UNIT 1 NATURE, NEED AND PLACE OF MATHEMATICS IN SCHOOL CURRICULUM

Structure

1.1 Introduction
1.2 Objectives
1.3 Nature of Mathematics
   1.3.1 Human Needs as a Basis of Growth in Mathematics
   1.3.2 Pure and Applied Mathematics
   1.3.3 The Role of Intuition and Logic in Mathematical Thinking
   1.3.4 Axiomatic Nature of Mathematics
   1.3.5 Language of Mathematics
1.4 Need and Importance of Mathematics in School Curriculum
   1.4.1 Social Aspects
   1.4.2 Mathematical Aspects
   1.4.3 Applications of Mathematics
1.5 Recent Trends and Principles of Formulating Mathematics Curriculum
   1.5.1 Factors Affecting Change in Curriculum
   1.5.2 Principles of Formulating Mathematics Curriculum
1.6 Contribution of Indian Mathematicians
1.7 Let Us Sum Up
1.8 Unit-end Activities
1.9 Answers to Check Your Progress
1.10 Suggested Readings

1.1 INTRODUCTION

Throughout the centuries, mathematics has been recognised as one of the central strands of human intellectual activity. From the very beginning, mathematics has been a living and growing intellectual pursuit. It has its roots in everyday activities and forms the basic structure of our highly advanced technological developments. It comprises intricate and delicate structures which have a strong aesthetic appeal. It also offers opportunities for opening the mind to new lines of creative ideas and channeling thought. Undoubtedly, the mechanism of resolving an intractable problem offers the most intense of all intellectual pleasures. At the same time, it is reputed to be, and rightly so, the most hypothetical of all sciences. It exhibits connections between things which can be visualized only through the agency of human reason. “Mathematics is thought moving in the sphere of complete abstraction from any particular instance of what it is talking about.” – A. N. Whitehead.

But what is mathematics? What does it encompass? Let us try to understand something about the enigma that mathematics is.

The word “mathematics” has been used in two distinct and different senses, i.e., one as a method used to solve the problems of quantity, space, order, etc., and the second as a set of laws or generalizations of truths that are discovered. Most teachers think of it as a tool in science, commerce and industry and are not able to appreciate its nature as a
way to think and what connects its various branches into a single logical framework of proof structure. Many questions which usually arise in the minds of teachers need to be resolved. In what terms does one justify the introduction of a new syllabus or a new topic? What is the relationship between mathematical laws and their applications? What, in fact, are the aims and purposes of teaching mathematics? What are the new methods and new ways of thinking about mathematics?

This unit gives careful thought to resolving some of these questions so that teaching in schools would rise above the mediocre and teachers would have the courage to break away from practices based on tradition and convention.

1.2 OBJECTIVES

At the end of this unit, you will be able to:

- acquire a clear perspective of the nature of mathematics;
- explain the social and practical influences which have affected the growth of mathematics;
- acquire an understanding of some elementary mathematical concepts;
- see the relevance of mathematics as an essential part of the school curriculum;
- enumerate the far-reaching changes that have taken place in mathematics and in the teaching of mathematics;
- appreciate the changes in curriculum and evolve new approaches to teaching; and
- get an insight into the various aspects such as patterns of intuitive thinking, experimental thinking, and rigorous and deductive thinking which have helped in the growth of mathematical knowledge.

1.3 NATURE OF MATHEMATICS

1.3.1 Human Needs as a Basis of Growth in Mathematics

Mathematics, like everything else that man has created, exists to fulfill certain human needs and desires. It is very difficult to say at what point of time in the history of mankind, and in which part of the world, mathematics had its birth. The fact that it has been steadily pursued for so many centuries, that it has attracted ever increasing attention and that it is now the dominant intellectual interest of mankind shows that it appeals very powerfully, to mankind. This conclusion is borne out by everything that we know about the origin of mathematics. More than 2,000 years before the beginning of the Christian era, both the Babylonians and the Egyptians were in possession of systematic methods of measuring space and time. They had the knowledge of rudimentary geometry and rudimentary astronomy. This rudimentary mathematics was formulated to meet the practical needs of an agricultural population. Their geometry resulted from the measurements made necessary by problems of land surveying Units of measurement, originally a stone or a vessel of water for weight, eventually became uniform over considerable areas under names which are now almost forgotten. Undoubtedly similar efforts occurred in early times in the southern part of Central Asia along the Indus and Ganges rivers and in Eastern Asia. Projects related to engineering, financing, irrigation, flood control and navigation required mathematics. Again a usable calendar had to be developed to serve agricultural needs. Zero was defined and this at once led to positional notations, for whole numbers and later to the same notation for fractions. The place value system which eventually developed was a gift of this period. These achievements and many more of a similar nature are the triumph of the human spirit. They responded to the needs of human society as it became more complex. primitive men can hardly be said to have invented or discovered their arithmetic they actually lived it. The men who shaped the stones in erecting the Temple of Mathematics were widely scattered, a few
in Egypt, a few in India, and yet others in Babylon and China. These workmen confronted nature and worked in harmony with it. Their products, therefore though scattered in time and space, partook of the unity of nature.

Mathematics is something that the man has himself created to meet the cultural demands of time. Nearly every primitive tribe invented words to represent numbers. But it was only when ancient civilizations such as the Summerian, Babylonian, the Chinese and the Mayan developed trade, architecture, taxation and other civilized contracts that the number systems were developed. Thus, mathematics has grown into one of the most important cultural components of our society. Our modern way of life would hardly have been possible without mathematics. Imagine trying to get through the day without using a number in some manner or the other. If a person lacks the ability to compute, he is as good as crippled. For instance, we need to know the time and tell the same. Telling the time is difficult and yet nearly everyone learns it. Soon, we shall lose an important experience of looking at the old fashioned clock with rotating hands, as we shall all be using digital readings to read time. A degree of estimation, not only in money but in weights and measures, is very important. Many of our daily routine chores involve sorting, ordering and organizing processes. We handle many mechanized devices which require geometrical or spatial skills. For travel, reading of maps, diagrams, interpreting scales becomes an essential part of our intellectual equipment. A knowledge of mathematics is useful to understand and interpret matters such as income tax and read information presented to us by the mass media in numerical form or in the form of graphs and understand the use of phrases such as rising prices, index, per capita income, inflation, stock market index etc. in ordinary day to day language. It is not necessary to provide an exhaustive list to prove the case in favour of “mathematics for survival” or “useful mathematics”.

Check Your Progress

Notes: a) Write your answers in the space given below.
    b) Compare your answers with those given at the end of the unit.

Given below are two important needs of any person. List the type of contribution mathematics can make to meet these needs.

1) To know how to spend money wisely and to determine the economic consequence of his/her actions.

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2) To develop the habit of rational thinking and to express his thoughts clearly and with understanding.

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1.3.2 Pure and Applied Mathematics

The study of the history of mathematics does not answer the question “what is mathematics”? However, it provides a valuable perspective to understand the nature of mathematics. Mathematics has a cumulative growth from prehistoric times. This growth
Nature, Objectives and Approaches to Teaching of Mathematics

has been of two kinds: extrinsic in the form of primary discoveries and intrinsic development of the subject.

The primary discoveries have been those of essential basic ideas, most of them gained by trial and error. Primarily, they were responses to human needs consistent with the body of knowledge already existing before the emergence of the new ideas. They are true accretions. Secondly, in addition to accretions motivated by human needs, the cumulative development of mathematics has been due to its inner growth.

As Nunn has so well said, 'Mathematical truths always have two sides or aspects. With the one they face and have contact with the world of outer realities lying in time and space, with the other they face and have contact with each other. Thus, the fact that equiangular triangles have proportional sides enables me to determine by drawing or by calculation the height of an unscaleable mountain peak twenty miles away. This is the first or the outer aspect of that mathematical truth. On the other hand, I can deduce the truth itself with complete certainty from the assumed properties of congruent triangles. This is its second or inner aspect.'

In brief, historically, mathematics has grown largely as a result of

i) social needs, as shown in everyday life, commerce, science and technology,

ii) the intellectual need to connect together existing mathematics into a single logical framework or proof structure.

Thus, the word mathematics can be used in two distinct and different senses, i.e.,

i) the truths that are discovered, and

ii) the methods used to discover truths.

This distinction leads us to explore the question of pure and applied mathematics. In a classroom much of the mathematics we teach is applied mathematics in the sense that it relates directly to life's activities connected with buying, selling, trade, business, consumer applications, weighing, measuring etc. These applications of mathematics to the world around us can be extended to more technical ones.

Mathematics has helped in analyzing motion and in doing so, Newton created the calculus which became known as applied mathematics.

More recently mathematical growth has been in areas such as operational research, linear programming system analysis, statistics, all involving processes to handle numerical information in an increasingly technologically advanced world. The mathematical ideas we teach in schools develop over many years of study and become associated in our minds with all the applications and illustrations presented to explain them. It is always easier to explain what we can do with a concept in mathematics than to say what it is. A teacher has to answer questions such as "what is the use of this to us this?, or "why do we have to learn this?" If he/she fails to do so, there will be many children who will not be able to see the point.

In pure mathematics we start from certain rules of inference, by which we can infer that if one proposition is true, then some other proposition is also true. These rules of inference constitute the major part of the principles of formal logic. For instance, we all know the axiom that in real numbers if \(a > b\) and \(b > c\), then \(a > c\). Thus, from given propositions we conclude that some other proposition is true. We then take any hypothesis that seems amusing, and deduce its consequences. If our hypothesis is about anything, and not about some (one or more) particular person or thing, then our deductions constitute mathematics. Thus, mathematics may be defined as "the subject in which we never know what we are talking about, nor whether what we are saying is true."

– Bertrand Russell
These ideas point out the abstract nature of mathematics. Mathematics deals with the application of arbitrary rules in an arbitrary situation which may or may not have significance in the world outside. It is a network of logical relationships. In school mathematics Euclidean geometry is essentially pure mathematics. A set of axioms and postulates are given and from them a body of definitions, theorems and propositions are derived. All pure mathematics is built up by combinations of primitive ideas of logic; its propositions are deduced from the general axioms of logic, such as the syllogism and the other rules of inference.

There is a very thin line dividing pure and applied concepts. On the one hand concepts of pure mathematics are formulated because of the need to apply them and on the other, every discovery or formulation has some application somewhere.

Check Your Progress

Notes:  
 a) Write your answers in the space given below.  
 b) Compare your answers with those given at the end of the unit.

3) Try to give a definition of mathematics based on school mathematics.

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4) Do you agree with the view that “algebra is generalized arithmetic”? Justify your answer.

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5) What is the role of structure in learning mathematics? Select an example from modular arithmetic.

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6) Give an example to distinguish between pure and applied mathematics.

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1.3.3 The Role of Intuition and Logic in Mathematical Thinking

Mathematics is creative and intuition is the first step towards creativity. The analytical approach is needed to validate the new discoveries of intuition in a rigorous manner.

According to the dictionary, intuition consists of the immediate apprehension, without the intervention of any reasoning process or knowledge or mental perception. Intuition is a mental act, a guess which gives a formulation or conclusion without going through a step by analysis. An intuitive thinker arrives at an answer with very little awareness of how he/she has reached it. In learning mathematics, the ability to visually dissect a pattern or a structure and guess a tentative generalization should be encouraged. When pupils are presented with the finished product which has been already formalized then their intuition suffers and they lose the opportunity to use an important tool in problem solving. No doubt, the intuition of pupils may not always be correct, nevertheless the teacher should have an open mind to the mistakes pupils make. This is because intuition is the essence of any non-rigorous method of solving problems.

In geometry, intuition helps to discover the proof of a result and the nature of mathematical proof is the next important question to consider. All propositions which mathematicians enunciate can be deduced one from the other by the rules of formal logic. This is one of the many methods of proving results in geometry.

We now discuss some types of proofs in geometry.

**Direct proof:** It proceeds from propositions already accepted (axioms or theorems) via a chain of syllogism to the desired conclusion. Example \((-a) \rightarrow (-b) \rightarrow ab\).

**Indirect proof:** (Or reductio ad absurdum) If \(p\) is to be proved, assume \(\neg p\) is true and hence derive a contradiction. Then, the assumption \(\neg p\) has to be false. So \(\neg \neg p\), i.e., \(p\) is true. Example: \(\sqrt{2}\) is irrational is proved in this way.

**Proof using contrapositive:** If \(p\) is of the form "\(a\) implies \(b\)”, then prove \(\neg b\) implies \(\neg a\).

**Example:** If two lines are cut by a transversal so that a pair of interior alternate angles are equal, the lines are parallel. The contrapositive is "If two lines cut by a transversal are not parallel, then the interior alternate angles are not equal".

**Disproof by counter example:** Used when the proposition conjectured is of universal form Simply exhibit a counter example.

**Example:** The sum of any two odd numbers is odd. This proposition is false, since \(3 + 5 = 8\) which is even.

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**Check Your Progress**

**Notes:**

a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the unit.

7) Identify two statements in mathematics which are true under certain conditions but false under certain other conditions.

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8) Give an instance of drawing inferences by analogy and by patterns in mathematics.

9) Give one example of a) direct method of proof, b) reductio ad absurdum, c) disproof by a counter example.

1.3.4 Axiomatic Nature of Mathematics

The idea of an axiomatic structure in mathematics concerns the very foundation of mathematical reasoning. The origin of axiomatics can be traced to Euclid's Elements. Euclid introduced rigour by deducing his geometrical theorems from clearly stated assumptions embodied in his axioms and postulates. The essential components of an axiom system are;

1. **Undefined terms**: These are derived in some way from experience and depend on intuition or imagination.

2. **Axioms or postulates**: These are assertions derived from undefined terms. They are formulated in terms of relationships for which no proofs are expected.

3. **Propositions**: Propositions derived from the axioms by logical reasoning.

Euclid's execution of this idea was defective as he inadvertently omitted to state all the assumptions which he subsequently utilized in his demonstrations.

Peano's axiom system for deriving the natural numbers is another example of axiomatics.

**Undefined terms:**  "0", "number", "successor"

**Axioms/Postulates:**

<table>
<thead>
<tr>
<th>Axioms/Postulates</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0 is a number.</td>
</tr>
<tr>
<td>A2</td>
<td>The successor of a number is a number.</td>
</tr>
<tr>
<td>A3</td>
<td>No two numbers have the same successor.</td>
</tr>
<tr>
<td>A4</td>
<td>0 is not the successor of any number.</td>
</tr>
<tr>
<td>A5</td>
<td>If P is a property such that (i) 0 has the property P, and (ii) whenever n has the property P, the successor of n has the property P, then every number has the property P.</td>
</tr>
</tbody>
</table>

The 5th axiom is called "the principle of mathematical induction".

1.3.5 Language of Mathematics

In teaching mathematics, the teacher uses ordinary language to communicate mathematical concepts and to clarify thoughts. Language is a means of gradually internalizing experience to the point where actions can proceed in imagination without recourse to their physical repetition. For learning mathematical concepts children are initially engaged in activities
with concrete materials, then encouraged to make audible descriptions and instructions - the concrete aids being withdrawn gradually until, finally, the concepts are internalized in verbal form. Thus language becomes a means of storing experience and facilitating problem solving.

Effective learning of mathematical concepts does not result from mastery over activities alone. It depends on how far teachers are successful in developing language or other symbolic representations, building links with past experiences to formulate corresponding abstractions or laws. The transition from concrete to abstraction depends upon explanations written in mathematical terms. Today a physicist (for that matter, any scientist) cannot pursue his or her studies without extensive use of mathematical language. Even subjects like biology, psychology, etc., which, used to be descriptive, are increasingly using mathematical notions. Persons studying the form and structure of language have also applied mathematics to explore it.

Roger Bacon said "Mathematics is the gate and key of the sciences. Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot view the other sciences or the things of the world. And what is worse, men who are thus ignorant are unable to perceive their own ignorance and so do not seek a remedy."

Mathematics, thus, may be seen as a tool or a means of communication.

Interesting studies of language difficulties experienced by children in mathematics have been made. Some features of mathematical language need special mention:

1. Mathematical language distinguishes between things and names of things. Number and numeral, and fraction and fractional numbers are a few examples.

2. Some common spoken words are used as technical terms and sometimes even in different contexts. For example, "variable" is used both as a noun and as an adjective; "root" is used as a root of an equation and as in square root, cube root, etc.

3. There are a variety of ways of calling a thing. For example, addition can be referred to as "find the sum", "find the value", "find the whole", "how many in all?" etc.

4. Abbreviations (or labelling) are used. This usually helps in sustained thinking but sometimes they may not be in standard form or may be used only to avoid some steps in an algorithm.

   For example, using gm. for gram is wrong; using cms. is also not correct

5. Frequently, auxiliary figures and markings are taught when new topics or operations are introduced. For example: to write carrying figure in addition; => or "or" in writing equations. 5 m x 4 m = 20 sqm. is not correct because the multiplier is simply a number, it cannot be concrete. The correct way is (5 x 4) sqm.

6. Mathematical solutions emphasize a specific arrangement of steps in the solution, i.e., an algorithm to develop accuracy of thought and precision in quantitative matters.

7. Like all other languages, the language of mathematics has its own grammar. It has its own nouns, verbs, adjectives, etc.

The main characteristics of mathematical language are simplicity, accuracy and precision in contrast to ordinary language which can be ambiguous, vague and emotive. Special care is needed in formulating definitions. A good definition should satisfy the following conditions:
1. A definition should be consistent, i.e. it should convey the same meaning of the terms in all possible situations of the system.

2. A definition should not only consist of undefined terms or other previously defined terms, but also the common articles and connectives.

3. A definition should be stated clearly and precisely without redundancy.

1.4 NEED AND IMPORTANCE OF MATHEMATICS IN SCHOOL CURRICULUM

Mathematics is an important component of school education. Its influence has been so fundamental and widespread that being numerate is becoming more important than being literate. The following values justify its position.

1.4.1 Social Aspects

- The routine activities of daily life demand a mastery of number facts and number processes. To read with understanding much of the materials in newspapers requires considerable mathematical vocabulary. A few such terms are percent, discount, commission, dividend, invoice, profit and loss, wholesale and retail, taxation, etc. As civilization is becoming more complex, many terms from the electronic media and computers are being added.

- Certain decisions require sufficient skill and understanding of quantitative relations. The ability to sense problems, to formulate them specifically and to solve them accurately requires systematic thinking.

- To understand many institutions and their management problem, a quantitative viewpoint (modelling) is necessary. It is illuminating to hear from an economist, an architect, an engineer, an aviator, or a scientist what in mathematics is helpful to them as workers.

- Many vocations need mathematical skills.

- The child should gain an appreciation of the role played by mathematics in many fields of work. Since, scientific knowledge and technology are linked with the progress and prosperity of a nation, we should be able to appreciate the role of mathematics in acquiring these.

- Mathematics has helped in bringing together the countries of the world which are separated from each other physically.

- Mathematics helped man to discover the mysteries of nature and to overcome superstitions and ignorance.

1.4.2 Mathematical Aspects

- Mathematics teaches us how to analyse a situation, how to come to a decision, to check thinking and its results, to perceive relationships, to concentrate, to be accurate and to be systematic in our work habits.

- Mathematics develops the ability to perform necessary computations with accuracy and reasonable speed. It also develops an understanding of the processes of measurement and of the skill needed in the use of instruments of precision.

- Mathematics develops the ability to
  a) make dependable estimates and approximations,
  b) devise and use formulae, rules of procedure and methods of making comparisons,
Nature, Objectives and Approaches to Teaching of Mathematics

c) represent designs and spatial relations by drawings, and
d) arrange numerical data systematically and to interpret information in graphic or tabular form.

1.4.3 Applications of Mathematics

- The history of mathematics is the story of the progress of civilizations and culture. “Mathematics is the mirror of civilization”.
- A country’s civilization and culture is reflected in the knowledge of mathematics it possesses.
- Mathematics helps in the preservation, promotion and transmission of cultures.
- Various cultural arts like poetry, painting, drawing, and sculpture utilize mathematical knowledge.
- Mathematics has aesthetic or pleasure value. Concepts like symmetry, order, similarity, form and size form the basis of all work of art and beauty. All poetry and music utilizes mathematics. Quizes, puzzles and magic squares are both entertaining and challenging to thought. Hence, the teaching of mathematics is inevitable in our schools.

1.5 RECENT TRENDS AND PRINCIPLES OF FORMULATING MATHEMATICS CURRICULUM

1.5.1 Factors Affecting Change in Curriculum

Educators are aware that changes have taken place in school mathematics in the past two to three decades. These changes have brought about a near revolution in the content, methods and instruction of mathematics.

Reasons for changes

- The rapid advance of knowledge in mathematics makes increasingly greater demand on an enlightened citizenry.
- The need for more effective articulation from one grade to the next and from elementary to secondary school.
- The recognition that the traditional mathematics programme, limited mainly to emphasis on computational skills and divided into traditional compartments viz. arithmetic, algebra and geometry, is somewhat lacking in a few fascinating and interesting aspects of mathematics.
- The need for a better understanding of the structure of mathematics and the mathematical process, its language and methods of proof.
- The need for the utilization of more effective media (technology and aids) for adapting mathematics learning to the needs of different abilities.

New emphasis in curriculum: As a result the following changes have taken place in recent years:

1. Concern for the child as an individual and as a learner caused educators to question the grade placement of certain topics in elementary school mathematics. Topics in arithmetic believed to have little use in daily life were omitted. Examples of this are vulgar fractions, compound quantities, problems on carpeting floors, papering walls, etc. Many theorems in Euclidean geometry have been omitted.
2. The term “arithmetic” was shifted to “elementary school mathematics” to indicate a change in emphasis to a more generalized language, formulation of laws and integrating algebraic processes in computational work.

3. The drill method of teaching was replaced with methods emphasizing “meaning” and explaining the “whys” of the processes as related to computational procedures.

4. Psychologists emphasized the relatedness of learning and explored the process of learning pertinent to the development of fundamental mathematical ideas. They found that there are levels, that is, (a) the level of the concrete or the world of things; (b) the level of the semiconcrete where experiences are internalized and fit together; (c) the symbolic level where abstractions and generalizations are formulated; (d) the level of applications where the generalizations are tested and applied to situations. This led them to conclude that certain topics should be introduced much earlier than was formerly believed.

Bruner’s hypothesis that any subject can be taught effectively in some intellectually honest form to any child at any stage of development led to an explosion of new methods based on discovery and problem-solving.

5. The study of geometry was expanded far beyond Euclid’s elements. The basics of transformations, vectors and coordinate geometry were included. In algebra emphasis has now been given to equations by broadening the base to include ideas such as mathematical sentences, replacement set and solution set. Generalizations of the properties of the real number system and the introduction of the algebra of sets, groups, etc., provided an expansion of mathematical ideas in both depth and breadth. Basic concepts such as function, variable, relations, etc., gained greater importance.

6. The use of computers has further enriched the content and practices in mathematics education in schools.

1.5.2 Principles of Formulating Mathematics Curriculum

The curriculum is a tool to achieve the proposed objectives of teaching a subject. In broader terms it is the sum total of all the experiences of the pupil that he undergoes in school, home and in informal contacts between the teacher and the pupil. The principles of curriculum construction in mathematics can be listed as an outcome of the values and importance discussed above. These are briefly mentioned here.

1. A good mathematics programme should present suitable learning experiences to foster common needs as citizens and special needs as an individual. The main consideration should be given to desirable pupil growth within the overall purposes of elementary education. It should therefore:

   a) emphasize those behaviours which will fit pupils better for useful service to their community and for ethical living;

   b) gain an appreciation of the importance and power of mathematics in the development of society;

   c) discover vocational possibilities;

   d) develop the ability to analyse and to solve problems of everyday life situations; and

   e) develop proficiency in mathematics as a method of communication.

2. In addition to competence in arithmetic (computational skills in four fundamental processes), elementary ideas of algebra, proof and sets are being included.
3. Pure rote learning until facts are memorized almost mechanically is now giving way to discovering numbers as a property of the manyness of a set, ordering the numbers, mapping, building a system of numeration and using expanded notation for fostering conceptual understanding.

4. Greater attention is now being given to preparing pupils for the subsequent study of mathematics at higher levels.

5. Many curriculum developers use Piaget’s stages of learning for planning learning situations. These stages are:
   a) pre-conceptual stage from birth to age four or five years;
   b) intuitive stage from ages five to eight years;
   c) concrete operational stage from ages eight to thirteen;
   d) formal operational stage from ages twelve or thirteen onwards, where learning of the reasoning process, appraising, forming hypothesis, generalizing, deducing and proving is emphasized.

The Gestalt approach to learning places emphasis on relation and restructuring of mathematical situation. It is exemplified by the so-called "multiple embodiment" procedure where many different aspects or approaches to a mathematical concept are presented at the same period of learning so that by gradual reinforcement and restructuring, there results a more general yet precise mental formulation of the concept.

6. The laboratory approach is more popular in many places. Herein the classroom pupils try to “discover mathematics”. The classroom is equipped with all types of materials: electronic devices, measuring equipment, models, geoboards, blocks, cards, charts, etc. The children may work in groups or individually on projects. Experimentation and discovery are encouraged.

7. The elementary school is but one phase of education for living, and as such should provide an orderly sequence that will enable the pupils to maintain steady growth. Knowledge and familiarity with the work of succeeding grades and with higher education is necessary. Within the field of mathematics, the correlation between various parts should be maintained. Algebra and arithmetic can be correlated with geometry. Algebra and geometry can be correlated with trigonometry. The tendency to keep various parts in “watertight compartments” needs to be given up.

8. Attention must be paid also to the relationship of mathematics with other subjects such as physics, chemistry, biology, geography or social sciences.

9. a) Work must be child-centred, not teacher-dominated. The pupils must participate and perform. The teacher should motivate, organize and direct the children to learn.

   b) The activities should have significance to the learner and should be based on practical life situations.

   c) Problem methods are more favourable teaching situations.

10. Content should be decided on the basis of mental age, interest level, present usefulness to learner and future use to learner.

11. Provision should be made for the below-average learners. Each child should get functional experiences on his own level of ability and interests so that he can succeed in developing mathematical competence of value in practical life situations.
1.6 CONTRIBUTION OF INDIAN MATHEMATICIANS

The Hindu civilization had climbed to lofty heights in the knowledge of mathematics but the path of ascent is not traceable. Much of Hindu mathematics remained merely a servant to astronomy and it remained chiefly in the hands of the priests who put in verses all the mathematical results, which therefore became unintelligible to the common man. While the Greek mind was pre-eminently geometrical, the Hindu mind was arithmetical. Numerical symbolism, the science of numbers and algebra attained far greater heights in India than they had previously reached in Greece. Hindu geometry was merely mensuration, unaccompanied by demonstration.

Since ancient times, Indians have made very significant contributions to mathematics. The system of numeration and the concept of zero are a gift of Indians to the world of mathematics.

Mathematicians like Arya Bhatta, Bhaskaracharya and Shridharacharya gave many important concepts like the value of π (pai), the functions, solutions of quadratic equations and many others to the world. During recent times, mathematicians like Ramanujan and P. L. Bhatnagar have been shining stars on the horizon of mathematics.

He clearly stated the relationship between the sides of the right angled triangle in one of his verses. This statement was recognised as Pythagoras Theorem. Mahavira belonged to 9th century and wrote Ganit Sar-Sangrah. He gave the method of finding LCM, which is used even now. He also gave the formula for finding the area of a quadrilateral:

\[\text{The area of a quadrilateral} = \sqrt{S(S-a)(S-b)(S-c)(S-d)}\]

where a, b, c, d, are the sides and S is the semi-perimeter of the quadrilateral.

This formula is true only for the cyclic quadrilaterals. Mahavira has also given the formula for calculating the area of an ellipse. Sridhar Acharya wrote three books – Trisānāhika, Patiganit, and Algebra. He gave a method of solving quadratic equations. He also gave the geometrical interpretation of algebraic formula. Sripati wrote two books – Siddhanta-Shekhar and Ganit-Tilak. After Sripati, Bhaskaracharya’s contribution is
Nature, Objectives and Approaches to Teaching of Mathematics

notable. A work entitled Siddhantha Siromani was written by Bhaskaracharya (A.D. 1150). The two important chapters in this book are - the Lilvathi (the beautiful) and Vija-ganita (rod-extraction) devoted to arithmetic and algebra. Lilavathi was the lone daughter of Bhaskara and it is said that Bhaskara decided to teach his daughter, who became a widow at a very young age, to keep her mentally alert and occupied. The text is believed to be devoted to her. It contains problems stated in the form of verses in Devnagari script, on arithmetic, algebra and geometry and are posed to the intelligent. An example is stated below for the benefit of the reader.

Out of a swarm of bees, a number equal to the square root of half their number went to the Malati Flowers; $\frac{8}{9}$ ths of the total number also went to the same place. A male bee enticed by the fragrance of the lotus got into it. But when it was inside it, night fell, the lotus closed, and the bee was caught inside. To its buzz, its consort was replying from outside.

What is the number of bees?

For the solution of the problem, we have the equation

$$\frac{x}{2} + \frac{8}{9} x + 2 = 2 \therefore x = 72$$

He has corrected the concept of zero as laid down by Mahavira and Brahmagupta.

Post-Medieval Period (A.D. 1200 to A.D. 1800)

In this period, very little original work was done. There were foreign invasions in North India; therefore mathematical development was checked in this period. At this time, South India was considered as the centre of learning science and mathematics. During this period, Kamlakar (A.D. 1600) wrote Siddhanta-Tatva. Vivek Narain wrote Ganit-Kaumudy and Neel Kantha (in A.D. 1587) wrote Tajik Neel Kanthia. Pandit Jagannath (A.D. 1731) translated the works of Ptolemy and Euclid into Sanskrit. This book is known as Samrat-Siddhanta. The present terminology of geometry is mostly taken from this book.

Those who wish to know more about the Indian contributions to mathematics in early periods may consult the following source books:

i) The History of Hindu Mathematics by A.D. Singh and V.P. Dutta.


Ancient Period (3000 B.C. To 500 B.C.)

Ancient period may be further sub-divided into three main periods:

i) Vedic period (3000 B.C. to 1000 B.C.): In the period, the number system and the decimal system were invented. During this period, various interpretations were derived from Vedas and put into the form of Samhitas (सामहित). At one place in the thirteenth Samhita (6/2, 4, 5) they have given the following relationship:

$$39^2 = 36^2 + 25^2$$

This is a particular case of Pythagoras theorem.

ii) Sulva period (1000 B.C. to 500 B.C.): The knowledge of geometry was one of the important features of this period. It was the belief of Aryans that yajna is the only means for the attainment of supreme life. Yajna was performed at different geometrical altars. The yajna was useful only when performed during the specified period at the specified altar. This was not possible without the knowledge of
mathematics. The altar requires the knowledge of geometry. Different geometrical theorems are stated in *Sulva Sutra*. Sulva was considered as the rule of cord used in the construction of altars. The knowledge of Pythagoras theorem was also prevalent during this period. This is stated by Baudhayan (1000 B.C.).

iii) **Vedanta period (1000 B.C. to 500 B.C.):** This period is famous for astronomical work. Astronomy, as a branch of mathematics, studies the positions and movements of stars and planets.

**Early Period (500 B.C. to A.D. 500)**

Only Jain religion *Grantha Book* and some pages of *Bakhshali Ganit* of this period are available at present. The following inventions were made in this period:

a) Place value system of writing numbers.
b) Invention of zero.
c) Introduction of algebra.
d) Development of arithmetic (*Bakhshali Ganit*).
e) Development of astronomy and *Surya Siddhanta*.

In the sixth century, Varaha Mihira Acharya wrote his *Pancha Siddhantika* which gives a summary of *Surya Siddhanta* and four other astronomical works, then in use. The concepts of Interest and Per cent were also known in this period.

**Medieval Period (A.D. 500 to A.D. 1200)**

This period starts from Aryabhata and continues up to Bhaskar II. Aryabhata has consolidated the rules of mathematics into 33 verses in his book *Aryabhatiya*.

*Aryabhatiya* is considered a pioneer in the field of algebra. He gave the value of $\pi$ as $\frac{62832}{20,000}$ which is correct upto fourth decimal. He stated that the earth is moving while the stars are stationary. This is described in "Aryabhatiya Goal-pad".

In the 5th century an anonymous Hindu astronomical work was known as *Surya Siddhanta* which is regarded as a standard work. Brahmagupta wrote *Brahma-Sutra-Siddhanta* – the revised system of Brahma. It is believed that it is Brahmagupta who gave zero its status. He has given a concept of infinity in this book. According to him, if a positive or a negative number is divided by zero the quotient will be infinity. He also gave the formulae for finding out the volumes of the prism, cone and pyramid.

Volume of pyramid $= \frac{H}{3} \left[ \Delta + \Delta' + \sqrt{\Delta \Delta'} \right]$

Where $H$ is the height, $\Delta$ is the area of base and $\Delta'$ is the area of other surfaces.

He also gave the formula to find the sum of geometric series. If $a$ is the first term, $r$ is the common ratio and $n$ is the number of terms, then

$$S = a + ar + ar^2 + \ldots \ n \ terms = \frac{a(r^n - 1)}{r - 1}$$

He also stated the relationship between the two intersecting chords of a circle, if $AB$ and $CD$ are two chords intersecting at $E$, the mid-point of $CD$, then

$$4 \ AE \times BE = (CD)^2$$

This relationship is also described in the Euclidean Geometry. Only the chapter of Kuttaka (Indeterminate equation) of *Brahma-Sutra Siddhanta* is available and due to this method algebra was known as Kuttaka Ganit.

(Reproduced from 'Content -Cum-Methodology of Teaching Mathematics', published by N.C.E.R.T.)
1.7 LET US SUM UP

Any consideration of the nature of mathematics leads to finding out answers to two questions: i) What is the nature of mathematical proof; and ii) What is the nature of mathematical truth?

The first question is for psychologists to answer and the second for philosophers. This unit discusses a few elementary ideas about these two questions. The nature of arguments, types of proofs, symbolism and logic are ideas related to the first question. The utility of mathematics, axiomatic structure, undefined and defined notions, and the nature of mathematical laws are ideas related to the second question.

The unit also provides an insight into the value and importance of mathematics. The traditional approach of classifying values as social, disciplinarian, cultural, vocational and pleasure has been modified to reflect a broader perspective. The elements of courses of study (curriculum) has been mentioned. These include modern views on determinants of the course content in arithmetic, algebra and geometry, flexibility, emphasis on methods of discovery, provision for individual differences etc.

It is suggested that the study of the unit may be supplemented by an actual analysis of school curricula and a study of the recommendations about mathematics education in the Education Policy Statement of the Government of India.

1.8 UNIT-END ACTIVITIES

1. In demonstrative geometry, make appropriate rearrangement of material or draw an appropriate drawing to arrive at fruitful conjectures mentioned below:
   a) The area of a parallelogram is equal to the product of its base and its altitude.
   b) In a triangle (i) the medians, and (ii) the altitudes are concurrent.

2. Prepare a home-made device or aid to suggest conjectures relating an inscribed angle to its intercepted arc or the angles in alternate segments.

3. Discuss the axiomatic nature of Euclidean geometry. Find out the gaps in axioms and the reasons for which some people wanted Euclidan geometry to go out of the course.

4. Take a book of mathematics and analyse the vocabulary to find out loose statements, faulty definitions and conventions.

5. Analyse errors such as the following and discus the mistakes.
   i) \[ \frac{1}{4} = \frac{1}{4}, \quad \text{ii)} \quad \frac{4+3}{4+10} = \frac{3}{10}, \quad \text{iii)} \quad \frac{3+2}{3+5} \]
   iv) \[ 8^2 - 2^2 = 82 - 22 = 60 \]

6. Analyse a textbook of mathematics and find out if the branches of mathematics are genuinely integrated.

7. What mathematics should be taught to school children? Comment on the probable answers:
   a) What does the child needs now?
   b) What will the child need most when he grows up?
   c) What is required to fit the child into a job?
   d) What is likely to give the ability to the child to think, explore and solve problems?
8. List the uses of mathematics in other areas: social studies; the sciences; art; music; health and physical education.

9. Study from books on history of mathematics
   a) the growth of the numeration system;
   b) the growth of measurement; and find-out how mathematics is related to culture.

1.9 ANSWERS TO CHECK YOUR PROGRESS

1. Mathematics helps the person in budgeting and rationalising expenses besides performing computations.

2. Mathematics trains the mind in seeking logic and reason which in turn helps in rational thinking and understanding.

3. According to the nature of mathematics taught in our schools, mathematics can be defined as discipline in which deductions are made on the basis of facts or truths called axioms and these deductions are applied to solve daily life problems.

4. Yes, algebra is generalized arithmetic because algebra uses arithmetic operations and the rules exactly as they are applied on numbers in arithmetic and the only difference being that the basis of all operations in algebra are literal numbers or variables which can take any specified value from the number system.

5. The natural number system which forms the basis of almost all arithmetic is the most fundamental structure in mathematics. Once we have understood the structure of natural numbers, it is quite easy to understand the structure of the other number systems which follow.

6. Euclid's postulates form pure mathematics but trigonometry applied to solve problems in physics or astronomy forms applies mathematics.

7. a) If in two triangles, two sides and an angle not included between them are equal then the two triangles may or may not be congruent. But in case of two right angle triangles, if two sides and an angle not included between them of one triangle are equal to the corresponding parts of the other triangle then the triangles are congruent.

b) The inequality ab ≥ ac ⇒ b ≥ c holds good if a > 0 but becomes false if a < 0.

8. Consider, the sequence a, a + d, a + 2d, a + 3d, ............

From the pattern of the terms we conclude that the nth term = a + (n−1) d.

9. a) Direct Proof

   The diagonals of a rectangle are equal.

   Proof: In Δ DAB and Δ CBA,
   AD = BC (opposite sides of a rectangle)
   AB = AB (common)
   ∠ DAB = ∠ CBA = 90°
   ∴ Δ DAB ≅ Δ CBA (SAS rule)
   ∴ DB = AC (cpctc)
Nature, Objectives and Approaches to Teaching of Mathematics

b) If a line drawn from the midpoint of one side of a triangle is parallel to another side, then it passes through the midpoint of the third side.

Proof: Let the line \( l \) through the midpoint \( D \) of side \( AB \) of \( \triangle ABC \) parallel to \( BC \) intersect \( AC \) at \( E \) and if possible let \( E \) not be the midpoint of \( AC \). Let \( F \) be the midpoint of \( AC \).

Then \( DF \) is a line segment drawn through the midpoints of two sides of a triangle. Hence \( DF \parallel BC \).

\[ \iff \]
\( l \) and \( DF \) are two different lines passing through \( D \) and both parallel to \( BC \). But this is not possible. Hence, we have a contradiction.

\[ \iff \]
\( E \) is the midpoint of \( AC \).

c) Consider the statement: \( 1/x \) is a real number for all real numbers, for \( x = 0 \), this statement is not true since \( \frac{1}{0} \) is not a real number. Hence, the statement is not true.

10. The two reasons for the need to change the school curriculum may be:

i) Rapid advances of knowledge makes it necessary to include some new concepts in the curriculum.

ii) With the advance of technology, better teaching aids have been developed. Some changes in the curriculum are required to make use of the better teaching aids.

11. The three important principles for deciding a syllabus may be:

i) The mental age and interest level of the learner.

ii) Usefulness of the material in developing proficiency in mathematics and its future use.

iii) The behavioural changes it will be able to produce in the learner.

1.10 SUGGESTED READINGS

Buxtom Laurie; Mathematics for Every Man; J.M. Dent & Sons Ltd., London.

Wilder, R.L.; Evolution of Mathematical Concepts; Transworld Publishers Ltd.


N.C.T.M.; The Growth of Mathematical Ideas, Grade K-12, 24th Year Book; Washington, USA.


NCERT; Content-Cum-Methodology of Teaching Mathematics; NCERT, New Delhi (India).