearing stress theoretically occurs on a plane which makes **an** angle of 45° with the **axis** of the member and this stress has a magnitude of one-half perpendicular to the axis. Thus, **highest** average shearing unit stress in bar in Figure 6.1 is 1000 kg/cm^2. These **ideas can** be used to classify materials. If the material is completely weak in shear, the shearing stresses which are build-up in objects subjected to tensile or **compressive** loads may cause object to fail in shear at a relatively low load, by sliding along the plane or planes **on** which developed rather than **in tension** or compression at a higher load as would be expected if the shearing stress **were** ignored.

Materials which are weaker in shear than in tension are **known** as **ductile materials**, **while** those which are comparatively weak in tension are called brittle materials.

If the cylindrical specimen is twisted, the specimen loaded **in** torsion the maximum shearing stress is developed on planes at right angles to axis of objects, and the **maximum** tensile and compressive stresses are developed on planes inclined, at an angle of 45° with the axis. **Therefore,** a ductile material loaded in torsion will fail on the plain of maximum shearing stress. **The** effect of the stress does not always result in failure of the material but may result in change of shape **and dimensions.**

6.2.2 Strain

The change in the dimensions of the material is called strain. This is also called **deformation. The** bar in Figure 6.1 will stretch as the **load** is applied **and** the amount which is lengthens is called total strain. Unit strain, also called unit deformation or simply strain is the total change in length divided by the length over which that change occurs. **This** can be written in the form of equation

$$\varepsilon = \frac{e}{L}$$

where,

ε = unit strain,

e = total elongation, and

L = length over which elongation **occurs.**

Since e and L have **same dimensions (i.e.** length), **ε is dimensionless.**

6.3 HOOKE'S LAW AND ELASTIC CONSTANTS

Many technically important materials; for example metals, ceramics. crystalline polymers and wood behave elastically at relatively small stresses and at ordinary temperatures. Elastic deformation is simple because time dependence is negligibly small, and the temperature aspect (discussed in Unit 8) is partially included in the constants. Hooke's law quantitatively describes the linear relationship between stress and strain characteristics of elastic deformation. Hooke's law is mathematically expressed as $\sigma = E\,\varepsilon$ for tensile deformation. The proportionality constant E is known as **Young's modulus.**

To see how the equation is used, let us calculate the strain in different materials produced by the same stress 10×10^7 N/m^2.

Using the literature value for steel we find that

$$\varepsilon = \frac{\sigma}{E} = \frac{10 \times 10^7}{20 \times 10^{10}} = 0.50 \times 10^{-3}$$

whereas in a specimen of Nylon because of its lower Young's modulus the strains are much more.

$$\varepsilon = \frac{\sigma}{E} = \frac{10 \times 10^7}{0.3 \times 10^{10}} = 33 \times 10^{-3}$$

A material is said to be anisotropic if the value of a given property depends on the direction in which the testing is performed. Equation is continued to be valid for a direction provided that the value of E is appropriate for a given direction. The anisotropy of the material arises from differences in atomic distribution for various directions of testing, The anisotropy of the material arises from differences in atomic distributions for various directions of testing, Similar anisotropy exist for most of the crystals such as copper, but ordinary copper is polycrystalline, that is it consist of a multitude of tiny crystals. Because these crystals are usually oriented at random a polycrystalline specimen is generally isotropic and value of E is same for all directions. These directional properties is treated on the basis of simple geometry and discussed in the next section.

We have defined stress as force divided by area, We know that a force is a vector quantity. When force vector is normal to the area, we call such a stress a "normal stress" and it is either tensile (+) or compressive (–). We all have some understanding of vector quantities and therefore it is easy for us to understand that a force vector can be resolved into number of components.

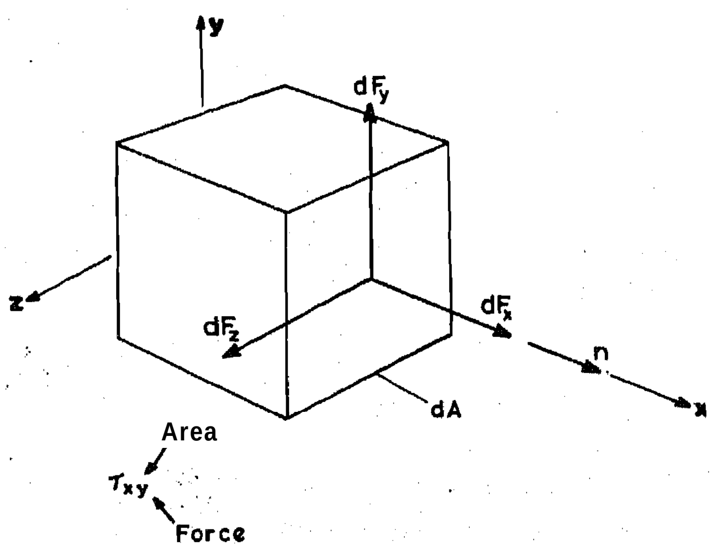

Figure 6.3 : The Total Force dF is Resolved into Three Components Normal and Tangential to dA

The simplest way for us to understand the action of an arbitrarily directed force is to resolve it into orthogonal xyz components such that one of the components is normal to the area while other two are tangential to it. The force component normal to the area produces normal stress (tensile or compressive). The tangential components, which exert shearing type actions, produce shear stresses. Thus the total stress on the X face of Figure 4.3 is replaced by three stress components

Normal stress, $\frac{dF_x}{dA_x} = \sigma_{xx}$ (as $dA \rightarrow 0$)

Shear stress, $\frac{dF_x}{dA_x} = \tau_{xy}$ (as $dA \rightarrow 0$)

Shear stress, $\frac{dF_z}{dA_x} = \tau_{xz}$ (as $dA \rightarrow 0$)

where,

a denotes normal stress while τ denotes shearing stress.

For each stress component there are two pertinent directions, that of the area normal and that of the force component. The double-subscript system used here identifies these two directions. The first subscript refers to the area normal (i.e. these areas), second subscript refers to the force direction. The sign of stress is determined by the signs associated with the area normal and the force component. Area normal and force component can be either positively (+) or negatively (–) directed. A stress is positive when both signs are positive $(+x+)$ or both are negative $(-x-)$ and the stress is negative when one sign is positive and the other is negative. Above three stresses can be now defined for generalised case and the stress at a point conveniently represented as a "Stress matrix" which often labeled $[\sigma_{ij}]$ or nearly, σ_{ij}. The stress at a point is shown in Figure 6.4.

$$\begin{array}{cc} & \text{direction} \\ & \begin{array}{ccc} X & Y & Z \end{array} \\ \text{Face} \begin{array}{c} X \\ Y \\ Z \end{array} & \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} = [\sigma_{ij}] = \sigma_{ij} \end{array}$$

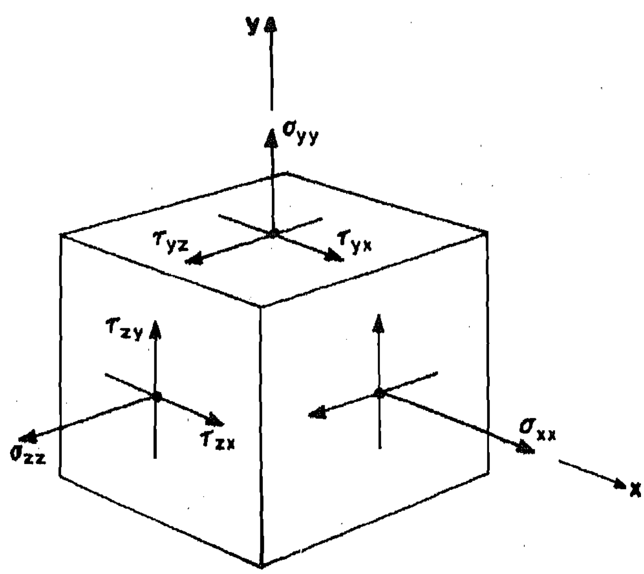

Figure 6.4 : Stress at a Point
Although Shown on a Cube, this Cube has Zero Volume. The Length of each Side has been Reduced to Zero

Equilibrium Equations

Whenever we want to study the stress in a structural member, we must establish a co-ordinate system. We chose a co-ordinate system that allows us to calculate stresses due to load. Generally this problem is not easy to solve unless proper mathematical formulation is done. Fortunately, the concept of "equilibrium" provides all the information we need and makes it easy to analyze problem mathematically. Equilibrium is applied to forces and moments not to stress. Hence we must multiply stresses by the areas over which they act to gel forces. We will not give detail proof here and write the equations obtained in equilibrium condition. These equations are

$$\frac{\partial \sigma_{xx}}{\partial x} dx\, dy + \frac{\partial \tau_{yx}}{\partial x} dy\, dx = 0,$$

$$\frac{\partial \sigma_{yy}}{\partial x} dy\, dx + \frac{\partial \tau_{xy}}{\partial x} dx\, dy = 0$$

Here we assumed that the only forces acting in the element were those due to the stresses acting on the surface of the element. In general there may also be force acting on the mass within element, force due to external fields of gravity, magnetism, or acceleration. These forces, which act directly on the body of the element as opposed to acting on the surface of element are called body forces. We simply denote them as x, y, Z. The body force per unit volume acting in the x, y, z directions respectively. Now the following equations can be written for equilibrium.

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + X = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + Y = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + Z = 0$$

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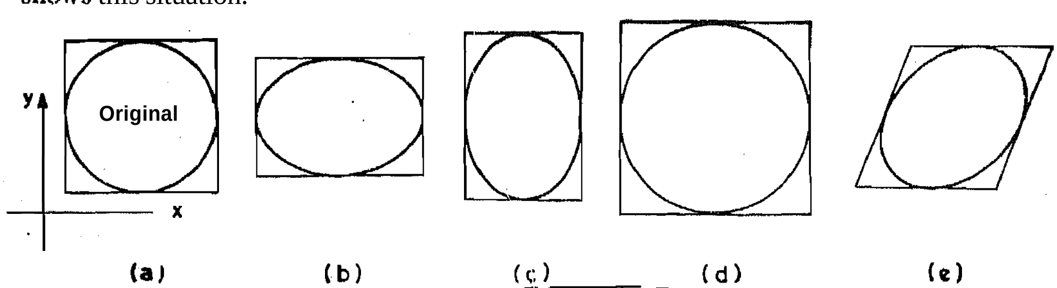
Is it possible to determine the stress in a body if one does not know the corresponding strain?

Strain

Strain, in general, is associated with motion. Strain at a point must be associated with the motion of that point. However, if every point on the body undergoes the same motion or displacement then displacement of a single point cannot define strain at that point. Strain involves the relative motion between two or more point?. It is only when two points undergoes different motion then we have deformation or strain. We have defined strain as the change in length divided by the original length. In a more general sense, we must consider the overall distortion of a volume of a material. One of the best ways to visualise strain is to consider the change in shape and size of a minute cubic element. Figure 6.5 shows this situation.



Mgure 6.5 : Shows How an Originally Square Element can be Dislortcd into Various Sizes and Shapes

For the simplicity we will consider the two-dimensional case for the strain tirst. For our analysis we will consider that there is zero strain in the Z direction and we will consider this "plain strain".

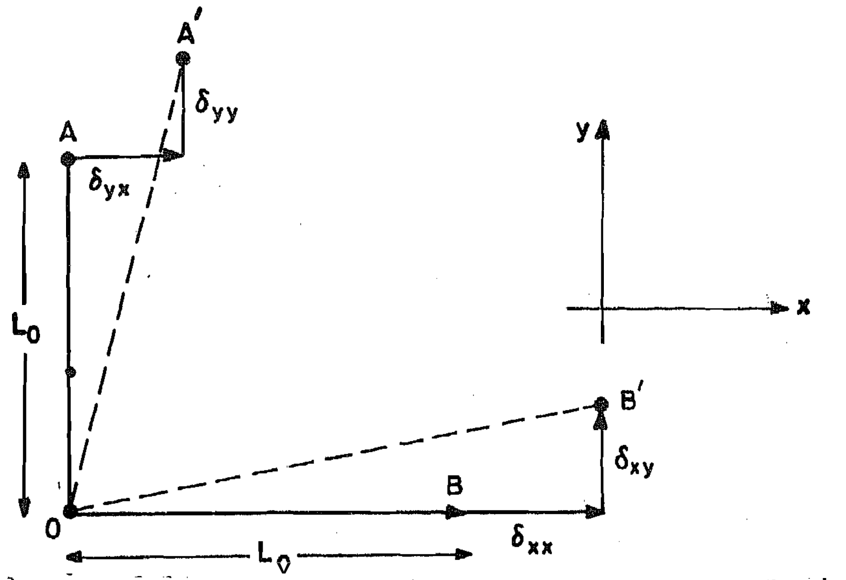


Figure 6.6 : Strains Defined in terms of the Motion of the Ends of Two Orthogonal Vectors

Consider now the deformation relative to the xy axis. Figure 6.6 shows there are two vectors **OB** and OA in the x and y directions respectively. This could represent side of the cube in the figure. Let the original length be L. Following deformation, point **A** has moved to **A′** and **B** has moved to B′. Each point has undergone a displacement in the x direction by d_{xx} and in the y direction by d_{yy}. In each case, one displacement vector is parallel to the original vector and one is at right angles to it and this is represented by d_{xy} and d_{yx}. The first subscript defines the direction of the original vector and second defines the direction. Thus displacement with the like subscript (xx, yy) produce normal strain or extension or compression.

Thus Normal strain, $\varepsilon_{xx} = \dfrac{d_{xx}}{L}$ and $\varepsilon_{yy} = \dfrac{d_{yy}}{L}$ as $L \rightarrow 0$

The displacement having unlike subscript (xy or yx) produce shear type deformation of the material and/or gross rotation of the material. We will define shear strain as follows.

$$\gamma_{xy} = \gamma_{yx} = \frac{d_{xy}}{L_o} + \frac{d_{yx}}{L_o} \text{ as } L \rightarrow 0$$

When we consider a strain it should be geometrically compatible. The only requirement for geometrical compatibility is that voids or such defects are not produced. Thus we must have continuity.

If we consider a body in the xyz co-ordinate system, we can define the motion of every point (x, y, z) in the terms of component of displacement in the x, y, z directions

$u\,(x, y, z)$ = displacement of point x, y, z in the x direction

v (x, y, z) = displacement of point x, y, z in the y direction

w (x, y, z) = displacement of point x, y, z in the z direction

Geometric compatibility requires that the displacements u, v, w be continuous functions of x, y and z. Any discontinuity will represent either fracture or evaporation of mass. Since strain is involved, the relative motion between two or more point strain (ε) as a change in length divided by the original length. In order that the compatibility be of use, we must formally relate the displacements u, v and w to the strains in the material. Returning to the two-dimensional case, Figure 6.7 shows two vectors **OA** and OB of initial length dy and dx. After deformation, the vectors have moved to the primed positions. In terms of displacement functions for u and v we see that point O has moved to u to the right and v upwards. If A and B also moved u and v, there would be no deformation, only translation,

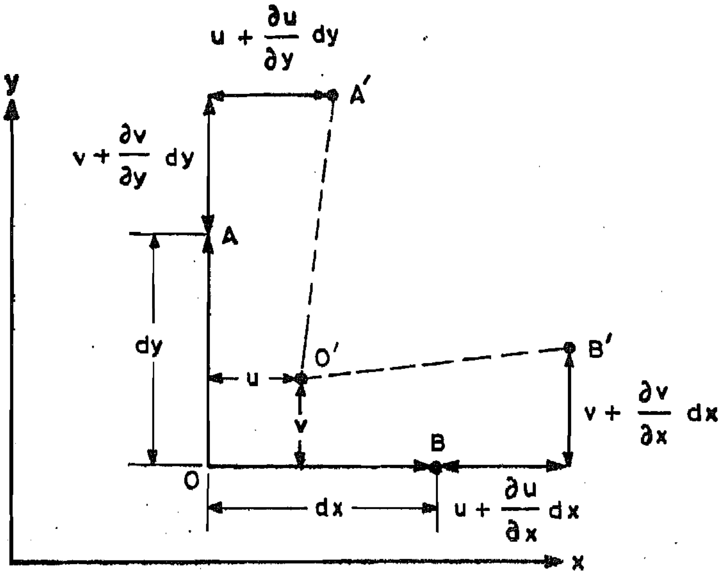


Figure 6.7 : Geometry to **Determine** Strain **in Terms of Displacement Functions**

However, point B moves to the right not only u, but in addition it moves an amount equal to the variation in u associated with the length dx (the partial derivative term). If we compare Figures 6.6 and 6.7, we can immediately write the strains in terms of displacements.

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}$$

$$\varepsilon_{xy} = \frac{\partial v}{\partial y}$$

$$v_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

Thus we can calculate three strain quantities from only two initial equations of motion (u and v).

In general, one cannot find a satisfactory solution because of the mathematical complexity and approximate solutions must be used. The most recent efforts have involved use of Finite Element Method (FEM); together with large computational capability to obtain solutions that are approximately correct.

6.4 PLASTICITY

Strain and stress usually, but not always, accompany each other. Strain may be produced without stress, for example by temperature change. Similarly, if a bar has its ends firmly clamped so that it cannot contract, and is cooled, axial stress will be produced without axial strain. Strains as well as stresses, are tensile, compressive or shearing. If the strain which accompanies a stress vanishes upon removal of the stress, elastic action is said to occur.

However, if a residual strain remains after the stress is removed, the action is said to be plastic or Inelastic. Most of the materials are nearly elastic at low stresses and are inelastic at higher stresses. However, probably no material would be found to be absolute elastic in any range of stress if sufficiently sensitive devices were used to measure the strains. If a material is loaded part way to failure and the measurements of load and deformations are taken for both increasing and decreasing loads characteristics are obtained. These are called stress-strain diagrams and they reveal whether or not the action was elastic. If the unloading curve come back to the origin action was elastic. This is shown in Figure 6.8.

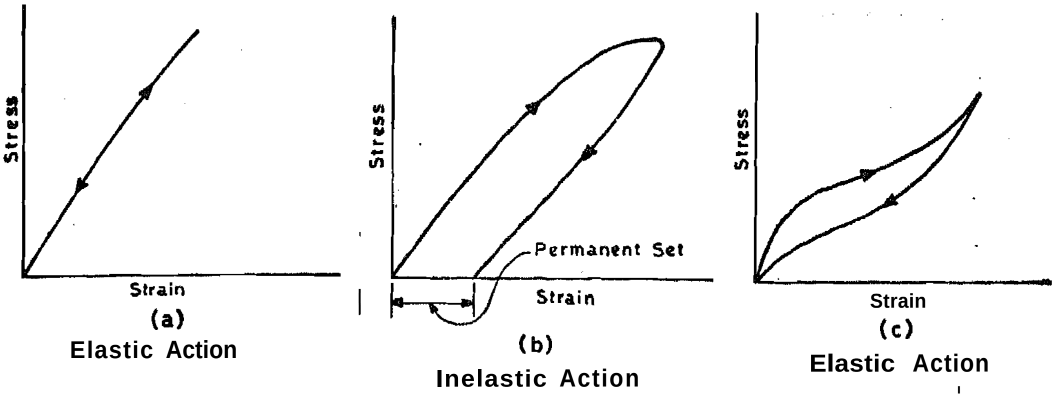


Figure 6.8 : Stress - Time Curves tor

a) **Elastic Material** where Loading **and Unloading Curves** Match
b) **Non-elastic** Action
c) A **Possible** Type **of Elastic** Action which **is Typical** for Some **Rubbers**

In general, plasticity denotes the capacity of a material for being molded or worked into shape under pressure and related to viscosity. Metals become more plastic as the temperature,increasestowards the melting point and concrete and similar materials are plastic before they harden.

Plasticity of the **material** is used in many ways. A typical example is the forming of part of an automobile such as its roof. In the process a powerful hydraulic press bends a flat sheet of steel over the contours of suitable massive steel called as die. When the pressure is removed the steel sheet retains essentially the form that was imparted by die.

The elastic limit of a materials is the highest unit **stress** to which the material may be subjected without permanent deformation,The exact elastic limit of a material cannot be determined. Its apparent value will depend upon **the** sensitivity of the measuring instruments used, and the length of **time** the load is applied. Usually stress and strain are proportional. Also each of the manifestationsof non-elastic action requires a different technique of measurement, so no single property **has** been devised for **comparing** the **plasticity** of materials. **This** is Hooke's law based on observation and is approximately true for **many** materials at low stresses, but in general it is not true at higher stresses. The range of stress over which **Hooke's** law applies is evident on the stress-strain diagram as the range of stress with in **which the** diagram is straight line.

As mentioned earlier most of **the materials** show inelastic characteristics if load is applied for **very** long periods. In this case stress types of inelastic actions have been observed. These are slip, creep and fracture. **The** term fracture is used to **denote** rupture, or complete separation of the materials into two or more **parts.**

This distinction between slip and creep may be **made** on the basis of time dependence of **inelastic** action. If the inelastic strain takes place within a few minutes after the stress is applied and if no increase in **strain** occurs with time while stress remains, the action *is* called slip. However, if the strain continues to increase so long as the **stress** is applied creep is said to **occur.** This is illustrated in Figure 6.9. The characteristics ace obtained as **follows :**

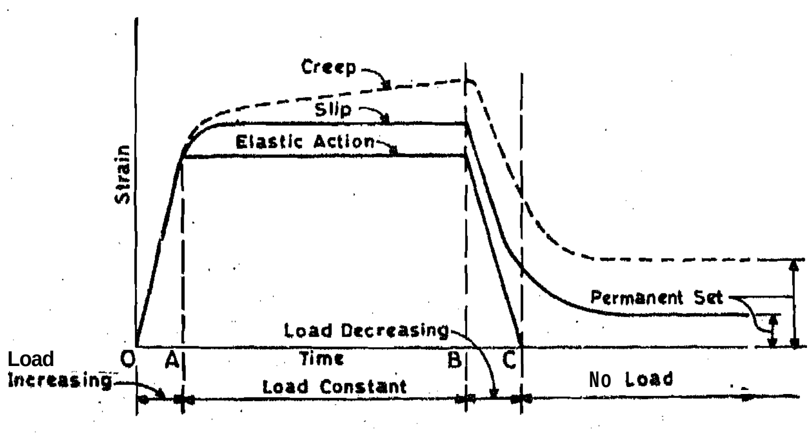


Figure 6.9 : Strain-Time Diagrams Indicating Elastic and Inelastic Behaviour

A specimen is loaded and the deformation is noted at certain intervals. For example, to determine the characteristics of material, under tension, a vertical rod of the material is held at the lower end. The elongation of the rod is measured at definite intervals and values are converted into unit strains. Values of unit strain are plotted as a function of time as shown in Figure 6.9. This figure shows that as soon as loading is stopped, there is marked difference between behaviour in three cases. The lower curve indicates elastic action, in this case the strain remains constant during the interval in which stress is constant and then remain constant for the duration of the loading period.

In the case of slip which is the middle curve, the strain increases for a short time after the stress becomes constant and then remains constant for the duration of the loading period. After the load is removed, there remains a permanent deformation, which is equal to (approximately) the additional deformation under stress.

In the case of creep the strain or deformation continues to increase under constant stress, resulting in a larger permanent deformation upon removal of load. Thus, if load is not removed then fracture may result. Thus, the fundamental distinction between creep and slip is the rate of deformation under constant stress. If the rate becomes zero shortly after the maximum stress is applied the phenomena is called slip, if it does not become zero then it is called creep. Naturally, creep is more dangerous as it may result in failure.

To understand this phenomena one has to examine the material at microscopic level. This is done in the next section. Before that we will overview some more points/terms in this phenomena. It has been observed (after physical characterisation) that slip occurs by the sliding of adjacent layers of particles within the materials. This movement takes place along certain definitely oriented planes in the materials. The intersections of these planes with a polished surface of the materials appear as lines known as slip lines.

Crystallographic studies show that these planes have weak structure in space lattice. Also there is a frictional force which opposes this motion of planes and thus deformation is finite. For most materials there is a limiting stress below which slip is not appreciable, this of course is the elastic limit. However, this stress limits cannot be set for creep, although is known that rate of creep increases with an increase in stress.

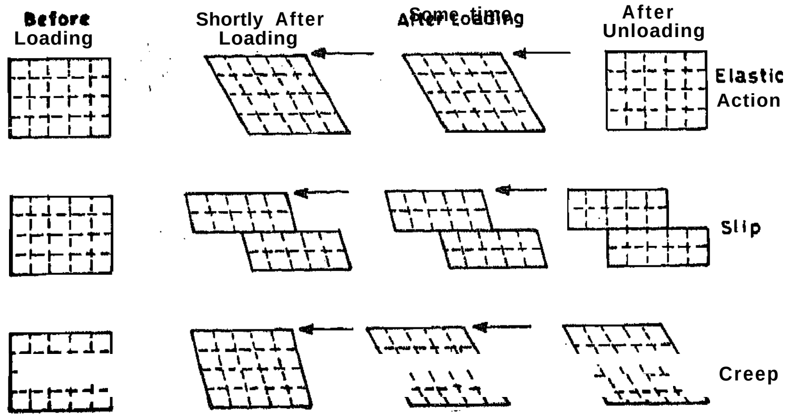


Figure 6.10 : **Diagrammatic** Representation of Elastic **Action, Slip** and Creep

Normally one would expect creep as just an extension of slip phenomena in the crystal. But examinations of such crystals do not show slip lines in the small or well distributed that no slip deformation included initial slip followed by long-time creep. Figure 6.10 gives representations of all these microscopic structure. Most metals will creep at elevated temperatures or temperatures approaching their melting points. Some of the metals with low melting points, such as lead, tin and their alloys will creep at ordinary temperatures.

Twins

In the last section we saw that material is deformed by applying forces. We also saw that macroscopically deformation is the movement of atoms lying in certain planes past each other. The result of the atomic displacements depends on the amount by which the atoms have moved. Now, consider a situation in which a crystal is deformed by applying a shear couple to the material. If all the displaced atoms move into the adjacent plane the material has same structure after deformation as it had before deformation. If it is possible, however, to arrest the movement of atoms in one plane after the atoms have moved into the adjacent but unequivalent position, then a deformation twin is produced. This can also occur under some other circumstances. Slippings are energetically favourable compared to small displacement. For example, at low temperatures atoms can "shift" slightly under stress to produce new arrangements such as twin. The process of slip would require a much higher stress. The mechanism of twinning results in appreciable macroscopic shear even though the individual atoms shift less than an interatomic distance.

It should be noted that the atom lying further away from twin junction must be displaced by larger amounts than those lying close to the junctions. In order for twinning to occur, the energy barriers separating the adjacent sites must be relatively small, lest the deformation stresses cause the crystal to rupture. It follows from this that deformation twinning can occur only parallel to energetically favoured planes in the crystal.

Typically twinning has the following characteristics:

1) The stress is large compared to slip.
2) A tiny "nucleus" of twin structure must form.
3) Growth then occurs by dislocation mechanism (discussed in the next section)
4) Rate of formation of twins may be extremely high even at low temperature.

Thus, even though the stress for twinning is higher than that for slip, special circumstances can favour twinning. An important instance is the deformation of hexagonal metal at low temperature. The twinned region has orientation more favourable for slip and deformation can continue by slip in the region. In most of the cases both twinning and slip occur as deformation of the metal proceeds. In many cases twinning is favoured by higher strain rates and/or low temperatures. Most metal form twins when deformed at high rates (for example when fonned by explosive charges) especially at subzero temperatures.

6.5 VISCOUS DEFORMATION

Rate of deformation is proportional to the applied stress. We can write expression for strain rate $\frac{d\varepsilon}{dt} = \frac{\sigma}{\eta}$ where η is the coefficient of viscosity. This equation with suitable values for η describes deformation of a wide variety of materials. It applies virtually to all liquids including liquid metals, liquid glass and typical molten polymers. The viscous bebaviour of an amorphous polymer is different from that observed in glass because of the difference in structure. Silica glass has network in which each atom is strongly bonded in its position. Individually bonds are occasionally broken by thermal energy, in presence of an applied stress the bond reforms with different neighbours and thus permit deformation to *occur.* In a polymer, on the other hand, the thermally activated process is not the breaking of strong bond in the chain itself by not changing of the angle between carbon atoms in the chain (which would result in the elastic deformation) and the slipping of the chain relative to its neighbours (permanent deformation). Consequently, when a stress is applied to amorphous polymer the resulting strain rate has both elastic and viscous component. This is called **viscoelastic deformation.** Even crystalline material such as steel can deform in a viscous manner but only under low stress at high temperature approaching the melting point. Creep obeys law of viscous flow. The rate of deformation is proportional to the stress during the major part of deformation by creep,

The term viscoelastic deformation aptly describes the combined effect of the two mechanisms in determining observed behaviour. Viscoelastic behaviour is particularly evident in amorphous polymers. If an elastic material is given an instantaneous strain ε and (after any arbitrary time) the stress σ, in deformed state is measured, the ratio is Young's modulus

$$E = \frac{\sigma}{\varepsilon}.$$

The result is independent of time. When the same type of experiment is done with glassy polymer slightly above its glass transition temperature the corresponding modulus $M(t)$ must be written as

$$M(t) = \frac{\sigma(t)}{\varepsilon}$$

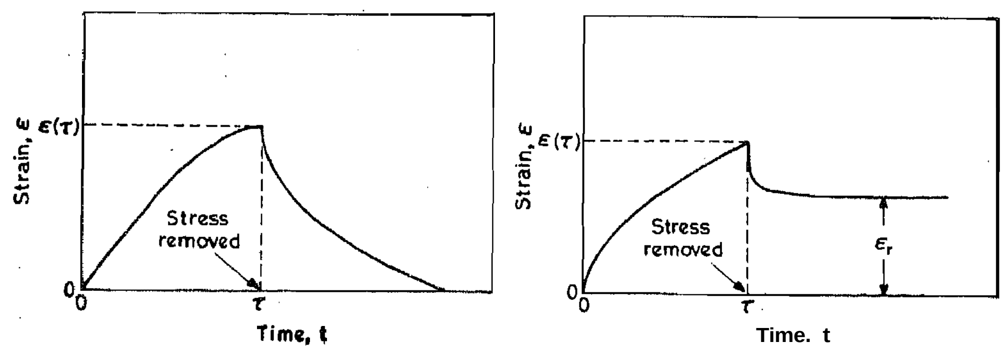
As time *t* of testing is increased, M (t) gradually decreases because the stress a decreases as the polymer gradually changes its internal structure in response to the constant strain ε. The memory of viscoelastic material for its past history can significantly affect behaviour. Shear stress (rather than tensile stress) are usually employed to measure this property and thus the shear relaxation niodulus is the quantity usually determined.

$$3\,G\,(t) = M\,(t)$$

The "memory" of a viscoelastic material for its past history can significantly affect its behaviour. The consequence of previous history is shown in recovery following a period of deformation under constant stress (creep). The initial creep strain ε (t) after a time *t* under stress a can be expressed as

$$\varepsilon\,(t) = \frac{J\,(t) \times \sigma}{3}$$

where *J* (*t*) is shear-creep compliance and is roughly inverse of G (t). Even though the deformation after given time *t* of stressing is partly viscous and partly elastic, it can be essentially completely recovered under suitable conditions (i.e. cross linking in the polymers), known as the **elastic aftereffects.** However recovery is only partial and residual permanent deformation remains (Figure 6.11b).



(a) Complete

(b) **Partial** Recovery of M Initial Deformation *E* (*t*)after Removal of Stress at Tie, *T*.

Figure 6.11 : Schematic Representation of

SAQ 2

Explain why G (*t*) is known as a modulus analogous to Young's modulus'?

SAQ 3

Give an example of

a) plastic deformation, and

b) elastic deformation that occurs during the use of a bicycle.

6.5 DISLOCATION THEORY

In most of our discussions above we have assumed that crystal is perfect. The applied load may be resolved into a force normal to the slip plane and two shearing forces in the slip plane – one in the slip direction and one normal to it. Studies of single crystals have shown that there is a critical **ultimate** strength in shear **which** is independent of the normal stress. **The** shearing strength is dependent of the **temperature,** becoming less **as** the temperature is increased. This finding is reasonable in the **atoms** at high temperatures. Thus favouring movements of **atoms** under applied force.

Two-dimensional model of a simple cubic lattice for which **the** slip plane is assumed to be **perpendicular** to the page and the slip direction along d-d if a force is applied to the group of atom above a-a, the group **will tend** to be displaced to the sight and resistance will he developed as a result of the changes in all inter-atomic **distances** which cross the line a-a. **Of** the many changes which are involved two are shown in Figure 6.12. As a result of the increase **in** distance b-c, the **inter-atomic** force may change by amount F, . **and** as result of **the** decrease in distance b-d, the inter-atomic force may change hy amount F_2.

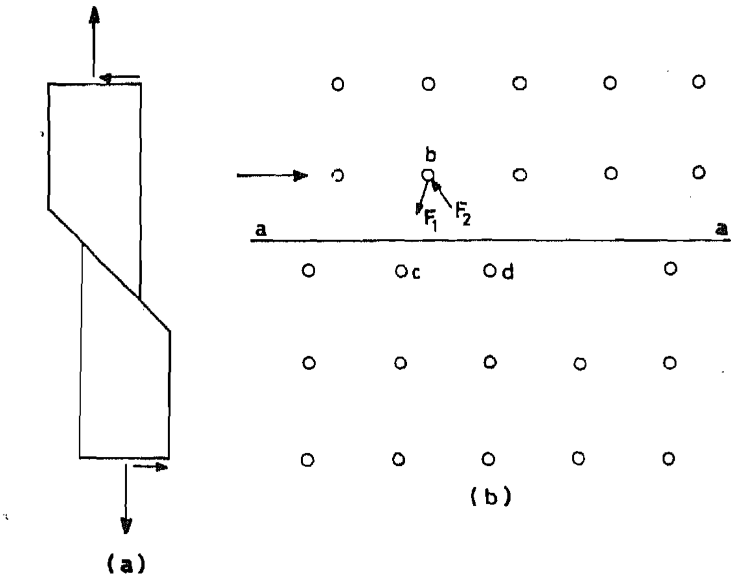


Figure 6.12 : Two-dimensional Models of Slip

By evaluating these forces in general terms **as** functions of the displacements and equating them to the applied force, a theoretical stress-strain diagram can be developed far the material. In **its** simplest form **and** with certain approximations this becomes

$$S = A \sin \frac{2\pi x}{d}$$

where,

S = Shearing stress,

A = Ultimate Strength,

x = Displacement, and

d = Atomic Spacing.

Thus when $x = d$ the stress is zero, which corresponds to the condition of the upper **block** of atoms being displaced one atom distance to the right, where they would again be in equilibrium. The **quantity** $\frac{x}{d}$ is nothing **but** unit strain ε. Hence

$$S = A \sin 2\pi \varepsilon$$

If this is differentiated with respect to ε, **results**

$$\frac{ds}{d\varepsilon} = 2\pi A \cos 2\pi \varepsilon$$

$\frac{ds}{d\varepsilon}$ for small values of ε is defined as modulus of elasticity **in shear.** Hence for ε equal to zero

$$E = 2\pi A$$

$$\text{or} \quad A = \frac{E}{2\pi}$$

This equation indicates that the ultimate strength in shear should be about 16% of the modulus of elasticity. For example, iron has a theoretical ultimate strength is shear of approximately (1,750,000 psi). This value is much higher than normally obtained. In fact most of bulk the commercial metals have strength of the order of one percent of the theoretical strength. However, experiments on tiny wires of materials have given strengths approaching their theoretical strength. This discrepancy between calculated and actual strength can be explained on the basis of imperfections. Among all imperfections which we have studied in the last unit dislocation dominates this phenomena. The dislocations are of several kinds, but each is a point of weakness in the lattice relatively large strain at a relatively low stress level. It would be expected that strengths would agree with the theoretical values if the dislocation could he eliminated from the structure.

One can explain this on the basis of plane model of hard balls as an atomic structure. Figure 6.13 (a) shows this model with imperfection which will result in dislocation. Looking at the rows of balls A & B we notice that ball **A,** fits exactly in the hollow between B_1 and B_2. So would any ball preceding A_1 (not shown in figure). In like manner, ball A_7 fits exactly in the hollow between B,, B_9 and so would any following **A,** (not all shown). At the approach to the center, however. there is a growing misfit, because row B has one ball more.

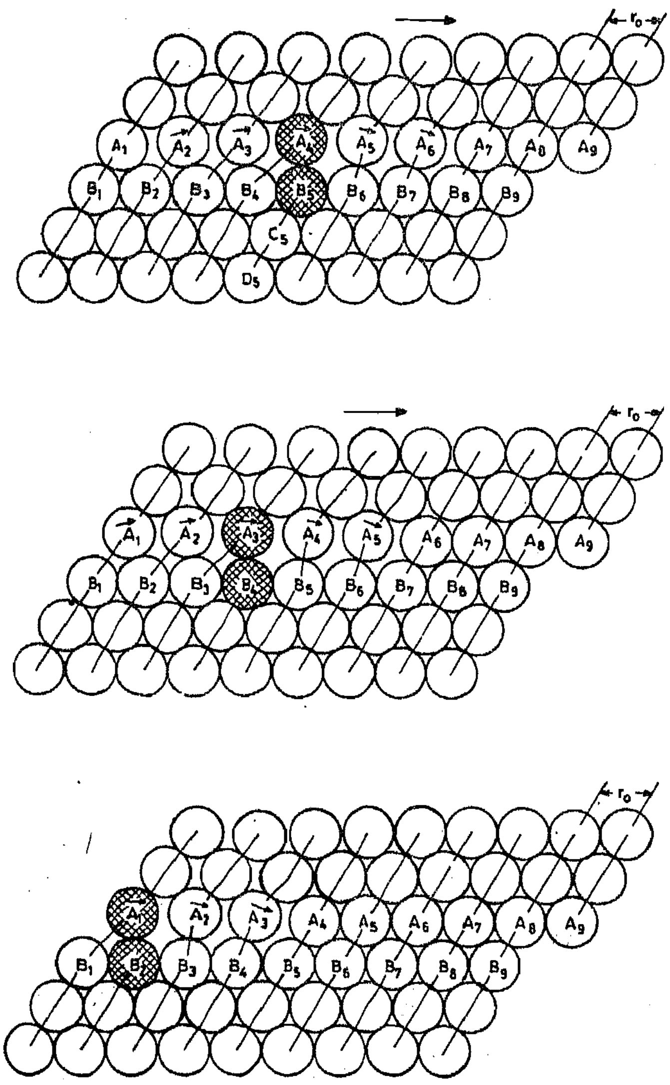


Figure 6.13 : Slip in a Plane Model Containing a Dislocation

a) Position of the Dislocation before Slip

b) Position of the Dislocntion after Slip of One Atomic Movement

c) Position of the Dislocation before Leaving the Crystal

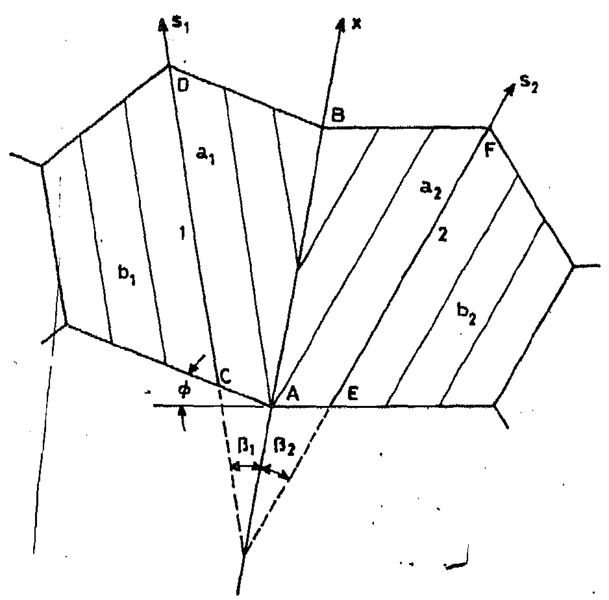
This misfit is greatest at the and of extra row when: ball A, is atop ball B_8. This kind of misfit is generally called a dislocation. These figures can explain why dislocation can induce stop to occur at much lower shear stress than theoretical value. We observe in Figure 6.13 (b), if all balls A were atop balls B, no stress would be necessary to achieve slip. Row A will try to go to equilibrium position of stable equilibrium. Due to this d l balls would simultaneously reach new positions of stable equilibrium. Now of resume slip a shear stress of theoretical magnitude will be necessary.

However, situation is quite different in the presence of dislocation as shown in Figure 6.13 (a), only one ball A, is atop another ball. Ball A_5 and A_6 are already on there way down and ball A_2 and A_3 are still on their way up. The process of slip is now local (at atom A) instead of going on simultaneously everywhere. Here every ball will move, instead of, as is true for perfect closed packed aggregate. Thus, at each step dislocation moves one atomic distance to the left involving next pair. Every ball A has now slipped past the underlying ball B and has slid down into new position of stable equilibrium Row A has undergone a complete shear movement and the upper portion of the aggregate has been displaced exactly one atomic distance or the right. Figure 6.13 (c) shows that dislocation is moved by one atomic distance and not vanished thus favouring slip under shearing stress.

6.7 DEFORMATION OF POLYCRYSTALLINE MATERIALS

When polycrystalline materials are deformed the aftereffects are different. When these materials are deformed in elastic limits the properties are the averages of the individual properties of crystallites comprising it. One of the principle factors that complicates the situation is that the crystallites usually are not completely at random.

Another factor that affects the elastic properties of polycrystalline materials is that the grain boundaries have different mechanical properties than the crystallites. The large concentration of imperfections at grain boundaries also means that some plastic deformation invariably occurs under an applied stress. The consequence of this is that the only meaningful calculations possible are based on averaging processes which take the various factors into amount in an approximate way. When these materials are subjected to harder shear slip mechanism discussed above does not hold true. Although most of the industrial metals are coherent aggregate of crystals. In such aggregates the cohesion along the grain boundaries forces the individual grain to slip in several directions. In other words, plastic deformation in polycrystalline aggregates involves a multiple rather than a single slip mechanism. Figure 6.14 shows two grains having boundary AB aligned with the direction of pull.



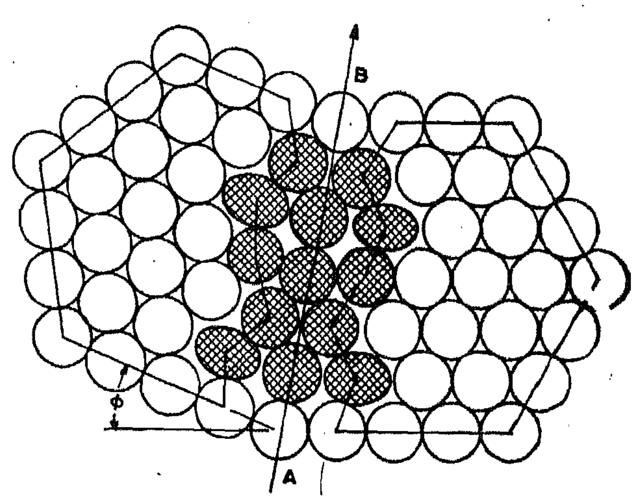


Figure 6.14 : Configuration of Two Adjacent Grains (Schematic)
a) Slip Planes and Directions
b) Boundary Atoms in a Closed-packed Aggregate

Let *CD* and *EF* be the braces of the planes, normal to the plane of the figure. S_1 & S_2 are the directions along which the grains would slip if they were isolated crystals. In the absence of a coherent boundary AB, the two crystals would part ways. They would rotate through an angles S_1 & S_2 with respect to each other because of the tendency of S_1 and S_2 to align them with direction of pull (see Figure 6.14 (a)). It can be shown mathematically that plastic deformation of only 1% would be sufficient to pull the edges A_1 and A_2 more than one hundred atomic distances apart (and complete failure of material). However with a coherent boundary such a course of action is not possible. A graphical proof for the two-dimensional model is shown in Figure 6.14 (b).

Each grain has now slipped in two directions and in such way that the angular change caused by one slip is offset by the other. The grains match again along the boundary AB. The aggregates has been extended in the direction of pull and contracted perpendicularly to it. The extension of this argument to three-dimensional polycrystalline crystal shows that each grain now slips in at least five directions to maintain the fit with its neighbours. This proof is not given because of complications involved. The actual mechanism of plastic deformation in polycrystalline materials appears to be a great deal more complicated.

A polycrystalline metal or ceramic also deform by slip, but in this case each individual crystal experiences two types of strains. First a dislocation is stopped by grain boundary as soon as it moves across grain. Only after many dislocations have piled up at the boundary, does the local stress become high enough to produce slip in the adjacent grain.

The second restrain arises from the fact that at least five independent slip systems must operate to allow a grain to deform into arbitrary shape imposed on it by neighbouring grains. In some materials such as MgO, the number of active slips systems increases to five or more at sufficiently high temperatures. These polycrystalline materials tend to be brittle at low temperatures but become more ductile at high temperatures.

SAQ 4

When you pick up an object, such as a ruler, each of its points experiences a displacement, why is there no corresponding strain in this case ?

SAQ 5

When you bend a steel paper clip it deforms what is the mechanism by which it deforms? By what mechanism would it deform if you first cool it to –150° C and hit it with a hammer?

6.8 SUMMARY

In this unit we discussed number of mechanical properties of materials. Concepts of stress and strain were introduced. Stress is a measure of an applied mechanical load or force, normalised to take into account the cross-sectional area. Strain represents the amount of deformation induced by a stress.

Mechanical characteristics of materials can be determined from simple stress-strain tests. On stressing a material first undergoes elastic or non-permanent deformation where stress and strain are proportional. The constant of proportionality, is called modulus of elasticity , in tension and compression and is the shear modulus when the stress is shear. The deviation from linearity in the stress-strain diagram signals the onset of plastic or permanent deformation of the materials. On the microscopic levels the plastic deformation corresponds to the motion of dislocations in response to an externally applied shear stress, a process known as slip. Slip occurs on specific crystallographic planes and within these planes only in certain directions. A slip system represents a slip plane-slip direction combination, and operable slip systems depend on the crystal structure of the material.

The critical value of the shear stress is required to initiate motion of dislocations. The strength (at which deformation will occur) for single crystal depends on both the magnitude of the shear stress and the orientation of the slip components with respect to direction of applied stress. For polycrystalline materials, slip occurs within each grain along the slip systems that are most favourably oriented with the applied stress. During deformation grains change shape to maintain the coherency at the grain boundaries.

Viscoelastic behaviour is intermediate between elastic and totally viscous is displayed by number of materials sucli as glasses and amorphous polymers. These materials are characterised by a time dependent modulus of elasticity.

6.9 KEY WORDS

Brittle Materials : Materials which are weak in tension are called brittle materials.

Creep : The time-dependent permanent deformation that occurs under stress; for most materials it is important only at elevated temperatures.

Creep Rate The rate at which a material continues to stretch as fuuction of time when stress is applied at a high temperature.

Ductile Material Materials which are weaker in shear than in tension are known as ductile materials.

Elastic Deformation Deformation of the material that is fully recovered when applied stress is removed.

Engineering Strain The amount by which a material deforms per unit length in tension.

Engineering Stress The applied load or force, divided by the cross-section of the material.

Hooke's Law The relationship between stress and strain in ttie elastic portion of the stress-strain curve.

Modulus of Elasticity Young's modulus or the slope of the stress-strain curve in the elastic region.

Shear A force applied so as to cause or tend to cause two adjacent parts of the same body to slide relative to each other, in a direction parallel to ttieir plane of contact.

Plastic Deformation Deformation that is permanent or non-recoverable after release of the applied load. It is accompanied by permanent atomic displacements.

Slip Plastic deformation as a result of dislocation motion and also the shear displacement of two adjacent planes of atoms.

Strain Elongation of the specimen divided by the original length.

Stress Instantaneous load applied to a specimen divided by its cross sectional area before any deformation.

Viscoelasticity A type of deformation exhibiting the mechanical characteristics of viscous flow and elastic deformation.

6.10 ANSWERS TO SAQs

SAQ 1

Yes, it is. The stress is defined as force per unit area.

SAQ 2

The definition of $M(t) = 3G(t)$ as shown in the equation is an exact analog of the definition of Elastic modulus.

SAQ 3

a) elastic deformation of the frame, caused by the weight of the rider.

b) bending of the front wheel, caused by a hard bump against the curbs.

SAQ 4

Strain is displacement relative to neighbouring points in the body. If all points are displaced equally the strain is zero.

SAQ 5

Twinning (because this mechanism is favoured at low temperatures and high strain rates).