UNIT 13 THE NATURE OF MATHEMATICS

Structure

13.1 INTRODUCTION

If someone asks you – when does a child begins to learn mathematics, you are most likely to say that once she joins school her knowledge and understanding of mathematics begins to develop. There is no doubt that one's understanding of concepts of mathematics develops as a result of effective school education. However, this does not mean that:

a) those who have never been to school or have attended school for only a short duration, have no understanding of mathematics, or do not use mathematics in their lives; and

b) children have no prior knowledge of mathematical concepts when they join the school and they learn it only as a result of formal learning that takes place in the school.

Most of us hold the above-mentioned wrong assumptions because we have a limited or incorrect understanding of what mathematics is. We think that mathematics is used only for formal study and that it is relevant for us only as a subject taught in school. However, that is not true. Mathematics is anintegral part of our daily lives and we make use of it right from the simplest and everyday tasks to the most complex and demanding tasks. Also, while the learning of mathematics takes place within the school, it happens outside of school as well, in our everyday life.

This Unit will help you reflect upon your existing understanding of mathematics and understand how mathematics is integrated with our daily life experiences. This is important to understand since how a teacher will teach mathematics in the school will be influenced by the teacher's understanding of and attitudes towards mathematics.

Objectives

After reading this Unit, you will be able to:

- describe various everyday activities that involve the use of mathematics;
- understand the nature of mathematics and how mathematical concepts develop; and
- differentiate between the correct and incorrect ways of teaching mathematics to young children.

13.2 MATHEMATICS IN OUR DAILY LIVES

What is the first thing you do when you get up? Probably, you make yourself a cup of tea or coffee. If so, then you're using mathematics! Do you agree? You would be putting in a certain amount of tea leaves, sugar, milk, and water. You would also keep an eye on the duration of time to boil the tea leaves. Don't you think that this anything needs some use of maths, even if not in the conventional way of solving an equation or using algorithms? (An algorithm, in maths, is a procedure, a description of a set of steps that can be used to solve a mathematical computation. For example, the step-by-step procedure used in long division is an example of a mathematical algorithm.)

Consider a carpenter making a table. Does he use mathematics in any way? Think of a tailor, a vegetable buyer, or a mason. Do they use mathematics in any way? When we use public transport or drive our own vehicle, or pay our child's school fees, we are using mathematics. Making a chair, sending a satellite up into orbit, building roads and bridges — can any of these activities be done without using mathematics?

What about various sports activities? A cricket captain once said that if he got his field placement right, half the job of getting the other team out would be done. What does field placement require? It requires an understanding of space! *Kho-Kho, Kabaddi*, Football, Basketball, etc. – all require an awareness and utilization of space. If we were to say it in a mathematical language, we would say, that playing these games is based on an understanding of geometry, though the player herself/himself and the audience do not realize this. Similarly, what about board games like chess? While playing, you need to think of a winning strategy (problem-solving). Good chess players employ simple geometric rules to figure out, with a simple glance at the chess board, what would be the result of the encounter. For every game in which a queen was captured, there are four different possibilities:

1) The white queen was captured first and the winner was white (w_white).
2) The white queen was captured first and the winner was black (w_black).
3) The black queen was captured first and the winner was white (b_white).
4) The black queen was captured first and the winner was black (b_black).

Here, the concept of probabilities is evident.

The "rule of the square" in a board game such as chess helps you to construct the possible movements at any instant, given the conditions under which the different pieces are allowed to move. In *Ashta Changa*, Ludo, *Chaupad*, and other such games, the players use mathematics.

All the above examples are related to adults. However, children also continuously use mathematics in their day-to-day activities. For example, if you give sweets or a piece of chocolate to two children, they immediately compare if they have been given equal shares or if one has more. This means they are doing mathematics.

You must have seen children playing hopscotch, seven stones (*pitthu*), marbles, and various other games. Do children use maths while playing these games – what do you think?

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Example 1

Let us take an example!

Some children are playing Ghar-Ghar (make-belief play). One child takes up the role of mother, another one plays the role of the father, one is the grandfather, another is the neighborhood aunt, and three children, younger than the rest, play the role of children. Here are some of their dialogues while playing:

"*Get up! It's morning. We have to make breakfast. You take this money and go and buy some vegetables."(Two children go to the market and the rest clean the house.)*

"We have got the vegetables. Take them, they cost 50 rupees. Let us cook now!" (Everyone starts cooking.)

"We are hungry, give us food!"

"Cooking takes time, have patience… The food is ready. Give me a plate and a spoon."(Everyone takes a plate and spoon.)

"Oh no! We are one plate short. You will have to eat from the pot."

"It's night. Let us go to sleep."

In this example, children are displaying a sense of:

a) Time (*It's morning; It's night; Cooking takes time; have patience.*)

b) Quantity (*We are one plate short; they cost 50 rupees.*)

These are mathematical concepts.

Example 2

Now let us look at another situation.

Lata wants to put up a swing on the tree in front of her house and wants to enjoy swinging on it. Does she need or use any mathematics for this? To put up the swing, she needs a rope and an appropriate branch of the tree. There are several questions that she needs to ask, such as :

- *How high should the branch be?*
- *How sturdy should it be?*
- *Will any other branches obstruct the ropes when she uses the swing?*
- *How long should the rope be? Does the length of the rope have any relationship with the height of the branch?*
- *How thick should the rope be?*
- *Can she and her friend swing together?*

To answer all these questions, Lata has to have a sense of mathematics. For example, to answer the third question, she needs to have a sense of geometry. She has to mentally visualize the sweep of the ropes when the swing is going up or down, and decide if any branch of the tree would interfere with the swing. To answer the last question she has to think of the thickness of the rope, the diameter of the branch (thickness),and their own combined weight to swing safely.

Suppose Lata succeeds in putting up her swing. With no one to push her, she can move the swing by pulling and pushing the ropes and shifting her weight on the swing rhythmically. As the swing starts going up and down, she has to match the rhythm and pattern of the movements of the swing and apply force in a way that makes the swing go higher and higher.

So, in putting up and using the swing, Lata is using her experience to estimate a lot of mathematically calculable quantities. She works through the estimates, checks them, and decides to use them or take up another set of estimates. She does all this mathematics without realizing that she's doing it. And therefore, she does it without stress, boredom, and frustration!

Thus, the use of maths is quite evident and rather inevitable in the lives of children right from the early years. In none of the examples above, neither the adults nor children solved an equation or used an algorithm, which is what we usually associate with doing mathematics, but they were doing mathematics, nonetheless!

Can you think of some more examples wherein children make use of maths in everyday life?

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Fun with Maths

Solve the problems given below. You can do these in your spare time, and maybe you'll enjoy working on them!!

a) Take any three-digit number and multiply it by 10. Now subtract the first number from your product. Now divide it by 9. What do you get? Do this with different numbers.

b) *Reshma is a brilliant math student. However, on Sundays, she prefers not to touch her textbooks. She spends this day selling the eggs laid by her pet hens. She loves to keep a record of the money earned and makes up mathematical games about the eggs.*

One day, she was going to the market, as usual, dreaming about her eggs. Suddenly, she banged into a stationary cycle. All the eggs broke. The cyclist was a kind man and he thought the little girl should be compensated. He asked Reshma how many eggs were there in her basket.

Being a mathematician, Reshma answered with a riddle. "I don't remember the exact number. But I do know that whether I divide with 2, 3 or 4, the remainder is always 1. But when I divide with 5, no eggs are left."

Can you tell what is the smallest number that fulfills Reshma's condition?

It was fun working on these riddles was it not?

Children also love to challenge their thinking as long as the puzzles are not beyond their level. Now, can you think of some similar mathematical riddles/questions?

13.3 THE NATURE OF MATHEMATICS AND MATHEMATICAL CONCEPTS

In this Section, we will understand the nature of mathematics and mathematical concepts. We will also discuss what is the implication of the nature of mathematics for teaching the subject.

13.3.1 Mathematical Statements are Unambiguous

Consider any mathematical concept that you are familiar with, say, a sphere. The definition of a sphere is clear and precise. Given any object, you can very definitely say whether it is a sphere or not. Similarly, the definition of any concept in mathematics, or any mathematical statement, is unambiguous, leaving no place for doubt. This is because we formally construct this abstract world of mathematics by first accepting a certain set of axioms/ statements that are consistent. Then, based on these axioms/statements, we formally define certain concepts clearly and precisely. Once the axioms/ statements are chosen and the situation defined, there is no scope for ambiguity.

Very often, of course, when we apply mathematical terms to real-life situations, we use the terms loosely, and not according to their mathematical definition. For example, we often refer to objects in the shape of cuboids (3-D) as rectangular shaped objects (2-D). That is when the term can become imprecise; we use the terms 'cuboid' and 'rectangle' synonymously to mean the same thing, which is not correct.

13.3.2 Mathematical Concepts Proceed from Concrete to Abstract

Mathematics, like all human knowledge, grows out of our concrete experiences. Let us take the example of three-dimensional shapes. Think about how you came to understand the concept of 'roundness' and 'sphere'. Was your mental process something like the following?

We see all sorts of objects around us. While dealing with them, we notice some of these objects – such as a ball, an orange, a tomato, and a *laddu*—all have the same kind of regularity, namely, 'roundness'. And so, the notion of 'roundness' gradually develops in our mind. We can separate the objects that are round from those that are not. We also realize that the property of 'roundness', common to all round objects, has nothing to do with the other specific properties of these objects – such as the substance they are made of, their size, or their colour. We gradually separate the idea of 'roundness' from many other concrete attributes of objects. Based on the essential property of 'roundness', we develop the concept of a 'sphere'. Once we have formed this concept, we donot need to think of a particular round object when we are talking of a sphere. This means we have successfully abstracted the concept of 'roundness' from our concrete experiences.

Implication for Teaching: Since mathematical concepts develop from concrete to abstract, the teacher, needs to ensure that during the early years, children are given enough concrete experiences to imbibe mathematical concepts. Simply introducing children to abstract signs and symbols used in mathematics and teaching them formulas or algorithms will hamper their learning, since they will not understand the concepts to which these signs, symbols, and algorithms relate. Without concrete experiences, they will not be able to relate mathematics to their daily life, and they may start to find it difficult and boring, gradually losing interest in it.

13.3.3 Mathematical Concepts Proceed from Particular to General

When you hear the word 'tail', what do you think of? Do you think of the tail of a horse or a monkey? Or do you think of the tail of your pet dog?

The tail of a particular animal has many features that are not a part of the concept of 'tail'. For instance, a particular horse has a dark tail, two feet long. One could describe the thickness of its hair, its colour, the angle at which it is inclined to the body, and so on. But would this description fit the tail of any horse? Wouldn't some of these features change from horse to horse? So, if one wants to apply this concept to all horses, one would need to form an image of a tail that is not bound by the particular properties of one particular horse's tail.

Now, you may notice that cows and dogs also have tails. So, you may further generalize the concept of the 'tail' to include the tails of all animals.

So, while generalizing from a particular case to cover more and more cases, we leave out some features of the specific example and pick out what is common to the various examples. In other words, we abstract their common aspects and form a general concept — we move from particular to general.

Isn't this the same way we form the concept of a quadrilateral in mathematics? This concept is the result of examining squares, rectangles, trapezia, etc., and picking out their common properties, namely, that they are all closed figures with four sides. So, we form a general concept of a closed four-sided figure, and call any such figure a 'quadrilateral'.

To summarize, as we move from one particular example/instance to more and more instances, we generalize the concept/ idea; in this way, we move towards more and more abstraction.

Implication for Teaching: Children need experiences with a variety of specific objects to generalize a concept. For instance, a child would find it difficult to abstract the idea of 'four-ness' unless the child is shown a variety of objects in the quantity of 4. Therefore, the teacher needs to ensure that one concept is represented and learnt with various specific examples so that the child may generalize and abstract the idea of that mathematical concept.

13.3.4 Mathematical Concepts are Related

You have understood that once we abstract a mathematical idea, it exists in our mind, independent of the concrete experiences that it grew from. This abstract mathematical idea can generate many more related abstract concepts and we can also generate abstract relationships between these ideas. The edifice of ideas and relationships keeps growing, making our world of abstractions larger and larger. For example, related to the concept of a sphere, we generate the concepts of radius, centre, surface area, and volume of a sphere. Thus every mathematical concept gives rise to more mathematical concepts.

Further, there are abstract and formal relationships between the related concepts. For instance, examine the relationship between a sphere and its volume. Irrespective of the size of a sphere or the material it is made of, the relationship is the same. The volume of a sphere depends on its radius in a certain way, regardless of how big or small the sphere is.

Implications for Teaching*:* Since mathematical ideas are related, the teacher needs to support the child to form a clear understanding of any mathematical concept. If a particular concept is not clear it will affect the understanding of other concepts. For example, if the idea of the radius is not clear, the child will not be clear about the concept of the diameter.

13.3.5 Mathematical Concepts are Hierarchically Structured

As the abstractions from concrete objects and materials become more and more general, they represent wider and wider ideas. If we put down each step of the process of generalization, we would have a series of ideas, each contained in the generalized idea following it. So, to understand each of these abstract concepts, one needs to understand every concept that comes before it in the stepwise build-up, or hierarchy of ideas.

For example,

- Fractions cannot be taught before whole numbers.
- Multiplication and division cannot be introduced before addition and subtraction.
- Number concepts and patterns are necessary to develop algebraic thinking.

Similarly, consider the number system.

a) From the counting of concrete objects, we abstract the set of natural numbers, namely, N = (1,2,3,4...).

b) If in this set, we include zero, we get the set of whole numbers, namely, W = (0, 1, 2, 3, 4...).

c) This set can be further enlarged to include negative numbers, and we get Z = (...-3, -2, -1, 0, 1; 2, 3...) as the set of integers.

d) To the set of integers, we can add positive and negative fractions to get the set of rational numbers, Q, and so on.

However, if one does not understand what natural numbers mean, one will certainly not be comfortable with whole numbers. Similarly, if one does not grasp what negative numbers mean, it will be difficult to understand what a rational number is.

Implication for Teaching: If you look at the historical development of any discipline, hierarchically lower concepts are usually identified before hierarchically higher concepts. The learning of concepts by children is also, broadly, along the same pattern. Therefore, it is usually better to introduce the child to a hierarchy of ideas the way they developed. Unfortunately, this does not always happen.

For example, a square is a particular type of rectangle, and a rectangle is a particular case of a parallelogram. However, many children in Grade 2 are taught these concepts at the same time, without even relating these to each other. What is the result? Even two years later, many children will say that a square is not a parallelogram.

So, to understand a new mathematical idea, a person requires a proper understanding of the mathematical concepts that come before it. This is what we mean when we say mathematics is a hierarchically structured discipline. If the child does not have a clear understanding of the earlier idea, then she will have difficulty in forming the latter idea.

13.3.6 Use of Symbols

"*Multiply seven thousand six hundred and fifty-three by four thousand nine hundred and eighty-one.*" Try doing this without writing the numbers in the form of numerals, i.e., without using the digits 0 to 9. Even understanding the problem is a problem! On the other hand, if we write "7653 x 4981 = ? 7", isn't the question easier to understand? Could it be the use of symbols that brought this ease?

Take another example: Read the given statement: *Ram has 8 apples. Mohan gave him 4 more but then Farida took away 2. How many apples are left with Ram?*

8 + 4 – 2 = 10: This is the same statement, written with symbols. This symbolic representation makes the statement brief and clear, provided one understands what each symbol means and can read statements and understand the mathematics involved in them. This implies that the teacher should remember that the child knows how to read and can understand what she has read, so that she can convert the given statement into a mathematical equation.

You know that mathematics deals with abstract ideas, which are precise and unambiguous. For working with these concepts, and for communicating these efficiently, we need to use common systems of notation (signs and symbols) with rules for manipulation. These systems are what add to the power of mathematics, and allow us to easily visualize whether a mathematical argument is valid or not.

Implications for Teaching: Using symbolic notation for various operations makes it easy to apply the algorithms for solving problems involving the operations. However, a note of warning here! Although notations, symbols, and algorithms make operations simple and fast, they also make them mechanical. While teaching/learning mathematics, it is very easy to fall into the trap of developing the ability to do the operations mechanically without knowing what is being done and why. To give just one example, consider the following problem: 5132 ÷ 5. Most of us would usually do it as shown below:

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5) 5132 (1026
   5
    13
    10
     32
     30
      2
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But how many of us ask:

a) Why is 5 written below 5132 in that manner only?

b) How did I get 13 in the third line, and why?

c) While writing 13 there, why do I need to write zero in the dividend?

One may not be able to answer these questions or many others of this nature. And yet, one can do the sum correctly.

This poses one of the most serious problems in teaching mathematics. Most teachers will be happy if the child has mastered the algorithm, even if she has no idea why the algorithm produces the correct answer. This means that the child has not understood the concept on which the algorithm is based. The approach of focusing on the teaching of the algorithm and ignoring the teaching of the mathematical concept on which it is based makes it more difficult for the child to acquire mathematical concepts later on, and sometimes it may block further learning completely. This should be avoided and more meaningful ways should be adopted to teach mathematics. You will read about how to teach mathematics meaningfully in the various Units of Blocks 3 and 4.

Check Your Progress Exercise 1

1) Which mathematical concepts/skills is the child using/exhibiting in the following situations?

   a) Telling the mother that her friend lives far away

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   b) Making a tower with blocks higher than the one made by one's friend.

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2) Which of the following statements is NOT true?

   a) The use of symbols is not a part of the field of mathematics.

   b) Mathematical statements may or may not be definite.

   c) Mathematics is relevant only as a formal subject taught in the classroom.

   d) If a child can use an algorithm correctly, it means she has understood the concept on which it is based.

   e) Giving many opportunities to engage with concrete objects helps the child to identify abstract concepts.

13.4 HOW TO TEACH AND HOW NOT TO TEACH MATHEMATICS

Children often do not like mathematics as a subject. Usually, this aversion is rooted in the way it is taught right from the early years. So let us see how mathematics is usually taught during the early years, what are implications of this incorrect way of teaching, and how one can improve one's ways of teaching mathematics during the early years.

13.4.1 Typical Maths Classroom: A Few Examples

The first lesson given to children in maths as soon as they join the school is concerned with counting which usually involves learning number names orally in the correct order and then being able to write these in the correct sequence as well. Being able to do this is believed to be the foundation of a child's mathematical learning. The use of signs and symbols (<, >, =, +, -) is also much practiced by the time the child reaches Grade 2. Let us look at examples of some maths classrooms to see how mathematics teaching usually takes place during these early foundational years.

Example 3

The teacher enters the classroom and asks the children to take out their copies/slates. One by one, she calls them and writes down the numbers from 1 to 10 in their copies. Then, she asks the children to copy each number 10 times. The entire period is spent doing this exercise.

Example 4

The teacher writes the following on the board:

$$\begin{array}{r} 36 \\ +\ 25 \\ \hline \end{array}$$

Teacher: (*pointing to the addition sum she has written on the blackboard*) *First, look at this sign. What is it?*

Children: *'Plus' sign.*

Teacher: *The sign is on the left side and from where do we start adding? From the right.*

(Putting her finger on 6 of 36). *What is written here?*

Children:*6*

Teacher:*(Putting her finger on 5 of 25) And this?*

Children:*5*

Teacher: *So first we will draw 6 lines and then 5* (*she draws* | | | | | | + | | | | |). *How many are there?*

Children:*11*

Teacher: *Good! We will write '1' of the '11' below 5 and 6 and carry over 1 to the tens column. Now we will add the tens column and then the hundreds.*

The teacher wrote 5-6 similar questions on the board and asked children to copy these in their notebooks and solve them. Only 2-3 children understood what was explained and the rest simply copied the numbers in their notebooks but could not do the addition. Later, the teacher solved each question on the blackboard and the children just copied the answers.

Most of our mathematics classrooms work similarly. Since children do not understand what is to be done and do not get the concept, they often lose interest or begin to fear the subject when such kind of mathematics teaching takes place on a daily basis.

Let us analyze the above examples to try and understand how mathematics could have been taught so that children neither lose interest nor end up fearing it.

If we analyze Example 3, it is clear that the teacher has not paid attention to the given aspects:

- What is meant by counting?
- What is involved in the process of counting?
- Does counting mean getting the children to write down numbers or is it something more? (You may recall from Unit 16 of MCD-002 that knowing number names and reciting number names in the correct order or even writing the numerals correctly does not automatically lead to the correct counting of objects. Being able to count correctly requires the foundational abilities to match, classify, seriate, and set objects in one-to-one correspondence.)

Before teaching a concept, a teacher must understand what is implied by it and what is included in it. For example, counting means:

a) To be able to identify and separate the objects to be counted from those not to be counted (involves the ability to match and classify)

b) Being able to say out numbers from 1-10 in order;

c) To see the relation between bigger and smaller numbers;

d) To be able to understand that one member has to be spoken on touching one object (involves one-to-one correspondence);

e) To be able to relate a number with a group of objects.

Now you can see how the teacher's understanding influences the way she teaches. When a teacher feels that 'counting' means being able to speak out numbers and write these down, then, in her classroom, you will only find children reciting numbers from 1-10 or 20 and practicing writing these, as the teacher did in Example 3, described above. However, on the other hand, if the teacher feels that counting involves the ability to match, classify, seriate, and set objects in one-to-one correspondence, then she will teach in a way that her learners engage in activities related to developing these concepts which will finally lead to learning to count.

In Example 4, the teacher did not explain the concept of addition. She simply demonstrated the use of the algorithm. Children did not grasp in which situation they are to apply the concept of addition or if they are already using addition in their daily lives. Before introducing the algorithm if the teacher had spent a few sessions giving examples where addition is used in their everyday lives, and then involved children in doing addition using concrete objects, then they would have been able to understand and also enjoy doing this addition sum using the algorithm.

Instead of doing this, the teacher in Example 4 focused on the written expression (algorithm) of addition in formal mathematics. The teacher began by using numerals and the addition sign to teach counting and addition. Now you may say – what is wrong with it?

Children indeed need to ultimately know the algorithm of addition and the sign of addition, but if these are the first things that are introduced to children without concrete examples that help children to visualize the process of addition, then the algorithm and the sign are only an arbitrary and abstract procedure for them, with no connection to real life. Taught this way, the process of addition holds no meaning for them and thus affects their learning of and interest in mathematics.

Thus, in both cases, there is a commonality – instead of first explaining what is meant by 'counting' or 'addition', the teacher has directly started with the use of mathematical symbols and algorithms for teaching.

13.4.2 A Constructivist Approach to Teaching Mathematics

You may recall from reading MCD-001 that Piaget and Vygotsky, the two famous psychologists uphold that a child is an active being who constructs and discovers knowledge on her own through her activities. The child's mind does not simply copy the information that is presented to her. Instead, the child actively interacts with her environment, manipulates objects, selects the environmental information, interprets it, and constructs her unique understanding of objects, events, and people.

According to Piaget, activities that allow discovery and exploration are essential for children to learn. Therefore, using the lecture method to teach where the teacher provides knowledge to students in its readymade form is not an appropriate method of teaching as the child is reduced to being a passive learner.

Furthermore, a Vygotskian classroom goes beyond learning through 'independent discovery' and promotes 'assisted discovery'. According to Vygotsky, every child can do many tasks and acquire many concepts independently (which indicates the child's actual developmental level) and can do many tasks with the assistance of adults (parents/teachers) or in collaboration with more expert peers (which indicates the child's level of potential development). These tasks which the child cannot do alone but can do with help were referred by Vygotsky to be within the child's 'Zone of Proximal Development (ZPD)'.

These ideas of Piaget and Vygotsky have significant implications for the teaching-learning of mathematics as well. As you teach mathematics to children during the early years, there are certain points that you may incorporate to ensure effective teaching-learning. These are shared as follows:

a) **Use of Concrete Materials for Teaching Concepts:** You may have come across teachers who believe that mathematical concepts should be first introduced using actual (concrete) objects. They are right! **This is because these concrete objects help the children to visualize an abstract concept.** To introduce and teach foundational concepts such as counting, addition, and place value, they use materials such as marbles, pebbles, sticks, and bundles to the maximum extent.

Let us elaborate on what is meant by '**abstract'.** For example, all of us adults know what '5' is. The image formed in our minds when we think of '5' is that of the number 5 or the meaning of 5, i.e., it stands for the quantity of 5. We do not imagine 5 chairs or 5 trees. It is because we have internalized this concept so that we do not need to use actual objects to understand it. Now think of a 4 or 5-year-old child who is in preschool. This child as yet cannot relate to 5 (or any other number or any other mathematical concept) without using a solid/concrete experience. Therefore, the child needs to get opportunities to relate numbers to different objects. **Once she starts understanding how much quantity a particular number represents, it is equally important to reduce the use of concrete material so that children start understanding the concept of that number at an abstract level as well (i.e., even in the absence of actual objects).**

b) **Consider the Child's Prior Knowledge during the Learning Process:** Another aspect that a teacher needs to consider while teaching mathematical concepts to young children is to acknowledge the prior knowledge and experiences of the children. Most of us believe that learning starts when the child first enters school and she does not know anything before coming here. Is it right to assume this?

Much before coming to school, the child can understand several mathematical concepts. For example, 'less and more', 'small and big', 'nearand far', etc. Let us see an example in which a teacher took into account the child's prior knowledge and experience to help the childlearn.

Example 5

Sumit, a second-grade child in a rural school, was introduced to the formal procedure of addition and subtraction in the following manner. The teacher tried to first find outhow much he knew before she taught him the formal method.

Teacher: *How much is 8 plus 11?*

Sumit: *19*

Teacher: *How did you do it?*

Sumit:*I counted. I took 11 and added on 8.*

(Sumit had used the strategy of "counting on" from the larger number, and could even describe his method in words.)

Thereafter, he was given the written problem:

$$\begin{array}{r} 18 \\ +\ 5 \\ \hline \end{array}$$

His answer was: 41

How did he get this answer? He added the 8 and 5 in the units column correctly to get 13, put 1 below them, and "carried over" the 3. Then, he added 3 to 1 in the tens column to get 4.

He was quite convinced that his answer was right.

The teacher then decided to pose the question differently. She said, "If you had eighteen marbles and you got five more, how many would you have altogether?" Sumit counted on his fingers and said 23. When the teacher pointed out his written answer to him, he slowly agreed that it was wrong.

In this example, Sumit demonstrates a well-developed skill of using appropriate and efficient strategies to add numbers. However, he finds the formal manipulation of symbols difficult, perhaps due to various reasons. It could be that Sumit has yet to develop an understanding of 'place value'. It could also be that Sumit does not find the given task of the addition of numbers with the algorithm meaningful. The moment the teacher posed the problem in a context and with reference to concrete objects (counting marbles), Sumit was able to understand it, and hence solve it.

The above example demonstrates that Sumit had evolved his own strategies of doing addition intuitively (i.e., without being formally taught) by 'counting on'. The example also shows how the teacher tried to assess Sumit's prior knowledge and used this knowledge to make the problem comprehensible to him by giving it a relevant context of a real-life situation which made the problem was concrete for him, even though no actual objects were given to him for counting. **Thus, a maths problem can be made concrete by giving actual objects to solve the problem or by connecting it to the child's real-life (lived experience)**.

c) **Encourage Questioning and Reasoning:** As you know, the emphasis of mathematical learning needs to be on the process of finding the answer, and not merely on getting the correct answer. Therefore, you as a teacher need to encourage children to observe, question, explore, and move logically toward an answer. You need to encourage them to systematize their reasoning. How would you do this? Let's find out!

One of the ways to do so is to ask children open-ended questions and encourage them to explore the answers. You could think of several types of activities for guiding and encouraging children to systematize their reasoning. For example, they could be asked to select criteria for sorting a set of objects and then helped to apply the criteria consistently. Or, they could be asked to make hypotheses regarding the different ways their schoolmates travel to school. Then they could collect, record, and analyze relevant data to prove or disprove their hypotheses.

Another example is to ask children questions like, "In how many different ways, can you fold this paper into the shape of a square?" and then give them a piece of paper to try it out. They can be given the opportunity to frame their own questions as well. Such opportunities help make learning less rigid and allow children's minds to unfold their potential. It would also help the children realize that there can be several solutions to a problem.

d) **Ensure Learning through Play:** Children can learn many basic mathematical concepts through games. You have read that when children of preschool age play a game of dividing coloured beads amongst themselves, they are matching one bead with one person, and developing the idea of one-to-one correspondence. When they play with blocks, they are experimenting with different shapes. When they sing about "five little monkeys", they are learning number names.

Learning mathematics through games is also very relevant for older children in primary school. New ideas and concepts can be introduced through games and situations that young children find familiar, enjoyable, and non-threatening.

You could devise several games to teach any mathematical concept. These games can be played with the whole class, or in smaller groups. The games could also be designed so that children learn the related mathematical language as well. Here are a few examples of some team games. The teams can be small (1-3 children) or big (13-20 children). We start with simpler games and move on to games based on concepts relevant to Grade 2.

i) The team members of one team place several stones in front of their group. The other team has to:

   1st game ((Preschool) — place as many stones, or

   2nd game (Preschool) — count and say how many they are, or

   3rd game (Grade 1 — add as many stones as necessary to make 13 stones, or

   4th game (Grade 2) — take away some stones to leave 3, etc.

ii) (For Grades 1 and 2) One team throws two dice (with dots or with numerals on them) and picks up, from the collection of stones placed in the centre, as many stones as the sum (or difference, or product depending upon the child's abilities) of the numbers shown on the two dice. The other team does the same. After two turns whoever has more stones wins. During the play, children could be introduced to the language like 'six plus two equals eight'.

iii) (For Grades 1 and 2)With stones, twigs, dice, cards, or beads, you can design games to teach 'place value'. With 10 stones (for base 10) being equivalent to one card or one bead, exchanges can be made and records can be kept. Once children can grasp the notion of tens and ones in concrete form, then they can be exposed to games using numerals, without the use of concrete objects.

   For example, you could take two sets of ten cards each, numbered from zero to nine, to be used by two groups. The children shuffle the cards and place them face down on the table. Then they take turns to select one card at a time and place it on the board, in the column marked 'units' or 'tens'. The aim is to make the largest possible number, and once a card is placed, it cannot be removed. As they play, they loudly say the number they are making. For example, if the first group turns over 3, and places it under 'tens', they should say 'thirty', and so on. This game can also be played with two dice, instead of cards.

iv) (For Grades 1 and 2) Children also enjoy word games. They are usually good at detecting verbal patterns. For example, you may give a particular word such as 'pony' and ask them to make rhyming words. Children will likely form words such as 'tony', 'boney', 'sony' etc. It is okay if children invent their own spellings, as the idea is to continue with the pattern. Since pattern recognition lies at the heart of mathematical thinking, children are doing mathematics, while at the same time, they are also developing their language skills.

e) **Repetition can be Fun and Helps in Learning:** To learn, children repeat an activity till they master it. In MCD-001, you have read that right from an early age during infancy, during the sensorimotor stage of cognitive development, children repeat their actions, such as dropping and picking up things, opening and closing boxes, repeating words, playing 'peek-a-boo' repeatedly, urging adults to repeat the same stories, and so on. Would you call any of these activities rote-learning? Not at all! Learning through repetition and rote learning are not the same thing. In rote learning, the 'information' is repeated mechanically without understanding (for example, memorizing an answer without understanding the concepts, or memorizing the procedure to solve a maths question without understanding the concept) whereas, in repetition, the same concept is dealt with again and again through a variety of activities with the aim of understanding the concept. In these repetitions, the participating children observe and experience something new and different each time. Thus, repetition concerning mathematics learning is imaginative and requires involving children in enjoyable activities, which could even be initiated by the children themselves.

If you look around you, you will notice that repetition happens quite naturally in a child's natural environment. However, it has to be consciously created in a formal learning environment, with enough variety to sustain the interest of children. How would you meet this challenge? Maybe, the following example can give you some idea.

Example 6

Children often consider multiplication tables to be the source of pain and misery. You may say mindless memorization of these tables during the early years has helped us to actually remember these during the later years of our lives. However, is it really necessary to learn these only through mechanical repetition? And does this memorizing by rote help a child understand what the tables mean? Is it not true that learning by rote usually stays at the superficial level of repeating tables in a given order and the child hardly recognizes that multiplication tables involve the concept of repeated addition ($2+2+2+2=2\times4$)? The fluency in using them is absent, which you can observe if you ask them to find the multiples in a different order. For example, while they may tell you that $2\times5=10$, because they know the table of 2, but they would find it difficult to tell you what 5×2 is, because they have not yet learnt the table of 5. One of the reasons is that they do not know how the concept of multiplication works.

Instead of only focusing on memorization of tables, is it not better to help the child to see the underlying pattern? You could think of several activities to enable children to establish the notion of multiplication and recognize patterns in the multiplication tables. For example, children can be asked to identify groups of 4 apples in a lot of 20 apples, and then answer simple questions like 'How many groups of 4 apples each are there?' 'How many groups of 5 apples are there?' and 'How many apples in all is that?'This can be done with a variety of objects.

This way, learning multiplication tables will be done by connecting the concept to real life and its abstract /conceptual meaning will become clearer to the children and they will find the learning sensible and meaningful.

f) **Offer a Motivating and Supportive Environment**: Let us understand this with the help of two situations as follows:

Example 7

Children were working in their notebooks on a maths problem that the teacher had asked them to solve. The teacher walked around the class, stopping to ask individual children how they got a particular answer. Most children reacted by reaching for a rubber, to erase whatever they had done even if they were proceeding correctly.

Example 8

You may have seen that the adults normally give their opinion as the truth, something which is to be accepted without doubt or question. If the child appears to doubt it or looks uncertain, the adults repeat what they have said patiently, then with a sense of irritation and, finally, angrily. The attitude of the adults is, "Why can't" you understand it!" The child, who is already overwhelmed by the authority of the adult, accepts this opinion, does not question the adult and starts losing confidence in his own ability to think.

Have you also ever come across such behaviours? What do these behaviours show?

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These behaviours show that when adults use their authority in a threatening way to get children to do certain tasks, children begin to lose confidence in their own ability to solve problems. Adults need to reflect on how they themselves might also feel a lack of confidence when faced with an authority figure in their own situation. This realization must lead them to create/offer a non-threatening environment to the children so that children can learn without any fear or negative remarks. If we, adults find it difficult to speak or perform our designated tasks in front of an authority figure, how much more overwhelmed must young children be feeling in the presence of adults!

It is necessary to show to children that you have confidence in them as they might often lack it. Insensitive remarks or behaviour may further affect their confidence level and they may find it difficult to develop this ability later. So, it is necessary to create a learning environment that builds confidence in children.

In order to create a non-threatening learning environment, keep the following aspects in mind:

First of all, do not present yourself as the prime authority figure who knows it all. If children are scared of you or they can sense that you are going to dismiss their learning in some or the another way, they will hardly participate. So, **you need to ensure that children look up to you as a knowledgeable but supportive educator**.

Second create opportunities for children to interact with each other. This is because students as teachers are very good learners – i.e., we learn best when we teach others. This applies to the young children as well. If you look closely at children playing in groups, you may get a clue. While playing together, children check each other’s thinking. They may be found making strategies to win over the opposing team and telling each other how they can increase their score, etc. Games and activities give children the opportunity to interact with each other in a non-threatening, autonomous, and easy atmosphere. Children give feedback to each other, in the form of an answer or a suggestion. A child takes such input from another child as just another opinion to be viewed, examined, accepted, or ignored. Thus, non-threatening peer interaction is very important for children’s learning.

The important thing is that these peer learning interactions should be informal, joyful, and non-threatening. Telling a child to teach another, by such statements as “Why don’t you teach your neighbour/friend/ brother/ sister this?” doesn’t usually work. This is because the child-tutor then tries to imitate the adult, and the learner becomes as defensive as with an adult.

Setting up child-to-child learning situations that are natural, and therefore, productive is not easy. You might have to decide how to pair children with each other. You may also need to observe children, without making it obvious, and see how they naturally interact. This will give you an idea of how to simulate peer learning in the formal classroom.

Third, peer learning does not mean that the teacher has no role play. You would also need to determine how would you offer help in a peer-based learning environment.

g) **Errors are Useful**: While teaching children, you must have found them making mistakes in the process of learning something. How do you respond to the errors? What do the errors tell you about the child – do you see these as the child's failure to learn, or the child's attempt to understand and internalize?

Children's errors are a natural and inevitable part of their process of learning. In the process of grasping new concepts, children apply their existing understanding to new knowledge, which may not match the method and content of formal instruction.

Children's errors are also a reflection of how children think and learn. The errors are a window into the child's world. For example, the earlier example where the child obtained the answer to 18+5 as 41 instead of 13, shows that the child has not yet grasped the concept of place value, and needs a lot more practice with grouping.

This kind of analysis of a child's error can play a highly constructive role in helping the teacher guide the learner to develop mathematical thinking. Making errors, and learning from them, is part of the process of developing a sound understanding. In fact, this is more important than producing the right answer. Unfortunately, the traditional teacher still tends to view learning as having occurred only when correct responses are given.

All in all, we, adults, need to keep these points in mind while building an environment favourable for learning. You will read more about how to create an appropriate teaching-learning situation throughout the various units of Blocks 3 and 4.

h) **Use the Four-Block approach in teaching:** To become mathematically proficient, children must be given a variety of experiences. For this, use the four-block approach in the daily classroom process.

Block 1 - Oral Math Talk: At the beginning of class, for 5-10 minutes, children could sing a poem with numbers or discuss their experiences with mathematics or problems they encounter in their lives. Discussion can also be on oral calculation, concepts, strategies, and reasoning. It works as a warm-up activity before going into the formal teaching process.

Block 2 - Skills Teaching (combining all strands): This refers to teaching mathematical concepts, problem-solving, and communication through concrete experiences, systematic activities, and instruction using the Gradual Release of Responsibility approach, about which you have learnt in Unit 9 of Block 2 of this Coures (MCD – 003), in context of language and literacy learning. Teachers could also set up a mathematics tasks and let children solve these independently before providing guidance and support. Every child must get an opportunity to do the activity learn, explain, and be given feedback.

Block 3 - Skills Practice: Providing children with various kinds of rich mathematical tasks based on concepts, processes, problem-solving, reasoning, and communication for practicing mathematical skills. This can be through a workbook, textbook, or a teacher-created task set.

Block 4 - Math Game: Children enjoy playing games. There could be various kinds of mathematical games which help children to strengthen their learning in various ways. These games must be based on problem-solving, concepts as well as reasoning. Group games can also be planned according to the learning levels of the children.

Check Your Progress Exercise 2

1) Fill in the Blanks
   a) Mathematical concepts progress from ………………….. to general and concrete to …………………….. .
   b) The emphasis of mathematical learning needs to be on the process of ………………… the answer, and not merely on the ………. answer.
   c) Mathematics is a ……………………. structured discipline - so to understand a new mathematical idea, an understanding of the mathematical concepts that come before it is required.
   d) Children's ……………. are a reflection of how they think and learn.
2) Which of the following is NOT appropriate while teaching mathematics to children?
   a) Devise various games that allow children to practice mathematical concepts.
   b) Teach them to recite number names as the first activity in school.
   c) Make use of materials such as leaves, beads, stones, and blocks to introduce and practice mathematical concepts.
   d) Do not encourage repeated practice of a mathematical concept.
   e) Ask children open-ended questions and encourage them to explore the answers.

13.5 SUMMING UP

The use and learning of mathematics take place within and beyond the premises of the school. It is an integral part of our lives – right from the most trivial and mundane tasks to the most efficient or demanding tasks of our lives involves some or the other use of mathematics. Not only adults but children also continuously use maths in their day-to-day activities.

The usual ways in which maths is taught in school right from the early years are quite mechanical as these do not incorporate children's prior knowledge and do not provide them enough opportunities to explore and understand mathematical concepts. Therefore, the teacher needs to adopt a constructivist approach to teaching of mathematics and incorporate the following aspects in her teaching:

Use of concrete materials; basing new knowledge on child's prior knowledge; encouraging questioning and reasoning; learning through play; making connections; encouraging learning through play; using repetition (not rote-learning); giving children opportunities to learn from one another; and seeing their errors as a part of their learning journey.

Mathematical ideas and concepts progress from concrete to abstract, from particular to general, and they form hierarchical structures. The nature of mathematics as a discipline is such that mathematical statements are unambiguous, and make use of symbols.

13.6 ANSWERS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress Exercise 1

1) a) concept of distance; skill of estimation
   b) concept of size (height)
2) a, b, c, and d

Check Your Progress Exercise 2

1) a) particular, abstract
   b) finding, correct
   c) hierarchically
   d) errors
2) b and d