UNIT 8 FORCE AND POWER REQUIREMENTS

Structure

8.1 INTRODUCTION

Design of the machine tools is based on many considerations like power requirements, rigidity of the machine tool elements to achieve the desired accuracy, and the size to accommodate workpiece having different dimensions. In this unit, the methods to calculate power requirements for the machining of the given workpiece will be discussed. The calculation of average power requirements may be useful, but for design purpose maximum power requirement should be used. Power requirements of a machine tool depends on the machining conditions, the type of the workpiece material being machined and the type of tool employed. Further, the power requirements for rough machining are always different from the power requirements for finish machining due to the difference in machining conditions.

In this unit, procedure for the power calculations are discussed for the case of single-point cutting and multi-point cutting. In case of milling, the forces acting are of impact type, which should be considered at the time of designing the machine tools.

Objectives

After reading this unit, you should be able to

- calculate power requirements for single-point cutting operations,
- calculate average and maximum force and power required in case of multi-point cutting operations like milling and drilling, and
- compute machining time in case of turning, milling and drilling operations.

8.2 SINGLE-POINT CUTTING

In this section force analysis and power requirements are going to be discussed.

8.2.1 Force Analysis

Let us analyse the forces acting on the chip in orthogonal cutting. These are shown in Figure 8.1(a) and are as follows : Force F_s is the resistance to shear of the metal in forming the chip. F_s acts along the shear plane. Force F_n is normal to the shear plane and is a backup force on the chip provided by the workpiece. Force N acting on the chip is normal to the cutting face of the tool and is provided by the tool. Force F is frictional resistance offered by the tool to the chip flow. The latter force acts downwards against the motion of the chip as it slides upwards along the tool face.

Figure 8.1(b) shows the free body diagram of the forces acting on the chip. Forces F_s and F_n are represented by the resultant R, and F and N are replaced by the resultant R'. This means that only two combined forces are acting on the chip, i.e., R and R'. There are external couples on the chip which curl it, and they may be negated in this approximate analysis. If equilibrium is to exist when a body is acted upon by two forces, they must be equal in magnitude, and be collinear. Hence, R and R' are equal in magnitude, opposite in direction and **collinear** (Figure 8.1).

Figure 8.1 : (a) Force Components Acting on a Chip, (b) Free Body Diagram of a Chip

Figure 8.2 shows a composite diagram in which the two force triangles of Figure 8.1, have been superimposed by placing the two equal forces R and R' together. Since the angle between F_s and F_n is a right angle, the intersection of these forces lies on the circle with diameter R as shown. Also, F and N may be replaced by R to form the circle diagram (Figure 8.2).

Figure 8.2 : Force Circle Diagram

The horizontal cutting force F_c and vertical force F_t can be measured in a machining operation by the use of a force dynamometer. The electric strain gauge type of transducer is used in the dynamometer. After F_c and F_t are determined, they can be laid off as in Figure 8.2 and their resultant is the diameter of the circle. The rake angle α can be laid off, and the forces F and N can then be determined. The shear plane angle ϕ can be measured approximately from a photomicrograph or by measuring t_c and t_u, or length of chip and corresponding length of unmachined chip (discussed elsewhere).

From Figure 8.2, the following vector Eqs. can be written

$$\vec{R}' = \vec{F} + \vec{N}$$

$$\vec{R} = \vec{F}_s + \vec{F}_n = \vec{F}_c + \vec{F}_t = \vec{R}'$$

Merchant represented various forces in a force circle diagram in which tool and reaction forces have been assumed to be acting as concentrated at the tool point instead of their actual points of application along the tool face and the shear plane. The circle has the diameter equal to R (or R') passing through tool point.

After F_c, F_t, α and ϕ are known, all the component forces on the chip may be determined from the geometry. For instance, the average stress on the shear plane can be determined by using force F_s and the area of the shear plane. Another useful quantity is the coefficient of friction (μ) between the tool and chip. Using force circle diagram, it can be shown that

$$F = F_t \cos\alpha + F_c \sin\alpha \qquad \ldots (8.1)$$

and,

$$N = F_c \cos\alpha - F_t \sin\alpha \qquad \ldots (8.2)$$

Then, the coefficient of friction (μ) is calculated as

$$\mu = \tan\beta = \frac{F}{N} = \frac{F_t \cos\alpha + F_c \sin\alpha}{F_c \cos\alpha - F_t \sin\alpha} \qquad \ldots (8.3)$$

where, β is the friction angle.

or,

$$\mu = \frac{F_t + F_c \tan\alpha}{F_c - F_t \tan\alpha} \qquad \ldots (8.4)$$

We can also write :

$$\beta = \tan^{-1}(\mu)$$

$$\beta = \tan^{-1}\left\{\frac{F_t + F_c \tan\alpha}{F_c - F_t \tan\alpha}\right\} \qquad \ldots (8.5)$$

From Figure 8.2, we get :

$$F_s = F_c \cos\phi - F_t \sin\phi \qquad \ldots (8.6)$$

$$F_n = F_t \cos\phi + F_c \sin\phi$$

$$F_n = F_s \tan(\phi + \beta - \alpha) \quad \ldots (8.7)$$

Also, from Figure 8.2,

$$F_c = R\cos(ß - a)$$

$$F_s = R\cos(\phi + ß - a)$$

$\therefore$

$$\frac{F_c}{F_s} = \frac{\cos(ß - a)}{\cos(\phi + ß - a)}$$

or,

$$F_c = F_s \frac{\cos(ß - a)}{\cos(\phi + ß - a)} \quad \ldots (8.8(a))$$

Shear plane area is equal to :

$$A_s = \frac{t_u b}{\sin\phi} \quad \ldots (8.9)$$

If τ be the shear strength of the work material, then,

$$F_s = \frac{t_u b}{\sin\phi}\tau \quad \ldots (8.10)$$

Substituting in Eq.(8.8 (a)), we get

$$F_c = \left(\frac{t_u b\tau}{\sin\phi}\right)\frac{\cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)} \quad \ldots (8.11)$$

hence,

$$R = \frac{t_u b\tau}{\sin\phi\cos(\phi + \beta - \alpha)} \quad \ldots (8.11(a))$$

From Figure 8.2,

$$F_t = R\sin(\beta - \alpha)$$

$$= \frac{t_u b\tau}{\sin\phi\cos(\phi + \beta - \alpha)}\sin(\beta - \alpha) \quad \text{(From Eq. 8.11(a))} \quad \ldots (8.12)$$

From Eqs. (8.11) and (8.12), we can write :

$$\frac{F_t}{F_c} = \tan(\beta - \alpha) \quad \ldots (8.13)$$

From the above analysis, unknown forces in the force circle diagram and the value of coefficient of friction can be calculated provided F_c, F_t, α, t_u and t_c are known measured.

During machining operations, chips are formed as a result of plastic deformation. Hence, chips experience stresses and strains. At shear plane, two normal forces simultaneously act, i.e., F_s and F_n. Shear stress (τ) can be found as

Mean shear stress

$$(\tau) = \frac{F_s}{A_s} = \frac{(F_c\cos\phi - F_t\sin\phi)}{bt_u}\sin\phi \quad \ldots (8.14)$$

$$\text{Mean Normal stress } (\sigma) = \frac{F_n}{A_s} = (F_t \cos\phi + F_c \sin\phi)\ \frac{\sin\phi}{b t_u} \quad \ldots (8.15)$$

where, A_s = Shear plane area = $b\, t_u / \sin\phi$.

8.2.2 Power Requirements

Power requirements for any cutting process should be known in advance, and should always be lower than the available machine tool power. Power (P_c) required for cutting process is given by Eq. (8.16).

$$P_c = F_c \,.\, V_c \quad \ldots (8.16)$$

where, F_c is cutting force and V_c is cutting velocity. This power is utilized mainly in chip formation in the primary shear deformation zone (PSDZ) and shear deformation of chip in the secondary shear deformation zone (SSDZ). Here, the friction between the tool and workpiece is assumed negligible. The power utilized in primary shear zone (P_s) and secondary shear zone (P_f) can be given by Eqs (8.17) and (8.18), respectively.

$$P_s = F_s \,.\, V_s \quad \ldots (8.17)$$

$$P_f = F.\ V_f \quad \ldots (8.18)$$

where, F_s is shear force, F is friction force, V_s is shear velocity and V_f is chip flow velocity.

The total specific energy (U_t) is thus given by the sum of the energies required in PSDZ (U_s) and SSDZ (U_f) as

$$U_t = U_s + U_f \quad \ldots (8.19)$$

where,

$$U_s = F_s \,.\, V_s / b\ t_u\ V_c \quad \ldots (8.20)$$

$$U_f = F \,.\, V_f / b\ t_u\ V_c \quad \ldots (8.21)$$

where, b is the width of cut and t_u is the uncut chip thickness.

The required specific energy per unit volume removal of material depends on many factors like work material, cutting conditions (V_c, f, t_u), cutting tool conditions (sharp or dull, tool angles), friction between tool and chip, and friction between tool and workpiece.

For gross estimation of the cutting power (P_c), cutting force (F_c) can be expressed in terms of material constant (K), feed (f) and depth of cut (d) as follows:

$$F_c = K\, d^{\,a} f^{\,b} \quad \ldots (8.22)$$

where, a and b are constants and depend upon the cutting conditions.

8.3 MULTI-POINT CUTTING

This section discusses the method for the computation of cutting power in multi-point cutting, viz., in the following two types of operations (milling and drilling) only.

8.3.1 Milling

In milling operation, the cutter (rotating tool) consists of a number of cutting edges (or teeth) each of which removes material from the workpiece during its rotation. In this process, feed is given to the table on which the workpiece is mounted. Milling operations can be classified in two categories based on the direction of rotation of the milling cutter and direction of feeding the workpiece. They are known as *up milling* (conventional milling) and *down milling* (climb milling).

In **up milling**, the work is fed toward the cutter against the direction of rotation (Figure 8.3). This technique prevents the work from being dragged towards the cutter, but it tends to lift the work off the machine table (force is acting upward). This technique

Figure 8.3 : (a) Up Milling, (b) Down Milling

is prone to chatter (self generated vibration) and rapid blunting of the cutting edge. In case of **down milling**, the workpiece is fed into the cutter in the same direction in which the cutter is rotating. Here, the cutting force keeps the workpiece pressed down against the machine table. Further, the risk of chatter is also low.

The total force during milling is made up of two components as discussed in earlier section on single-point cutting.

(i) Force required for plastic deformation of the workpiece to form the chips, and

(ii) Force needed to overcome friction between the chip and face of the tooth of the milling cutter, and between the workpiece and the cutter.

The force required for milling a workpiece can be determined if the specific cutting pressure (p) (i.e., force (in kg) required per square millimeter of chip cross-section being removed) is known. Specific cutting pressure depends not only on the workpiece material but also on the maximum thickness of the chip being removed from the workpiece. Force ($F = F_c$) required during milling a workpiece is given by

$$F_c = p\,A \text{ (kg)} \qquad \ldots (8.23)$$

where, A is the area of cross-section of the chip. To evaluate the maximum force (F_{max}) acting during milling, A should be replaced by A_{max} (maximum cross-sectional area of the chip)

$$F_{max} = p\,A_{max} \text{ (kg)} \qquad \ldots (8.24)$$

The ratio of F_{max} and F usually lies between 1.2 to 1.8.

If the force and cutting speed are known, the power can be calculated using following equation.

$$HP = \frac{F_c V_c}{60 \times 75} \quad \text{(metric)} \qquad \ldots (8.25)$$

where, $V (= V_c)$ is the cutting speed in m/min. To calculate maximum power required, replace F_c by F_{max}.

From above discussion, it is not possible to find out the specific cutting pressure unless A_{max} is known. To determine A_{max}, two cases can be analyzed: plain milling and face milling.

Plain Milling

Figure 8.4 shows a chip (shaded part) produced during plane milling by one tooth (also shaded) of the rotating cutter. The linear feed is being given to the workpiece. Feed per tooth is expressed as f_t, d as depth of cut, D is cutter diameter and V is peripheral speed of the cutter. O is the original position of the center of the cutter and O' is the new center position after imparting a feed equal to f_t.

The equation to determine A_{max} is mainly derived using the geometry of the chip as follows :

From Figure 8.4

(i) $OC = \left(\frac{D}{2} - d\right) \quad (OA = D \text{ and } AC = d)$

$$OB = \frac{D}{2}$$

$$\therefore BC = \sqrt{OB^2 - OC^2} = \sqrt{\frac{D^2}{4} - \left(\frac{D}{2} - d\right)^2}$$

$$= \sqrt{d(D - d)}$$

$$\therefore \sin \phi_c = BC/OB = \frac{2\sqrt{d(D-d)}}{D} \qquad \ldots (8.26(a))$$

The tooth contact angle ($\angle$ OCB) ϕ_c is given as

$$\phi_c = \cos^{-1} ((D - 2d)/D)$$

Figure 8.4 : Chip Cross-section during Slab Milling by Single Tooth

(ii) The chip thickness (t) at an instant (say, at section x) of the tooth can be evaluated as

$$t = f_t \sin\phi \quad \ldots (8.26(b))$$

(ϕ is shown in Figure 8.4. Note that at $\phi = \pi/2$, $t = f_t$).

Let Z be number of teeth in a cutter and N be the rotational speed of cutter (RPM). Then the feed rate (f) per minute is given by

$$f = f_t N Z \quad \ldots (8.26(c))$$

Therefore, $t = \frac{f}{ZN} (\sin\phi)$ (From Eqs. 8.26(b) and 8.26(c)) . . . (8.26(d))

Maximum chip thickness (t_{max}) will occur when $\phi = \phi_c$. Hence,

$$t_{\max} = \frac{f}{NZ} (\sin\phi_c)$$

hence,

$$t_{\max} = \frac{f}{NZ} \left(\frac{2\sqrt{d(D-d)}}{D} \right) \quad \ldots (8.27)$$

(iii) In plain milling with straight teeth cutters (helix angle = 0°), the width of the cut (W) is equal to the width of the workpiece (assuming the cutter width greater than the workpiece width). Hence, the maximum cross-sectional area of the chip A_m is given by

$$A_{max} = W\, t_{max} \quad \ldots (8.28)$$

As can be seen from Figure 8.4, the chip thickness varies from zero to the maximum. Hence, mean cross-sectional area ($\overline{A}$) can be taken as approximately half of A_{max}.

$$\overline{A} = 0.5\, A_{max} \quad \ldots (8.29)$$

In case of helical cutters (helix angle > 0), several teeth of the cutter are engaged with the workpiece simultaneously. In this case, the mean cross-sectional area of the chip can be evaluated by volumetric material removal rate (MRR_v, m^3/min) and cutter speed (m/min).

$$\overline{A} = \frac{Wdf}{V_c} \text{ m}^2 \quad \ldots (8.30)$$

Face Milling

Figure 8.5(b) shows the cross section of the undeformed chip produced during symmetrical face milling (Figure 7.23(b) and Figure 8.5(a)). It is assumed that only a single tooth is engaged in machining. For a milling cutter with D as diameter, workpiece width as W, and feed per tooth as f_t, the undeformed chip thickness (t_u) at any position of the cutter can be evaluated as follows:

Figure 8.5(a) : Face Milling Cutting in Operation

Figure 8.5(b) : Cross-section of the Chip Produced during Face Milling

For the geometry shown in Figure 8.5(b), the undeformed thickness (*t*) of the chip at an angle ϕ is given by

$$t_u = f_t \sin \phi$$

The minimum and maximum values of 't_u' would occur at $\phi = \left(\frac{\pi}{2} - \frac{\phi_c}{2} \right)$ and $\phi = \frac{\pi}{2}$, respectively.

$$t_{u_{\min}} = f_t \sin\left(\frac{p}{2} - \frac{\phi_c}{2}\right) = f_t \cos\left(\frac{\phi_c}{2}\right) \quad \ldots (8.31)$$

where, $\sin \phi_c = W/D$.

$$t_{u_{\max}} = f_t \sin \frac{\pi}{2} = f_t \quad \ldots (8.32)$$

To evaluate the mean undeformed chip thickness $(\bar{t}_u)$ integrate the chip thickness expression between the lower limit and the upper limit of ϕ, and divide the same by the total angle of tooth engagement, ϕ_c.

$$t_u = \int_{\frac{\pi}{2} - \frac{\phi_c}{2}}^{\frac{\pi}{2} + \frac{\phi_c}{2}} f_t \sin \phi \; d\phi$$

$$t_u = 2 f_t \sin \left(\frac{\phi_c}{2} \right)$$

Therefore, the mean undeformed chip thickness $(\overline{t_u})$ is given by

$$\bar{t}_u = \frac{2 f_t \sin\left(\frac{\phi_c}{2}\right)}{\phi_c} \quad \ldots (8.33)$$

The approximate mean cross-sectional area of the chip can be calculated using the following equation.

$$\overline{A} = W \overline{t_u} = \frac{2 W f_t \sin(\frac{\phi_c}{2})}{\phi_c} \qquad \ldots (8.34)$$

(To be more accurate, W should be replaced by the mean arc length $[\{(D - f_t)/2\}\phi_c]$, where ϕ_c is in radians.)

Machining Time in Milling

Consider a workpiece of length '*l*' milled by a slab milling (arbor mounted milling cutter, Figure 7.22(a), Unit 7) using a cutter of diameter '*D*' and depth of cut '*d*'. Then the distance to be traversed by the milling cutter is equal to $(l + 2\, A_p)$ where 'A_p' is known as approach over-run allowance. It is taken at entry as well as exit side of the cutter. From the geometry given in Figure 8.6,

$$A_p = \sqrt{\left(\frac{D}{2}\right)^2 - (D/2 - d)^2}$$

or, $$A_p = \sqrt{d\,(D - d)} \qquad \ldots (8.35)$$

Figure 8.6 : Slab Milling

In case of face milling operation, two cases arise :

(i) If width of cut (W) < D/2, then, $A_p = \sqrt{W(D - W)}$ . . . . (8.36(a))

(ii) If $D/2 \leq W \leq D$, then $A_p = D/2$. . . . (8.36(b))

Time to complete one pass,

$$T_p = \frac{l + 2A_p}{f_t\, N\, Z} \qquad \ldots (8.37)$$

8.3.2 Drilling

Force and Torque

During drilling, the drill (Figure 8.7) is subjected to a twisting couple (M) and an axial thrust (T). The values of M and T are governed by workpiece properties (hardness, strength), as well as the drill geometry (diameter, helix angle, chisel angle, point angle and number of cutting edges) and cutting conditions (feed rate, depth of cut, and cutting fluid).

Figure 8.7 : Couple (or Torque) and Thrust Acting during Drilling

The torque acting on the drill is obtained from an empirical relationship as given below :

$$M = C_1 D^{1.9} f^{0.8} \quad \text{kg-mm} \qquad \ldots (8.38)$$

where, C_1 is a constant whose value depends on the workpiece material and cutting conditions, D is drill diameter and f_r is feed rate in mm/rev.

The thrust force (T) acting on the drill is evaluated using the empirical relationship :

$$T = C_2 D f^{0.7} \qquad \ldots (8.39)$$

where, C_2 is another constant whose value depends on the workpiece material and cutting conditions.

Power required in drilling can be calculated if specific energy (U) required for drilling one unit volume (MRR) of workpiece material is known. Then, the power is given by Eq. (8.40).

$$\text{Power} = U \times MRR \qquad \ldots (8.40)$$

Here, MRR is the material removed per unit time (= ($\pi D^2 f_r N) / 4$) where, f_r is feed rate per revolution, D is drill diameter and N is spindle speed in RPM).

Power is also given as

Power = Torque $\times$ Rotational speed in radians/seconds.

$$= M \times (\pi \times N_s)$$

where, N_s is revolution per second of the drill.

Drilling Time

As shown in Figure 8.8, a drill requires an approach distance (= a) before it can actually start removing material from the workpiece. This approach distance is needed for positioning the drill above the hole and it is usually about 2 mm or so. As can be seen in Figure 8.8, the conical part of the drill should completely come out of the hole before the drilling is stopped. This additional distance A to be traversed is known as *breakthrough distance* and can be expressed as follows with the help of geometry of Figure 8.8 :

$$A = (D/2) \tan \alpha \quad \ldots (8.41)$$

Thus, total length of the drill travel (L) is given by the following equation :

$$L = l + a + A \quad \ldots (8.42)$$

If the value of α for the drill is not given then its normal value of 59° can be used.

Suppose the drill is being fed at 'f_r' (mm/rev) and it rotates at N RPM, then the time (t_d) for drilling a given hole is expressed as

$$t_d = L / f_r \,.\, N \quad \ldots (8.43)$$

and the volumetric material removal rate can be evaluated as :

MRR_v = Cross sectional area of the hole × Depth of the hole drilled in unit time

$$MRR_v = \frac{\pi}{4} D^2 (f_r N) \quad \ldots (8.44)$$

Figure 8.8

8.4 EXAMPLES

Example 8.1

During orthogonal turning operation, following data were observed : undeformed chip thickness, t_u = 0.125 mm, deformed chip thickness, t_c = 0.225 mm, cutting velocity, V_c = 133 m/min, back rake angle, α = 10°, width of cut, b = 6.25 mm, cutting force, F_c = 42.0 kgf, and tangential force, F_t = 17.0 kgf.

Calculate the percentage of total energy consumed in friction at the tool-chip interface.

Solution

Percentage of total energy consumed in the friction (η_f) at the tool-chip interface can be expressed as

$$\eta_f = \frac{\text{Friction energy}}{\text{Total energy}} \times 100$$

$$= \frac{F\,V_f}{F_c\,V_c} \times 100 \quad \text{(From Eqs. (8.16) and (8.18))}$$

$$V_f = V_c \cdot r_c$$

$$\therefore \eta_f = \frac{F\,r_c}{F_c} \times 100 \quad \ldots \text{(A)}$$

In Eq. (A), r_c and F both are not known. Let us evaluate both of them as follows.

(a) The chip thickness ratio (r_c) is given by

$$r_c = \frac{0.125}{0.225} = 0.555$$

$$r_c = 0.555$$

(b) Friction force, $F = R.\sin\beta$ (From Figure 8.2) . . . (B)

where, $R = \sqrt{F_c^2 + F_t^2}$ (From Figure 8.2) . . . (C)

$$= \sqrt{42^2 + 17^2}$$

$$= \sqrt{1764 + 289}$$

$$= 45.31 \text{ kgf}$$

Also, $F_c = R\cos(\beta - \alpha)$ (From Figure 8.2) . . . (D)

$$42.0 = 45.31\cos(\beta - 10)$$

$$\therefore \cos(\beta - 10) = 0.9269$$

$$\beta = 32°$$

Substitute the value in Eq. (B)

$$\therefore F = 45.31 \sin\beta$$

$$F = 24 \text{ kgf}$$

Substitute the value in Eq.(A), we get

$$\eta_f = \frac{F.r}{F_c} \times 100 = \frac{24 \times 0.555}{42} \times 100$$

$$\eta_f = 31.7\ \%$$

Note : Thus, it is seen that in this problem (process) the substantial percentage of total energy is utilized in overcoming the friction.

Example 8.2

A steel workpiece is being milled (Plain milling) by a milling cutter of 75 mm diameter having total number of teeth as 8. The feed velocity is 75 mm/min while cutting speed is 25000 mm/min. The depth of cut is 5 mm and width of cut is 100 mm. The specific pressure for this steel is assumed to be equal to 300 kg/mm^2, and maximum chip thickness is equal to 0.0442 mm. Calculate the mean power required by assuming (a) single tooth in contact, (b) several teeth in contact, (c) also calculate the maximum power required. Comment on the results obtained in sections (a) and (b).

Solution

Following data are given in the problem :

D = 75 mm, Z = 8, f = 75 mm/min, V_c = 25000 mm/min (= 25 m/min), d = 5 mm, W =100 mm, p = 300 kg/mm^2, t_{max} = 0.0442 mm.

(a) Eq. (8.25) is used to determine power required

$$\text{Power} = \frac{F_c\ V_c}{60 \times 75} \text{HP} \qquad \ldots (8.45)$$

Here, cutting speed, V_c = 25 m / min is given.

Value of average force is determined from Eq. (8.46)

$$\overline{F} = p\overline{A} \qquad \ldots (8.46)$$

Here, $\overline{A}$ is not known. p is given in the problem for the maximum undeformed chip thickness as p = 300 kg/mm^2.

The mean cross-sectional area of the chip is given as

$$\overline{A} = W\,\overline{t} \quad \text{(From Eq. (8.34))}$$

W = 100 mm is given in the problem. And,

$$\overline{t} = \frac{f}{N\ Z}\left(\frac{\sqrt{d\,(D-d)}}{D}\right) \qquad \ldots (8.47)$$

Here, except the value of N, all the other parameter and values are given.

(f = 75 mm/min, d = 5 mm, D = 75 mm, Z = 8)

Spindle speed (N) can be determined using the relationship between N and V_c.

$$V_c = \pi\ D\ N$$

or, $$25000 = \pi \times 75 \times N$$

$$\therefore\ N \approx 106 \text{ RPM}$$

Now, substitute the values in Eq. (8.47)

$$\overline{t} = \frac{75}{106 \times 8}\left(\frac{\sqrt{5\,(75-5)}}{75}\right)$$

$$\overline{\boldsymbol{t}} = 0.022 \text{ mm}$$

The average cross-sectional area is determined from Eq. (8.29)

$$\overline{A} = 100 \times 0.022$$

$$\overline{\boldsymbol{A}} = 2.20 \text{ mm}^2$$

$$\overline{F} = p\overline{A} = 300 \times 2.20 = 660 \text{ kg.}$$

Now, average Power with single tooth in contact can be found from Eq. (8.25)

$$\overline{\text{Power}} = \frac{660 \times 25}{60 \times 75} = 3.67 \text{ H.P.}$$

$$\overline{\text{Power}} = 3.67 \text{ H.P.}$$

(b) **Notes :** Above procedure is good enough when it is assumed that a single tooth is cutting at a time. In actual practice, more than one tooth are cutting simultaneously and such teeth are helical teeth (Figure 7.22(a)). In such cases, $\overline{A}$ should be calculated using Eq. (8.30) as follows :

$$\overline{A} = \frac{Wdf}{V_c}$$

$$= \frac{100 \times 5 \times 75}{25000}$$

$$\overline{\boldsymbol{A}} = 1.5 \text{ mm}^2$$

$$\therefore \quad \overline{F} = 300 \times 1.5 = 450 \text{ kg}$$

Hence, $$\overline{\text{Power}} = \frac{450 \times 25}{60 \times 75}$$

$$\overline{\text{Power}} = 2.5 \text{ HP}$$

(c) However, for design purposes one should use maximum cross-sectional area (A_{max}) of the chip in place of mean cross-sectional area ($\overline{A}$) calculated earlier. Hence,

$$A_{max} = \frac{2Wf}{NZD}\sqrt{d\,(D-d)}$$

$$A_{max} = 4.41 \text{ mm}^2$$

$$F_{max} = 300 \times 4.41$$

$$F_{max} = 1323 \text{ kg}$$

$$\text{Power}_{max} = \frac{1323 \times 25}{60 \times 75} = 7.35$$

$$\text{Power}_{max} = 7.35 \text{ HP}$$

Comments : From above three results, it is clear that there is a substantial difference in the calculated power if the requirements are not clearly stated. This difference can be as high as three times in case of Power_{max} as compared to average power for helical teeth. Further, depending upon the cross-sectional area of the chip, the value p would vary which has not been taken care of in this example.

Example 8.3

In the above Example, take depth of cut as 2.5 mm and feed velocity as 150 mm/min (the material removal rate will remain same). Calculate the mean power requirement for slab milling (helical teeth).

Solution

In this case, to calculate power from Eq. (8.25), F_c and V_c should be known. V_c is given as 25000 mm/min. F_c can be calculated if pressure at maximum chip thickness is known. So, t_{max} is calculated from Eq. (8.27).

$$t_{max} = \frac{2 \times 150 \times \sqrt{2.5(75-2.5)}}{106 \times 8 \times 75}$$

$$t_{max} = 0.0636 \text{ mm}$$

Corresponding to this maximum chip thickness, approximate specific cutting pressure can be taken as 280 kg/mm^2 (from the standard tables). Then, the peripheral cutting force can be taken as

$$F_c = 280 \ \overline{A}$$

$$= 280 \times 2.20 \ (\overline{A} = 2.20 \text{ mm}^2 \text{ from Example 8.2})$$

$$F = 616 \text{ kg.}$$

Therefore, $$\overline{\text{Power}} = \frac{616 \times 25}{75 \times 60}$$

$$\overline{\text{Power}} = 3.42 \text{ HP}$$

Example 8.4

Details of slab milling of steel workpiece are given in Figure 8.9. Calculate the cutting time for reducing the height of the workpiece by 12 mm.

Figure 8.9

Solution

Time for one pass is given by

$$T_1 = \frac{l + 2A_p}{f_t \ N \ Z}$$

where, $l = 250$ mm, $f_t = 0.18$ mm/tooth, $Z = 16$ teeth. N and A_p are not known.

To know rotational speed of cutter (N),

$$V_e = \frac{\pi D N}{1000}$$

$$\therefore N = \frac{V \times 1000}{\pi D}$$

$$= \frac{60 \times 1000}{\pi \times 150} \qquad \text{(Data from Figure 8.9)}$$

$$= 127.3 \text{ RPM}$$

This RPM (= 127.3) is not available on the machine. Therefore, let us assume that the RPM available on the machine is 120 hence it can be taken in place of 127.3 RPM.

$$N = 120 \text{ RPM}$$

To evaluate A_p, we know

$$A_p = \sqrt{d(D-d)}$$

$$= \sqrt{6(150-6)}$$

$$= \sqrt{6 \times 144}$$

$$A_p = 29.4 \text{ mm}^2$$

The time required for one cut is given by T. Then,

$$T = \frac{250 + 2 \times 29.4}{0.18 \times 120 \times 16}$$

$$T = 0.90 \text{ min}$$

As can be seen from Figure 8.9, the width of the milling cutter is 15 mm. Hence, total time required to remove 6 mm deep layer of material from the workpiece surface is T_1. Then,

$$T_1 = \frac{105}{15} \times 0.90$$

$$T_1 = 6.3 \text{ min.}$$

To reduce the height by 12 mm, two complete passes are required. Then,

$$T_2 = 2 \times 6.3$$

$$T_2 = 12.6 \text{ min}$$

SAQ 1

(a) Choose the most appropriate answer.

(i) The specific energy per unit volume of the workpiece material removed during metal cutting does not depend on

(a) work material properties, (b) cutting conditions, (c) tool material properties.

(ii) Up milling does not result in

(a) Chatter, (b) dragging of the work in the cutter, (c) fast blunting of the cutting edge.

(iii) Frequent impact loading of the cutting tool takes place during

(a) turning, (b) drilling, (c) milling, (d) shaping.

(iv) The chip thickness in slab milling is directly proportional to

(a) cutter's rotational speed, (b) angle ϕ, (c) number of cutter's teeth, (d) workpiece feed rate.

(v) In case of milling using helical cutter, number of teeth in contact with the workpiece at a time is

(a) one, (b) more than one, (c) either of these two options.

(vi) In case of face milling, number of teeth in contact with the workpiece at a time is

(a) one, (b) more than one but not all, (c) all of them.

8.5 SUMMARY

During metal cutting, power is consumed in removal of material from the workpiece (or chip formation), to overcome friction between the chip and the tool face, and to overcome friction between the tool and workpiece. The contribution by the last factor (friction between the tool and work) being small, is usually ignored. Knowing the cutting force (F_c) and friction force (F_f), the power required during single-point cutting can be calculated if V_c and V_f are known. Invariably, in all such cases the orthogonal cutting conditions are assumed.

In case of multi-point cutting, only milling and drilling have been analysed. In case of milling, only plain slab milling and symmetrical face milling cases are considered to evaluate the average and maximum chip thickness and cross-sectional area. After this, the average and maximum power required during milling can be calculated if the specific pressure for the given workpiece material is known. During the design of the milling machine, the maximum power required should be considered in place of the average power.

In case of drilling, the torque (or moment) and the thrust force are evaluated with the help of empirical relationships. Once these quantities are known, the power required during drilling is calculated.

8.6 KEY WORDS

Chip	:	It is the material which is separated from the workpiece when the tool moves into the workpiece.
Primary Shear Deformation Zone (PSDZ)	:	Finite zone (thin or thick depending upon various governing parameters) within which deformation takes place.
Secondary Shear Deformation Zone (SSDZ)	:	Deformation which takes place at tool-chip interface.
Orthogonal Cutting	:	Two dimensional (2-D) cutting in which cutting edge is straight, parallel to the original plane surface of the workpiece and perpendicular to the direction of cutting.
Oblique Cutting	:	Cutting operations are 3-D in nature. In this type, cutting edge at the tool is inclined to the line normal to the cutting direction.

8.7 ANSWERS TO SAQs

SAQ 1

(a) (i) c

(ii) b

(i) c

(ii) d

(vi) b

(vii) c

8.8 EXERCISES

Exercise 1

(a) A soft steel workpiece 20 mm wide is being machined by a straight slab milling cutter of 25 mm radius having 20 teeth. The table feed velocity is 15 mm/min and the cutter rotates at 1 revolution per second (RPS). If the depth of cut is 1 mm then calculate the average power consumption. Use specific cutting pressure as 300 kg /mm^2.

(b) A steel (medium) workpiece having 50 mm width and 150 mm length is being face milled by 25 mm radius milling cutter at 25 m/min peripheral speed with 5 mm as depth of cut. The cutter has total 10 teeth. The table feed is equal to 1.25 mm/sec. Take specific cutting pressure as 430 kg/mm^2. Calculate

(i) maximum chip thickness,

(ii) mean cross-sectional area of chip, and

(iii) power required.

Exercise 2

(a) During plain milling, prove the below given expression for the average chip thickness

$$A_m = \frac{fW}{NZ}\sqrt{d/D}$$

(b) A hole of 30 mm diameter (= D) is to be drilled in a workpiece having its height equal to 2 D. The drill is fed at 0.20 mm/rev and the cutting speed equal to 60 m/min. Calculate total time taken for drilling and material removal rate.

8.9 NOMENCLATURE

ϕ_c	Tooth contact angle
A	Area of the cross-section of the chip
$\overline{A}$	Mean cross-sectional area of the chip
A_m	Maximum cross-sectional area of the chip
A_p	Approach allowance
b	Width of cut
d	Depth of cut
D	Cutter diameter
F	Friction force
F_{max}	Maximum Force
f	Feed rate
F_c	Cutting force
F_s	Shear force

f_r	Feed rate per revolution
K	Material constant
N	Spindle speed
P_c	Power required for cutting process
P_s	Power utilized in primary shear zone
P_f	Power utilized in secondary shear zone
r_c	Chip thickness ratio
t	Chip thickness
$\bar{t}$	Mean chip thickness
t_u	Uncut chip thickness
$\bar{t}_u$	Mean undeformed chip thickness
V_c	Cutting velocity
V_f	Chip flow velocity
V_s	Shear velocity
W	Width of cut
U_f	Energy required in the secondary shear deformation zone
U_s	Energy required in the primary shear deformation zone
U_t	Total specific energy
Z	Number of teeth in a cutter

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