# UNIT 10 MACHINING ECONOMICS

## Structure



## 10.1 INTRODUCTION

Design and operation of a manufacturing system must be based on economic considerations. It is always desirable to perform a machining operation at minimum possible cost but satisfying all requirements of the machined components. Sometimes under certain circumstances (say, war like situation), one may be more interested in highest possible production rate even at higher cost per component to meet the crucial demands of certain commodities. However, one may be interested in profit rate rather than minimum cost per component, or maximum production rate. All such problems have been studied by optimization techniques. Majority of these problems are dynamic in nature.

In real life, a component undergoes many kinds of operations like drilling, milling, etc. However, in this unit, to understand the procedure, a simple case of single pass turning has been described. Although more practical case is of multipass turning. The procedure followed is simple one. Develop a model (or equation) for the given kind of problem, differentiate it with respect to feed rate or cutting speed, and then solve it to evaluate optimum cutting parameters (feed or speed).

### Objectives

After reading this unit, you should be able to

- differentiate between variable costs and fixed costs involved in any manufacturing operation,
- understand the machining situations in which a particular criterion should be employed,
- evaluate optimum machining conditions for the given criterion, and
- modify the evaluated optimum machining conditions accounting for the various kinds of constraints applicable to the type of operation under consideration.

## 10.2 ECONOMICS OF MACHINING

It is not enough to suggest a feasible procedure to manufacture the desired component, but the procedure should also be economically justified. It is a difficult problem because there are several variables that affect the economics of a machining operation. These variables are :

(i) tool material and tool geometry,

(ii) machine tool capacity (power, force, size, etc.), and

(iii) cutting conditions (speed, feed and depth of cut).

Optimum selection of these variables can seldom be achieved by a machine operator, rather it is done by the process planning engineer who has access to all relevant data. Real life problem is difficult to solve because it involves process interactions, variation in anticipated sales, etc. Such problem may be studied using dynamic programming, but the solution becomes complex. Instead of full optimization, a procedure that is usually adopted is to select conditions at each operation to yield a sub-optimized solution. These conditions can then be modified, if necessary, after reviewing the process interactions by inspection of the whole production program. This latter phase is carried out continuously using some form of flow chart or production planning chart to organize the data.

Three objective functions frequently used in sub-optimization of machining operations are :

(i) minimum cost per component,

(ii) maximum production rate, and

(iii) maximum profit rate for the operation.

In general, the lowest cost per component consideration leads to lower production rate. Sometimes, optimization process may give the machining conditions which may be beyond the capabilities of the available machine tool. Hence, in selecting the economic operating conditions, machine tool capacities must be taken into account. If the selected conditions are not available on the machine tool proposed for a particular operation, it is necessary to either change the operating conditions or review the machine tool selection by cost comparison. One should not select the machine tool of the capacity higher than the desired one. The capacity limits of a machine tool include feed, speed, power and maximum allowable cutting force (or thrust force). Further, there may be feed and speed constraints to achieve the desired surface finish on the component.

A component usually requires more than one pass of cutting for completion. For simplicity of analysis, we will analyze only a simple case of single pass turning operation.

### 10.2.1 Cost per Component, Production Rate and Profit Rate Criteria

The machining cost per component is made up of a number of different costs. The total cost ($C$) of making one component (excluding material cost) is given by

$$C = xT_l + xT_c + xT_d \left( \frac{T_{ac}}{T} \right) + y \left( \frac{T_{ac}}{T} \right) \quad \text{. . . (10.1)}$$

or,

$$C = C_1 + C_2 + C_3 + C_4 \quad \text{. . . (10.2)}$$

(Fixed charges are not taken into account because they will not affect optimization.)

where,

$C_1$ = non-productive cost per component (cost of loading and unloading the component, idle time costs and other non-cutting time costs),

$C_2$ = cost of machining time,

$C_3$ = tool changing time cost,

$C_4$ = tool cost per component,

$T$ = tool life,

$T_{ac}$ = actual cutting time;

$\left(\frac{T_{ac}}{T}\right)$ = number of tool (or cutting edge) changes per component,

$T_c$ = machining time,

$T_d$ = time required to change a cutting edge,

$T_l$ = sum of all non-productive times,

$x$ = cost rate including labor and overhead cost rates, and

$y$ = tool cost per cutting edge.

For a brazed tool tip, the cost / cutting edge

$$y = \frac{\text{Cost of tool}}{\text{No. of resharpening} + 1} \qquad \text{. . . (10.3)}$$

For the throw away tips,

$$y = \frac{\text{Cost of insert}}{\text{No. of cutting edges}} + \frac{\text{Cost of tool holder} + \text{Accessories}}{\text{No. of cutting edges over the life of the tool holder}} \qquad \text{. . . (10.4)}$$

From Eq. (10.1), it is evident that cost per component can be reduced by decreasing the loading time, unloading time, idle time and tool changing time (by employing improved fixtures, jigs, inspection gauges, tool holder, etc). Improved tool materials and tool geometry which give longer tool life values and hence would reduce the number of tool replacements and grinding costs. Increasing the cutting speed has opposing effects on the cost per component because $C_2$ decreases while the total tool costs ($C_3 + C_4$) increase (Figure 10.1). The production rate is inversely proportional to the production time per component. The total production time per component ($T_t$) is given by

$$T_t = T_l + T_c + T_d \left(\frac{T_{ac}}{T}\right) \qquad \text{. . . (10.5)}$$

(a) (b)

**Figure 10.1 : Variation of (a) Time per Component, (b) Cost per Component, with Cutting Speed [*Armergo and Brown, 1969*]**

As for minimum cost, decrease in $T_l$ and $T_d$ will increase the production rate. Increase in cutting speed will reduce $T_c$ but it will increase the tool changing time per component (tool life decreases at higher cutting speed); a minimum time per component ($T_T$) (or maximum production rate) will therefore result as can be seen in Figure 10.1(a).

The profit rate ($P_r$) is expressed by

$$P_r = \frac{I - C}{T_t} \qquad \text{. . . (10.6)}$$

where, $I$ is income per component excluding material cost and $C$ is cost per component excluding material cost.

Using Eqs. (10.1), (10.5) and (10.6), profit rate can be written as

$$P_r = \frac{I - x\left[T_l + T_c + \left(\frac{T_{ac}}{T}\right)T_d\right] - y\left(\frac{T_{ac}}{T}\right)}{T_l + T_c + T_d\left(\frac{T_{ac}}{T}\right)}$$

or,
$$P_r = \frac{I - y\left(\frac{T_{ac}}{T}\right)}{T_l + T_c + T_d\left(\frac{T_{ac}}{T}\right)} - x \qquad \ldots (10.7)$$

The variables which reduce the cost per component and increase the production rate will increase the profit rate. In general, the speed for maximum profit rate will differ from those for minimum cost per component and maximum production rate.

The generalized tool life equation for a turning operation is given by

$$T = \frac{K}{V^{1/n} f^{1/n_1} d^{1/n_2}} = \frac{A}{V^{1/n} f^{1/n_1}} = \frac{B}{V^{1/n}} \qquad \ldots (10.8)$$

where,

$T$ = tool life in minutes,

$V$ = cutting speed in m/min,

$f$ = feed in m/rev,

$d$ = depth of cut in m, and

$K, A, B$ = constants.

$1/n$, $1/n_1$, $1/n_2$ are exponents of speed, feed and depth of cut, respectively.

In the following analysis, it is assumed that machine tool, tool and work material have been selected. The three criteria discussed above will be applied only to a single pass turning operations.

## 10.3 MINIMUM COST PER COMPONENT CRITERION

Procedure to achieve optimum cutting speed and optimum feed rate for minimum cost per component involves the following steps :

(i) Write the total cost equation (Eq. (10.1)) in terms of the two variables ($f$ and $V$) only (depth of cut is assumed to remain constant for a single pass case).

(ii) Differentiate the total cost equation with respect to cutting speed $V$ and feed $f$ separately, and solve them.

(iii) From the equations obtained in Step (ii), derive an equation for tool life for minimum cost per component.

(iv) Consider various necessary constraints and modify the selected optimum machining conditions.

The values $x$, $T_l$, $T_d$ and $y$ in Eq. (10.1) are found from the cost data and standard times hand book. Since the depth of cut is usually fixed, the speed and feed must be chosen to minimize the cost per component. Machining time (for single pass) $T_c$ is, generally, approximately equal to the actual cutting time $T_{ac}$ , and is found from

$$T_c = \frac{l}{f\ N} = \frac{l}{\lambda V f} \cong T_{ac} \qquad \ldots (10.9)$$

where,

$f$ = feed (m/rev),

$l$ = distance traveled by the tool in making one turning pass (m),

$N$ = spindle speed (revolutions per second),

$V$ = cutting (or peripheral) speed (m/s), and

$\lambda = 1 / \pi D$.

($D$ work diameter).

**Step 1**

Substituting Eqs. (10.8) and (10.9) in (10.1), we get cost per component in terms of speed and feed.

$$C = xT_l + x\frac{l}{\lambda V f} + xT_d\,\frac{l}{\lambda A}V^{(1/n-1)}f^{(1/n_1-1)} + \frac{yl}{\lambda A}V^{(1/n-1)}f^{(1/n_1-1)} \qquad \ldots (10.10)$$

**Step 2**

Following two cutting conditions should be satisfied to get minimum cost per component (i.e., to get optimum cutting speed and optimum feed).

$$\frac{\partial C}{\partial V} = 0 \qquad \ldots (10.11(a))$$

$$\frac{\partial C}{\partial f} = 0 \qquad \ldots (10.11(b))$$

Partially differentiating Eq. (10.10) with respect to cutting speed $V$, we get

$$\frac{\partial C}{\partial V} = 0 + x\frac{l}{\lambda f}(-1)V^{-1-1} + xT_d\,\frac{l}{\lambda A}f^{1/n_1-1}\left(\frac{1}{n}-1\right)V^{1/n-1-1} + \frac{yl}{\lambda A}f^{1/n_1-1}\left(\frac{1}{n}-1\right)V^{1/n-1-1}$$

or, $$\frac{\partial C}{\partial V} = -x\frac{l}{\lambda f}V^{-2} + \frac{l}{\lambda A}f^{1/n_1-1}\left(\frac{1}{n}-1\right)V^{1/n-2}\left(xT_d + y\right) \qquad \ldots (10.11(c))$$

Using Eq. (10.11(a)), we may write Eq. (10.11(c)) as :

$$\frac{xl}{\lambda f V^2} = \frac{l}{\lambda A}f^{1/n_1-1}\left(\frac{1}{n}-1\right)V^{1/n-2}\left(xT_d + y\right)$$

or, $$1 = \frac{1}{A}f^{1/n_1}\left(\frac{1}{n}-1\right)V^{1/n}\left(\frac{xT_d + y}{x}\right)$$

or, $$1 = \left(\frac{1}{n}-1\right)\frac{f^{1/n_1}V^{1/n}}{A}\left(\frac{xT_d + y}{x}\right) \qquad \ldots (10.12)$$

Eq. (10.12) gives the condition to determine optimum cutting speed.

Now, after partiatially differentiating Eq. (10.10) (i.e., $\frac{\partial C}{\partial f} = 0$) with respect to $f$, we get

$$\frac{\partial C}{\partial f} = 0 + x\frac{l}{\lambda V}(-1)\,f^{-1-1} + xT_d\,\frac{l}{\lambda A}V^{1/n-1}\left(\frac{1}{n_1}-1\right)f^{1/n_1-1-1} + \frac{yl}{\lambda A}V^{1/n-1}\left(\frac{1}{n_1}-1\right)f^{1/n_1-1-1}$$

or, $$\frac{xl}{\lambda V f^2} = \frac{l}{\lambda A}\left(\frac{1}{n_1} - 1\right) V^{1/n-1} f^{1/n_1 - 2} \left(x T_d + y\right)$$

or, $$1 = \left(\frac{1}{n_1} - 1\right) \frac{f^{1/n_1} V^{1/n}}{A} \left(\frac{x T_d + y}{x}\right) \qquad \ldots (10.13)$$

Eq. (10.13) gives the condition to determine optimum feed rate.

Eqs. (10.12) and (10.13) cannot be satisfied simultaneously and unique minimum does not occur.

**Step 3**

Eq. (10.12) is often represented in the form

$$\frac{A}{V^{1/n} f^{1/n_1}} = \left(\frac{1}{n} - 1\right) \left[\frac{x T_d + y}{x}\right] = T_{vm} \qquad \ldots (10.14)$$

or, $$V = \frac{A^n}{T_{vm}^n f^{n/n_1}} \qquad \ldots (10.15)$$

where, $T_{vm}$ is known as tool life for minimum cost per component (while optimizing cutting speed).

In the same way, equation for tool life ($T_{fm}$) for minimum cost per component (while optimizing feed rate) can be derived.

**Step 4**

Eq. (10.14) shows that when the cutting conditions are found in terms of the tool life ($T_{vm}$), the corresponding optimum speed ($V_{opt}$) is found from Eq. (10.15) where, $f$ is given the highest possible value.

*Machine Tool Speed and Feed Restriction*

The final choice of cutting conditions must satisfy a number of restrictions. The method of selecting the cutting conditions for each restriction is now considered. In case the optimum feed ($f_{opt}$) is not available on the machine tool, select the highest feed available (closest to the $f_{opt}$) on the machine tool, and then re-calculate the optimum value of velocity from Eq. (10.12) corresponding to this selected feed. If optimum speed ($V_{opt}$) is not available on the machine tool, select the highest speed (closest to $V_{opt}$) available on the machine and then find out $f_{opt}$ from Eq. (10.13) corresponding to this chosen speed. Further, selection of $V_{opt}$ or $f_{opt}$ will be decided according to the type of operation, say, for finishing operation select high speed (maximum permitted) and low feed. For roughing operation, take maximum permitted feed and find out the optimum speed from Eq. (10.12).

*Machine Tool Maximum Power Restriction*

The cutting power may be expressed by

$$P = W V f^{\alpha} d^{\beta} \qquad \ldots (10.16)$$

where, $P$ is cutting power, $W$, $\alpha$ and $\beta$ are constants for a given tool-work combination.

If the values of selected speed and feed are such that the available cutting power is exceeded, then one or both must be reduced. It means a change moving the machining conditions away from the machining conditions for minimum cost. To make this change as small as possible, speed or feed should be adjusted by as little as is necessary to meet the power restriction. In other words, the maximum possible power should be used.

Now, put $\frac{\partial C_p}{\partial V} = 0$ to get optimum speed corresponding to the maximum power.

Similarly, find out optimum feed for maximum power ($P_m$) from $\frac{\partial C_p}{\partial f} = 0$.

After substituting the value of *f* from limiting power Eq. (10.16) in Eq. (10.10), we get

$$C_p = xT_l + \frac{xl}{\lambda}\left(\frac{Wd^{\beta}}{P_m}\right)^{1/\alpha} V^{[(1/\alpha)-1]}$$

$$+\left(xT_d + y\right)\frac{l}{\lambda A}\left(\frac{P_m}{Wd^{\beta}}\right)^{(1/n_1-1)(1/\alpha)} V^{\left[\left(\frac{1}{n}-1\right)-\frac{1}{\alpha}\left(\frac{1}{n_1}-1\right)\right]} \quad \text{. . . (10.17)}$$

Differentiate Eq. (10.17) and equate to zero $\left(\frac{\partial C_p}{\partial V} = 0\right)$. This leads to

$$V = \left\{\left[\frac{\left(\frac{1}{n_1}-1\right)-\alpha\left(\frac{1}{n}-1\right)}{1-\alpha}\right]\left[\frac{xT_d + y}{x}\right]\left[\frac{P_m}{Wd^{\beta}}\right]^{1/\alpha n_1}\frac{1}{A}\right\}^{(1/\alpha n_1)-1/n} \quad \text{. . . (10.18)}$$

Now, follow the similar procedure, as followed above, to solve for $\frac{\partial C_p}{\partial f} = 0$.

This leads to,

$$f = \frac{P_m^{1/\alpha}}{W^{1/\alpha} d^{\beta/\alpha}\left\{\left[\frac{\frac{1}{n_1}-1-\alpha\left(\frac{1}{n-1}\right)}{1-\alpha}\right]\left[\frac{xT_d + y}{x}\right]\left[\frac{P_m}{Wd^{\beta}}\right]^{1/\alpha n_1}\frac{1}{A}\right\}^{\left(\frac{n-\alpha n_1}{\alpha^2 n_1 n}\right)}} \quad \text{. . . (10.19)}$$

*Maximum Force Restriction*

In operations where workpiece dimensional accuracy is important cutting force '$F_c$' should not be too high otherwise tool may vibrate and may lead to form error. As a result, the dimensional accuracy may not be achieved as desired.

The cutting force may be expressed as :

$$F_c = E\, f^{\alpha} d^{\beta} \quad \text{. . . (10.20)}$$

Eq. (10.20) gives a maximum allowable feed for the given depth of cut. For this restriction, the feed chosen is such that it gives the maximum force (within permissible limits, i.e. $F_c \leq F_{max}$). Then, the corresponding speed is found from Eq. (10.12).

*Surface Finish Requirement*

In finish machining, the power requirement is not likely to be a constraint. This however, should be considered in case of rough machining. Hence, the following condition should be satisfied

$$P = W\, V f^{\alpha} d^{\beta} \leq P_{max}$$

where, $P_{\max}$ is the maximum power available on the machine tool.

Surface finish constraint must be taken into consideration during finish machining. For a round nosed tool turning, surface finish is expressed in terms of feed rate ($f$) and nose radius ($R$); peak to valley height ($h'$) in this case is expressed by :

$$h' = \frac{f^2}{8R}$$

And in case of CLA (center line average) value, the surface finish is expressed by :

$$h = \frac{f^2}{18R\sqrt{3}}$$

Therefore, the constraint on the feed for the specified surface finish ($h_{max}$) can be expressed as :

$$Cf^2 \leq h_{\max}$$

where, $C = 1/8R$ (for peak to valley height)

$$C = 1/(18\sqrt{3}R) \text{ (for CLA)}$$

These values of '$V$' and '$f$' should be checked for remaining restrictions whether they are violated or not.

*Feed and Speed Steps*

In any conventional machine tool, the speed and feed vary in steps. The cutting conditions are found by selecting the highest permissible feed and then determine the corresponding speed from Eq. (10.12). From the tool life point of view, choose the lower speed (RPM) from the two closest values available on the machine tool.

For example, suppose the calculated RPM is 310 but the available speeds on the machine tool are 320 and 275. Then, choose 275 in place of 320 from the tool life consideration. It is not necessary to apply all the restrictions discussed above, at a time. Which restriction should be applied, is decided by the requirements of the job. There may be other restrictions as well which have not been discussed here.

**Graphical approach for selection of minimum-cost-per component conditions when various restrictions apply :**

Figure 10.2 shows how the various machining constraints may govern the final selection of the optimum cutting conditions. The minimum permissible cost is tentatively set at 1 with the maximum available machine feed.

**Figure 10.2 : Selection of Minimum Cost per Component Conditions when Various Restrictions Apply [*Armergo and Brown, 1969*]**

- Point 1 satisfies feed and speed restrictions. With the power restriction introduced, the conditions are moved to point 2 and latter to point 3 to account for the force restriction. The surface finish restriction moves the conditions to point 4 and since the machine does not have the required step for point 4, the next lower feed step 5 is used; now, the available speed steps are checked for lower cost. In practice, the minimum cost cutting conditions can be arrived at fairly quickly, since all the restrictions may not apply at the same time.
- The cutting conditions are found by selecting the highest permissible feed and determining the speed from Eq. (10.12).

## 10.4 MAXIMUM PRODUCTION RATE CRITERION

Maximum production rate means the time required to make each component should be minimum. The time per component for a single-pass turning operation is found by substituting Eqs. (10.8) and (10.9) in Eq. (10.5). Thus,

$$T_t = T_l + \frac{l}{\lambda V f} + \frac{T_d l}{\lambda A} V^{1/n-1} f^{1/n_1-1} \qquad \text{. . . (10.21)}$$

Assuming that $T_l$ and $T_d$ are lowest possible, the conditions for maximum production rate are evaluated as follows :

For optimum cutting speed,

$$\frac{\partial T_t}{\partial V} = 0 \qquad \text{. . . (10.21(a))}$$

Following the same procedure as in Section 10.3, solution of the above Eq, (10.21(a)) will result in

$$1 = \left(\frac{1}{n} - 1\right) \frac{V^{1/n} f^{1/n_1}}{A} T_d \qquad \text{. . . (10.22)}$$

Similarly, for optimum feed rate

$$\frac{\partial T_t}{\partial f} = 0$$

Solution of the above equation will result in

$$1 = \left(\frac{1}{n_1} - 1\right) \frac{V^{1/n} f^{1/n_1}}{A} T_d \qquad \text{. . . (10.23)}$$

Eqs. (10.22) and (10.23) cannot be satisfied simultaneously hence an unique minimum time per component (i.e., maximum production rate) does not exist. Eq. (10.22) is often presented in the form

$$\frac{A}{V^{1/n} f^{1/n_1}} = \left(\frac{1}{n} - 1\right) T_d = T_{v_p} \qquad \text{. . . (10.24)}$$

where, $T_{v_p}$ is the tool life for maximum production rate when speed is optimized. The tool life $T_{v_p}$ is found first and speed is found when the feed is selected. In the same say, $T_{fp}$ using Eq. (10.23) can be evaluated.

## 10.5 MAXIMUM PROFIT RATE CRITERION

The criterion is based on maximizing Eq. (10.7). Substituting Eqs. (10.8) and (10.9) in Eq. (10.7), we get

$$P_r = \frac{I - \dfrac{yl}{\lambda A} V^{1/n-1} f^{1/n_1 -1}}{T_l + \dfrac{l}{\lambda V f}\left[1 + T_d \left(\dfrac{V^{1/n} f^{1/n_1}}{A}\right)\right]} - x \qquad \text{. . . (10.25)}$$

For maximum profit rate,

$$\frac{\partial P_r}{\partial V} = 0 \qquad \text{. . . (10.25(a))}$$

and,

$$\frac{\partial P_r}{\partial f} = 0 \qquad \text{. . . (10.25(b))}$$

Differentiate above Eq. (10.25) with respect to $V$ and simplify. Then, the condition for maximum profit rate with respect to speed is obtained as Eq. (10.26).

$$1 = \left(\frac{V^{1/n} f^{1/n_1}}{A}\right)\left[\left(\frac{yT_l}{I} + T_d\right)\left(\frac{1}{n} - 1\right) + \frac{yl}{In\,\lambda\, f\, V}\right] \qquad \text{. . . (10.26)}$$

Similarly, the condition for maximum profit rate with respect to feed is given by

$$1 = \left(\frac{V^{1/n} f^{1/n_1}}{A}\right)\left[\left(\frac{yT_l}{I} + T_d\right)\left(\frac{1}{n_1} - 1\right) + \frac{yl}{In_1\,\lambda\, f\, V}\right] \qquad \text{. . . (10.27)}$$

Eqs. (10.26) and (10.27) cannot be satisfied simultaneously hence no unique maximum profit rate exists. The conditions for the highest profit rate are found by selecting the maximum possible feed, and then speed corresponding to Eq. (10.26). The method of selecting the cutting conditions when various restrictions apply is similar as discussed in the case of minimum cost per component in Section 10.3.

## 10.6 EXAMPLES

**Example 10.1**

In a certain manufacturing company, a turning operation is performed under the following conditions :

Depth of cut = 0.00127 m, feed rate = $3.81 \times 10^{-4}$ m / rev, work dia. = $76.2 \times 10^{-3}$ m, axial length of cut = 0.1524 m, time required to load and unload components = 15 s/component, time required to change a tool = 4 min / tool. Average cost of reconditioning a worn tool is Rs. 2/- per cutting edge, machine operating cost is Rs. 10/hr, number of components required per year is 30,000. The average number of components produced are 620 at 330 RPM and 15 at 535 RPM during the life of tools for each speed.

Find annual minimum savings (if any) by modifying the drive to give some intermediate speed.

**Solution**

To solve the problem, the steps to be followed are as given below :

(a) Convert all the data in the same units as shown in step 1.

(b) Two spindle speeds are given in the problem and the third speed (optimum speed) can be calculated. Using the cost per component at these three speeds, the minimum savings can be calculated using following Eq. (10.1).

$$C = xT_l + xT_c + xT_d\left(\frac{T_{ac}}{T}\right) + y\left(\frac{T_{ac}}{T}\right)$$

Here, some of the quantities like $T_{ac}$ and $T$ are unknown.

(c) Calculate $T_{ac}$ and $T$ for both the given speeds using Eq. (10.9).

$$T_{ac} \approx \frac{l}{fN}$$

Tool life, $T$ = time per component ($T_{ac}$) × total number of components that can be produced.

(d) Now, optimum speed should be determined using the following equation

$$1 = \left(\frac{1}{n} - 1\right) \frac{V^{1/n}}{B} \left[\frac{x T_d + y}{x}\right]$$

- Here, $1/n$ and B are unknown. They should be determined using the following relationship.

$$T = \frac{B}{V^{1/n}}$$

- $T$ has been calculated (step (c)) for two speeds. Make two equations for $T_1$ and $T_2$ and then solve them to evaluate $B$ and $1/n$. Now, substitute the values of $B$ and $1/n$ to determine optimum speed (or optimum spindle speed) from Eq. (10.12).

(e) Calculate cost per component for three cases ($C_1$, $C_2$ and $C_3$) from Eq. (10.1) and then calculate the minimum savings per year.

**Step 1**

Given : $d = 1.27 \times 10^{-3}$ m, $f = 3.81 \times 10^{-4}$ m / rev, D = $76.2 \times 10^{-3}$ m, $l = 0.1524$ m, $T_l$ = ¼ min /comp, $T_d$ = 4 min per tool, $y$ = Rs. 2/- per cutting edge = 200.0 paise per cutting edge, $x$ = Rs 10/- per hour = 16.67 paise / min, $N_1$ = 330 RPM, $N_2$ = 535 RPM.

**Step 2**

The equation that gives the optimum speed for minimum cost per component is given by

$$1 = \left(\frac{1}{n} - 1\right) \frac{V^{1/n}}{B} \left[\frac{x T_d + y}{x}\right] \quad \text{. . . (10.12)}$$

To know optimum $V$, $1/n$ and $B$ should be known. These two unknowns can be determined from the following relationship.

$$T = \frac{B}{V^{1/n}} \quad \text{. . . (10.8)}$$

where, $T$ = tool life in min, and $V$ = velocity in m/min.

Let $V_1$ be the cutting speed at $N_1$ and $V_2$ be the cutting speed at $N_2$. Then,

$$V_1 = \pi D N_1 = \pi \times 76.2 \times 10^{-3} \times 330$$

or, $$V_1 = 78.998 \text{ m/min.}$$

$$V_2 = \pi D N_2 = \pi \times 76.2 \times 10^{-3} \times 535$$

or, $$V_2 = 128.07 \text{ m/min.}$$

$$T_{c_1} \approx T_{ac}$$

$$= \frac{l}{f\, N_1}$$

$$= \frac{0.1524}{3.81 \times 10^{-4} \times 330}$$

$$T_{c1} = 1.21 \text{ min}$$

Therefore, $T_1$ = time per component × total number of components

$$= 1.21 \times 620$$

or, $$\boldsymbol{T_1 = 750 \text{ min}}$$

$$T_{c2} = \frac{l}{f\, N_2} = \frac{0.1524}{3.81 \times 10^{-4} \times 535}$$

$$T_{c2} = 0.748 \text{ min}$$

$$T_2 = 15 \times 0.748$$

or, $$\boldsymbol{T_2 = 11.22 \text{ min}}$$

**Step 3**

On putting the values of $T_1$ and $T_2$ in Eq. (10.8), we get

$$750 = \frac{B}{(78.889)^{1/n}} \qquad \ldots \text{(E1)}$$

$$11.2 = \frac{B}{(128.07)^{1/n}} \qquad \ldots \text{(E2)}$$

Divide first one (E1) by second (E2),

$$\frac{750}{11.2} \, (= 66.845) = \left( \frac{128.07}{79.0} \right)^{1/n}$$

or, $$(1.62)^{1/n} = 66.845$$

$$\therefore \; \frac{1}{n} = 8.71$$

From Eq. (E1),

$$750 = \frac{B}{(79)^{8.71}}$$

$$\boldsymbol{B = 2.53 \times 10^{19}}$$

Putting these values in Eq. (10.12), we get,

$$1 = (8.71 - 1) \frac{V^{8.71}}{2.53 \times 10^{19}} \left[ \frac{16.67 \times 4 + 200}{16.67} \right]$$

$$V^{8.71} = \frac{2.53 \times 10^{19}}{7.71} \times \frac{1}{15.9976}$$

$$= 2.05 \times 10^{17}$$

$$V_{opt} = 97.18 \text{ m/min}$$

After having obtained the optimum cutting speed, the corresponding spindle speed ($N_3$) can be obtained as follows :

$$N_3 = \frac{V_{opt}}{\pi D} = \frac{97.18}{\pi \times 0.0762}$$

$$N_3 = 405.95$$

$$\therefore \; N_3 = 406$$

**Step 4**

Now, let us calculate $C_1$, $C_2$ and $C_3$ to compute minimum savings per year.

Let, $C_1$ = cost/component at speed 330 RPM.

$$C_1 = xT_1 + xT_{c_1} + xT_d \left( \frac{T_{ac_1}}{T_1} \right) + y \left( \frac{T_{ac_1}}{T_1} \right)$$

$$T_{c_1} \approx T_{ac} = 1.21 \text{ min (from step 2)}$$

$$T_1 = 750 \text{ min (from step 2)}$$

$$C_1 = 16.67 \times \frac{1}{4} + 16.67 \times 1.21 + 16.67 \times 4 \times \left( \frac{1.21}{750} \right) + 200 \left( \frac{1.21}{750} \right)$$

$$C_1 = 24.68 \text{ paise / component}$$

Similarly, cost per component at spindle speed 535 is $C_2$. Then,

$$C_2 = xT_1 + xT_{C2} + xT_d \left( \frac{T_{ac2}}{T_2} \right) + y \left( \frac{T_{ac2}}{T_2} \right)$$

$$T_{c_2} = 0.748 \text{ min. (from step 2)}$$

$$T_2 = 11.2 \text{ min. (from step 2)}$$

Therefore,

$$C_2 = 16.67 \times \frac{1}{4} + 16.67 \times 0.748 + 16.67 \times 4 \times \left( \frac{0.748}{11.22} \right) + 200 \left( \frac{0.748}{11.22} \right)$$

$$C_2 = 34.34 \text{ paise / component}$$

Let the cost per component at speed $V_{opt}$ be $C_3$. Then,

$$C_3 = xT_1 + xT_{c3} + xT_d \left( \frac{T_{ac3}}{T_3} \right) + y \left( \frac{T_{ac3}}{T_3} \right)$$

$$T_{c3} \approx T_{ac3} = \frac{1}{fN_3} = \frac{0.1524}{3.81 \times 10^{-4} \times 406}$$

$$T_{c3} = 0.985 \text{ min.}$$

$$T_3 = \frac{B}{(V_{opt})^{1/n}} = \frac{2.53 \times 10^{19}}{(97.18)^{8.71}}$$

$$T_3 = 123.4 \text{ min}$$

$$C_3 = 16.67 \times \frac{1}{4} + 16.67 \times 0.985 + 16.67 \times 4 \times \left( \frac{0.985}{123.4} \right) + 200 \left( \frac{0.985}{123.4} \right)$$

$C_3 = 22.72$ paise / component

Minimum profit anticipated = $C_1 - C_3$

= 24.680 – 22.716 (while calculating profit from ($C_2 - C_3$), it will be more)

=1.96 paise / component

$$\text{Total profit / year} = \frac{1.96 \times 30000}{100}$$

$$= \text{Rs. } 589.22$$

Minimum profit anticipated = Rs. 589.22 per year

**Example 10.2**

It has been found that for 18/8 stainless steel turned on a lathe machine with a carbide tool, the following relations apply :

Cutting force, $F_c = 41384418\, d\, f^{\,0.76}$ (kgf); tool life (min), $T = \dfrac{18.636}{V^5 f^{2.15} d}$.

where, $V$ = cutting speed in m/min, $f$ = feed in m/rev, $d$ = depth of cut in m.

For the tool and work material combination, determine speed and feed for the lowest cost when turning at a depth of cut as $2.54 \times 10^{-3}$ m in a lathe machine with a 5 HP motor and 75% drive efficiency. The following data apply :

(i) Machine operating cost = Rs. 12/- per hour,

(ii) Throw away tips are used and cost Rs. 2.50 per cutting edge,

(iii) Time taken to replace worn edge is 3 min,

(iv) To achieve a satisfactory finish the feed is not to exceed $1.016 \times 10^{-3}$ m/rev, and

(v) Maximum allowable force is 136.2 kgf.

**Solution**

To determine speed and feed for the lowest cost, following procedur should be followed :

(a) Convert all the given data in the same units.

(b) Check whether the given depth and feed satisfy the force condition. If not, modify the feed value such that the selected value of '$f$' satisfies both the force condition as well as surface finish condition.

(c) Using the above selected value of '$f$' and the given tool life equation, compute the optimum cutting speed $V_{opt}$ using Eq. (10.12).

(d) Now, check for power restriction using $V_{opt}$. If it is violated, then modify '$f$' or '$V$' or both.

**Step 1**

The data converted into uniform units are as follows :

$x$ = Rs. 12 / hour = 20 paise / min, $y$ = Rs. 2.50 / cutting edge = 250 paise /cutting edge, $T_d = 3$ min , $d = 2.54 \times 10^{-3}$ m, $F_{cmax} = 136.2$ kgf, $f_{c\,max} = 1.016 \times 10^{-3}$ m/rev.

**Step 2**

The force restriction is checked as follows :

$$F_c = 41384418\, d\, f^{0.76}$$

$$= 41384418 \times 2.54 \times 10^{-3} \times (1.016 \times 10^{-3})^{0.76}$$

$$= 558.35 \text{ kgf}$$

The value of $F_c$ (= 558.35 kgf) is higher than the maximum allowable force of 136.2 kgf. Hence, let us decrease the feed to reduce the force to the value of 136.2 kgf as follows :

$$136.2 = 41384418 \times 2.54 \times 10^{-3} f_1^{\,0.76}$$

$$f_1^{\,0.76} = 1.2957 \times 10^{-3}$$

$$f = 1.587 \times 10^{-4} \text{ m/rev}$$

This feed value ($1.587 \times 10^{-4}$ m/rev) satisfies the force condition as well as it is lower than the feed required for the satisfactory surface finish ($1.016 \times 10^{-3}$ m/rev).

**Step 3**

For minimum cost per component, we wish to know the optimum value of speed, $V_{opt}$ from the following equation :

$$1 = \left(\frac{1}{n} - 1\right) \frac{V^{1/n} \, f^{1/n_1}}{A} \left[\frac{x T_d + y}{x}\right] \qquad \ldots (10.12)$$

Tool life, $$T = \frac{18.636}{V^5 \, f^{2.15} \, d} \text{ min}$$

From the above given relationship for $T$, the following values of constants of tool life equation can be written as

$$K = 18.636, \ \frac{1}{n} = 5, \ \frac{1}{n_1} = 2.15, \ \frac{1}{n_2} = 1.$$

Now, substitute the value of $d$ in the tool life equation

$$T = \frac{18.636}{V^5 \, f^{2.15} \, 2.54 \times 10^{-3}}$$

or, $$T = \frac{7337.17}{V^5 \, f^{2.15}}$$

From this equation, the value of $A$ is obtained as

$$A = 7337.17$$

After substituting the above evaluated values of different parameters in Eq. (10.12), the value of $V_{opt}$ is evaluated as

$$1 = (5 - 1) \, \frac{V^5 \, (1.587 \times 10^{-4})^{2.15}}{7337.17} \left[\frac{20 \times 3 + 250}{20}\right]$$

$$1 = 5.729 \times 10^{-11} \, V^5$$

or, $$V = \left(\frac{10^{11}}{5.729}\right)^{1/5}$$

Solution of this equation gives the value of $V_{opt}$ as

$$V_{opt} = 111.78 \text{ m/min}$$

**Step 4**

Motor power constraint should also be checked as

$$F_c \times V = 5 \text{ HP}$$

$$= 5 \times \frac{75}{100} \times 76 \text{ Watt}$$

or, $V = \frac{5 \times 0.75 \times 76}{136.2}$ m/s (using maximum permissible force, $F_{cmax}$ = 136.2 kgf)

$$V = 125.55 \text{ m/min}$$

$V_{opt}$ is smaller as compared to $V$ =125.55 m/min, calculated above. Also, the calculated feed $f_1$ = 1.587 $10^{-4}$ m/rev (in Step 2), is smaller than the max. allowable feed.

Hence, for minimum cost/component, we have optimum values of feed and speed, for given conditions as $f = 1.587 \times 10^{-4}$ m/rev and $V$ = 111.78 m/min, respectively.

## SAQ 1

Time for turning ($T_m$) is given in terms of length of turning $L$ (mm), feed rate $f$ (mm/rev), spindle speed $N$ (RPS), peripheral speed $v$, and work diameter, $D$ (mm) as

(a) $T_m = L/fN$

(b) $T_m = \pi DL / fv$.

(i) (a) is correct

(ii) (b) is correct

(iii) Both are correct

(iv) None of them is correct.

## SAQ 2

Life of a cutting edge for the given machining conditions and tool-work combination is 2.3 hrs and one component requires 30 min 25 seconds for machining. Tool bit is square in shape. One tool bit can machine the number of components as

(i) 5

(ii) 4

(iii) 3

(iv) none

## SAQ 3

In SAQ 2, the cost of the throw-away tool bit is Rs. 100/-. The tool cost of machining a component is

(i) 25

(ii) 20

(iii) 22

(iv) none

## SAQ 4

Machine tool power constraint is to be applied in case of

(i) Rough machining

(ii) Finish machining

(iii) Both

(iv) None

**SAQ 5**

Surface roughness obtained on the workpiece depends on

(i) Feed rate

(ii) Tool radius

(iii) Both of them

(iv) None of them.

**SAQ 6**

Manufacturers of cutting tools recommend that $f_{max}$ should not exceed 0.4 to 0.5 times the nose radius for triangular inserts. It is done to take care of

(i) Material removal

(ii) Surface finish

(iii) To safeguard the tool

(iv) None of these

## 10.7 SUMMARY

The objective of a manufacturing engineer should be to produce an object at the minimum possible cost accounting for the constraints given. To solve such problems, all the possible variable costs incurred in the manufacturing activity should be included, for example, manpower cost, machine tool operating cost, tool cost, etc. A total cost equation in terms of $V$ and $f$ variables is written. It is partially differentiated with respect to $V$ to get optimum cutting speed and with respect to $f$ to get optimum feed. The sub-optimum values calculated in this fashion are checked for the various constraints like limiting feed, speed, power, force, etc. If a constraint is violated, the cutting conditions need to be modified.

Under many circumstances, to deliver the product as per schedule becomes more important than minimum cost. Hence, sometimes maximizing the production rate overrules the minimum cost per component criterion. For this purpose, total time taken to make a component is represented in the form of an equation in terms of $V$ and $f$ variables and some constants. This equation is then partially differentiated separately with respect to $V$ and $f$, and then solved. It minimizes the time taken in making the parts. Then the computed values of the variables are checked for various constraints and modified if necessary. As a result one may end up with sub-optimized results.

However, the ultimate target of an entrepreneur could be to maximize the profit rate rather than the other two criteria stated above. In this case, the profit rate is written in terms of variables $V$ and $f$, and constants. This equation is then partially differentiated with respect to $V$ and $f$. The computed values are checked for various restrictions and modified, if necessary. This problem of optimization requires enough data to solve it.

Above three criteria have been applied only for a single-pass turning operation, although they could be applied for multi-pass cutting and other categories of operations as well.

## 10.8 KEY WORDS

| | | |
|---|---|---|
| **Maximum Production Rate** | : | It means that the time required to make each component should be minimum. |
| **Tool Life** | : | Useful life of a tool, expressed in terms of the time from the start of a cut to some termination point defined by the failure criterion. |

## 10.9 ANSWERS TO SAQs

**SAQ 1**

(iii)

**SAQ 2**

(ii)

**SAQ 3**

(i)

**SAQ 4**

(i)

**SAQ 5**

(iii)

**SAQ 6**

(ii)

## 10.10 EXERCISES

**Exercise 1**

(a) What are the three optimization criteria generally used in metal cutting? Which one requires more information in terms of costs that may not be available with the process planner?

Prove that the number of pieces produced per regrind $P_g$ are given by

$$P_g = \frac{f C^{1/n} V^{(1-1/n)}}{\pi D l}$$

Assume that the Taylor's tool life equation applies.

(b) Diameter ($D$) of a job is 150 mm and its length is 4D, the depth of cut is D/100 while the feed rate is given by 5 mm / min (N = 200 RPM). Labor cost is Rs. 12/- per hour and m/c overhead is Rs. 40/- per hour. Grinding cost and grinding m/c overhead are Rs. 15/- per hour and Rs. 50/- per hour, respectively. Idle time can be taken as 5 min. The constants of Taylor's tool life equation are $n$ = 0.22 and $C$ = 475. Suppose tungsten carbide brazed tool is used. Initial tool cost is Rs. 60/- , grinding time for each cutting edge is 5 min, tool change time is 2 min and total 9 regrinds per tool are permitted. Calculate optimum cutting speed, tool life and cost of operation while applying following criteria (a) minimum cost per component, and (b) maximum production rate.

(c) Solve the above problem (Exercise 2) for the case of throw away tips. Use total cutting edges on the tip to be four. Initial tool cost is reduced to Rs. 40/- and tool change time is reduced to 1.5 minutes.

(**Hint :** Throw-away tips are not reground.)

(d) Derive an equation for tool life for minimum cost per component using the equation for optimum feed rate for minimum cost per component.

(**Hint :** Use Eq. (10.13))

**Exercise 2**

(a) Derive equations for optimum cutting speed and optimum feed rate for the case of minimum cost per component, satisfying the machine tool maximum power restriction.

(**Hint :** Derive Eqs. (10.17) and (10.18) giving all details.)

(b) Derive equations for optimum cutting speed and optimum feed rate for the case of maximum production rate.

(**Hint :** Derive Eqs. (10.21) and (10.22) giving all details.)

(c) Derive equations for optimum cutting speed and optimum feed rate for the case of maximum profit rate.

(**Hint :** Derive Eqs. (10.26) and (10.27) giving all details.)

# BIBLIOGRAPHY

Armarego, E. J. A. and Brown R. H., 1969, *The Machining of Metals*, Prentice Hall, Englewood Cliffs, NJ.

Pandey P. C. and Singh C. K., 1998, *Production Engineering Science*, Standard Publishers Distributions, Delhi.

Rao P. N., 2000, *Manufacturing Technology : Metal Cutting and Machine Tools*, Tata McGraw Hill Publishing Co. Ltd., New Delhi.

G. Boothroyd, 1975, *Fundamentals of Metal Cutting and Machine Tools*, McGraw Hill Kogakusha Ltd., Tokyo.

# PRINCIPLES OF METAL CUTTING

This block consists of four units. In the previous blocks, we have already studied the Principles of Metal Casting and Forming. In this block, we are going to study the Principles of Metal Cutting.

In Unit 7, Tool Geometry has been introduced. This unit specifies the classification of cutting tools. After classification, different types of cutting tools have been elaborated.

Unit 8, Force and Power Requirements, discusses various kinds of consideration like power requirements, rigidity etc. for the design of the machine tool. In this unit, methods to calculate power requirement, average power requirement and various criterion have been described.

Unit 9 describes the important variables to be considered for a given machining operation. Various cutting parameters and their effects depending upon the specifications of the workpiece have been explained.

Finally, in Unit 10, Machining Economics, different aspects of cost involved in machining have been described. In this unit, concept of variable cost, fixed cost and concept of optimum machining conditions for the given criterion have also been explained.