EXPERIMENT 8

TO STUDY AN OFF-BALANCE WHEATSTONE BRIDGE AND TO INVESTIGATE ITS USE IN THE MEASUREMENT OF STRAIN

Structure

8.1 INTRODUCTION

You must be familiar with the Wheatstone bridge right from your school physics courses. Recall that it is a simple arrangement for measuring an unknown resistance, when it is balanced for zero current. This technique can be put to interesting uses when it is off-balance, to measure such physical quantities, the changes in which can be related to changes in resistance. From the physics courses you have studied so far, can you recollect instances where detection or measurement of the change in a physical quantity was done by first converting it into a change in electrical resistance? If you have taken the course Physics Laboratory-II [PHE-08(L)], you would have used a thermistor to measure temperature. There, the change in temperature was being converted into a change in the resistance of a wire. By measuring the change in the wire's resistance, you could measure the change in the temperature. In this laboratory course also, you have investigated the temperature dependence of black body radiation in an analogous manner.

Apart from temperature, light intensity and strain are some other physical quantities which can be converted into resistance through a sensor. We can then measure this resistance (or the change in it) by the familiar Wheatstone bridge, when it is off-balance.

In this experiment you will carry out measurements of resistance with the help of an off-balance Wheatstone bridge and use this technique to measure the strain produced in a cantilever beam as the load is increased.

Objectives

After performing the experiment you should be able to

- use Wheatstone bridge in an off-balance condition
- linearise the bridge output, and
- measure strain of a cantilever using an off-balance Wheatstone bridge.

8.2 BASIC PRINCIPLES

The measurement of any physical quantity by converting it into an electrical quantity has three basic elements. These are indicated in the block diagram given ahead :

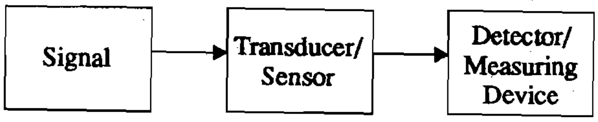

Fig.8.1: Block diagram for a typical measurement process using the electrical method

Devices which accept an input signal in one form of energy (e.g. mechanical thermal, fluid) and convert it into an output signal in another form of energy (e.g. mechanical, electrical) are called transducers or sensors. In many cases, the sensor or transducer converts the physical quantity of interest into a proportionate current or a resistance that changes with it. Thus, by measuring the change in the electrical signals, we can measure the corresponding change in the physical quantity.

The **sensor** converts the physical quantity into an electrical signal, which could be a voltage, current or a resistance. The signal is then suitably transformed for further measurement in a desired range by analog or digital techniques and displayed accordingly. Depending on how you design the experimental set up, transforming the signal could involve, say, a conversion from resistance to voltage, or from current to voltage or merely an amplification of the signal. For these purposes you could use a resistive bridge or the versatile operational amplifier. You have many choices!

SAQ 1

a) Make a list of the various sensors you have encountered so far. What is the nature of the output produced by each of these?

b) In Laboratory II, you used a thermistor to measure temperature. Specify [illegible] elements in the measurement process.

Signals from resistive sensors are usually measured using a Wheatstone bridge, which transform the change in resistance to a change in voltage. You may be familiar with the Wheatstone bridge having worked with it in PHE-03(L) as well as PHE-08(L). Recall that normally you balance the bridge using a null detector. Then you compute the unknown resistance of the sensor from the relationship between the resistances. Thus the precision of the measurement you make depends upon the precision of the standard resistances. Null methods are, however, slow and can be used reliably only for static or slowly varying quantities.

SAQ 2

What problems did you encounter in calibrating the thermistor with temperature using a balanced Wheatstone bridge? What was the accuracy of your method?

An alternative technique involves balancing the bridge once at some initial value of the physical observable. All subsequent changes in the transducer signal are then measured as small differential changes in the bridge output. The off-balance bridge is extremely convenient to use and particularly suited for dynamic systems wherein the physical observable undergoes rapid changes.

We shall now present a brief discussion of the Wheatstone bridge used as an off-balance bridge.

8.2.1 Wheatstone Bridge

As you know, the Wheatstone bridge provides a basic and precise technique for measuring resistance. The well known four-arm circuit of the Wheatstone bridge is shown in Fig.8.2.

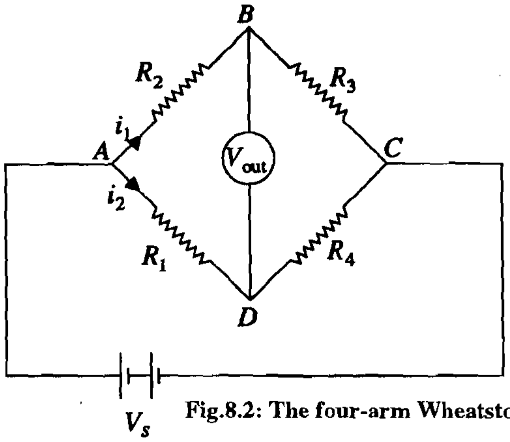

Fig.8.2: The four-arm Wheatstone bridge

To study an Off-Balance Wheatstone Bridge and to Investigate its use in the Measurement of Strain

Here V_S is the power supply voltage and R_1, R_2, R_3 and R_4 are the four resistive elements. Using Kirchoff's laws you can quickly determine the output voltage of this bridge, measured at the junctions B and D, which is given by

$$V_{out} = V_S \left[\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right] \quad (8.1)$$

Let the currents in arms ABC and ADC of the bridge be i_1 and i_2, respectively. Then Kirchoff's Law applied to loop ABD yields.

$i_2 R_1 - i_1 R_2 = V_{out}$

Also

$i_1 (R_2 + R_3) = i_2 \ (R_1 + R_4) = V_S$

Therefore,

$V_{out} = V_S \left[\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right]$

If we now replace R_1 by a resistive sensor keeping R_2, R_3 and R_4 fixed, V_{out} will vary with R_1 and the physical quantity at the input of the sensor may be measured in terms of the bridge output. Since

$$V_{out} = V_S \left[\frac{1}{1 + (R_4/R_1)} - \frac{1}{1 + (R_3/R_2)} \right] \quad (8.2)$$

the absolute values of R_2 and R_3 are not important; the output essentially depends upon the three parameters V_S, R_4 and the ratio $r = R_3/R_2$.

Balance Condition

You know that the bridge is said to be "balanced" when R_4 is adjusted till the output voltage is zero. In this configuration.

$$\frac{R_4}{R_1} = \frac{R_3}{R_2}$$

and the sensor resistance is readily given by

$$R_1 = rR_4$$

Let us see what happens when the bridge is off-balance.

Off-Balance Bridge

Let the bridge be balanced initially at some reference point at which the resistance of the sensor is denoted by $R_1 = R$. Let us now examine how the output will vary as the resistance is changed by a small amount ΔR. Choosing $r = 1$ and expressing $R + \Delta R = R(1+x)$, where $x = \Delta R/R$ is the fractional change in resistance, we may write

$$V_{out} = V_S \left[\frac{R(1+x)}{R(1+x)+R} - \frac{R}{R+R} \right] = V_S \left(\frac{x}{4}\right)\left(1 + \frac{x}{2}\right)^{-1}$$

since $r = 1$ and $R_1 = R_4 = R$. Using binomial expansion, we can rewrite it as

$$V_{out} = \frac{1}{4} V_S \, x \left[1 - \frac{1}{2}x + \left(\frac{1}{2}x\right)^2 - \right]$$

$$= \frac{\Delta R}{4R} V_S \left[1 - \frac{\Delta R}{2R} + \left(\frac{\Delta R}{2R}\right)^2 - .... \right] \quad (8.3)$$

It is important to note that Eq.(8.3) is non-linear and is rather cumbersome to use for defining a calibration curve or fitting data. Hence, the bridge is usually designed so that the output can be approximated by a linear function of the fractional change in resistance x.

Linearizing Bridge Output

When $x<<1$, the quadratic and higher order terms in x can be dropped. Then the bridge output takes the form

$$V_{out} = \frac{\Delta R}{4R} V_S \tag{8.4}$$

That is, V_{out} varies linearly with ΔR, the change in resistance! You may wonder how good this approximation is and how much departure from linearity will occur for large changes in x?
This is best seen by computing the contribution from the various terms. As x changes by 1%, the departure from linearity is merely 0.5%. You can get an idea of how much variation from linearity your measurement can tolerate by answering the following SAQ.

SAQ 3

Complete Table 8.1. How much should the fractional change in resistance be to ensure that the bridge output is linear to within 10%?

Table 8.1

Sl. No.	x	1st term ($x/4$)	2nd term ($x^2/8$)	% departure from linearity	$v = V_{out}/V_S$
1.	0.01	0.0025	0.0000125	0.5	0.0024875
2.	0.02				
3.	0.05				
4.	0.10				
5.	0.20				

In the discussion so far we have chosen $r = 1$. Therefore, r does not appear in Eq. (8.3) when the bridge is balanced. However, the resistance ratio r plays a significant role in the design of the bridge circuit. Let us investigate the effect of choosing $r = 10$ at balance. The off-balance voltage is then given as

$$V_{out} = V_S \left[\frac{R(1+x)}{R(1+x)+10R} - \frac{R}{R+10R}\right]$$

$$= V_S \left(\frac{10x}{11(x+11)}\right)$$

$$= \frac{10}{121} V_S\, x \left[1-\left(\frac{x}{11}\right)+\left(\frac{x}{11}\right)^2+....\right] \tag{8.5}$$

You should now re-do SAQ 3 and complete Table 8.2 to get an idea about the linearity of the bridge for the new set of parameters.

Table 8.2

Sl. No.	x	1st term	2nd term	% departure from linearity	$v = V_{out} / V_S$
1.	0.01				
2.	0.02				
3.	0.05				
4.	0.10				
5.	0.50				
6.	1.00				

Bridge Sensitivity

In addition to linearity, it is essential that the off-balance bridge is sensitive to small changes in the resistive element. A measure of bridge sensitivity is provided by the slope of the V_{out} / V_S versus x curve in the region of linearity, i.e. $(\partial V/\partial x)_{x=0}$.

Design Criteria

You should now be able to appreciate that the value of the ratio r decides

i) how linear the bridge output is; and

ii) how sensitive the bridge output is to small changes in resistance.

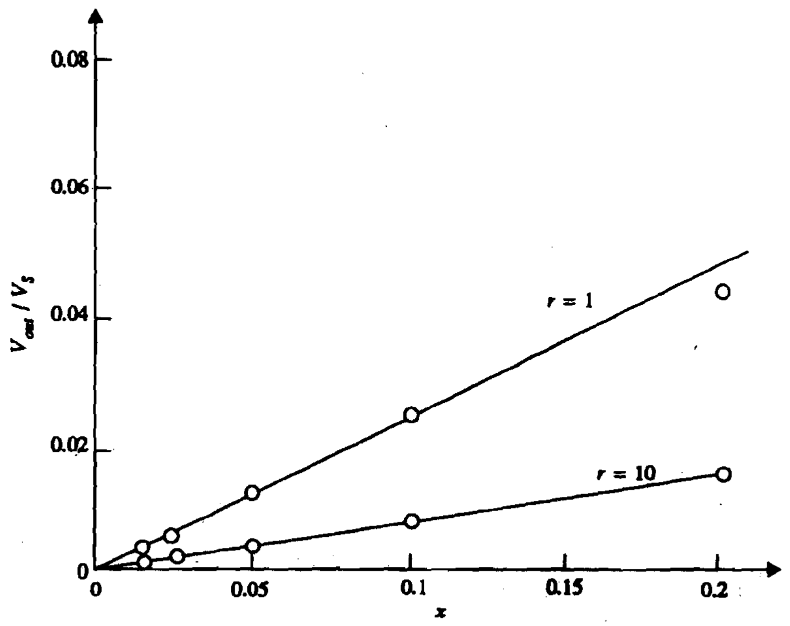

Fig. 8.3: The linearity and sensitivity of the off-balance bridge for $r = 1$ and $r = 10$

Fig. 8.3 shows a plot of V_{out}/V_S versus x for $r = 1$ and $r = 10$. The graph shows that for $r = 1$, the bridge output is quite sensitive but has large departures from linearity. Such a bridge is useful when the changes in x to be detected are small. On the other hand, for $r = 10$, the linearity improves tremendously but at the cost of the sensitivity of the bridge, which is rather low.

The value of the bridge output also depends on the power supply voltage and can be increased by choosing a larger value of V_s. However, this also increases the current in the circuit. It is important to take care and never let the electrical power $i_1^2 R$ dissipated in the sensor exceed the ratings quoted by the manufacturer.

Now that you understand the basic principles involved in an off-balance Wheatstone bridge, you can learn how it is used for the measurement of strain for a cantilever.

8.2.2 Measurement of Strain

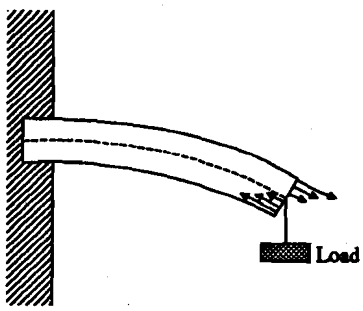

Fig. 8.4: A cantilever

Fig. 8.4 shows a cantilever which is a narrow beam rigidly clamped at one end. A load is attached to the free end causing the beam to bend. The state of such a system is described in terms of linear stress which is the restoring force per unit area (F/A) and linear strain, which is the ratio of the change in length to the original length ($\Delta l / l$). For a given material, the quantity

$$E = \frac{\text{Stress}}{\text{Linear Strain}}$$

is a constant and is called **Young's modulus of Elasticity.**

Obviously, stress induces strain in longitudinal as well as transverse directions. An increase in the length in the longitudinal direction (tensile strain) is invariably accompanied by a decrease in length along the transverse direction (compressive strain). The ratio of longitudinal to transverse strain is constant for a given material and is called the Poisson's ratio (γ).

The strain of the cantilever can be measured by using a strain gauge. Commercial strain gauges are usually made from a fine metal wire or foil arranged in a zig-zag pattern, designed to increase sensitivity in one particular direction. This is bonded on to an insulatory substrate like plastic film or paper and finally glued or cemented to the surface in which strain is to be measured. The principle on which the strain gauge works is briefly explained below.

Working Principle of Strain Gauge

You know that the resistance of a conducting element is directly proportional to its resistivity (ρ) and length (l) and inversely proportional to its area of cross-section (A). If the conductor is stretched, its length will increase and the cross-sectional area will decrease. On both accounts, the resistance will increase and provide a measure of the strain. The defining relationship is simply obtained by evaluating the differential change in resistance. Since $R = \rho l/A$, we can write

$$\Delta R = \left(\frac{\partial R}{\partial l}\right)_{\rho,A} \Delta l + \left(\frac{\partial R}{\partial A}\right)_{\rho,l} \Delta A + \left(\frac{\partial R}{\partial \rho}\right)_{l,A} \Delta \rho$$

Since $\frac{\partial R}{\partial l} = \rho/A$, $\frac{\partial R}{\partial A} = -\frac{\rho l}{A^2}$, $\frac{\partial R}{\partial \rho} = \frac{l}{A}$ and $R = \frac{\rho l}{A}$, this yields

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} - \frac{\Delta A}{A} + \frac{\Delta \rho}{\rho}$$

Using the definitions of longitudinal and transverse

strain $\Delta l/l = e_L, \frac{\Delta A}{A} = \frac{\Delta b}{b} + \frac{\Delta t}{t} = 2e_T$ and the relation $e_T = -\gamma e_L$, we get

$$\frac{\Delta R}{R} = (1+2\gamma)\, e_L + \frac{\Delta \rho}{\rho}$$

The relationship between resistance and strain is usually expressed as

$$\frac{\Delta R}{R} = G e_L \qquad (8.6)$$

where $G = (1+2\gamma) + \frac{1}{e_L}\frac{\Delta \rho}{\rho}$ is the so-called Gauge factor. For most metals, G has a value of around 2. You should ascertain the exact values for both R and G from your Counsellor. (Usually the manufacturer provides these for every strain gauge.)

SAQ 4

A strain gauge of Gauge factor $G = 2.1$ is fixed on a steel cantilever of length $l = 50$ cm at a distance $x = 30$ cm from the fixed end. Compute the fractional change in resistance when a 1000N load produces strain in the cantilever. Young's modulus for steel, $E = 2.1 \times 10^{11}$ Nm^{-2}.

The answer to SAQ 4 will have given you some idea of the change in resistance you will be measuring in the experiment.

We can now introduce the strain gauge in a Wheatstone bridge to carry out strain measurement.

Strain Gauge Bridge

The change in resistance of a strain gauge is extremely small. Thus the measurement bridge must be designed for maximum sensitivity. Choosing $r = 1$ and $R_2 = R_3 = R_4 = R$, (which is the value of resistance of the unstrained gauge), the off-balance voltage is given by the linear relation $V_{out} = \frac{1}{4} G e V_S$

To increase the sensitivity of the bridge, you should use two identical resistive elements. Such a bridge is said to be two-element bridge. Fig. 8.5 shows such a configuration with $R_1 = R_3 = R(1+x)$ as the variable resistances and $R_2 = R_4 = R$.

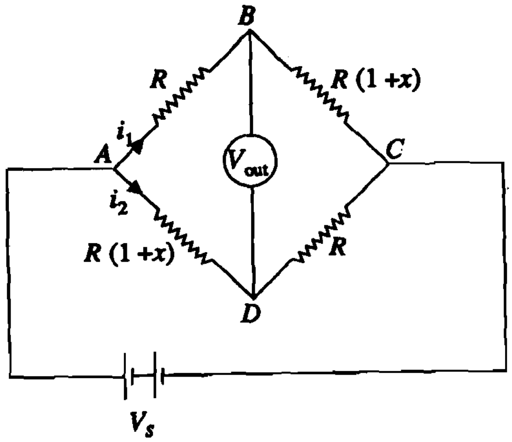

Fig.8.5: Two element bridge

The off-balance voltage for this is given by (Eq. 8.1)

$$V_{out} = V_S \left[\frac{R(1+x)}{R(1+x)+R} - \frac{R}{R+R(1+x)} \right]$$

$$= V_S \left(\frac{x}{2+x} \right) = \frac{V_S \, \Delta R}{2R + \Delta R}$$

$$V_{out} \cong \frac{\Delta R}{2R} V_S \text{ for } \frac{\Delta R}{R} << 1 \qquad (8.7)$$

You will note that the degree of nonlinearity is the same as before but the sensitivity has doubled.

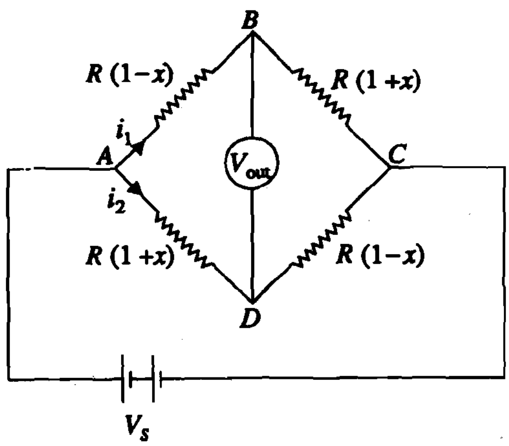

Fig. 8.6: Four Element bridge

The sensitivity can be further enhanced if four identical elements are configured as shown in Fig. 8.6. In a four-element bridge, two of these resistances are made to increase [$R_1 = R_3 = R\,(1+x)$] and the other two are made to decrease ($R_2 = R_4 = R\,(1-x)$] simultaneously. This is what happens when four strain gauges are mounted symmetrically on the cantilever such that R_1 and R_3 are on the top surface and experience tensile strain while R_2 and R_4 are directly below and are subjected to a compressive strain. For this four-element bridge,

$$V_{out} = V_S \left[\frac{R(1+x)}{R(1+x)+R(1-x)} - \frac{R(1+x)}{R(1+x)+R(1-x)} \right]$$

$$= . x\, V_S = Ge_L V_S \qquad (8.8)$$

An important consequence of the complementary nature of changes in the resistances placed in opposite arms is that the output of this bridge is linear. This arrangement also compensates the changes that arise in the gauge resistance due to heating and thus avoids the usual problem of drifts.

SAQ 5

The resistance of a metal strain gauge at temperature T is given by $R(1+\alpha T)$. Will the output of the four-element bridge be independent of temperature? Discuss.

Having understood the basic principles you can now experimentally study the off-balance bridge and use it to measure strain. But before doing the experiment you need to know the apparatus.

Apparatus

Assortment of fixed resistors (values 100 Ω, 1kΩ, 10kΩ, 100kΩ. Variable resistance boxes in the range 0–100Ω, 0–1kΩ. Power supply with V_S = 10 V. Instrumentation amplifier, Digital multimeter, Cantilever with fixed strain gauges, and Slotted weights (50 – 500 gm-wt)

8.3 EXPERIMENTS WITH AN OFF-BALANCE WHEATSTONE BRIDGE

Experiment 1 : To design an off-balance Wheatstone bridge for performance in a given range and to investigate the dependence of the bridge linearity and bridge sensitivity on circuit parameters.

The design of an off-balance bridge depends upon the specific characteristics of the resistive sensor element to be used. In this experiment, we shall not use an actual sensor but will simulate the behaviours of a strain gauge using a variable resistance box.

Procedure

1. Set up the resistive bridge with V_S = 10 V and $R_1 = R_2 = R_3 = R_4$ = 350 Ω. Use the standard 1% precision metal film resistors or a variable resistance box to configure the bridge.
2. Note the output voltage. Is the bridge balanced?
3. Put another resistance box in series with the resistance R_1.
4. Note the off-balance voltage as R_1 is increased from the initial value of R by a small amount ΔR and complete the Observation Table 8.3 as x changes from -1 to +1.

Observation Table 8.3: Change in output voltage with a change in R

$V_S =$ V $R_1 = R_2 = R_3 = R_4 = 350\,\Omega$

S.No.	R (Ω)	$x = \Delta R/R$	V_{out}	$v = V_{out}/V_S$

5. Depict this data graphically by plotting a graph between v and x. You should obtain an almost linear curve.

6. Calculate the slope in the region of linearity.

7. Repeat steps 1–6 for the following choice of the circuit parameters.

Set	R_1 (Ω)	R_2 (Ω)	R_3 (Ω)	R_4 (Ω)
A	350	35	350	35
B	350	3500	350	3500
C	350	35000	350	35000

8. In each case, make your observation table in the space given below and plot a graph between v and x for $|x| < 1$. You will have to make your own decision about the change ΔR and how many observations you want to include.

Summarize the characteristics of the various bridge configurations by entering the required information about sensitivity and linearity of the bridge in the Calculation Table 8.7.

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Calculation Table 8.7 : Sensitivity and linearity of the bridge

Set	$r = R_4/R_1$	Slope $(\partial v/\partial x)_x = 0$	Region of linearity x_{min}	x_{max}
A	0.1			
B	1.0			
C	10.0			
D	100.0			

When is the sensitivity maximum?

In which case is the linearity best?

State your conclusions below:

....

....

....

....

....

Experiment 2 : Set up a four-element strain bridge to measure strain in a cantilever beam. Evaluate the performance of the bridge and calibrate the bridge output in terms of strain.

You are now well equipped to undertake an actual measurement of a physical variable using a resistive sensor and get some hands-on experience of modern day instrumentation. Since care must be taken in handling strain gauges and these are a bit expensive. we are providing you with an already assembled set up. Four gauges are aligned as required and fixed on the top and bottom surface of the beam at a distance *x* from the end that is clamped. These are configured into the circuit shown in Fig.8.7. The power supply *S* that provides the excitation voltage is also fixed on the board.

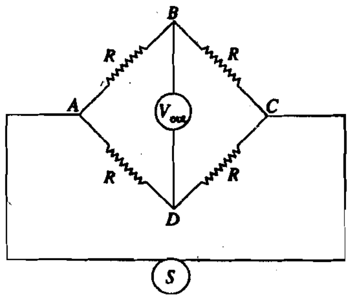

Fig. 8.7: Four-element strain bridge configuration

Before you start the experiment, note the following values.

Length of cantilever l =

Distance of strain gauge from fixed end ; x =

Width of cantilever w =

Thickness of cantilever t =

Young's modulus of elasticity E =

Power supply voltage V =

Resistance of strain gauge R =

Gauge factor G =

(You should ask the Counsellor to provide you information about the values of E and G. Make sure that measurement of resistance is always made with the power supply switched off!)

The strain is simply related to the load by the expression

$$e = \frac{6(l-x)F}{w^2 E l} \tag{8.9}$$

Thus, the output $V_{out} = GeV_S$ is simply proportional to the load F. We expect you to determine the exact form of this relationship for the given setup experimentally.

To start the experiment, switch on the power supply. Since the four elements are identical, in the absence of any load, the bridge should be balanced. If V_{out} has a finite value at the outset, this can be taken as a constant offset. Increase the load in small steps, noting the bridge output in the Observation Table 8.8 below:

Observation Table 8.8: Bridge output for changing loads

No.	Load (gm - ωt)	V_{out} (V)	$v = V_{out} / V_S$
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Choose an appropriate scale, plot v with respect to F and report

1. Range of linearity of the calibration curve : ____________________
2. Sensitivity of the bridge: ____________________