UNIT 16 LEARNING TO ADD AND SUBTRACT

Structure

16.1 INTRODUCTION

Addition and subtraction are two basic operations that we all use throughout our lives. These are some of the most fundamental concepts that children are to be taught during the foundational stage. How do you think children should be introduced to the concepts of addition and subtraction?

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This Unit will help you answer this question.

In this Unit, we will discuss how and when to introduce the operations of addition and subtraction to young children. We usually begin to teach children '+' and '-' problems using the algorithm. **This Unit will highlight that it is necessary to introduce children to the symbols and algorithm but it should not be the first step. The first step should be to help the children develop a conceptual understanding of what it means to add and subtract.** With a special emphasis on the use of concrete objects to introduce the concepts of addition and subtraction, the Unit will discuss various types of addition and subtraction word problems as well. The concept of place value and how it is crucial in the development of the concept of addition and subtraction will also be discussed.

Objectives

After reading this Unit, you will be able to:

- explain the concept of addition and subtraction;
- understand common errors children make while solving addition and subtraction problems;
- explain the meaning of place value;
- understand the role of place value in the development of the concept of addition and subtraction;
- develop an understanding of how the algorithms used for addition and subtraction work;
- describe various types of word problems and be able to formulate them; and
- plan and conduct activities to teach addition and subtraction.

16.2 MEANING OF ADDITION AND SUBTRACTION

Let us quickly see what 'addition' and 'subtraction' encompass.

16.2.1 Addition

Addition is the process of adding two or more items together. Concerning mathematics, addition includes calculating the sum of two or more numbers. It is a primary arithmetic operation that is used multiple times daily in our everyday life. Some of the most common uses of addition are when we use money or calculate time.

16.2.2 Subtraction

The process of subtraction is the reverse of addition. A collection can be made bigger by adding a few more to it and it can be made smaller by taking a few away from it – these are two opposite processes. Subtraction is more complicated for children than addition. This is because they have to identify the quantity from which they have to remove (take away) the other quantity. This is an important difference. While solving addition problems,'2 + 3' is

the same as '3 + 2'. However, '9-2' is not the same as '2-9'. **This implies that we cannot change the order of numbers in subtraction**. In other words, subtraction is not a commutative process as addition is a commutative process.

16.3 LIMITATIONS OF THE TYPICAL METHOD OF TEACHING ADDITION AND SUBTRACTION

Any mathematical problem is solved by following a finite number of steps that frequently involve single or repetitive use of an operation. **This is called an algorithm**. Various symbols (mathematical notations) are used to represent the algorithm.

With reference to the addition and subtraction problems, children use the following symbols:

a) Use of digits to represent a number, such as 4, 7, 18, 23 etc.

b) Use of the '+' symbol to signify by the operation of 'addition'.

c) Use of the '–' symbol to signify the operation of 'subtraction'.

d) Use of the '=' symbol for representing 'is equal to'.

As adults, we find it very easy and natural to use the algorithm. Therefore, usually, when children are introduced to the concept of addition and subtraction in grades 1 and 2, it is typically done by teaching them to perform the algorithm. In other words, they are introduced to the abstract expression of addition and subtraction through the use of the algorithm as the very first step. However, most children perform the algorithm, by learning the series of steps, without actually understanding why they are applying the particular algorithm using these symbols. This means that they perform the algorithm mechanically, without developing a conceptual understanding of what it means to add and subtract. You may recall from Units 13 and 14 that children understand and engage with mathematical concepts better when they are introduced to maths concepts through concrete materials first and later the algorithm, which is abstract. This is because young children's conceptual knowledge and understanding progresses from concrete to abstract.

Even when children are introduced to addition and subtraction with the help of concrete objects such as beads, blocks or sticks, they are not given enough opportunities to see, understand and practice addition and subtraction using concrete objects. Either the teacher uses only one type of concrete material or uses it for a very short time and quickly comes to the use of the algorithm.

So while it is necessary to introduce children to these symbols and the algorithm and encourage them to use it while solving addition and subtraction problems, it should not be the first step in teaching addition and subtraction. Why are we saying this? This is because of the following reasons:

a) Children may use the algorithm correctly and give you the 'correct answer' but this does not mean that they have understood the concept of addition and subtraction.

b) Children commit some common errors when they are made to use algorithms mechanically and this clearly shows that they have not understood the concept of addition and subtraction.

Let us understand this point by seeing how a child solved addition problems.

a) *23+5*

$$\begin{array}{r} 23 \\ +\ 5 \\ \hline 73 \end{array}$$

b) *28+14*

$$\begin{array}{r} 28 \\ +\ 14 \\ \hline 312 \end{array}$$

You can see that the child has added the numbers in the first row with the numbers in the second row. As taught by the teacher, she counted them and wrote the answer. However, her answers are not correct. But what are the reasons for the answers being incorrect? Let us find out!

In the first problem, the child does not recognize 23 as 20+3 as a single number. Instead, according to her, '2' and '3' are two separate numbers. In other words, she does not know that the place value of '2' is tens and '3' is ones. Since she does not have a fair understanding of place value, she has also placed '5' whose place value is ones under '2' whose place value is tens. The correct way is to place digits with the place value of ones under each other. The same rule applies to digits with the place value tens, hundred, etc. **So before proceeding to addition, the child should know place value**.

In the second problem, the child had added 8 and 4 and written '12' without carrying over '1' whose place value is tens. Here as well, the concept of ones and tens is not clear to the child. **She has not understood the concept of carry-over, which is based on the concept of place value.**

Now, let us look at some subtraction problems performed by this child to further understand some common errors.

a) *203-21*

$$\begin{array}{r} 203 \\ -\ \ 21 \\ \hline 222 \end{array}$$

b) *512-415*

$$\begin{array}{r} 512 \\ -\ 415 \\ \hline 103 \end{array}$$

Here, in the first case, the concept of 'zero' is not clear. It can be that the child has been told that '0' is 'nothing' and so she writes down the digit '2' as such. In the second case, the child did not read the numbers as one entity but treated the digits separately. Since the child does not understand that 512 and 415 are complete numbers wherein each of the digits holds the place value of ones, tens, and hundreds, she simply subtracted the smaller digit from the bigger digit in each column.

To summarize, **the point we are making is that while children need to learn how to use the algorithm of addition and subtraction, they need to first understand the meaning of what it means to add and subtract**. The teacher should introduce the concept of addition and subtraction in such a way wherein children first get a lot of experience in practicing addition and subtraction through a variety of concrete objects, then practice it through

pictorial representations. Once they have enough opportunities of addition using concrete objects and pictorial representation, they should be introduced to symbols and algorithms which are abstract representations. In section 16.4 we have given an example of how the concepts of single-digit addition and subtraction can be introduced to children.

The examples discussed in this section clearly show that one of the concepts that teachers need to teach children before introducing the algorithm of addition and subtraction is that of place value. The concept of 'place value' and how it can be taught to children has been discussed in Sections 16.5 and 16.6.

16.4 TEACHING ADDITION AND SUBTRACTION FOR SINGLE DIGIT NUMBERS: AN EXAMPLE

Let us understand how this progression from concrete to pictorial to abstract in the learning of maths concepts can actually be done in the classroom by reading about how Jagdish, a primary school teacher, introduced and taught the concept of single digit addition and subtraction to children in class 1.

16.4.1 Level 1: Teaching Addition Using Concrete Materials

Jagdish made several colourful cut-outs of birds. Then he tied a string across the front wall of the classroom and then, started the addition lesson with a story as shared further:

Several birds lived on a tree in the jungle. They would go here and there in search of food and return to the tree.

Jagdish recited a short poem using the cut-outs of birds. He placed a cut-out of one bird on the string and sang:

Ek chidiya bhai ek chidiya
Jungle me rehti ek chidiya.

The children repeated these lines after him.

Then, he placed one more cut-out of a bird next to the first bird and sang:

Kya jaane kidhar se
Ek chidiya aagayi phur se.

Now he asked the children – how many birds were there now?

Children responded quickly – two birds.

He continued this way till there were cut-outs of five birds on the string, and asked question, "How many are there now?" each time.

Now, thereafter, Jagdish started flying the birds away. He sang:

Paanch chidiya bhai Paanch chidiya

Jungle me rehti Paanch chidiya.

Kya jaane ki dhar se

Ek chidiya ud gayi phur se

Bolo Bolo reh gayi kitni chidiya?

The answer to the question – 'How many birds are left?' was given by children by counting the remaining birds. In this way, Jagdish flew away the birds one by one and asked the question, 'Now how many remain?', every time.

In the next round, Jagdish made a slight change and began to add or remove two to three birds at one time from the string and asked the children, 'How many are there now and how many are left now?'

Jagdish repeated the same activity in the next few days wherein birds were replaced by fish, monkeys, and rabbits. He added questions such as, 'How many more have come?', 'Now, how many have gone?', and 'How many remain?' etc.

Gradually, a stage came when the children no longer had to count the cut-outs. A mere glance was enough for them to give the correct response. After much practice, children could easily answer questions such as – If five monkeys were sitting on a tree and three monkeys ran away, then only two monkeys remained, and so on.

You may have observed the following in this example:

a) Jagdish did not introduce to the concept of addition to the children using an algorithm. He began by using concrete objects (cut-outs of birds and other animals) and contextualization (storytelling) to help children understand what addition encompasses. Jagdish's strategy of weaving a story using concrete objects and using these objects to teach the concept of addition and subtraction helped children to understand what is involved in addition without much difficulty and they rather enjoyed it. This is not to say that children should not be introduced to symbols and algorithm at all; however, to do so at the initial stage is not a good idea as the symbols may seem arbitrary and meaningless to many children.

b) Jagdish used a ***variety*** of concrete materials (monkey, bird, fish cut-outs) to ensure that children generalize the concept of addition and subtraction; in other words, this helped them to understand that addition can be applied to all sorts of items.

Note*:* Refer to Section 16.6 to get more ideas for different kinds of concrete objects that can be used to develop the concept of addition and subtraction.

16.4.2 Level 2: Teaching Addition and Subtraction Using Pictorial Representation

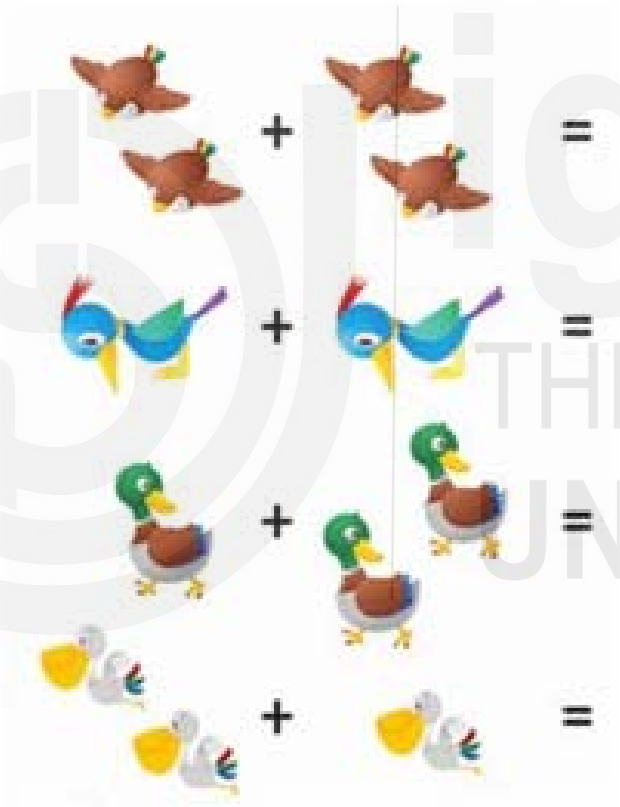
As children acquired ample experience with addition and subtraction using concrete objects, Jagdish introduced some pictures and asked various questions that required children to add and subtract. For example, look at the following image:

Using this picture, Jagdish asked how many birds would there be if the bird on the right (the one not sitting on the wire was:

a) Flying away

b) Landing on the wire

To further strengthen children's understanding of addition and subtraction, he introduced some pictorial worksheets in the class as follows:



Worksheet 1

In Worksheet 1, for each set of birds, he asked the children how many birds were there of a particular type and asked them to draw the same number of birds on the right side. As you can see, he also used this pictorial worksheet to introduce children to the symbols '+' and '='. This way, children could be easily made to understand that '1 and 2 ducks are 3 ducks', which can also be represented as '1 duck plus (+) 2 ducks are equal to (=) 3 ducks'.

You can also create or locate similar worksheets and use these with children to develop their understanding of addition and subtraction.

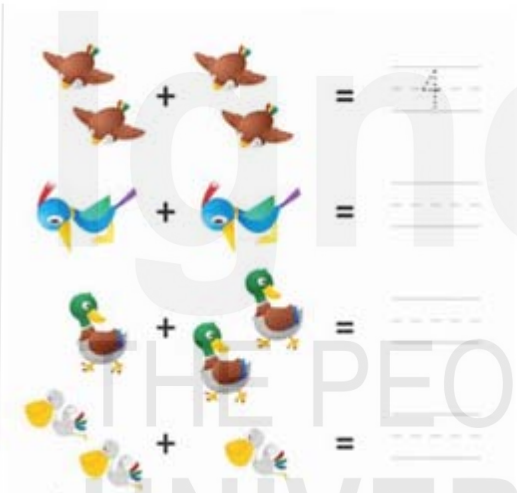
16.4.3 Level 3: Moving towards Abstract Representation of Addition and Subtraction

The next step in Jagdish's strategy was to introduce children to numerals while counting and speaking the number name. The following activity was conducted using the blackboard to help children move towards abstract representation. He wrote the following:

How many were there?		*How many have come?*		*Now how many are there?*
4	+	*1*	=	*5*

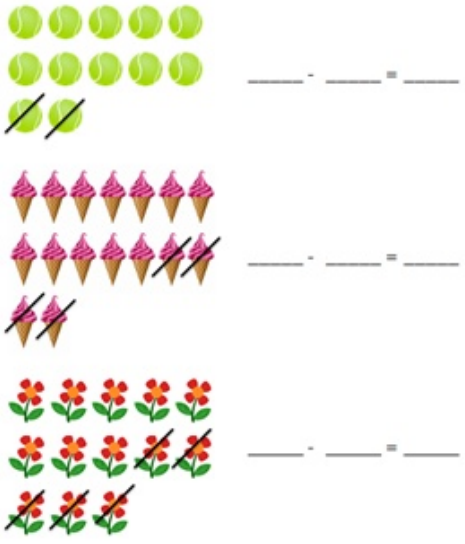
In Worksheet 2, Jagdish asked the children to write the sum of the birds represented on the left side using the numeral.

In this worksheet, he used pictures and symbols together. This way, he introduced children to the symbolic representation of addition and subtraction through the pictorial representation of addition and subtraction, which the children were already familiar with.



Worksheet 2

For subtraction, he introduced similar worksheets (with both pictures and symbols) such as Worksheet 3.



Worksheet 3

To introduce this worksheet he wrote the following on the blackboard:

How many were there?		*How many were removed?*		*Now how many are there?*
12	-	*2*	=	*10*

As children are repeatedly asked questions such as – 'How many were there?', 'How many have left?','Now how many are there?' along with making them familiar with the symbolic representation using signs and numerals, children will soon be able to write the numerals according to the statement and use the correct operational sign. However, you may ask – Can the children learn this easily? Let's find out what Jagdish has to say about his experience.

Children make mistakes. To address this, I talk to them. Gradually, they start using the signs by understanding the given situation. They begin to understand when something is being added or taken away. Once they understand this difference, they make fewer mistakes while using the signs.

You may further ask – Is Jagdish's method in any way different from the typical method we discussed in Section 16.3? Ultimately, he is also using numbers and signs. Isn't he? Let us see how Jagdish would respond to your query. Here is what Jagdish has to say!

The biggest difference is that what others do in the beginning, I do at the end. I start with what children are familiar with. For example, they know how to count; they can establish one-to-one correspondence and seriate numbers. So I keep different types of things before them to count – birds, rabbits, pebbles, seeds, etc. I also ask them to add different things. Children feel that they are counting while actually, they are adding. They enjoy this and along with old experiences, new ones also are formed.

After this, they start using mathematical language. Children say, "There were four monkeys, two more came so there were six monkeys. When two is added to four, it is six. Out of six, three have left and so three remain. So six minus three is three". Gradually, children start writing these as numerals and signs. Once this stage is reached, then, they are given mathematical problems only in the symbolic form; concrete objects are no more provided for reference. They have earlier added these numbers through the use of concrete objects and also spoken out the addition they are doing. In this way, they start solving the problems when given in the following manner:

5	5	7
$\underline{-\ 2}$	$\underline{-\ 2}$	$\underline{+\ 2}$
	3	9

So, as you can see through Jagdish's example, the use of concrete materials supports the acquisition of the concept of addition and subtraction. It serves as a strong foundation to introduce children to the algorithm. In the absence of concrete experiences, children struggle and do not understand the logic behind performing a certain algorithm and end up calculating the wrong answers to addition and subtraction problems.

16.4.4 Understanding the Concept of Equality and '=' Sign

When teaching the concept of addition and subtraction, one important concept that is also crucial is that of equality, represented by the '=' sign. Let us see with the help of an example, how a teacher introduced her children to the same.

A teacher asked children in second grade to compare single-digit addition problems, such as – compare 5+3 and 4+2 or compare 6-2 and 5-3 or compare 2+5 and 3+4. The task was to find whether they were equal or more/less. She encouraged children to give reasons for their answers.

Many children considered the '=' sign as an instruction to write the answer to its right side. But we know that the '=' sign indicates equality in the value of both sides of the '=' sign. Another objective of this task was to guide children's attention to the relationship between numbers and operations, rather than only focusing on answers.

In response to the task, most children calculated the answers for both pairs and concluded whether they were equal or more/less. For example, for the pair 6-2 and 5-3, children calculated that 6-2=4 and 5-3=2; since 4 is more than 2, so, 6-2 is more than 5-3. However, a few gave other reasons for their answers. For example, for the pair, 5+3 and 4+2 Raju said that "Both the numbers are big here [5+3] and both the numbers are small here [4+2] and so 5+3 is greater than 4+2". Similarly, Susie saw the equality relation between 2+5 and 3+4 through a quick manipulation of coloured cubes (Figure 15a).

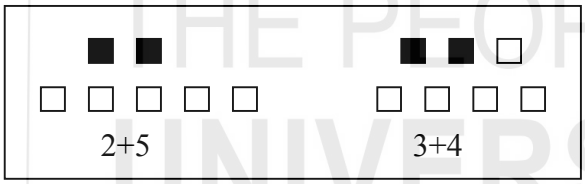


Fig. 16a: Susie's solution for showing the equality between 2+5 and 3+4

You would notice that the numbers are very small and many of the children would have memorized them as number facts. The objective of the task was not to give practice in simple addition or subtraction but to create opportunities to understand the meaning of equality and the '=' sign. **This shift in understanding from "answers" to "relationships" will happen only if children are exposed to such tasks.**

Check Your Progress Exercise 1

1) Why should the use of symbols/algorithms be avoided as the first step with children to the concept of addition and subtraction? Give two reasons.

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16.5 USING COUNTING TO ADD AND SUBTRACT

16.5.1 Using Counting to Add

When we count, we are actually adding – each number is one more than the previous number. Add one more to 3 and it becomes 4, add one more to 4 and it is 5, and so on. While adding, we are combining two or more groups – such as 5 erasers and 2 erasers, or 4 erasers and 3 erasers. **Counting can be used in two ways to add numbers.**

a) **Count All:** Usually, children use the count-all strategy to add when they cannot visualize what number a group or set represents. Let's see how one can count using this method.

   For example, if you are to teach children to add 3 and 2 using manipulatives (such as beads), you would do it in the given manner. You would place three red beads in front of them and count out the pieces. "One, two, three. There are three red beads." Next, you would place another two yellow beads separately and count out the pieces. "One, two. There are two yellow beads." Then you would say "Let's add the two sets by moving all the beads together." You would then line up the two sets of beads in a row and count all of the pieces. "One, two, three, four, five. There are five beads in all. So, three plus two is equal to five."

b) **Count On/Count Up**: Using this technique, a child starts with one number and 'counts on' with the other to get to the sum. For example, if the child is to solve 4 + 3, she will identify 4 as the first number and count on three more —"4 … 5, 6, 7". The answer or the sum is 7. Using the technique of 'count on', children have to be able to hold the second number in their mind and then add by counting forward as many numbers. The child has to hold the number '3' in her mind and count forward by saying out aloud three numbers. The final number is the answer to the addition.

You can begin by demonstrating the technique of 'counting on' by using concrete materials, as explained further.

Place five red beads and three yellow beads in a row on the table. Use your hand or a piece of paper to hide the yellow beads and say "five" while pointing toward the red beads. Then unveil the yellow beads and count on. Point to each yellow bead to emphasize the counting of each one: "Six, seven, eight. There are eight beads in all. Five plus three equals eight." Explain that with this strategy you do not count all of the beads. Instead, you name the quantity in the first set and count on from that number.

Once children have sufficient experience of 'counting on' using concrete objects, you can introduce the 'Count on' technique on the open number line. Do you know what an open-number line is? Write your thoughts in the space below

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An **open number line** is used to express what comes before or after a particular number. This is known as the ordinal relations between numbers. The open number line need not start with '0' nor end with a particular number. It is an open line like the one below (Figure 16b). As you can see in Figure 16b, the distances between the numbers are not proportional to the actual difference between the two numbers.

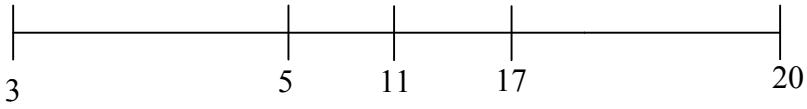


Fig. 16b: The open number line for representing numbers between 0 and 20

Figure 16c shows how to use the open number line for 'count on' technique.

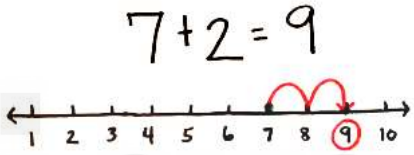


Fig.16c: Representation of '7+2=11' using the open number line

16.5.2 Using Counting to Subtract

Counting can also be used to help children to subtract. Let us see how!

a) **Count Back:** This technique includes subtraction by counting backward from the larger number. Here, we remove one number in every step. For example: Suppose that there are 13 lollipops and you eat 5 of them; how many lollipops will remain? You would count back 5 numbers beginning from 13 – 13, 12, 11, 10, 9. So you are left with 8. You can use concrete objects, pictures, or even a number line to demonstrate the technique of counting back as shown in the Figures 16d and 16e given below.

Fig. 16d: Representation of '15-4=11' using concrete objects

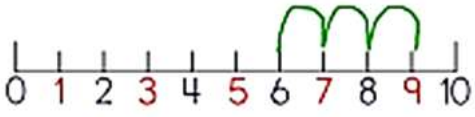
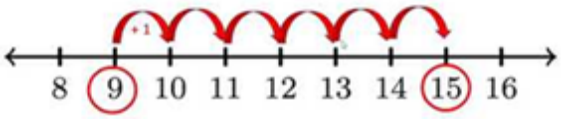


Fig.16e: Representation of '9-3=6' using the number line

b) **Count Forward:** This technique includes subtraction by counting forward from the smaller number to the larger number. Let's understand with an example. If you are to subtract 16 from 39, count forward after 16 until you reach 39. Therefore, you would need to count – 17, 18, 19, 20, 21, 22, 23, 24,25,26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39. Now if we count all the numbers, we get 23. So, we get 39 – 16 = 23. You can demonstrate this using number line, and pictures as well, as

shown in Figure 16f.

15 – 6 = ?



15 – 6 = 9

Fig. 16f: Representation of '15–6 = 9' using the number line

16.6 WHAT IS PLACE VALUE?

In the decimal system, any number can be written using one or more digits from the series 0 to 9. The characteristic of the decimal system is that the value of a number depends completely on the place in which each digit is written in relation to the other digits in it. Therefore, the biggest number can be written in a small form using the decimal system. For example, ten, hundred, and thousand can be written using only 1 and 0. The only difference is in their place in the number. This is possible because we have decided on a different value for each place. We call this 'place value'. For example, in the number 12, the place value of 1 is tens, and 2 is ones.

16.6.1 Teaching Place Value

While it seems a difficult concept to teach, learning aids such as *Ganitmala* or bundles of matchsticks are quite effective in helping children to understand place value as these help the children to see the idea of ones, tens, and hundreds in a concrete form. *Ganitmala* is a chain of beads with groups of 10 beads of the same colour. For example, look at Figure16g.

Fig.16g: 100 beads *Ganitmala* by Jodogyan

This is an effective teaching-learning aid that can be used with children in Grades 1 and 2. You can use it in several ways:

a) Children can count upto 100.

b) They can see that each colour block (red and white in this case) is made up of 10 beads (which can be conceptualized as either 10 ones or 1ten). This helps them to understand grouping by tens.

c) Children can represent numbers using the knowledge of ones and tens as shown in the Figure 16h.

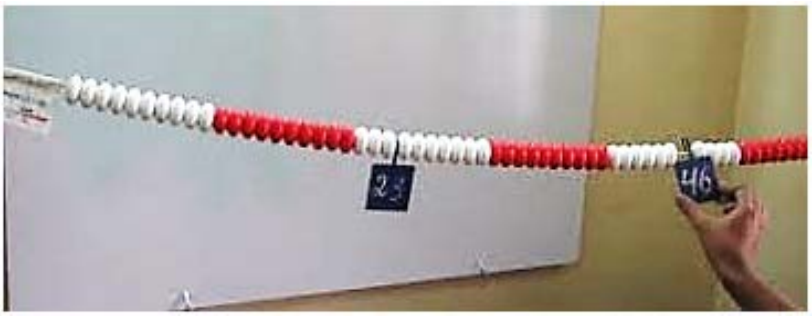


Fig. 16h: Representation of numbers using the *Ganitmala*

Most of us are familiar with using bundles of matchsticks for teaching place value (ones, tens, and hundreds). In Section 16.6, we have given examples of how to use matchstick bundles and the open number line to teach addition and subtraction. The open number line is a pictorial representation of numbers and the relationship among numbers. The use of these materials and aids as discussed in Section 16.6 will further help you to understand how you can support the development of the concept of place value in children.

16.6.2 Role of Place Value in Carry-Over (Addition) and Borrowing (Subtraction)

In the previous Section, you understood that children make frequent errors while solving problems that involve carry-over and borrowing. This is because children do not know the concept of place value and do not take it into consideration while applying the algorithm. Thus, the prime issue is that of not knowing what is place value and how it works.

So let us now see how the algorithm for borrowing and carry-over works and how developing and understanding of place value is of utmost priority in this regard.

When asked to add 126 to 325, most people will write it as follows:

$$\begin{array}{r} {\scriptstyle 1} \\ 325 \\ +\ \underline{126} \\ 451 \end{array}$$

We get 11 on adding 6 and 5. Why do we carry-over the '1'?

In standard addition algorithm, we start the process of addition from the smallest unit i.e., the ones place, and then move to the left. We add the 'ones' and as the addition gives the result as number 10 or more, then we divide that number into ones and tens, and write the ones under the 'ones' column and keep the number in the tens place separate as carry-over. Then, the digits in the tens place are added and the carry-over is added to this sum – the entire algorithm is based on this process. Children need to be made to realize that the carry-over '1' is 'one tens' or 'ten ones'. Before moving to the formal algorithm for addition, children must know ones, and tens. For the child to develop this understanding, the child needs to work sufficiently with concrete objects to understand grouping by tens and place value.

16.7 TEACHING TWO-DIGIT ADDITION AND SUBTRACTION

In this Section we have given various examples of how to introduce two-digit addition and subtraction. **Example 1** is the typical way many teachers introduce addition and **it should be avoided**. **Examples 2, 3 and 4** describe different ways of teaching addition and subtraction **which are appropriate**.

Example 1: A typical Algorithm based Method of Teaching Addition

The teacher, Sheetal had just shown on the blackboard how to do the addition of two numbers with carry-over using the typical algorithm. She then gave the children a problem to find the answer to 26+15. She ensured that children wrote the digits in separate columns of tens and ones in their notebooks so that they did not make obvious errors. After a while, she called one of the children in the class to work out the sum on the blackboard. What happened in the class is given below.

The teacher asked Akhil to come up to the board and solve the given addition problem. She drew the columns indicating Units and Tens for Akhil to write on the board. She also gave him matchsticks, in case he needed them to count and add.

	T	U
	1	
	2	6
+	1	5
	4	1

Akhil comes up to the board and fills the columns of Ones and Tens by the number given in the sum.

Sheetal: Okay, good. You have written the numbers. What would you do next? We always move from right to left. Add up the units column first, isn't it?

Akhil: He takes out the matchsticks corresponding to the numbers 6 and 5. 6 and 5, (counting the matchsticks) seven, eight, nine, ten, eleven.

Sheetal: How will you write the 11 in the Ones place? When we have ten matchsticks, we group them and make a bundle of 10. Remember to write only one digit there. How would you write that?

Akhil: I will write 1 here and a small 1 up in this column (tens column).

Sheetal: Very good. Now, you must add the numbers in the tens column

Akhil: 2 and 1 is 3.

Sheetal: You must remember to add the small 1.

Akhil: 3 and 1 are 4.

Sheetal: So your answer is four (pause) one, 41.

This is the typical way children are taught addition and subtraction using the algorithm. This is a mechanical way of teaching and does not build any understanding of what is involved in addition and subtraction. **We do not advise that you begin the teaching of addition or subtraction in this way**. So what are the alternatives? **Let's read the following two examples to see how two-digit addition and subtraction should be taught to the children**.

Example 2: Learning Addition Using Matchsticks

Let us see how Chetan developed the understanding of place value in his seven-year-old daughter Medha by using matchstick bundles.

a) *Chetan gave his 7-year-old daughter, Medha plenty of matchsticks and asked her to make bundles of ten matchsticks each. When he asked her to give him 10 matchsticks, she counted out 10 from a bundle and gave these to him. Gradually, she realized that one bundle meant the same as giving 10 matchsticks and that she did not have to count the matchsticks in the bundle again. In other words, she understood that 10 matchsticks make a bundle.*

b) *Then, Chetan asked her to represent 35 and 16 using the bundles and individual matchsticks. Medha displayed these numbers using a matchstick in the following way:*

   35 – 3 bundles and 5 single matchsticks

   16 – 1 bundle and 6 single matchsticks

c) *Then, Chetan asked Medha – if these bundles and sticks are added, what will be the number? She kept all the bundles together and formed another bundle using the loose matchsticks. She made 1 bundle out of the 11 loose match sticks and kept 1 match stick separately. Then, she counted all the bundles and remaining matchsticks and said – 5 bundles and 1 matchstick, so 51.*

Example 3: Learning Addition and Subtraction Using Strings of Beads

Let us take another example of how a teacher helped 7-year-old Nazakat to develop the understanding of addition and subtraction using strings and beads.

Nazakat was given several beads and asked to make strings with ten beads in each string. She was told that each string denoted 'one tens' and a single bead denoted 'ones'. After this, when asked to give 33 beads, she would hand over 3 strings and 3 single beads. Then, she was also asked to write down these numbers. To provide further clarity, her teacher drew two columns as follows with the words 'strings' and 'beads' written in the columns along with 'tens' and ones as follows:

Strings (tens)	***Beads (ones)***
3	***3***

When the teacher saw that Nazakat had understood how to represent numbers correctly according to their place value, she asked Nazakat to add two two-digit numbers using the strings of beads. The teacher began by demonstrating how to add using the strings and beads. For example, the teacher showed how to add 25 and 18. She first took out 2 strings and 5 beads for 25 and 1 string and 8 beads for 18. Then the teacher counted the loose beads and said 13. The child knew that the number of loose beads could not exceed 9. At this point, the teacher engaged the child in the following conversation:

Teacher: What will you do now?

Nazakat: I will make a string of 10 beads out of the 13 beads.

Teacher: Very Good! This means that you can exchange one string for 10 beads.

After this, Nazakat counted the number of strings and beads that she had and got 43 as the final answer. She found it to be very simple. The teacher, then said,"Let us write down what we have done in columns." The teacher created columns as below and helped the child to represent the process that had been done. After some help, she could do the following:

Strings (tens)	***Beads (ones)***	
2	***5***	
+ 1	***8***	**= 3 strings + 13 beads**
3+1	***3***	**= 3strings + 1 string + 3 beads**
4	***3***	**= 4 strings + 3 beads**

Throughout the activity, the teacher kept asking Nazakat to explain what was being done. Soon, Nazakat was able to use mathematical language quite naturally. After a few days, the teacher moved on to the next stage of adding using an algorithm and adding numbers such as 65 and 28 without using strings and beads. Gradually, Nazakat could add any 2 two-digit number and got used to carry-over as well.

Similarly, for subtraction, the teacher demonstrated several problems to children using strings and beads. As they got familiar with the steps to be followed, she asked children to subtract 18 from 37. This is how the child solved the sum:

So, first, the child picked up three strings and 7 individual beads. To subtract 18, the child removed one string. Now she was left with 2 strings. To subtract 8, the child removed 7 individual beads and 1 bead from one of the strings. As a result, she was left with one string (tens) and nine individual beads (one). So the answer was 19.

Example 4: Learning to Add and Subtract using the Open Number Line

Let us look at Gurpreet's pedagogy for teaching addition and subtraction using concrete material as well as open number line.

Gurpreet teaches Maths in class 2. In her class, children had the following experiences:

- *The children were familiar with numbers up to 100. Gurpreet used matchsticks to emphasize grouping by tens and to understand that numbers are made of groups of tens and ones (for example, 23 is made of two tens and three ones). This is how most of us would introduce two-digit numbers to children.*
- *Along with this, she also used other means to help the children understand splitting a number into two (e.g., 9 = 5+4, 12 = 6+6).*
- *She also used the open-number line and its concrete representation – the Ganitmala to understand the relationship between numbers, and help children develop number sense.*

Gurpreet asked children to represent numbers on the open number line (Figure 16i).

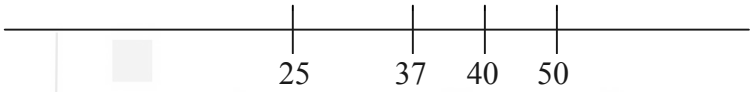


Fig.16i: The open number line

She asked questions like, "Where would 10, 8, 5, 15, 12, 18 be on this line?" (Figure 15e). As long as the numbers marked by the children followed the ordinal relation, the teacher considered it correct.

Once children were familiar with numbers up to 100, Gurpreet used a variety of tools to explain adding and subtracting tens as well as adding and subtracting numbers between 1 and 10. Using the open number line, Gurpreet asked the children questions like, "What is two steps to the left of 10?" or "What is 3 steps to the right of 6?" to give them practice with simple addition and subtraction. The children also learned skip-counting by 10 from any number on the open number line and the Ganitmala, like 10 steps to the right of 17 is 27 and 10 steps to the left of 17 is 7. In this way, she was building children's capabilities for doing simple mental calculations.

The open number line or its concrete representation *Ganitmala*, is useful in helping children learn these ideas and move forward to more complex addition and subtraction.

Think of some more tasks that you can give children on the open number line to help them understand place value. Why do you think these tasks would be interesting for children?

Through all these activities, children in Gurpreet's class could learn the ordinal relationship between numbers up to 100 and practice the addition and subtraction of numbers. They were more familiar with grouping by tens (through matchstick bundles, Ganitmala, and open number line. They could also do grouping by 50s. This means making one group of 50 and then two of 50s without knowing the number name for it, yet knowing that three 50s is more than two 50s. Since children in Gurpreet's class had the experience of doing all this, by the time Gurpreet began teaching subtraction with

borrowing, the children could do simple mental calculations involving addition or subtraction.

Encouraging Multiple Methods to Add and Subtract

To introduce subtraction using borrowing, Gurpreet made available a variety of materials – matchstick bundles of 10, Ganitmala, open number line, cards that could be used as 10s, stones, etc.

As the first step towards the more complex subtraction using borrowing, she gave the children the task of finding the answer to the following problem: "Ajit had 17 colour pencils. He gave 8 colour pencils to his younger sister, Madhu. How many colour pencils is Ajit left with?" Children were free to solve the problem using any material they preferred – bundles of 10 matchsticks, Ganitmala beads, open number line, or any other way. All these materials were available in the class. After the children had worked on the task for a while, she invited them to share their responses with everyone. This is what happened in the class.

Gurpreet: Who will tell us the answer to the question?

(Many children raise their hands.)

Gurpreet: Okay! All of you will get a chance to give your answers. Only one of us will speak while others will listen. After that others will share their solutions, one by one. Pay attention to what is being said and if you do not think it is correct, please help your friend correct the solution. Come, Vishal, tell us your answer. Also, remember that you have to tell us how you got it.

Vishal: The answer is 9.

Gurpreet: How do you know that?

Vishal: (Displays 17 as 1 bundle of 10 matchsticks and 7 single matchsticks)17 is 1 bundle of matchsticks and 7 single matchsticks. I have to take away 8. I took away the 7 single matchsticks. Then, I opened the bundle and took 1 matchstick from there (Displays taking away the 7 single matchsticks and opens the bundle and takes 1 from there). Now 9 are left, one, two, three…nine (confirming by counting).

Radha: Can't we take 17 single matchsticks and remove 8 from them? Then, we do not need to open the bundle. I did it like that. I got the same answer.

Gurpreet (to the whole class): What do you think of the solutions given by Vishal and Radha?

Some children agreed with Radha's solution and said that they had also done it that way. Some others had followed Vishal's method.

Another child, Meena, said that her answer was 11.

Gurpreet: Can you tell us how you did the sum?

Meena (speaking out loud): 17-8. 8-7=1, and 1-0=1. So, 11.

Gurpreet: Okay, can anyone explain which is correct; the answer 9 or the answer 11?

Saleem: I used the open number line (Figure16j). I too got 9. See what I did. I jumped 8 places to the left of 17 and reached 9.

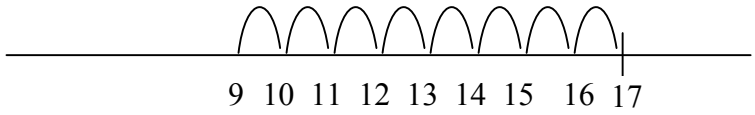


Fig. 16j: Saleem's solution for 17-8 on the open number line

Krishna: I have done it another way (Figure 16k). Subtract 7, got 10 (checking on the Ganitmala). Subtract 1 more, got 9.

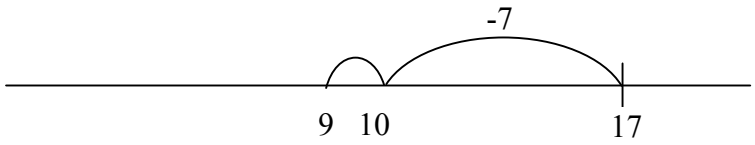


Fig. 16k: Krishna's solution to 17-8 on the open number line

Gurpreet: Very good all of you. You have found many different ways of solving it. But while most of you have got 9, some have got 11.

How do we know which is correct? Let us consider 17 and find out how many places we have to jump for 11 as the answer. How many matchsticks have you removed if your answer is 11?

Children (after counting the matchsticks and the jumps on the open number line): Six.

Gurpreet: If removing 6 from 17 gets us to 11, then clearly, 17 - 8 cannot give us 11. It has to be 9.

Gurpreet then wrote 17-8 as:

$$\begin{array}{r} 17 \\ -\ 8 \\ \hline \\ \hline \end{array}$$

She then said, "Let us see what Meena did. Meena says that in the ones column since we can only subtract the smaller numbers from bigger hence 8-7 = 1. In the tens column, 1-0 is 1. We would then have 1 in the units place and one in the tens place. If the answer 11 is wrong, then how would you explain why it is wrong?"

A few children raised their hands. One of them said Meena was wrong because "You cannot subtract the numerals of the lower row from the numerals of the upper row."

Radha: "We need to break the bundle of 10 matchsticks in the number 17. Then we get 10 single matchsticks which we add to 7. We get 17 single matchsticks. Now we can subtract 8 single matchsticks from 17 and get the answer.

Gurpreet then asked the children to do some more sums and talk about their strategy for solving these. She also asked them to formulate similar sums for

their friends and some even for the teacher. Children enjoyed making and solving problems.

Features of Gurpreet's Classroom

Based on this example, can you identify the features of the classroom?

In Gurpreet's class, children are thinking; they are given opportunities to articulate their thinking and converse with each other.

It was quite common for children in Gurpreet's class to share their solutions and give explanations of their answers. Some other strategies that the teacher used were:

- She asked the children to explain how they got their solution.
- She asked the children to differentiate correct from incorrect solutions based on mathematically valid arguments.
- Most importantly, Gurpreet recognized that the children in her class were at different levels of mathematical thinking. So, she made available various kinds of materials and tools with which children could think about problems. She also introduced new things as and when opportunities arose. The classroom environment allowed the children to freely share their ideas and come up with different solutions. Some children used matchsticks to find the solution to the problem – they were at the stage of using concrete materials to solve problems. Some others used the open-number line which is a comparatively abstract pictorial way of representing numbers. The children who used the *Ganitmala* to verify their answers after using the number line were on their way to abstract thinking but still needed the help of concrete materials to feel confident.

Differences could also be seen in the ways children used the matchsticks and the open-number line. Some children only counted in 'ones' and took away 8 from 17 (like Radha's solution with matchsticks and Saleem's solution on the open-number line) while some others could carry out a more sophisticated computation, like taking away 7 from 17 and then taking away one more (like Vishal's solution with matchsticks and Krishna's solution on the open-number line).

Thus, these examples suggest that learning abstract concepts such as place value, addition, and subtraction should be made engaging for the children. **Simply introducing them to symbols (algorithm as the first step), as we also discussed in Section 16.3, should be avoided. The pedagogy should be developmentally appropriate.**

Check Your Progress Exercise 2

1) How does the concept of place value help in developing the concept of addition? Explain with the help of an example.

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2) Refer to the Example 1 in Section 16.6. Why do you think Sheetal's teaching is not effective for teaching addition and subtraction?

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16.8 WORD PROBLEMS

In the previous Sections, we discussed how to introduce children to the concept of addition and subtraction to children in a developmentally appropriate manner. However, you should remember that this is not enough. Knowing what addition and subtraction are, and how to solve addition and subtraction problems be it using concrete material, pictures, or symbols, is necessary. Along with this the child also needs to know when to use addition and subtraction in a particular situation. For this, you must introduce them to a situation that involves the use of addition and subtraction. Word problems are very effective in this regard.

Given below are two types of word problems related to addition that the children have to deal with.

a) **Aggregation:** Under this, one group has to be formed after counting and involves aggregating more than two things like pencils, money, distance, volume, etc. For example – Neetu has two pencils and Ravi has three. So how many pencils are there in all?

b) **Augmentation:** Under this, the numbers of a particular group are increased and the final number has to be found or the amount by which the numbers in a group have increased has to be found. For example – Neetu has two pencils and Ravi gave her three more. Now how many pencils does Neetu have?

Now let us examine some word problems related to subtraction that the children have to deal with. There are four main types:

a) **Partitioning**: This consists of finding out the number of remaining things after some are removed. For example –There were fifteen birds on a tree. If ten flew away, then now many remain?

b) **Reduction**: In this type, children need to find out the number of objects taken, when the original number and the remaining number of objects are given. For example – There were fifteen birds on a tree and now five are there. How many flew away?

c) **Comparison**: This includes finding out the difference between the two groups of numbers. For example – There are five birds on one tree and ten on the other. How many birds are more on the second tree?

d) **Complementary Addition**: This type includes questions such as – How

much has to be added to a certain number to get another given number? Fifteen girls can sit on a bench. If five girls are sitting right now, how many more can be seated?

We often carry out *complementary addition* when we purchase things in the market. If you give a Rs.100/- note to the shopkeeper after purchasing goods worth Rs.67/-, then the shopkeeper returns the balance in this manner: He holds the packet of goods and says – Rs.67/-. Then he gives Rs.3/- and says 67 and 3 are Rs.70/-. Then gives Rs.10/- notes and says – ten …eighty, ten …ninety, ten…. hundred. This is an example of *complementary addition.*

Of all the types of word problems given above, children face the most difficulty in recognizing *complementary addition* questions. Such types of questions usually ask *'how many more'*? Usually the words 'and' and 'add' are used in these problems and children associate these with addition and add all the numbers given in the problem.

When children start solving word problems, assess whether their concept of addition and subtraction is clear. This can be done in several ways. Firstly, they can be assessed by continuously observing them when they are performing activities with concrete objects or doing pictorial activities. Secondly, ask them to form word problems that require addition or subtraction of numbers.

16.9 USING ESTIMATION TO STRENGTHEN THE CONCEPT OF ADDITION AND SUBTRACTION

In Section 16.3, we discussed the errors committed by a child as the algorithm was used without understanding. If the child could have estimated the range in which her answer should lie, then she could have easily spotted any obvious error herself. Let us see how a teacher helped a child to check her answer for an addition problem by using estimation with the help of the given example.

Sheila, a primary school teacher strengthened her student Raju's estimation abilities and used it to develop an understanding of the concept of addition. For this, she performed the given activity.

She used bundles of ten matchsticks each and some loose matchsticks. She asked him to add 33 and 25 sticks. He took 3 bundles and 3 sticks, and then another 2 bundles and 5 more sticks. The given conversation followed:

Sheila: Would there be more than forty matchsticks?

Raju:Yes, there are 5 bundles. So it will be more.

Sheila: Good! Will it be more than 70?

Raju: No, there are 5 bundles. So it has to be around 50.

Sheila: And the loose sticks? Can they be more than 20?

Raju: No!

Sheila: Why?

Raju: Because, I can see that these sticks together are not looking like a bundle, so they will be less than 10.

With more such examples, he slowly realized what Sheila was trying to get him to see — that ***when adding two-digit numbers, one can estimate the quantity by adding the digits in the 'tens' place first.*** *After many such experiences with estimating the sum using bundles of sticks, he could use estimation when the addition problem was given in the form of symbols such as '24+32', '25+17', etc. Sheila helped him to see how he could use estimation to check if the exact answer he got was correct.*

If the child commits a similar error while solving an addition problem using an algorithm, as shown in the given example, you may engage the child in a conversation and encourage her to estimate if their answer is correct. Let's read on to further understand.

The teacher asked the children to solve the given problem:

$$\begin{array}{r} 23 \\ +\ 17 \\ \hline \end{array}$$

Some children came up with an answer – 310. She asked them to explain how they reached this answer. One of them explained, "I added 2 and 1 and got 3, and similarly, I added 3 and 7 and got 10. Together, they make 310." In response, she asked her, "If you have 23 sweets and someone gives you 17 more sweets, would you now have more than 300 sweets?" Children said, "Of course not!" She further asked, "So how many sweets would you have?" The child made a rough estimate and said around 40.

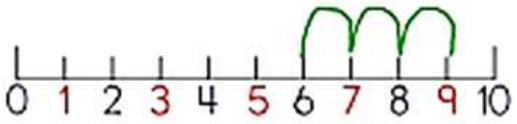
Thus, you can make use of the child's estimation abilities to strengthen the concept of subtraction as well in a similar manner.

Check Your Progress Exercise 3

1) Match various types of word problems with their respective examples.

a) Complementary Addition	i) There were ten birds on a tree. If seven flew away, then now many remain?
b) Aggregation	ii) Rama has seven toys and Joy gives two more. Now how many toys does Rama have?
c) Comparison	iii) Five boys can sit on a bed. If two boys are sitting right now, how many more can be seated?
d) Partitioning	iv) Rama has three toys and Joy has five. So how many toys are there in all?
e) Augmentation	v) There are five children in one room and ten in the other. How many more children are in the second room tree?

2) How can you teach both addition and subtraction using the given figure? Mention the specific strategy.



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16.10 SUMMING UP

In this Unit, we tried to develop an understanding of the concept of addition and subtraction and the challenges of teaching related algorithms to children. We also discussed how the use of concrete objects and the experiences of children help in concept formation. Exposure to different kinds of word problems should also be ensured. We discussed some examples of questions/problems solved by children illustrating the errors generated when algorithms are applied mechanically. These errors are steps in the learning process and are important clues as to how children learn and their learning levels. To teach carry-over and borrowing while teaching children algorithms for addition and subtraction needs to develop estimation skills in children.

16.11 ANSWERS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress Exercise 1

1) a) Children may use the algorithm correctly and give you the 'correct answer' but it does not mean that they have understood the concept as well.

   b) Children may make certain errors while applying the algorithm and give you the 'incorrect' answer, which clearly shows that they have not understood the concept. The use of the algorithm as the first step does not help children develop an understanding of addition and subtraction since children at this age need many experiences with actual objects (concrete materials) to understand what is involved in addition and subtraction.

Check Your Progress Exercise 2

1) The concept of place value essentially helps a child to understand that the value of a digit in a number is based on the location of the digit. For example, 2 represents 'ones' in the number 12 and 'tens' in the number 21. As a result, the child understands when asked to add 23 and 49, the

answer would not be 612 (which children get because they do not carry over). If the child understands place value, she would add the ones digits, i.e. 3 and 9, and get the sum 12. She would understand that since '1' represents tens in the number 12, it will be carried forward to the tens place and added with the tens digits, i.e. 2 and 4.

An understanding of place value also helps the child to estimate. She can estimate that 23 is nearer to 20 and 49 is nearer to 50. Since 50 and 20 make up 70, the answer cannot be more than 100.

2) Sheetal's method of teaching addition is not effective for several reasons. First, it is not clear whether children have learned to add using concrete material or pictures. Introducing children to the symbols/algorithm as the first step to teach addition is not developmentally appropriate. In addition, children were mindlessly engaged in performing the algorithm; they did not understand why they were to follow certain steps. Also, the concept of place value was not established at all which would leave children clueless that the carry over '1' is not one but rather a ten, and hence, would result in a wrong understanding of addition.

Check Your Progress Exercise 3

1) a-iii
   b-iv
   c-v
   d-i
   e-ii

2) Addition can be taught using the strategy of counting on wherein the teacher teaches the addition of 6+3=9. Subtraction can be taught using the strategy of counting back wherein the teacher teaches 9-3=6.