<table>
<thead>
<tr>
<th>Block 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONTENT BASED METHODOLOGY-II</td>
</tr>
<tr>
<td><strong>UNIT 14</strong></td>
</tr>
<tr>
<td>Statistics and Probability</td>
</tr>
<tr>
<td><strong>UNIT 15</strong></td>
</tr>
<tr>
<td>Parallel Lines, Parallelograms and Triangles</td>
</tr>
<tr>
<td><strong>UNIT 16</strong></td>
</tr>
<tr>
<td>Trigonometry and its Application</td>
</tr>
<tr>
<td><strong>UNIT 17</strong></td>
</tr>
<tr>
<td>Mensuration and Coordinate Geometry</td>
</tr>
</tbody>
</table>
EXPERT COMMITTEE

Prof. I. K. Bansal (Chairperson)  Prof. Anuj Sehgal Gupta
Former Head, Department of Elementary  School of Humanities
Education, NCERT, New Delhi  IGNOU, New Delhi

Prof. Shridhar Vashistha  Prof. N. K. Dash (Director)
Former Vice-Chancellor  School of Education
Lal Bahadur Shastri Sanskrit  IGNOU, New Delhi
Vidyapeeth, New Delhi

Prof. Parvin Sinclair  Prof. M. C. Sharma
Former Director, NCERT  (Programme Coordinator- B.Ed.)
School of Sciences  School of Education
IGNOU, New Delhi  IGNOU, New Delhi

Prof. Aejaz Mashih  Dr. Gaurav Singh
Faculty of Education  (Programme Co-coordinator-B.Ed.)
Jamia Millia Islamia, New Delhi  School of Education

Prof. Pratyush Kumar Mandal  IGNOU, New Delhi
DESSH, NCERT, New Delhi

SPECIAL INVITEES (FACULTY OF SOE)

Prof. D. Venkateswarlu  Dr. Bharti Dogra
Prof. Amitav Mishra  Dr. Vandana Singh
Ms. Poonam Bhushan  Dr. Elizabeth Kuruvilla
Dr. Eisha Kannadi  Dr. Niradhar Dey
Dr. M. V. Lakshmi Reddy

Course Coordinators  Prof. M. C. Sharma, SOE, IGNOU
  Dr. Anjuli Suhane, SOE, IGNOU

COURSE TEAM

Course Contribution

Unit 14
Adopted from ES-342 and major transformation by
Dr. Anjuli Suhane

Unit 15
Adopted From ES-342

Unit 16
Ms. Vandita Kalra
PGT (Maths), Govt. Sr. Sec. School, New Delhi

Unit 17
Ms. Vandita Kalra
and
Sh. Ajith Kumar
Assistant Professor, SOE, IGNOU, New Delhi

Content Editing
Prof. K.K. Vashistha
Former Dean
Department of Elementary Education
NCERT, New Delhi

Language Editing
Prof. Amitav Mishra
Assistant Professor
SOE, IGNOU, New Delhi

Format Editing
Dr. Anjuli Suhane
Assistant Professor
SOE, IGNOU, New Delhi

Proof Reading
Dr. Anjuli Suhane
SOE, IGNOU, New Delhi

PRODUCTION

Prof. Saroj Pandey  Mr. S.S. Venkatachalam
Director  Assistant Registrar (Publication)
SOE, IGNOU, New Delhi  SOE, IGNOU, New Delhi

April, 2017
© Indira Gandhi National Open University, 2017
ISBN-
All rights reserved. No part of this work may be reproduced in any form, by mimeograph or any other means, without permission in writing from the Indira Gandhi National Open University.
Further information on the Indira Gandhi National Open University courses may be obtained from the University’s Office at Maidan Garhi, New Delhi-110068.
Printed and published on behalf of the Indira Gandhi National Open University, New Delhi by Director, School of Education, IGNOU, New Delhi.
Printed at:
Course: BES-143 Pedagogy of Mathematics

BLOCK 1: UNDERSTANDING THE DISCIPLINE OF MATHEMATICS
Unit 1 Nature and Scope of Mathematics
Unit 2 Aims and Objectives of Teaching -Learning Mathematics
Unit 3 How Children Learn Mathematics
Unit 4 Mathematics in School Curriculum

BLOCK 2: TEACHING -LEARNING OF MATHEMATICS
Unit 5 Approaches and Strategies for Learning Mathematics
Unit 6 Organizing Teaching-Learning Experiences
Unit 7 Learning Resources and ICT for Mathematics Teaching-Learning
Unit 8 Assessment in Mathematics
Unit 9 Professional Development of Mathematics Teacher

BLOCK 3: CONTENT BASED METHODOLOGY-I
Unit 10 Number System, Number Theory, Exponents and Logarithms
Unit 11 Polynomials: Basic Concepts and Factoring
Unit 12 Linear Equations, Inequations and Quadratic Equations
Unit 13 Sets, Relations, Functions and Graphs

BLOCK 4: CONTENT BASED METHODOLOGY-II
Unit 14 Statistics and Probability
Unit 15 Parallel Lines, Parallelograms and Triangles
Unit 16 Trigonometry and its Application
Unit 17 Mensuration and Coordinate Geometry
BLOCK 4  CONTENT BASED METHODOLOGY -II

Block Introduction

The course BES-143: Pedagogy of Mathematics contains four blocks. This is the fourth block which is titled Content based Methodology-II. The third and fourth blocks focus on content/concepts. In both the blocks, teaching-learning process and different modes of evaluation of these concepts have been discussed. Different aspects of Statistics, Probability, Geometry, Trigonometry, Coordinate Geometry and Mensuration are discussed in this block. This block consists of four units.

Unit 14: Statistics and Probability

Unit 15: Parallel Line, Parallelogram and Triangles

Unit 16: Trigonometry and its Application

Unit 17: Mensuration and Coordinate Geometry

We can not imagine any news bulletin without reference to the inflation rate, index numbers, industrial and agricultural growth rates ,etc. We all come across some tables, graphs, charts etc., while going through newspaper and news channel of television. Graph and charts are the pictorial presentation of data and this is the one of the key aspect of statistics. Thus the Unit 14 discusses the important concepts of statistics and probability and its application in day-to-day life.

The systematic study of geometry helps in developing logical thinking and increase the power to analyze things. So the Unit 15 is devoted to develop geometrical concepts by the children. Parallel lines, parallelograms, concepts of congruence and the similarity of geometrical figures are dealt in this Unit.

Unit 16 presents a brief discussion of the basic concepts of trigonometry and their application in solving problems of 'heights and distances'.

Unit 17 'Mensuration and Coordinate Geometry' basically have two sections one is Mensuration and other is Coordinate Geometry. Mensuration is the branch of Mathematics which deals with length of lines, area of surfaces and volume of solids. This Unit is devoted to the study of mensuration. The formulae for finding the area of different plane figures and the volume of different types of solids discusses with their application. In the other section of this Unit basics of coordinate geometry; distance and section formula and its applications are discussed.

After going through this block, you will be able to describe and explain the need and ways of teaching-learning Statistics, Probability, Geometry, Trigonometry, Coordinate geometry and Mensuration to the tender minds and you will also be able to appreciate the practical applications of these branches of Mathematics.
UNIT 14  STATISTICS AND PROBABILITY

Structure

14.1  Introduction
14.2  Objectives
14.3  A Brief Look at Statistics
   14.3.1  What are Statistics?
   14.3.2  Why do We Use Statistics?
14.4  Collecting Data
   14.4.1  Data and its Sources
14.5  Organising Data
   14.5.1  Pictorial Presentation
   14.5.2  Handling Large Data
14.6  Interpreting Data
   14.6.1  Measures of Central Tendency
   14.6.2  Measures of Dispersion – Range, Mean Deviation , Standard Deviation
14.7  Probability and its Applications
14.8  Let Us Sum Up
14.9  Unit End Activities
14.10  Answers to Check Your Progress
14.11  References and Suggested Readings

14.1  INTRODUCTION

Modern age is known as information age. Lots of information comes in numbers. Information involving numbers which we come across is known as data. Statistics is a branch of Mathematics which deals with collection, organization and interpretation of data. By using Statistics we get specific trend or pattern of data. From these trends or patterns, we derive some predictive inferences which may or may not be completely true. On the basis of these inferences we hope that an event occur in certain way. And probability says simply how likely something is to happen. Whenever we’re unsure about the outcome of an event, we can talk about the probabilities of certain outcomes—how likely they are. Mathematics teachers have special role in this field. It is not just a matter of teaching few topics listed under the title ‘Statistics and Probability’. Children need to understand these topics in relation to whatever is happening in society around them. So the unit provides opportunities to involve students in enjoyable and challenging activities – collecting data, presenting them in the form of graphs and summarizing them in terms of average (mean), median and mode. Such data may be in respect of some activity in which students normally get interested. The unit also provides opportunities to involve students in various activities that help them to understand the concept of probability and its applications in day-to-day life.
14.2 OBJECTIVES

At the end of this unit, you will be able to:

• illustrate the meaning of Statistics and its importance in everyday life situations;
• help students in identification which graphic representation best suits a given set of data;
• help students in organization of given data in suitable class intervals;
• enable students to calculate mean, median and mode of raw as well as grouped data;
• understand the concept and importance of probability; and
• explain and design learning activities that help to transact various concepts of statistics and probability.

14.3 A BRIEF LOOK AT STATISTICS

14.3.1 What are Statistics?

Main Teaching Point: Statistics deal with the collection, presentation and interpretation of data.

Teaching-Learning Process: The topic should be introduced to students by raising questions which require the collection of data in a real life situation. The data collected would also need to be organized in some manner to be able to answer the questions raised. Here is an illustration:

Ask: Suppose we need the following information about the students in class:

a) the height of the tallest student
b) the height of the shortest student
c) the difference in their heights
d) the height which the maximum number of students have
e) the number of students who are taller than the height which the maximum number of students has
f) the number of students who are shorter than the height which the maximum number of students has.

The answer you may get is that they would make all the students stand in a line according to increasing or decreasing height and identify the tallest and shortest students. But then, can we answer questions (a) and (b) unless we measure their heights? Certainly, not. To answer all the above questions, we need to measure the heights of all the students in the class.

You, as a teacher, should point out at this stage that the information we get by measuring their heights is called data, that is in the language of Statistics. Data are a number of facts. Data is the plural of the Latin word ‘datum’ which means ‘fact’. Measuring and noting down the heights of the students is called the process of data collection.

Suppose as a result of collecting the data, we get the heights of 45 students in a class as under:
Statistics and Probability

Heights (in cm.) of 45 students of a class

<table>
<thead>
<tr>
<th>140</th>
<th>142</th>
<th>143.5</th>
<th>140.5</th>
<th>150</th>
<th>149</th>
<th>148.5</th>
<th>148</th>
<th>141</th>
</tr>
</thead>
<tbody>
<tr>
<td>148</td>
<td>148.5</td>
<td>152</td>
<td>153</td>
<td>150.5</td>
<td>147</td>
<td>146</td>
<td>142.5</td>
<td>148</td>
</tr>
<tr>
<td>152.5</td>
<td>154.5</td>
<td>155</td>
<td>153.5</td>
<td>152.5</td>
<td>149</td>
<td>147</td>
<td>146</td>
<td>156</td>
</tr>
<tr>
<td>152</td>
<td>148</td>
<td>143</td>
<td>149</td>
<td>144</td>
<td>150</td>
<td>152</td>
<td>153</td>
<td>154</td>
</tr>
<tr>
<td>156</td>
<td>153</td>
<td>154.5</td>
<td>154</td>
<td>147</td>
<td>148</td>
<td>146</td>
<td>153</td>
<td>154</td>
</tr>
</tbody>
</table>

Ask the students to answer the questions raised. It may still be difficult for them to answer those questions quickly. Lead the students to realize that they can answer the questions faster if the data are organized in some manner. Since the questions relate to a comparison of heights, the best way to organize the data is to write the heights in increasing or decreasing order. Then see the following table:

Heights (in cm.) of 45 students of a class
(arranged in ascending order)

<table>
<thead>
<tr>
<th>140</th>
<th>140.5</th>
<th>141</th>
<th>142</th>
<th>142.5</th>
<th>143</th>
<th>143.5</th>
<th>144</th>
<th>146</th>
</tr>
</thead>
<tbody>
<tr>
<td>146</td>
<td>146</td>
<td>147</td>
<td>147</td>
<td>147</td>
<td>148</td>
<td>148</td>
<td>148</td>
<td>148</td>
</tr>
<tr>
<td>148</td>
<td>148.5</td>
<td>148.5</td>
<td>149</td>
<td>149</td>
<td>149</td>
<td>150</td>
<td>150</td>
<td>150.5</td>
</tr>
<tr>
<td>151</td>
<td>152</td>
<td>152</td>
<td>152</td>
<td>152.5</td>
<td>152.5</td>
<td>153</td>
<td>153</td>
<td>153</td>
</tr>
<tr>
<td>153</td>
<td>153.5</td>
<td>154</td>
<td>154</td>
<td>154</td>
<td>154.5</td>
<td>154.5</td>
<td>155</td>
<td>156</td>
</tr>
</tbody>
</table>

Having organized the data as shown above, it is now easy to get the information sought. Answering the questions and inferring other results from the organized data comes under the domain of interpretation of data.

Thus, statistics is a branch of mathematics which deals with collection, organization, and interpretation of data. The teacher will do well to tell the students that interpretation of data involves much more than answering simple questions such as those related above. It includes prediction, testing of assumptions made etc.

Methodology Used: Mainly the discussion method is used to discuss the nature of statistics.

14.3.2 Why do We Use Statistics?

Main Teaching Point: Understanding the need to use statistics in various real life situations.

Teaching-Learning Process: Ask the students to visualize the situations in which some data are needed to answer the questions posed. Let them start exploring their immediate environment: the school, the home, the community, the market place, busy thoroughfares, industry, etc. Here are some situations:

1) Is the class doing better in Mathematics than in English?
2) If the school is co-educational, are the girls doing better in Mathematics than the boys in the class?
3) Is one section doing better than the other in some particular subject?
4) What is the average height of students in the class?
5) What is the average age of students in the class?

6) Has the school been improving its performance in the Board’s examinations in the course of past ten years? (This will require a determination of indicators of performance and weights to be attached to these indicators).

7) Which size of shoes are sold the most (data may have to be collected for at least 10 to 15 days from a shop).

8) The number of vehicles per minute passing at a busy intersection of roads during different parts of the day (the survey may be extended to include different types of vehicles such as passenger cars, auto-rickshaws, tempos, two-wheelers, trucks, buses).

9) The average salary of workers in a factory.

10) The expenditure on advertisement and the corresponding sale over a period of time (this leads to the study of regression that the business houses use to plan advertisement expenditure as a measure of sales promotion).

Listing various situations which require the use of Statistics (collecting, presenting and interpreting data) could be a useful group activity where a group proposes a number of situations, they discuss and modify.

**Methodology Used:** The discussion method is most suitable.

### 14.4 COLLECTING DATA

#### 14.4.1 Data and its Sources

**Main Teaching Point:** Meaning of the word data, understanding about collection of data and primary and secondary data

**Teaching-Learning Process:** By now students would have realized that the situations listed above or those proposed by them require collection of some information to be in order to answer the questions raised. Generally, the information collected would be quantitative in nature but sometimes it might be qualitative also such as the degree of darkness of films put on glass panes, shades of colour of different objects from violet to red.

The information collected in each case constitutes the “data” for that particular study.

Now explain to the students that the set of objects or persons from whom data are collected from the population for that study. When data are collected directly from the elements of the population, it is called ‘primary data’. However, many a times, the data already available as part of earlier data collection for some bigger project or as a matter of routine exercise, these data can be utilized and it is referred as ‘secondary source’, and not collected directly from the population.

If we are assessing the relative performance of the students of a class in English and Mathematics, then we may not go to the students to collect their marks because they would be available in the school records i.e. secondary source.

If we are collecting information about the sale of shoes of different sizes at a particular shoe store for a period of two weeks, we need not to sit for two weeks at the store and record each sale. Instead, we can get the information from the stock register maintained at the store.
If we are making a study of the rural-urban population shift in the country over a period of fifty years, it would be impossible to collect the data from the primary source, but the information may be collected from the records maintained with the Census Commissioner of India, i.e. from a secondary source.

These examples illustrate the importance of secondary data. In fact, in modern days, every government lays emphasis on data collection. The population census helps the government to observe the rural-urban shift, the birth rate and the death rate, the growth rate of population, infant mortality, life expectancy etc.

Data on tax collection, exports and imports, production, GDP (Gross Domestic Product), etc. help the government to plan strategies, the tax structure, incentives and taxation policies.

Divide the students in group of four or five each. Let each group undertake a project on data collection. You should help each group refine its project and determine the line of action to be followed. A number of projects arise out of questions posed under sub-section 14.3.2. ‘Why do we study Statistics’? Here are a few more possible projects.

1) The most common letter in five pages of your English textbook.
2) The most common length of words in seven pages of a book.
3) Rolling a die 100 times and noting the number of times each score is obtained.

**Methodology Used**: The discussion method is used.

### 14.5 ORGANISING DATA

#### 14.5.1 Pictorial Presentation

**Main Teaching Point**: Includes a) Bar Chart b) Pie Chart c) Pictogram

**Teaching-Learning Process**: Suppose a group of students undertake a project to find out how the students of their class travel to school. Their findings are:

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using the school bus</td>
<td>20</td>
</tr>
<tr>
<td>Using public transport</td>
<td>10</td>
</tr>
<tr>
<td>Cycling to their school</td>
<td>4</td>
</tr>
<tr>
<td>Taken by their parents in a car</td>
<td>2</td>
</tr>
<tr>
<td>Taken by their parents on a two wheeler</td>
<td>7</td>
</tr>
<tr>
<td>Walking to the school</td>
<td>2</td>
</tr>
<tr>
<td><strong>Total students</strong></td>
<td><strong>45</strong></td>
</tr>
</tbody>
</table>

Let’s see how best these data can be depicted pictorially.

**Bar Chart**: One way is to use a bar chart. As the name suggests, the bar chart consists of bars of equal thickness (why?), with the length/height being proportional to the quantity the bars represent.

Here is the bar chart depicting the above information.
Note that:

i) The bar-chart has a heading.

ii) Sub-divisions are made on the vertical axis, using a suitable scale (suitable means that the scale is so chosen that the space on the graph paper is optimally used by the given data).

iii) Bars are of equal thickness and do not touch each other (see next example).

iv) Each bar is assigned a heading to denote what it represents.

v) The height of each bar represents the number it is supposed to represent.

Bars could be drawn horizontally also. Then the numbering would be on the horizontal axis.

Sometimes, the numerical scale may not begin from zero. This is particularly so if the change in the values of the variable is not large yet we want to depict the data vividly. Consider the day to day price variation for 10 gm. of standard gold over a period of one week in a certain city.
Statistics and Probability

Date | Price (per 10 gm) (in Rs.)
--- | ---
11.1.15 | 26700
12.1.15 | 26900
13.1.15 | 27050
14.1.15 | 26700
15.1.15 | 26500
16.1.15 | 26600
17.1.15 | 26600

Ask the students to visualize what practical problem will arise if we were to start the scale from 0 instead of 26,000.

Sometimes, we need to make a comparative chart about two elements, such as exports/imports of a certain commodity over a period of time. Then, two bars may be drawn side by side for each unit of time.

The accompany diagram represents inflation rates in the consumer price index and the wholesale price index for a number of countries for June 2014 over June 2013.

<table>
<thead>
<tr>
<th>City</th>
<th>Consumer Price Index</th>
<th>Wholesale Price Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td>U.K.</td>
<td>2.6</td>
<td>2</td>
</tr>
<tr>
<td>JAPAN</td>
<td>0.8</td>
<td>-1.9</td>
</tr>
<tr>
<td>GERMANY</td>
<td>2.9</td>
<td>1.8</td>
</tr>
<tr>
<td>FRANCE</td>
<td>1.8</td>
<td>-1.5</td>
</tr>
<tr>
<td>INDIA</td>
<td>10.5</td>
<td>10.2</td>
</tr>
</tbody>
</table>

**Inflation Rates**

**Figures in Percentage for June 2014 over June 2013**
Wholesale prices are stable in the US, and are falling in Japan and France. In India, however, they have risen by 10.2 percent when computed on an annual basis. Sure, the weak rupee is still protecting exports, but the exchange rate is also under strain at the moment.

An accompanying ability to draw bar charts is the ability to read and interpret a given bar chart. Present a few bar charts in the class and ask the students to describe what the bar charts show and what information can be derived from the same.

**Pie Chart:** Explain to the students that another mode of representing data is through the use of a pie chart (also called circle graph). The entire circle is taken to represent one whole and all the constituents of the entire data are represented proportionally in the pie chart.

Ask the students to suggest a method to determine the proportion.

If an answer is not forthcoming straightaway, suggest that the angle around the centre of a circle is 360°. If we divide a circle into four equal parts by drawing two mutually perpendicular diameters, we get four quadrants. Each quadrant is 1/4 of the circle and the angle made by the two arms of a quadrant is also 1/4 of the entire angle i.e., it is 90°. Similarly, a semi-circle divides the circle into two equal parts and the angle of 360° is also divided into two halves. Encourage the students to use their intuition to devise a method to make proportional parts of a circle.

The following table depicts the ‘mode of transport used by students to reach the school’ problem.

<table>
<thead>
<tr>
<th>Mode of Transport</th>
<th>Number</th>
<th>Angle in the Sector</th>
<th>Approximate % of the whole</th>
</tr>
</thead>
<tbody>
<tr>
<td>School bus</td>
<td>20</td>
<td>160°</td>
<td>44.5%</td>
</tr>
<tr>
<td>Public transport</td>
<td>10</td>
<td>80°</td>
<td>22.2%</td>
</tr>
<tr>
<td>Cycle transport</td>
<td>4</td>
<td>32°</td>
<td>8.9%</td>
</tr>
<tr>
<td>Parent’s car</td>
<td>2</td>
<td>16°</td>
<td>4.4%</td>
</tr>
<tr>
<td>Parent’s two-wheeler</td>
<td>7</td>
<td>56°</td>
<td>15.6%</td>
</tr>
<tr>
<td>Walking</td>
<td>2</td>
<td>16°</td>
<td>4.4%</td>
</tr>
<tr>
<td>Total</td>
<td>45</td>
<td>360°</td>
<td>100%</td>
</tr>
</tbody>
</table>

A pie chart depicting the above information will look like the following figure.
Students using different modes of transport

Note that while we construct a pie-chart using the idea of angles (since protractor helps us in constructing it), the magnitude of each item is described in terms of percent as shown.

Constructing a pie chart involves the following steps:

1) Work out the angle of the sector and approximate per cent of each item in the data.
2) Draw a circle and construct sectors using the sectoral angle.
3) Label each sector and write the per cent of the whole.

Ask the students to practice drawing pie charts for given data.

Pictogram: Sometimes the picture of an object is used to display information. For example, if we want to depict pictorially the production of cycles in the country over a number of years, we choose a suitable scale such as one cycle representing 1 lakh cycles and draw the required number of cycles. The last cycle representing some fraction of a lakh will be drawn only partly in the same proportion. Sometimes, a dotted outline may be given to complete the picture. A pictorial representation of this type is called a pictogram.

Example: The production of motor cars in a certain country over a period of five years is shown by the following table:

<table>
<thead>
<tr>
<th>Year</th>
<th>No. of Cars (in ten thousand)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>3</td>
</tr>
<tr>
<td>2013</td>
<td>5.5</td>
</tr>
<tr>
<td>2014</td>
<td>6</td>
</tr>
<tr>
<td>2015</td>
<td>7.5</td>
</tr>
<tr>
<td>2016</td>
<td>8</td>
</tr>
</tbody>
</table>

Taking one car to represent ten thousands, we can depict the above information in a pictogram as shown:
Content Based Methodology-II

Production of Cars

= ten thousand

Methodology used: To draw figures is a skill which is developed by practice. A sufficient number of exercises should be provided to the students.

Check Your Progress

Notes: a) Write your answers in the space given below.
    b) Compare your answers with those given at the end of the Unit.

1) The expenditure on health care by the Government of India during the first six five year plans is shown below:

<table>
<thead>
<tr>
<th>Plan</th>
<th>Expenditure (in Crores Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>90</td>
</tr>
<tr>
<td>Second</td>
<td>160</td>
</tr>
<tr>
<td>Third</td>
<td>250</td>
</tr>
<tr>
<td>Fourth</td>
<td>400</td>
</tr>
<tr>
<td>Fifth</td>
<td>800</td>
</tr>
<tr>
<td>Sixth</td>
<td>1900</td>
</tr>
<tr>
<td>Total</td>
<td>3600</td>
</tr>
</tbody>
</table>

Construct a pie chart to depict the above information.
2) The number of schools of different types in Delhi in the year 2016 were as follows:

<table>
<thead>
<tr>
<th>Type</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>2500</td>
</tr>
<tr>
<td>Upper Primary</td>
<td>500</td>
</tr>
<tr>
<td>Secondary</td>
<td>1000</td>
</tr>
<tr>
<td>Higher Secondary</td>
<td>2000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>6000</strong></td>
</tr>
</tbody>
</table>

Illustrate by representing this information on a bar chart.

14.5.2 Handling Large Data

Main Teaching Points: Class intervals and Constructing grouped frequency distribution and their graphical representation.

Teaching-Learning Process: So far we have restricted ourselves to representation of data which are not very large. You may now confront students with large data and ask them to represent them by a bar chart or in some other manner.

Ask: Suppose that a factory has 200 workers. You are provided with data about their monthly pay packet. You are now asked to represent it by a bar chart. How will you do it?

The students would obviously realize that it is futile to think of constructing 200 bars to represent pay packets of 200 persons. Therefore, the next best option is to group together those persons whose salaries are close to some convenient figure, deviating from it by small amounts only. In other words, we choose a class of persons whose salaries lie in a small interval around that convenient figure. We make a number of such intervals. Now a host of questions arise. How do we go about constructing such class intervals? How many class intervals should we have? Once a person is included in a class intervals, can we still get the information about his individual packet? The teacher will need to discuss these points one by one with the students.

Suppose the highest salary that a worker of the factory gets is Rs.11,500 per month and the lowest salary a worker gets is Rs.8,000. We need to divide this salary range into a suitable number of class intervals. In this case, we have the range from Rs.8,000 to Rs.11,500. The number of intervals should not be too small or too large. Generally, this number should lie between 5 and 15.
The intervals should be equal in size. From Rs.8,000 to Rs.11,500, as a range of Rs.3,500, we may make seven intervals of class size of 500 each. The number of workers who fall in one particular class interval is called the frequency of that class interval. To determine the frequency of each class interval, we mark a tally for each person in that class, till all the data are exhausted. Explain to the students the mode of putting the tallies in groups of five tallies to facilitate counting.

**Class mark, Frequency, Cumulative frequency**

Suppose that the data about 200 workers of the factory is classified as below:

<table>
<thead>
<tr>
<th>Class Intervals</th>
<th>Tallies</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>8,000 – 8,500</td>
<td>33</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>8,500 – 9,000</td>
<td>30</td>
<td>63</td>
<td></td>
</tr>
<tr>
<td>9,000 – 9,500</td>
<td>32</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>9,500 – 10,000</td>
<td>28</td>
<td>123</td>
<td></td>
</tr>
<tr>
<td>10,000 – 10,500</td>
<td>25</td>
<td>148</td>
<td></td>
</tr>
<tr>
<td>10,500 – 11,000</td>
<td>27</td>
<td>175</td>
<td></td>
</tr>
<tr>
<td>11,000 – 11,500</td>
<td>25</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>200</strong></td>
<td></td>
</tr>
</tbody>
</table>

Note that:

i) the upper limit of each class interval appears again as the lower limit of the next interval. Where data are so classified, the overlapping limit is included only as the lower limit of the next interval so that there is no ambiguity.

ii) Cumulative frequency is the progressive total all the frequencies up to that class interval. Interpreted in physical terms, cumulative frequency such as 123 means that there are a total of 123 persons up to that class interval.

**Ease of calculation vs. loss of accuracy**

Having classified all the data as above, an important question arises, from the data can we tell the salary of any particular worker of the factory?

The only thing we can say is that his salary lies in some particular income bracket. If we are required to calculate the entire monthly salary bill of the workers, we cannot go so unless we make an assumption. This important assumption is that all the persons in any particular class interval are supposed to be drawing a salary midway between the upper and lower limits of that class interval. This mid-value is called the “class mark” for that class.

Thus, 33 persons are supposed to be drawing a salary of Rs.(8,000 + 8,500)/2 = Rs.8,250 each. 30 persons are supposed to be drawing a salary of Rs.(8,500 + 9,000)/2 = Rs.8,750 each, and so on.

Ask the students to guess how much inaccuracy this will lead in the total salary. It will be difficult for them to answer it. The teacher should then point out that 33 persons in the first class interval will include persons some of those salaries would be below the class mark and some whose salaries are above.
The negative and positive deviations may balance each other to a good extent and only a small inaccuracy may enter in the total bill. The inaccuracy may be negligible compared to the total bill and the advantage of each calculation may far outweigh the little error.

**Histogram, Frequency Polygon, Ogive**

There are alternative modes of representation of data other than the bar chart, the pie chart and the pictogram. The histogram is closest to a bar graph. If the bars in a graph are drawn touching each other, then we get a histogram. Highlight the major differences between a bar chart and a histogram. They are:

i) The histogram is drawn when a frequency distribution is given.

ii) In a bar graph the lengths of the bars are proportional to the frequency, whereas in a histogram the area of the rectangle is proportional to the frequency. Clarify this using an example with unequal class intervals.

In a mid-points of the upper side of the bars are joined by straight lines and the lies reach up to the base on either side, then we get a polygon. Each vertex of this polygon has coordinates equal to the mid-value of any interval and the corresponding frequency. In the adjoining diagram, B has coordinates (8250, 33).

**Histogram and Frequency Polygon**

![Histogram and Frequency Polygon](image)

This represents the statement that the number of persons drawing a salary of Rs.8250 per month (as per our assumption) is 33. The polygon ABCDEFGHI is called frequency polygon for this data (distribution).

**Note that:**

i) B has been joined to A and H to I in the polygon. To locate A and I, we take the mid-points of the preceding and succeeding class intervals, i.e. of 7500-8000 and 11500-12000. Since the frequencies in these two intervals are zero, A and I lie on x-axis.

ii) If the points from A to I are joined by a free hand curve, the curve is called the frequency curve.
If both the histogram and the frequency polygon are to be drawn, then it is advisable first to draw the histogram and then join the mid-points of the tops of the rectangles of the histogram to get the frequency polygon as shown above. However, if only the frequency polygon is to be drawn, then first represent the class marks along the x-axis and frequencies along the y-axis and then plot the corresponding points and finally join them.

**Ogive:** An ogive is a graph of cumulative frequency distribution. It is also called frequency distribution curve.

To draw a cumulative frequency distribution curve, we plot the points with the upper limits of the classes on the x-axis and the corresponding cumulative frequencies on the y-axis.

Ask the students to explain why we take the upper limits and not the class-mark (mid point of the class interval). Also ask them to explain why ogive is a rising curve.

The reason for taking the upper limit of each class is the fact that persons getting salaries within any class interval are actually spread over the whole class and we take off persons who are getting salaries upto the upper limit (cumulative frequency taken).

Ogive is a rising curve because the cumulative frequency goes on increasing with each class.

(*Note: In the foregoing diagram, the ogive looks like a straight line since the frequencies in each class are very close to each other. But this may not always be the case*).

Ask the students to practice drawing the Ogive in all problems where they were asked to draw a histogram. They should practice both with or without the histogram.

**Methodology Used:** The discussion method is used with numerous illustrations.
Check Your Progress

Notes: a) Write your answer in the space given below.
   b) Compare your answer with those given at the end of the Unit.

3) Draw histogram and an ogive for the following data. Explain the steps involved in plotting histogram and ogive.

<table>
<thead>
<tr>
<th>Marks</th>
<th>No. of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-100</td>
<td>10</td>
</tr>
<tr>
<td>100-200</td>
<td>18</td>
</tr>
<tr>
<td>200-300</td>
<td>30</td>
</tr>
<tr>
<td>300-400</td>
<td>35</td>
</tr>
<tr>
<td>400-500</td>
<td>32</td>
</tr>
<tr>
<td>500-600</td>
<td>20</td>
</tr>
<tr>
<td>600-700</td>
<td>15</td>
</tr>
<tr>
<td>Total</td>
<td>160</td>
</tr>
</tbody>
</table>

14.6 INTERPRETING DATA

14.6.1 Measures of Central Tendency

Main Teaching Points: a) Mean, b) Mode, and c) Median

Teaching-Learning Process: In the preceding sections 14.4 and 14.5, we discussed how we collect and display information. Let us now examine some of the ways of selecting a typical value to represent such information.

We go back to the first example in which we collected the heights of the 45 students of a class. Ask the students how they would choose one of the heights to represent all others.

There could be a number of suggestions such as:

i) Choose the height that most students have.

ii) Let all the students stand in increasing order and choose the middle most students, i.e. the 23rd from either side. This suggestion is equivalent to arranging the numbers representing the heights in increasing order and then choosing the middle-most value.
iii) Add all the heights and divide the sum by the number of students, i.e., 45.

There might be a suggestion to choose the greatest height or the smallest height but hopefully it will not find favour with most students. If we look at the organized data, we notice that:

i) The height of the most students is 148 cm.

ii) The middle most height is 149 cm.

iii) The sum of heights = 6709.5 and average height is 6709.5/45 = 149.1.

The teacher should explain that the value of the variable which occurs most number of times is called the ‘mode’ of the data or commonly occurring value in a data is called mode. The middle-most number after rearranging in increasing or decreasing order is called the ‘median’ and the sum divided by the number of observations is called the ‘mean or arithmetic mean’. Or we can say

Mean= Sum of observations / Number of observations

Let us consider an example to understand mean, median and mode:

Example1: The numbers of children in 15 families of a locality are as follows: 2,3,2,4,2,2,2,2,0,2,1,3,3,2,0.

Calculate mean, median and mode of above data.

\[
\text{Mean} = \frac{\text{Sum of observations}}{\text{Number of observations}} = \frac{30}{15} = 2
\]

On arranging above data in ascending order, we get

0,0,1,2,2,2,2,2,2,2,3,3,3,4

Here total observation is 15 and 2 lies in middle of observed data. So ‘2’ is the median value.

From the above data we can observe that 9 families have ‘2’ children. So ‘2’ is the mode value.

Now pose the following situations:

Situation (a): If there were an even number of observations such as 40, how would you find the middle-most value?

Explain: In case of even number of observations, we take the average of two middle-most values.

Let us take an example

Example 2: To find median of 3,5,2,6,9,7.

On arranging this data in ascending order, we get

2,3,3,5,7,9
Now we have even number of observations in the above data. So we take two middle values to find median.

Here two middle values are 3 and 5. Now there mean value is \((3+5)/2 = 8/2 = 4\)

So 4 is the median of the above collected data.

**Situation (b):** If there were more than one number occurring an equal number of times, which one would you call mode?

**Explain:** In case of most common values, we call it a biomodal distribution (bi mean two).

**Situation(c):** If the data was grouped in class intervals, how would you calculate the median of the data?

**Explain:** Let us see how to obtain the median in this situation. Consider a grouped frequency distribution of marks obtained, out of 50, by 50 students, in a certain examination, as follows.

<table>
<thead>
<tr>
<th>Marks in Hindi</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 10</td>
<td>8</td>
</tr>
<tr>
<td>11 – 20</td>
<td>14</td>
</tr>
<tr>
<td>21 – 30</td>
<td>12</td>
</tr>
<tr>
<td>31 – 40</td>
<td>9</td>
</tr>
<tr>
<td>41 – 50</td>
<td>7</td>
</tr>
</tbody>
</table>

**Step 1:** Construct the cumulative frequency distribution.

**Step 2:** Decide the median class interval.

The median class interval is the corresponding class where the median \((n/2)\) value falls.

**Step 3:** Find the median by using the following formula:

\[
\text{Median} = L_m + \frac{h \left( \frac{n}{2} - F \right)}{f_m}
\]

\(n\) = the total frequency  
\(F\) = the cumulative frequency before median class interval  
\(h\) = the class width  
\(f_m\) = the frequency of the median class interval  
\(L_m\) = the lower boundary of the median class interval

<table>
<thead>
<tr>
<th>Marks in Hindi</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 10</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>11 – 20</td>
<td>14</td>
<td>22</td>
</tr>
<tr>
<td>21 – 30</td>
<td>12</td>
<td>34</td>
</tr>
<tr>
<td>31 – 40</td>
<td>9</td>
<td>43</td>
</tr>
<tr>
<td>41 – 50</td>
<td>7</td>
<td>50</td>
</tr>
</tbody>
</table>
n/2=50/2=25 so the median class is 21–30.

\[ F = 22, \ h = 10, \ f_m = 12 \text{ and } L_m = 20.5 \]

\[
\text{Median} = L_m + \frac{h \left( \frac{n}{2} - F \right)}{f_m}
\]

\[
= 20.5 + \frac{10(25 - 22)}{12}
\]

\[
= 20.5 + 2.5
\]

\[
= 23
\]

**Situation (d):** If the data was grouped in class intervals, how would you calculate the mean of the data?

**Explain:** If \(x_1, x_2, \ldots, x_n\) are observations with respective frequencies \(f_1, f_2, \ldots, f_n\), then this means observation \(x_1\) occurs \(f_1\) times, \(x_2\) occurs \(f_2\) times, and so on. Now, the sum of the values of all the observations = \(f_1x_1 + f_2x_1 + \ldots + f_nx_n\), and the number of observations = \(f_1 + f_2 + \ldots + f_n\). So mean (which is denoted by \(\bar{x}\)) is given by \(\bar{x} = \frac{\sum x_if_i}{\sum f_i}\)

Let us see an example.

<table>
<thead>
<tr>
<th>Marks in Hindi (Class Interval)</th>
<th>Frequency (f_i)</th>
<th>Class mark (x_i)</th>
<th>f_ix_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 10</td>
<td>8</td>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>11 – 20</td>
<td>14</td>
<td>15</td>
<td>210</td>
</tr>
<tr>
<td>21 – 30</td>
<td>12</td>
<td>25</td>
<td>300</td>
</tr>
<tr>
<td>31 – 40</td>
<td>9</td>
<td>35</td>
<td>315</td>
</tr>
<tr>
<td>41 – 50</td>
<td>7</td>
<td>45</td>
<td>315</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>50</strong></td>
<td></td>
<td><strong>1180</strong></td>
</tr>
</tbody>
</table>

\[
\bar{x} = \frac{\sum x_if_i}{\sum f_i}
\]

\[
= \frac{1180}{50}
\]

\[= 23.6\]

Assign a few problems to the class to find mean, mode and median.

Explain the role of the assumed mean to facilitate calculation and explain

\[ X = a + \frac{h \sum x_if_i}{\sum f_i} \]

where \(a = \text{assumed mean}, \ h = \text{size of class interval and} \)

\[ u_i = \frac{x_i - a}{h} \]

\(x_if_i\) as general notations for different values of \(x\) and \(f\) should be explained when they are first introduced.
Ask the students about how to choose the assumed mean. Let them realize the importance of choosing a value somewhere mid-way in the grouped distribution data so that some $u$ are negative and others are positive giving $\sum u_i f_i$ equal to a very small number.

Let the students discuss the following problems:

1) If each observation of some data is increased (or decreased) by 5, what happens to the mean and median?

2) If each observation of some data is multiplied (or divided) by 2, what happens to the mean?

3) If one of the observations of a data consisting of 10 values is wrongly copied as 65 in place of 25, will the true mean increase or decrease and by how much?

4) Show that:
   
   Mean, mode and median are called ‘measures of the central tendency’ of a distribution.

**Methodology Used:** Mostly discussion method is used to transact the meaning and calculation of Central Tendencies. However, many examples should be given to illustrate the points.

### Check Your Progress

**Notes:**

a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

4) Find the mean of the first ten odd numbers.

5) Find the mode of the following data:

   6, 3, 5, 6, 4, 7, 4, 6, 8, 9, 6, 3
6) a) Find the median of the following data:
   36, 37, 41, 18, 17, 12, 21, 24, 33, 32, 28

b) Find the median of the following data:
   85, 68, 63, 70, 82, 78, 64, 60

7) Find the mean of the following data:

<table>
<thead>
<tr>
<th>Marks</th>
<th>No. of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>7</td>
</tr>
<tr>
<td>10 - 20</td>
<td>10</td>
</tr>
<tr>
<td>20 - 30</td>
<td>15</td>
</tr>
<tr>
<td>30 - 40</td>
<td>8</td>
</tr>
<tr>
<td>40 - 50</td>
<td>10</td>
</tr>
</tbody>
</table>

14.6.2 Measures of Dispersion – Range, Mean Deviation, Standard Deviation

Main Teaching Points: a) Range, b) Deviation Mean, and c) Standard deviation

Teaching-Learning Process: Another important characteristics of new distribution is the spread or range of data.
Let the students consider the performance of two students, A and B, in their monthly tests in Mathematics.

<table>
<thead>
<tr>
<th>Student/Test</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15</td>
<td>24</td>
<td>40</td>
<td>9</td>
<td>32</td>
<td>120</td>
</tr>
<tr>
<td>B</td>
<td>28</td>
<td>30</td>
<td>24</td>
<td>18</td>
<td>20</td>
<td>120</td>
</tr>
</tbody>
</table>

The mean score of each of them is 24.

The highest score of A is 40 and lowest 9.

The highest score of B is 30 and lowest 18.

Thus, the students are likely to observe that although A might be sharper, yet he is not consistent. B may not be as sharp as A but he is consistent.

Similarly, if two classes have the same mean score but the range (or spread) of the marks of one is largest compared to that of the other, than the latter may be treated as a better class.

If the per capita income of two countries A and B is the same but there is a wide variation in the highest and lowest income group of country A than in country B, then A has more poor people and the disparities in income in A are higher than in B.

Range or spread is an important indicator of any distribution. It is called a “measure of dispersion” (i.e. how much the data is dispersed). We shall consider two more measures of dispersion.

**Mean Deviation:** We have seen that the mean is an important representative of any given distribution. Some observations will be less than the mean and some will be more than if such that \( \sum (x_i - \bar{x}) = 0 \). To find out the average deviation from the mean we take the absolute deviation and find out its average. In case of ungrouped data.

\[
\text{Mean deviation} = \frac{\sum (x_i - \bar{x})^2}{n}
\]

**Standard Deviation:** Standard deviation is the most important and most frequently used measure of dispersion. It is denoted by \( \sigma \) (sigma)

\[
\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}
\]

However, the working formula is

\[
\sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}
\]

It can be easily shown that the first formula simplifies into the second.

To find \( \sigma \) we arrange the data in a tabular form as below.

**Example:** Find the standard deviation of the following distribution 6, 8, 9, 10, 12, 11, 7, 8, 10, 9
The teacher should discuss with the students the method of finding standard deviation for grouped data and the short cut method to minimize lengthy calculations.

**Methodology Used:** Again, we mostly depend on discussion with the students to teach the concept of dispersion and use the drill method to provide sufficient practice in the use of the formulae.

### Check Your Progress

**Notes:**

a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the unit.

8. List the steps involved in finding the spread and standard deviation for the following data:

5, 10, 12, 7, 8, 6, 15, 17, 1, 19


<table>
<thead>
<tr>
<th>$x_i$</th>
<th>$x_i^2$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>$\sqrt{\frac{840 - (90)^2}{10}}$</td>
</tr>
<tr>
<td>12</td>
<td>144</td>
<td>$\sqrt{84 - 9^2}$</td>
</tr>
<tr>
<td>11</td>
<td>121</td>
<td>$\sqrt{84 - 81}$</td>
</tr>
<tr>
<td>7</td>
<td>49</td>
<td>$\sqrt{3}$</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>81</td>
<td></td>
</tr>
</tbody>
</table>

| 90 | 840 |

$$\sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$
14.7 PROBABILITY AND ITS APPLICATIONS

Main Teaching Point: a) Concept of Probability, b) Formula for measures of Probability, c) Types of Events and Probability, and d) Applications of Probability in daily life

Teaching-Learning Process: You can start this topic through a discussion or by organizing an activity.

Ask to students that if a spinner has 4 equal sectors colored yellow, blue, green and red, what are the chances of landing on yellow after spinning the spinner?

The answer you may get that the chances of landing on yellow are 1 in 4, or one fourth. Then you can ask what are the chances of landing on red. Then students may tell that the chances of landing on red are also 1 in 4, or one fourth.

Now you can introduce them the concept of probability. Explain them whenever we’re unsure about the outcome of an event, we can talk about the probabilities of certain outcomes—how likely they are. Probability is simply how likely something is to happen. It is a branch of Mathematics that is capable of calculating the chance or likelihood of an event will occur. Probability has a scientific basis and if you have 10 likelihoods and you want to calculate the probability of 1 event taking place, it is said that its probability is 1/10 or the event has a 10% probability of taking place.

Probability of an event happening = \( \frac{Number\ of\ ways\ it\ can\ happen}{Total\ number\ of\ outcomes} \)

The probability of an event is shown using ‘P’ and P(A) means ‘Probability of Event A’.

Some words have special meaning in Probability.

<table>
<thead>
<tr>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>An experiment is a situation involving chance or probability that leads to results called outcomes.</td>
<td>In the activity 1, the experiment is spinning the spinner.</td>
</tr>
<tr>
<td>An outcome is the result of a single trial of an experiment.</td>
<td>The possible outcomes are landing on yellow, blue, green or red.</td>
</tr>
<tr>
<td>An event is one or more outcomes of an experiment.</td>
<td>One event of this experiment is landing on blue.</td>
</tr>
<tr>
<td>Probability is the measure of how likely an event is.</td>
<td>The probability of landing on blue is one fourth.</td>
</tr>
</tbody>
</table>
Let us take a few examples to understand concept of probability.

**Example:** What is the chances of rolling a "4" with a die?

Students may come like that

Number of ways it can happen: 1 (there is only 1 face with a ‘4’ on it)

Total number of outcomes: 6 (there are 6 faces altogether)

So the probability = \( \frac{1}{6} \)

**Example:** Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?

Students may come like that

Number of ways it can happen: 9 (event of getting a multiple of 3 or 5 = 3, 6, 9, 12, 15, 18, 5, 10, 20)

Total number of outcomes: 20

So the probability=\( \frac{9}{20} \)

Further you can explain that probability is quantified as a number between 0 and 1 where 0 indicates impossibility and 1 indicates certainty. The higher the probability of an event, the more certain that the event will occur. A simple example is the tossing of a fair (unbiased) coin. Since the coin is unbiased, the two outcomes (‘head’ and ‘tail’) are both equally probable; the probability of ‘head’ equals the probability of ‘tail’. Since no other outcomes are possible, the probability is 1/2 (or 50%), of either ‘head’ or ‘tail’. In other words, the probability of ‘head’ is 1 out of 2 outcomes and the probability of ‘tail’ is also 1 out of 2 outcomes.

You can assign individual project to students. Tell them throw a die 100 times, record the scores in a tally table. They can record the results in this table using tally marks and draw a bar graph and illustrate their results.

**Types of Events and Probability**

Now it is expected that students know that an event can include several outcomes and getting a Tail when tossing a coin is an event; rolling a ‘5’ is an event.

So you can further explain that an event whose chances of happening is 100 % is called a **sure event**. The probability of such event is 1. In sure event, one is likely to get the desired output in whole sample experiment. On the other hand, when there are no chances of happening an event, the probability of such event is likely to be zero. This is said to be an **impossible event**. On the basis of quality events, these are classified into three types which are as follows:

- Independent (each event is not affected by other events),
- Dependent (also called "Conditional", where an event is affected by other events)
- Mutually Exclusive (events can't happen at the same time)

Let's discuss at each of those types.

**Independent Events:** Two or more events are independent if the occurrence of one event does not affect the occurrence of any of the others. Example: You toss a coin three times and it comes up "Heads" each time ... what is the chance
that the next toss will also be a ‘Head’? The chance is simply 1/2, or 50%, just like any other toss of the coin.

**Dependent Events**: Two events are dependent if the occurrence of one of the events affects the probability of the occurrence of the other. Example: Drawing 2 king cards from a Deck. For the 1st card the chance of drawing a King is 4 out of 52. But for the 2nd card:
- If the 1st card was a King, then the 2nd card is **less** likely to be a King, as only 3 of the 51 cards left are Kings.
- If the 1st card was **not** a King, then the 2nd card is slightly **more** likely to be a King, as 4 of the 51 cards left are Kings.

**Note**:
- Replacement: When we put each card back after drawing it the chances don't change, as the events are **independent**.
- Without Replacement: The chances will change, and the events are **dependent**.

Now, you can ask them:

If 2 blue and 3 red marbles are in a bag. What are the chances of getting a blue marble?

Student: The chance is 2 in 5.

Teacher: But after taking one out the chances change!

Student: In the next time:
- if we got a red marble before, then the chance of a blue marble next is **2 in 4**
- if we got a blue marble before, then the chance of a blue marble next is **1 in 4**

**Mutually Exclusive Events**: Mutually exclusive events are two or more events that can not occur simultaneously. If one dice is thrown and comes up three, it cannot come up six or any other number at the same time. If a coin is tossed and comes up tails, it cannot come up heads on the same toss. The probability of one or the other of two mutually exclusive events happening is the sum of the separate probabilities of these events.

**Note**: What isn't Mutually Exclusive?
- Kings and Hearts are **not** Mutually Exclusive, because we can have a King of Hearts!

The following example will make the idea clear.

**Example: A Deck of Cards**

In a Deck of 52 Cards:
- the probability of a King is 1/13, so \( P(\text{King}) = \frac{1}{13} \)
- the probability of an Ace is also 1/13, so \( P(\text{Ace}) = \frac{1}{13} \)

When we combine those two Events:
- The probability of a card being a King **and** an Ace is 0 (Impossible)
• The probability of a card being a King or an Ace is \((1/13) + (1/13) = 2/13\)

Which is written like this:

\[
P(\text{King and Ace}) = 0
\]

\[
P(\text{King or Ace}) = (1/13) + (1/13) = 2/13
\]

The teacher should discuss with the students the method of finding probability for independent, dependent and mutually exclusive events.

**Methodology Used:** Again, we mostly depend on discussion with the students to teach the concept of probability and use the drill method to provide sufficient practice in the use of the formulae.

---

**Check Your Progress**

**Notes:**

a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the unit.

10. A die is thrown once. What is the probability that the score is a factor of 6?

11. From a well shuffled deck of 52 cards, one card is drawn at random. What is the probability that the draw card is queen?

12. What are various types of events?
This unit provides the teacher an opportunity to introduce the topic through everyday real life situations helping the students to develop a positive attitude towards the study of Mathematics. The student sees the importance of collecting and organizing data to be able to answer questions about the population from whom the data are collected. The students learn how to present data pictorially and to interpret such data appearing in books, newspapers, journals, etc.

The measure of the ‘Central Tendency’ and of ‘Dispersion’ is introduced to the students as tools to make an objective assessment of the characteristics of any data. They learn how to determine these characteristics.

You would have realized that mode is the most common observation, median is a point below which lie exactly fifth percent of cases and mean is a value which represents average of all observations. In the course of handling large data, the students learn that in order to make calculations less cumbersome, they have to make a sacrifice elsewhere (in accuracy). This is an attitude which often comes into play in real life situations also. They are confronted with decision-making situations such as in deciding the number of class intervals or in making a choice of assumed mean.

The unit also provides an opportunity to teachers to help students understand the concept of probability and its application in day-to-day life.

At the end of the unit let your students do the following:

1) Find out the age to the nearest month of each student in the class. Prepare a list in order of age and find the total. Take your own age as a reference point or zero. Write down the ages of everybody else as so many months older than or younger than yourself. Add these positive and negative values. Set up a formula to arrive at the previous total from this total.

2) Obtain the number of children in the families to which the students of your class belong. Make a bar chart with the number of children in the family along the x-axis and the number of families on the y-axis.

3) A family spends the following amounts monthly on the items listed below:

<table>
<thead>
<tr>
<th>Item</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>Rs.1,600/-</td>
</tr>
<tr>
<td>Clothing</td>
<td>Rs.1,000/-</td>
</tr>
<tr>
<td>Housing</td>
<td>Rs.2,400/-</td>
</tr>
<tr>
<td>Transport</td>
<td>Rs.600/-</td>
</tr>
<tr>
<td>Education</td>
<td>Rs.800/-</td>
</tr>
<tr>
<td>Misc. Expenditure</td>
<td>Rs.200/-</td>
</tr>
<tr>
<td>Savings</td>
<td>Rs.600/-</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>Rs.7,200/-</strong></td>
</tr>
</tbody>
</table>

Represent the above information in a pie chart. Express the expenditure as per cent.
4) The heights of 21 girls in centimeters were measured and the heights were:

<table>
<thead>
<tr>
<th>Height (cm)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>157</td>
<td>2</td>
</tr>
<tr>
<td>160</td>
<td>3</td>
</tr>
<tr>
<td>163</td>
<td>1</td>
</tr>
<tr>
<td>157</td>
<td>2</td>
</tr>
<tr>
<td>162</td>
<td>2</td>
</tr>
<tr>
<td>162</td>
<td>2</td>
</tr>
<tr>
<td>164</td>
<td>2</td>
</tr>
<tr>
<td>165</td>
<td>3</td>
</tr>
<tr>
<td>155</td>
<td>1</td>
</tr>
<tr>
<td>156</td>
<td>1</td>
</tr>
<tr>
<td>161</td>
<td>1</td>
</tr>
<tr>
<td>147</td>
<td>1</td>
</tr>
<tr>
<td>162</td>
<td>2</td>
</tr>
<tr>
<td>162</td>
<td>2</td>
</tr>
<tr>
<td>164</td>
<td>2</td>
</tr>
<tr>
<td>161</td>
<td>1</td>
</tr>
<tr>
<td>157</td>
<td>2</td>
</tr>
<tr>
<td>162</td>
<td>2</td>
</tr>
<tr>
<td>162</td>
<td>2</td>
</tr>
<tr>
<td>159</td>
<td>1</td>
</tr>
<tr>
<td>162</td>
<td>2</td>
</tr>
<tr>
<td>164</td>
<td>2</td>
</tr>
<tr>
<td>156</td>
<td>1</td>
</tr>
<tr>
<td>159</td>
<td>1</td>
</tr>
</tbody>
</table>

Answer the following:

i) What is the greatest height?
ii) What is the least height?
iii) What is the median height?
iv) What is the mode?
v) What is the arithmetic mean?

5) The following table gives the distribution of total household expenditure (in rupees) of factory workers in a city:

<table>
<thead>
<tr>
<th>Expenditure</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>600-700</td>
<td>24</td>
</tr>
<tr>
<td>700-800</td>
<td>40</td>
</tr>
<tr>
<td>800-900</td>
<td>33</td>
</tr>
<tr>
<td>900-1000</td>
<td>28</td>
</tr>
<tr>
<td>1000-1100</td>
<td>30</td>
</tr>
<tr>
<td>1100-1200</td>
<td>22</td>
</tr>
<tr>
<td>1200-1300</td>
<td>16</td>
</tr>
<tr>
<td>1300-1400</td>
<td>7</td>
</tr>
<tr>
<td>Total</td>
<td>200</td>
</tr>
</tbody>
</table>

Find the average expenditure per household

6) Calculate deviation from the mean for the data given in Q.5. Calculate the standard deviation also for this data.

7) A school has 4 sections in Class IX having 40, 35, 45, 42 students. The mean marks obtained in the chemistry test by three of the sections are 50, 60; and 55 respectively. If the overall average of marks per student for all the section is 52.3, calculate the mean marks obtained by the fourth section.

8) Select any two concepts from the following:
   a) Mean  b) Mode  c) Median  d) Standard Deviation
   How will you teach these concepts to your students? Illustrate the methodology to explain the context.

9) An urn contains 6 red, 5 blue and 2 green marbles. If 2 marbles are picked at random, what is the probability that both are red?
10) A bag contains 12 white and 18 black balls. Two balls are drawn in succession without replacement. What is the probability that first is white and second is black?

11) A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?

### 14.10 ANSWERS TO CHECK YOUR PROGRESS

<table>
<thead>
<tr>
<th>Plan</th>
<th>Expenditure (in Crores Rs.)</th>
<th>Angle of sector</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>90</td>
<td>9</td>
<td>2.5</td>
</tr>
<tr>
<td>Second</td>
<td>160</td>
<td>16</td>
<td>4.4</td>
</tr>
<tr>
<td>Third</td>
<td>250</td>
<td>25</td>
<td>7.0</td>
</tr>
<tr>
<td>Fourth</td>
<td>400</td>
<td>40</td>
<td>11.0</td>
</tr>
<tr>
<td>Fifth</td>
<td>800</td>
<td>80</td>
<td>22.1</td>
</tr>
<tr>
<td>Sixth</td>
<td>1900</td>
<td>190</td>
<td>53.0</td>
</tr>
<tr>
<td>Total</td>
<td>3600</td>
<td>360</td>
<td>100</td>
</tr>
</tbody>
</table>
3. Marks

<table>
<thead>
<tr>
<th>Marks</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-100</td>
<td>10</td>
</tr>
<tr>
<td>100-200</td>
<td>28</td>
</tr>
<tr>
<td>200-300</td>
<td>58</td>
</tr>
<tr>
<td>300-400</td>
<td>93</td>
</tr>
<tr>
<td>400-500</td>
<td>125</td>
</tr>
<tr>
<td>500-600</td>
<td>145</td>
</tr>
<tr>
<td>600-700</td>
<td>160</td>
</tr>
</tbody>
</table>

4. Mean = 10

5. Mode = 6

6. (a) Median = 28  (b) Median = 69
### Statistics and Probability

<table>
<thead>
<tr>
<th>Marks</th>
<th>x</th>
<th>f</th>
<th>(a = 25)</th>
<th>(h = 10)</th>
<th>(u = x - a/h)</th>
<th>u.f</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>5</td>
<td>7</td>
<td>-20</td>
<td>-2</td>
<td>-14</td>
<td></td>
</tr>
<tr>
<td>10 - 20</td>
<td>10</td>
<td>10</td>
<td>-10</td>
<td>-1</td>
<td>-10</td>
<td></td>
</tr>
<tr>
<td>20 - 30</td>
<td>15</td>
<td>15</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>30 - 40</td>
<td>25</td>
<td>8</td>
<td>10</td>
<td>1</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>40 - 50</td>
<td>35</td>
<td>10</td>
<td>20</td>
<td>2</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>-</td>
<td>50</td>
<td></td>
<td></td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{Mean} = a + \frac{h \sum u f_i}{f_i}
\]

\[
= 25 + \frac{4 \times 10}{50} = 25 + 0.8 = 25.8
\]

8. Range = 19 – 1 = 18

S.D.

\[
X = 10
\]

\[
X : 5, 10, 12, 7, 8, 6, 15, 17, 1, 19
\]

\[
d = X - \bar{X} : -5, 0, 2, -3, -2, -4, 5, 7, -9, 9
\]

\[
\text{S.D.} = \sqrt{\frac{\sum d^2}{N}} = \sqrt{\frac{294}{10}} = 5.42
\]

9. S.D.

\[
= \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}
\]

\[
= \sqrt{\frac{385}{10} - \left(\frac{55}{10}\right)^2}
\]

\[
= \sqrt{38.5 - 30.25}
\]

\[
= \sqrt{8.25}
\]

\[
= 2.87
\]

10. \(\frac{2}{3}\)

11. \(\frac{1}{13}\)
12. Independent (each event is not affected by other events), Dependent (also called ‘Conditional’, where an event is affected by other events), Mutually Exclusive (events can't happen at the same time)

14.11 REFERENCES AND SUGGESTED READINGS


- [https://www.mathisfun.com/data/probability](https://www.mathisfun.com/data/probability)

- [https://en.m.wikipedia.org/wiki;probability](https://en.m.wikipedia.org/wiki;probability)

- [https://www.probabilityformula.org](https://www.probabilityformula.org)
UNIT 15  PARALLEL LINES, PARALLELOGRAMS AND TRIANGLES

Structure

15.1 Introduction
15.2 Objectives
15.3 Basic Concepts
15.4 Parallel Lines
15.5 Parallelograms
15.6 Comparison of Two Figures
15.7 Similarity
15.8 Congruence
  15.8.1 Definition
  15.8.2 Line Segments and Angles
  15.8.3 Correspondence and Congruence
  15.8.4 Congruence of Triangles
15.9 Constructing Triangles with Available Information
  15.9.1 Three Sides (SSS)
  15.9.2 Two Angles and a Side (ASA, AAS)
  15.9.3 Two Sides and the Included Angles (SAS)
  15.9.4 The Hypotenuse and one Side of a Right Triangle (RHS)
15.10 Triangle – A Rigid Figure
15.11 Congruence of Triangles
15.12 Let Us Sum Up
15.13 Unit End Activities
15.14 Answers to Check Your Progress
15.15 Suggested Readings

15.1 INTRODUCTION

The notions of plane Geometry originated in ancient times. They arose out of the necessity to solve practical problems. Egyptians used Geometry to determine the lengths, areas or volumes of various objects. Ancient Hindus used it to design altars (Vedis) for worship. Later, Greeks formulated the logical or deductive aspects of Geometry and developed it as a discipline. Most of the ancient civilizations used Geometry only for practical purposes and not much was done to make it a systematic study.

We find ample application of Geometry in daily life. A learner can be helped to appreciate the use of geometrical forms in nature and architecture all around. He/she may also use the concept of Geometry to construct figures or develop designs. Geometrical propositions can be understood through practical applications and logical reasoning. These approaches are reflected in this Unit.

The Unit deals with the investigation and study of the concepts of elementary Geometry; correspondence and congruence in geometrical figures and construction of triangles. Practical methods, intuition and deduction have been freely employed to demonstrate their truth. The unit is designed to enable you
to get a greater understanding of the concept of parallel lines, parallelogram and triangles and to proceed to a logical treatment of the subject.

### 15.2 OBJECTIVES

After going through this Unit, you will be able to:

- explain the basic concepts of Geometry;
- create interest in studying parallel lines and parallelogram;
- demonstrate geometric discussions;
- explain the concept of equality and similarity together with illustrations;
- explain and apply the concept of congruence to geometric figures in general;
- understand the symbols for these concepts;
- put across the importance of 1-1 correspondence;
- understand that given SSS, SAS, ASA (AAS), RHS we always get triangles of the same size and shape; and
- develop an activity-based and problem-based approach for teaching-learning various concepts covered in this unit.

### 15.3 BASIC CONCEPTS

**Main Teaching Points:** a) Concepts of point, line, plane and angle, and b) Related axioms

**Teaching-Learning Process:** The topic should be introduced by reviewing fundamental concepts on points, lines and planes in such a manner that the practical aspect of Geometry is realized. You can organize following activities in your class to review fundamental concepts of Geometry.

**Ask:** Hold a matchbox in your hand. Tell students to examine the outside cover of the match box. And ask them how many faces are there in a matchbox?

Bring out that the box has six sides or faces, each of which is a rectangle.

**Explain:** The box may be represented by drawing it in two ways: In the first drawing (Figure 15.1(a)), we can see only three faces of the box. The other three faces are hidden from view.

If we were to construct a box of the same shape using a piece of wire for the outline of each face, it will look as in Figure 15.1(b). All faces of this box can be seen. The hidden part of Figure 15.1(a) has been shown by dotted lines.
Ask: Which faces have the same shape and size in Figure 15.1?

Faces ABCD and HGFE are two such faces. Similarly, AHED and BGFC is another pair of such faces. ABGH and CFED is a third pair. They are on opposite sides of each other.

Explain: In Geometry, we are concerned only with the study of the shape and size of objects. We are not concerned with the matter or material which they may contain.

A solid body is conceived as occupying space and the amount of this space is called its **volume**.

Ask: Where do the two adjacent faces meet?

Write: The two adjacent faces meet in a **straight line** which is called an edge.

Explain: The faces ABCD and CFED meet in a straight line CD. In the whole box there are 12 straight lines or edges.

Ask: What is the intersection of two edges called?

The intersection of **two edges is a point**.

Explain: The edges AD and DC meet at the point D. Explain that there are eight such points which are called corners. Each of these points indicates the meeting points of three edges. Thus, D indicates the intersection of edges DE, DA and DC.

Ask: What, in your opinion, should be taken as basic terms which we shall accept as undefined and proceed to define other terms using these basic terms?

There may be different suggestions made by learners, or may be there is no response.

Explain: The terms ‘point’, ‘straight line’ and ‘plane’ are difficult to explain in simple words or in terms of the relatively simpler notions. We take them as undefined. Explain that points such as D, E, F, etc., have corresponding positions in space. We frequently mark a position on a piece of paper, or a map, or on a picture by marking a small dot which indicates some particular position. **Thus, a point indicates a position in space and has no size or magnitude.**
Ask : How can we explain the formation of a line using points?

Mark P and Q as two points on the surface of the paper as shown in Figure 15.2. Suppose point P moves to position Q.

Explain : There are a number of paths which P may take such as PRQ and PSQ. Measure the length of different paths using thread. We will find these paths vary in length. The most direct path will be along the straight line PQ which joins the points. Thus, a straight line or a line may be described as: the shortest path between two points P and Q.

Explain : The straight line which marks the path of a moving point has length but no width. Hence, a line is said to have one dimension only.

Explain that from the geometrical point of view a line is a set of points and extends endlessly in both directions. The symbol PQ or QP is used for the line l as shown in Figure 15.3.

Ask : How many points are there on a line?

Explain : If you see points as:

P Q

Now you may keep on proceeding from P to Q or Q to P through points as

P ............. ...... ...... Q

In case you cover the whole distance you will find many points which you may not count if they are very close to each other and form a line.

Thus we may conclude that there are infinite number of points on a line. We may consider it as the first axiom on lines and points.

Ask : How may lines can pass through a given point?

Bring out that infinite number of lines can pass through a given point.
Explain : It is the second axiom on lines and points.

Ask : How many lines can be drawn through two given points?
Bring out one and only one line can be drawn through two given points.

Explain : It is the third axiom on lines and points.

Explain that as a consequence of the third axiom, it is deduced that if two lines intersect, this intersection is exactly one point.

Let us consider a line as a number line, then each point of the line corresponds to a real number. The number associated with a point is called the coordinate of the point, and the point associated with a number is called the graph (or graphical representation) of that number.

Explain : The correspondence described here is called a one-to-one correspondence between the points of a line and the set of real numbers. It is called the ‘Ruler axiom’.

Explain that if the coordinate of a point P is X and the coordinate of the point Q is Y, then the distance between P and Q is |Y – X| which is equal to |X – Y|. We will accept this as one of our basic assumptions.

Explain : This real number is called the distance between two points. This distance is the absolute value of the difference of the real numbers corresponding to the two points. Explain that symbol PQ is used to refer to the number which is the distance between points P and Q.

Ask : Can you think of objects with flat surfaces?
Bring out that table tops, mirrors, papers, walls, etc., are objects with flat surfaces.

Explain : Flat surfaces are examples of planes. But these are physical models of planes. A plane is a mathematical abstraction.

Explain that collinear points are points that lie on the same line. Similarly, coplanar points are points that are on the same place.

Explain the following pyramid as in Figure 15.4.
A₂ and A₃ are collinear; A₁, A₄ and A₂ are non-collinear; A₁, A₂ and A₄ are coplanar; A₁, A₂, A₃ and A₄ are non-coplanar.

Ask : How many planes pass through three non-collinear points?

Bring out that there is exactly one plane containing three non-collinear points.

Explain : This is the first axiom on lines and planes.

Ask : Think of any two points in a plane.

Bring out that the line containing them is contained in the plane.

Explain : This is the second axiom on lines and planes.

Ask : How many lines can be formed by the intersection of two planes?

Bring out if two planes intersect, then their intersection is exactly one line.

Explain : It is third axiom on lines and planes. Using these axioms many theorems can be proved.

Ask : Take two distinct points P and Q on line l (Figure 15.5).

Bring out that P and Q determine a line segment, or simply a segment.

Explain : The union of the set containing two points P and Q of line l and the set of all points of l between P and Q is called a segment denoted by PQ. The length or measure of PQ is the distance between P and Q and is denoted by PQ. Thus,
\[PQ = \{P\} \cup \{Q\} \cup \{\text{points between } P \text{ and } Q\}\]

**Explain:** A line may be named by any two of its points but a segment is always named by its end-points.

**Ask:** Take O on line \(l\) (Figure 15.6).

Bring out that O separates all the other points of the line into two sets of points. Point O does not belong to the set of points on either side of O.

![Fig. 15.6](C O B)

**Explain:** The set of all points on either side of O, excluding O, is called a half-line. To describe the two half-lines on either side of O consider two points B and C on either side of O. Consider P as a moving point on the line.

Denoting half-line on the side of B by \(\overrightarrow{OB}\).

\[
\overrightarrow{OB} = \{\text{all points } P \text{ between } O \text{ and } B\} \cup \{B\} \cup \{\text{all points } P \text{ such that } B \text{ is between } O \text{ and } P\}.
\]

**Ask:** What is the union of the set containing the point O and a half-line \(\overrightarrow{OB}\).

Bring out that it is a ray denoted by \(\overrightarrow{OB}\) (Figure 15.8). Point O is called the end point of the ray.

![Fig. 15.8](A O B)

**Explain:** Explain that the rays OA and OB are called opposite rays if and only if O is between A and B.

**Ask:** Draw Figure 15.9 on the board. Identify a line, several rays and several segments.

Ask what is \(\overrightarrow{OP} \cup \overrightarrow{OQ}\).

Bring out it is the union of two rays which is subsets of the same line.
An angle is the union of two non-collinear rays having a common end point. The two rays are called the sides or arms of the angle. The common end point is called the vertex of the angle.

**Ask:** What is Adjacent angles

**Explain:** Explain that two angles which have a common vertex, a common arm and which lie on opposite sides of the common arm form a pair of adjacent angles.

**Ask:** Think of a ray standing on a line.

Bring out that the sum of two adjacent angles so formed is $180^\circ$. Conversely, if the sum of two adjacent angles is $180^\circ$, the non-common arms of the angles are two opposite rays.

In figure 15.10 (a), **PQ is a line and a ray OS standing on it** which forms two angles at point O. The sum of these two angles $\angle SOQ$ and $\angle SOP$ is $180^\circ$. 

![Fig. 15.9 below shows OP $\cup$ OQ](image)

![Fig. 15.10](image)

![Fig. 15.10(a)](image)  ![Fig. 15.10(b)](image)
Parallel Lines, Parallelograms and Triangles

In Fig 15.10 (b), OX and OY are two lines perpendicular to each other and a ray ON generates at O. The two angles thus generated are such that \( \angle YON + \angle NOX = 90^\circ \).

This is an axiom on the angle.

Further explain that two angles are called supplementary angles if their sum is 180°, and two angles the sum of whose measures is 90° are called complementary angles. For example 70° and 110° are a pair of supplementary angles and 55° and 35° are a pair of complementary angles. Explain that ray \( \overline{PS} \) is said to be the “bisector” of \( \angle QPR \), and \( \angle QPS = \angle SPR \). Here, \( \angle RPS = \frac{1}{2} \angle QPR \). (Fig. 15.11).

Methodology Used: Intuitive, deductive, analytical and experimental approaches form the essential features of teaching geometry.

Check Your Progress

Notes: a) Write your answers in the space given below.
   b) Compare your answers with those given at the end of the Unit.

1) Take three collinear points A, B and C. How many triangles can you draw with A, B as the vertices?

2) How many edges does a cube have? Illustrate.
3) Explain how to find the measure of an angle which is equal to its supplement.

...................................................................................................................
...................................................................................................................
...................................................................................................................

4) How many lines can you draw passing through:
   a) One point
      ...........................................................................................................
      ...........................................................................................................
   b) Two points
      ...........................................................................................................
      ...........................................................................................................
   c) Three points
      ...........................................................................................................
      ...........................................................................................................

15.4 PARALLEL LINES

Main Teaching Point: The concept of parallel lines and related axioms

Teaching-Learning Process: We have seen in the preceding section that when two lines intersect, we get a number of geometric figures such as rays and angles. What happens if two lines do not intersect? In this section, we shall consider certain properties of non-intersecting lines.

Ask : Examine the ruled printed lines on an exercise book.
      Bring out two facts:
      a) the distance between any pair of lines is always the same;
      b) even if the lines are produced to any extent beyond the page of the exercise book, they never meet.

Explain : Such straight lines drawn in a plane are called parallel straight lines.
      Explain that two lines are parallel if and only if they are coplanar and they do not intersect.

![Fig. 15.12]
Ask: Think of two coplanar lines and a line which intersects each of the two given lines.

Bring out two distinct points.

Explain: A line which intersects two coplanar lines at two distinct points is called a transversal line. Explain that in Figure 15.13, \( l_1 \) and \( l_2 \) are coplanar lines and are cut by a transversal \( m \) at two distinct points \( A \) and \( B \). Although another line \( n \) intersects both \( l_1 \) and \( l_2 \) at the intersection point \( C \), \( n \) is not a transversal as it does not intersect the lines \( l_1 \) and \( l_2 \) at two distinct point.

![Fig. 15.13](image)

Explain that transversals and the lines they intersect form alternate angles, interior angles, corresponding angles, etc. These pairs of angles are of special importance in our investigation of geometric properties.

Ask: Think of a line \( l \) and a point \( A \) it on.

Ask how many lines can be drawn through \( A \) and parallel to \( l \).

Help them explore that there is one and only one line which passes through \( A \) and is parallel to \( l \). (Figure 15.14).

![Fig. 15.14](image)

Explain: This is the first parallel axiom.

Help them explore, that if a transversal intersects two parallel lines, then each pair of corresponding angles are equal. Conversely, if a transversal intersects two lines making a pair of corresponding angles equal, then the lines are parallel. This is the second parallel axiom.
Let them verify that if two lines are intersected by a transversal such that two alternate interior angles are equal, then the lines are parallel. It can also be proved using the parallel axioms.

Explain that, using parallel axioms, several theorems on parallel lines and constructions involving parallel lines can be proved. The theorems and their proofs may be seen in any standard book on Geometry.

**Distance between two parallel straight lines.**

Ask: Draw two parallel straight lines PQ and RS. (Figure 15.15). Take a point A on PQ. Take AB as a straight line which is perpendicular from A to RS.

Bring out that AB is the distance between the two parallel straight lines, which remains the same throughout.

![Fig. 15.15](image)

Ask: Give examples from their day to day life where they see parallel lines.

**Check Your Progress**

**Notes:**

a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the unit.

5) If \( l \parallel m \) in the following figure and \( \angle 1 = 60^\circ \) then find the value of \( \angle 8 \).

………………… ……………………
……… …… …………
………………… ……………………
………………… ……………………
………………… ……………………
………………… ……………………
………………… ……………………
………………… ……………………
………………… ……………………
………………… ……………………
6) Observe the given figure and answer the following:

Is $S_1 || S_2$? Give reason for your answer.

```
………………………………………………………………………..
………………………………………………………………………..
………………………………………………………………………..
```

**15.5 PARALLELOGRAMS**

**Main Teaching Point:** Different types of quadrilaterals and their interrelations.

**Teaching-Learning Process:**

**Ask:** What is a plane figure?

Bring out that a part of plane surface which is excluded by line segments or by a closed curve is called a region and the outline of the region (boundary) is called a plane figure.

**Explain:** If the boundary lines are all line segments, the figure is called a rectilinear figure. If four straight lines in a plane intersect in pairs, the figure formed is a quadrilateral. In Figure 15.16 the points P, Q, R, S are four vertices of the quadrilateral.

Explain lines that join two opposite vertices are called diagonals. In Figure 15.16 PR and QS are diagonals of quadrilateral PQRS.
The sum of the angles of any quadrilateral is equal to four right angles. i.e. \( \angle P + \angle Q + \angle R + \angle S = 4 \times 90° \) or 360°.

**Ask**: Think of a quadrilateral in which both pairs of opposite sides are parallel and one of its angles is a right angle.

Bring out that it is a rectangle (Fig. 15.17).

![Fig. 15.17](image)

**Ask**: Take the outer cover of an ordinary match box without its inner open box. Squeeze gently so that two opposite edges come closer. The rectangular shape of open ends changes. Angle between two edges is no longer a right angle but the opposite edges are still parallel. A quadrilateral with each pair of opposite sides parallel is called a parallelogram. (Figure 15.18)

![Fig. 15.18](image)

**Ask**: What about the shape of the open end of the match box if it had been a square instead of a rectangle? Bring out that it would still be changed to a parallelogram, but its sides will all be equal. It is called a rhombus which is again a special form of a parallelogram.

**Explain**: A quadrilateral with both pairs of opposite sides parallel, and both adjacent sides equal is called a rhombus. (Figure 15.19)

![Fig. 15.19](image)

**Ask**: Think of a quadrilateral whose both pairs of opposite sides are
parallel, one of its angles is a right angle and both adjacent sides are equal.

**Explain** : Bring out that it is a square (Figure 15.20)

![Figure 15.20](image)

**Ask** : What about a quadrilateral in which two opposite sides are parallel but the other sides are not parallel?

**Explain** : Bring out that it is a trapezium (Figure 15.21).

![Figure 15.21](image)

**Ask** : Can you think of a chart in the development of the properties of quadrilaterals.

Facilitate students so that they come out with the following definitions on their own.

**Quadrilateral** is a plane figure made up of four sides (line segments).

**Trapezium** is a quadrilateral whose one pair of opposite sides is parallel.

**Parallelogram** is a quadrilateral whose opposite sides are parallel.

**Rectangle** is a quadrilateral whose opposite sides are parallel and one angle is a right angle.

**Rhombus** is a quadrilateral whose opposite sides are parallel and adjacent sides are equal (or a rhombus is a parallelogram whose all sides are equal in length).

**Square** is a rectangle whose all sides are equal.

Using the above definitions let them draw a chart interconnecting different types of quadrilaterals mentioned above. Give them time to discuss these definitions with their peers. Appreciate them if they give some alternate definition or establish the relation between any two figures in some other way.
Ask : Can you think of a way of learning by using a geobard?

Explain : A geoboard maybe made of 66mm or 8 mm ply board of size 6”X6” is divided into 36 equal squares with one pin fixed vertically at the centre in each square.(Figure 15.22)

Explain : Using ‘rubber bands’ of different colours, various geometrical figures can be made on the board. Explain: Several geometrical patterns viz., Parallelogram, rectangle, square, rhombus etc. can be made on the geoboard (Figure 15.23).

Ask : Using a geoboard can you think of providing result like (a) diagonal of a parallelogram bisect each other; and (b)diagonals of rhombus bisect each other at right angle.

Bring out that answer is yes and draw result on geoboard. (Figures 15.24 and 15.25).

Methodology Used: While teaching Geometry, more stress should be on intuitive and experimental approach. Analytic and synthetic approach is suitable while doing proofs of standard theorems. Conclusion of proofs may be drawn through activity method.
Check Your Progress

Notes: a) Write your answers in the space given below.
    b) Compare your answers with those given at the end of the Unit.

7) State whether true or false.
   (i) Every rectangle is a square but the converse is not true.
   (ii) Every rhombus is a square but the converse is not true.
   (iii) Every parallelogram is a quadrilateral but the converse is not true.

15.6 COMPARISON OF TWO FIGURES

Main Teaching Points: a) Comparing shapes and sizes; and b) The size determined by area

Teaching-Learning Process: Before actually starting the topic, you may share learning experiences of the students by asking them which objects do they use in pairs in their day-to-day life. It may be discussed with them whether these objects have same shape and size or only their shape is same? Do they overlap each other? Hold in your hands two cut-outs of the same shape.

Ask : Look at these two cut-outs. What can you tell about these cut-outs? (In case there is no response in the beginning, you may bring in leading suggestions like : they look alike, their sizes are not the same, what else …. There may be response regarding their volumes, their corners, their sides, … these are ruled out).

Repeat this line of interaction with 3 or 4 pairs of cut-out.

Ask : When we look at two things or articles, what questions come to our mind regarding their shapes, size or any other characteristics?

Ask : Do we try to know which one is bigger? Which one is smaller?

Ask : What are we comparing?

When we say that one is bigger than the other, what are we comparing?
Is it their weight?
Or is it their length?
Or is it the space each over?
Ask : Which one is bigger?
   How do you know?

Explain : By covering them with squared transparent papers.

Repeat the same questions with the following figures:

Conclude:
Two figures can be different in shape and different in size.
Two figures can be different in shape and same in size.
Two figures can be same in shape and different in size.
Two figures can be same in shape and same in size.
Size of the figures is measured by their area.

Methodology Used: Demonstration combined with proper discussion is used to illustrate the point.

Note: We can use transparencies to show movement of one figure into another (super positions).

15.7 SIMILARITY

Main Teaching Points: a) Geometrical figures of the same shape are called similar; and b) The size of similar figures may be same or different.

Teaching-Learning Process: Here we are concerned with the shape of the figures. Hold in your hand two circles.

Ask : What can you say about the shapes of these two figures?
   Repeat with pairs of squares and equilateral triangles. You may even consider pairs of regular figures.
Ask : Do the two have the same size/area/ (the figures chosen must include those of different sizes and the same sizes).

Activity 1 : Draw two line segments of 5 cm and 7.5 cm length. Name them AB and XY.

On A and X draw angles of 60° with AB and XY respectively.

Similarly on B and Y draw angles of 75°.

Let the arms of the two angles (other than the common arm) intersect in C and Z respectively.

Ask : What shapes do you get?

Place the two triangles such that A falls on X and AB on XY. Next repeat the same thing with B falling on Y and BA on XZ.

What can you say about the positions of AC and XZ?

Do the two triangles have the same shape? Do they have same size?

Bring out the two triangles have the same shape but different sizes.

Ask : What will happen if AB and XY both are of the same length? Will you get two triangles of the same shape? What about their sizes?

Bring out the triangles are of the same shape and the same size.

Activity 2 : Hold a triangular template in front of a candle. Form an image on a plain sheet of paper.

Now move the template towards the flame and away from the flame.

Ask : What happens to the shadow? Do you have the same shape always? What care you have to take it get a sharp image?

Activity 3 : Repeat Activity 2 with a template of some different figures/shapes.

Ask : What can you say about your passport size photograph and the same in the post-card size?

Explain : We get figures of the same shape. But their sizes may be same or different.

When the shapes of two figures A and B are alike we say that figures A is similar to figures B.

In particular
If $\triangle ABC$ is similar to $\triangle XYZ$.

We write it as: $\triangle ABC \sim \triangle XYZ$.

**Conclusion:** Two figures are similar if they are of the same shape. Their sizes may or may not be the same.

**Methodology Used:** Learning by doing to be the best approach. Let the students draw the figures and help them to reach the conclusions.

### Check Your Progress

**Notes:**

a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

8) Draw a triangle of sides measuring 3 cm, 4 cm, 5 cm and another one of sides 6 cm, 8 cm, 10 cm. What can you say about their shapes? Measure their angles and compare. How will you demonstrate that the triangles are similar?

9) Draw a triangle having two sides of 4 cm and 5 cm and their included angle of 45°. Draw another triangle having sides measuring 6 cm and 7.5 cm and their included angle of 45°.

Compare the two triangles.

What can you say about their angles?

Find the ratio of the third side of one triangle with the third side of the other. Compare it with ratios of other two pairs of corresponding sides. How will you demonstrate that the triangles are similar?

---

**15.8 CONGRUENCE**

**15.8.1 Definition**

**Main Teaching Point:** Congruent figures have same shape and same size.
**Teaching-Learning Process:**

You have seen above that we can have two figures of the same size and also we can have two figures of the same shape.

![Fig. 15.28](image)

**Ask:** Are the triangles ABC and XYZ of the same shape? Can you confirm it by turning around Δ ABC or Δ XYZ and then comparing?

What can you say about their sizes?

Repeat with the following pairs of triangles:

![Fig. 15.29](image)

In all the above parts the size of one triangle is equal to the other and also the shape of one triangle is same as the shape of the other.

In other words the triangles have the same size and are also similar. You may note that if two figures completely cover each other when super imposed, these are of same size and similar too.

The concept of equality of size and similarity of shape is combined in congruence. We say that one figure is congruent to the other.

Symbol \( \cong \) is used to denote **congruence** of two figures.

**Methodology Used:** Demonstration-cum-discussion leading to the conclusion.

**15.8.2 Line Segments and Angles**

**Main Teaching Points:** a) Line segments of equal length are congruent; and b) Angles of the same measures are congruent.
Teaching-Learning Process:

Ask : Given two line-segment what information will you need to decide whether they are congruent?

Draw : Two line-segments of 3 cm each. Do they look alike (similar)? Do they have the same length?

Explain : So you have a pair of congruent line-segment. Try with as many pairs.

Ask : What do we conclude?

Line-segments of Equal Length are congruent

Ask : In the same way let us consider a pair of angles. What information will you need to say that the pairs of angles are congruent?

Draw two angles of 60° each.

Are these angles of the same measure?

Explain : So, you have a pair of congruent angles.

You may try with more pairs.

What do we conclude?

Two angles are congruent if they have the same measure.

Methodology Used: Demonstration-cum-discussion is used to lead the students to conclusion.

15.8.3 Correspondence and Congruence

Main Teaching Points: a) There are many correspondences possible between two figures; and b) Some of them may be congruences and others may not.

Teaching-Learning Process: In two sets (consider two teams participating in a hockey match) there is element-to-element (a man-to-man) matching or association. This association in mathematics is termed as 1 – 1 correspondence.

![Fig. 15.30](image)

Ask : In how many different ways can the vertices, A, B, C or ΔABC is set in 1-1 correspondence with vertices P, Q, R of ΔPQR?

We can set up 1 -1 correspondence in six different ways, namely:
Explain: 1-1 correspondence between vertices of the triangles determine the 1-1 correspondence between angles and sides of the two triangles.

e.g. in A – P

B – Q
C – R

∠A – ∠P and AB – PQ
∠B – ∠Q and AC – PR
∠C – ∠R and BC – QR

We also call this correspondences as ABC ↔ PQR

Activity 1: Let us superimpose ΔPQR on ΔABC according to the 1-1 correspondence described above.

Draw figures for other correspondences also. In how many cases a correspondence results in completely covering one figure over the other. (This happens only in ABC ↔ PQR).

Activity 2: Take a ΔABC in which AB = AC. Make a carbon copy of this triangle and name it ΔPQR. Superimpose and find out how many correspondences result in covering one figure over the other completely. In case of ABC ↔ PQR and ABC ↔ PRQ, one Δ covers the other completely.

Activity 3: Take a ΔABC which is equilateral.

Make Δ PQR which is a carbon-copy of ΔABC.

Superimpose and investigate in this case also as to how many correspondences result in covering one triangle over the other completely.

Explain: In all correspondences, one triangle covers the other completely.
Methodology Used: This is purely on activity-based method. Let the students do themselves and arrive at the conclusion.

15.8.4 Congruence of Triangles

Main Teaching Point: Defining congruence of triangles.

Teaching-Learning Process: This leads us informally to state that “If there exists at least one correspondence between two triangles such that if we superimpose one triangle over the other, and they cover each other completely, then the two triangles are congruent.”

Under activity (1) we found that $\triangle ABC \cong \triangle PQR$ which implies that

$\angle A = \angle P, \ \angle B = \angle Q, \ \angle C = \angle R, \ AB = PQ, \ BC = QR, \ AC = PR.$

Explain : We frequently use the symbol $\equiv$ in place of $\cong$ for line-segments and angles. This usage is widely prevalent and it does not cause any misunderstanding.

We name the triangles according to the correspondence which results in a congruence. This helps in immediately pointing out the pairs of corresponding angles and corresponding sides which are congruent (or equal). We conclude that, “If three angles of a triangle are congruent (equal) to three angles of another triangle each to each and their corresponding sides are congruent (equal), then the two triangles are said to be congruent”.

Including the converse of the above statement we define correspondence of triangles as below.

Definition : Two triangles are congruent if and only if their corresponding angles and corresponding sides are equal.

Reinforcement : Cut-out the following pairs of figures and discover the corresponding vertices. Write the corresponding part(s) that are equal.

Correctly name the triangles which are congruent. You may device more activities of this type.

Methodology Used: Deductive reasoning is used. From the activities of the previous sections, the students can deduce the definition.
Check Your Progress

Notes: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

10) Write the corresponding sides and corresponding angles in the following figures if $\triangle ABC \cong \triangle MNO$.

```
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
...................................................................................................................
```

15.9 CONSTRUCTING TRIANGLES WITH AVAILABLE INFORMATION

If a triangle is given, one can find out measures of angles and sides of the triangle. Conversely, if data about sides and angles of a triangle are given, let us investigate whether we can construct the triangle.

Ask : Do you think you need to know data about all sides and angles of a triangle to be able to construct it.

What data is required so that you can construct a unique triangle?

Explain : We can construct a triangle uniquely if we can fix its three vertices. Generally, this is possible if measurements of three parts of which one must be a side are given. (The teacher should remember that ambiguous case is an exception to this rule).

15.9.1 Three Sides (SSS)

Main Teaching Point: We get a unique triangle if we know its sides.

Teaching-Learning Process: Let us construct a triangle whose sides measure 3 cm, 4 cm and 5 cm using ruler and compass only.
Let every student draw $BC = 5\text{ cm}$

Now Ask: How can you locate $A$ such that $A$ is 3 cm from $B$ and 4 cm from $C$?

Lead the response till you have the answer. By drawing arcs (parts of a circle) with radius 3 cm and 4 cm and centres $B$ and $C$ respectively.

Ask: What do you observe about the triangle drawn by you and the one drawn by your neighbouring student?

Expected Response: They are all similar and equal in size, i.e., they are congruents.

Ask them to verify by measuring and writing the corresponding angles.

You may repeat by having another set of sides.

Conclusions:

1. Given the measures of the three sides of a triangle, we can construct the triangle uniquely.
2. Triangles drawn with the same measures of sides will be congruent.
3. Given two triangles measures of whose corresponding sides are equal, the triangles are congruent. This is called SSS Congruence.

Methodology Used: It is purely on activity-based method. Let the students work it out for themselves and reach the desired conclusion.

15.9.2 Two Angles and a Side (ASA, AAS)

Main Teaching Points: Two angles and an include side determines a triangle uniquely.

Teaching-Learning Process: Let us construct a triangle $ABC$ which has $BC = 4.5\text{ cm}$, $\angle C = 50^\circ$ and $\angle B = 70^\circ$. 
Every student draws a line-segment (BC) equal to 4.5 cm. Then he draws $\angle B = 70^\circ$ and $\angle C = 50^\circ$, the non-common arms of the two angles intersect at A.

Ask : What can you say about triangle drawn by you and the ones drawn by your neighbours?

Explain : They are congruent.

You may repeat by changing the measures.

**Conclusions:**

1) Given the measures of two angles and the included side, we can construct the triangle uniquely.

2) Triangles drawn with the same measures of two angles and the included side will be congruent.

3) Given two triangles such that the measures of one of their sides and the two angles on these sides are equal, the triangles are congruent.

This is called ASA congruence.

**Methodology Used:** It is purely an activity-based method. Let the students work it out for themselves and reach the desired conclusion.

**15.9.3 Two Sides and the Included Angle (SAS)**

**Main Teaching Point:** Two sides and an included angle determines a unique triangle.

**Teaching-Learning Process:** Let us construct a triangle XYZ which has XY = 3.5 cm, $\angle Y = 45^\circ$, YZ = 4.5 cm.

[Repeat the steps as in 15.9.1 and 15.9.2].

Draw XY = 3.5 cm, measure $\angle Y = 45^\circ$, on this arm from Y take 4.5 cm and mark the point as Z, join XZ.

**Conclusions:**

1) Given the measures of two sides and the included angle of a triangle, we can construct a unique triangle.

2) Triangles drawn with the same measures of two sides and the included angle will be congruent.

3) Given two triangles such that the measures of two of their sides and the included angle are correspondingly equal, the triangles are congruent.
This is called SAS congruence.

**Methodology Used:** It is purely an activity-based method, let the students work it out for themselves and reach the desired conclusion.

### 15.9.4 The Hypotenuse and one side of a Right Triangle (RHS)

**Main Teaching Point:** If in a right triangle, one side and hypotenuse are given then we get a unique triangle.

**Teaching-Learning Process:** Let us construct a triangle ABC right angled at B which has its hypotenuse AC = 5 cm and side BC = 4 cm.

![Fig. 15.35](image)

Every student draws a line-segment BC = 4 cm, then at B makes an angle $\angle ABC = 90^\circ$.

**Ask:** How can you locate A on BX such that CA = 5 cm?

**Lead the responses till you have the answer:** By drawing an arc (part of a circle) with centre at C and radius = 5 cm.

Now repeat the comparison and construction as in the case of previous sections.

**Conclusions:**

1) Given the measures of hypotenuse and a side of a right triangle we can construct the triangle.

2) Right triangle drawn with the same measure of hypotenuse and a side are all congruent.

3) Given two right triangles whose hypotenuse and one side each are equal, the triangles are congruent.

This is called RHS congruence.

**Methodology Used:** It is purely an activity-based method. Let students work out themselves and reach the desired conclusion.

**Check Your Progress**

**Notes:**

a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

11) Measurement of how many parts should be given for constructing the following triangles:
(i) Equilateral triangle
(ii) Isosceles triangle

Justify your answer.

15.10 TRIANGLE – A RIGID FIGURE

Main Teaching Point: A triangle cannot be deformed.

Teaching-Learning Process:

Ask : What is the geometric figure that you see when you look at a bridge, an electric pole or any other heavy structure.

Ask : Why do you think the triangle is used for this purpose?
Let us conduct an experiment.

Experiment : Take four strips of wood long enough to be joined to form a four-sided figure.

Fix these sticks, as shown in the figures with nails and screws.
Now try to flex the figure to change its shape.
What do you observe?
Now repeat by forming a five sided figures.

Fig. 15.36(a)

Fig. 15.36(b)
What do you observe?
Let us now form a triangle shape.

![Fig. 15.36(c)](image)

What do you find?
Now you must have understood why in structures we employ a triangle.

Explain: The three sticks cannot be moved even slightly. Therefore, the triangle is the most rigid structure which cannot be deformed.

**Methodology Used:** The students can learn this concept best by performing the activities themselves.

### 15.11 CONGRUENCE OF TRIANGLES

**Main Teaching Point:** To list out the four conditions under which two triangles are congruent.

**Teaching-Learning Process:** You have seen in section 15.9 that we have congruent triangles in four cases:

- **SSS:** When the three sides of one triangle are equal to the corresponding three sides of another triangle.
- **SAS:** When two sides and the included angle of one triangle i.e. equal to the corresponding parts of the other triangle.
- **ASA:** When two angles and the included side of one triangle are equal to the corresponding parts of the other triangle.
  OR
- **AAS:** When one side and any two angles of one triangle are equal to the corresponding parts of the other triangle.
- **RHS:** When the hypotenuse and one side of one right triangle are equal to the corresponding parts of the other right triangle.

**Methodology Used:** Deductive logic is used. The students have already reached the conclusions in section 15.9.
Check Your Progress

Notes: a) Write your answers in the space given below.
     b) Compare your answers with those given at the end of the Unit.

12) In the following diagrams are the triangles in each pair congruent? Why? Equal parts are shown by similar markings. Explain

15.12 LET US SUM UP

In this Unit you have learnt:

- Straight line is the shortest path between two points.
- An Infinite number of lines can pass through a given point.
- An angle is the union of two non-collinear rays having a common end point. The two rays are called the sides or arms of the angle. The common end point is called the **vertex** of the angle.
- If the boundary lines are all line segments, the figure is called a rectilinear figure.
- **Quadrilateral** is a plane figure made up of four sides (line segments).
- **Trapezium** is a quadrilateral whose one pair of opposite sides is parallel.
- **Parallelogram** is a quadrilateral whose opposite sides are parallel.
- **Rectangle** is a quadrilateral whose opposite sides are parallel and one angle is a right angle.
- **Rhombus** is a quadrilateral whose opposite sides are parallel and adjacent sides are equal (or a rhombus is a parallelogram whose all sides are equal in length).
- **Square** is a rectangle whose all sides are equal
- Figures of the same shape but not necessarily of the same size are similar.
- Similarity of triangles is dependent on the equality of the corresponding angles.
Figures of the same shapes may be of different sizes.

Triangles of the same shape and same size are congruent.

A unique triangle can be construct when we are given:
- measures of three sides SSS
- measures of two sides and the included angle SAS.
- measures of any two angles and one side AAS or ASA.
- a right triangle with hypotenuse and one side RHS.

A triangle is a rigid figure.

Two triangles are congruent when:
- their corresponding sides are equal SSS.
- they have two pairs of corresponding sides and the included angle equal
- two angles and a side of one triangle are equal to the corresponding parts of the other.
- the triangles are right angled and the hypotenuse and one side of one triangle are equal to the corresponding parts of the other.

### 15.13 UNIT END ACTIVITIES

1) Draw a picture of three collinear points, P, Q and R.
2) Draw a picture of three non-collinear points, A, B and C.
3) If P, Q and R are three non-collinear points.
   i) How many different lines are determined by choosing a distinct pair of points each time?
   ii) How many different planes would contain the three points?
4) We proved that if two lines intersect, their intersection is a point. Consider the possible intersection of two distinct planes. Can two planes intersect in:
   i) exactly one point?
   ii) exactly two points?
   iii) exactly three non-collinear points?
   iv) exactly one line?
5) If lines PQ, PR, PS and PT are parallel to a line \(l\), what can be said about the points P, Q, R, S and T?
6) Given two points P and Q, how many line segments do they determine?
7) Given three collinear points, P, Q and R, name all the line segments they determine.
8) Discuss each of the following briefly. Support your argument by appropriate axioms or theorems:
   i) Three points P, Q and R are in plane P. These same three points are in plane Q. Is it certain that plane P is equal to plane Q?
ii) If a line and a plane not containing the line intersect, then the intersect is a unique plane containing two intersecting lines.

iii) There is a unique plane containing two intersecting lines.

9) If $S$ lies in the interior of $\angle QPR$,

$\angle QPR = 80^\circ$ and $\angle QPS = 45^\circ$

What is the measure of $\angle SPR$?

10) An angle of $28^\circ$ more than its complement. What is its measure?

11) The measure of an angle is thrice the measure of its supplementary angle.

Find its measure.

12) In the following figure $L$ is the mid-point of side $PS$ of a trapezium $PQRS$, with $\overline{PQ} \parallel \overline{SR}$. A line through $L$ parallel to $PQ$ meets $QR$ in $M$. Show that $M$ is the mid-point of $QR$.

13) In the following figure of parallelogram $PQRS$ if $SL \perp PQ$ and $QM \perp SR$, prove that $\overline{SL}$ is congruent to $MQ$.

14) Prove that in a parallelogram, the sum of the squares of the lengths of the diagonals is equal to twice the sum of the squares of the lengths of any two adjacent sides.

15) Given the information in each part of the question, fill up the blanks:

For $\triangle ABC$ and $\triangle DEF$
<table>
<thead>
<tr>
<th></th>
<th>The corresponding is</th>
<th>Two triangle(s) are similar or congruent</th>
<th>Why</th>
</tr>
</thead>
</table>
| a) | $\angle A = \angle D$  
    $\angle B = \angle E$ | ABC ↔ ________ | ________ |
| b) | $AC = DE$  
    $\angle B = \angle E$ | ABC ↔ ________ | ________ |
| c) | $\angle A = \angle F$  
    $\angle B = \angle D$ | ________ ↔ DEF | ________ |
| d) | $\angle F = \angle B$  
    $\angle E = \angle C$  
    FE = BC | ________ ↔ DEF | ________ |
| e) | $\angle D = \angle A = 90^\circ$  
    EF = BC  
    DE = AB | ABC ↔ ________ | ________ |

16) Segments AB and CD insect each other at O. Prove that:
   a) $\triangle OAC \cong \triangle OBD$
   b) AC = DB

17) BP bisects $\angle ABC$. PE and PD make equal angles with BP and meet AB and AC at D and E respectively prove that $\triangle BPE \cong \triangle BPD$.

18) AE = ED  
    EF = EF  
    Prove that AB = CD

19) DA = DC, BA = BC  
    Prove that EA = EC
20. \[ \text{DC} = \text{AB} \]
\[ \angle 1 = \angle 2 \]
\[ \angle 3 = \angle 4 \]
Prove \( \text{AE} = \text{CF} \)

21. \[ \text{AD} = \text{BC} \]
\[ \angle 1 = 50^\circ \]
\[ \angle 2 = 50^\circ \]
Prove \( \triangle \text{ABC} \cong \triangle \text{BAD} \)

22. \[ \text{AB} = \text{AD} \]
\[ \text{AC} = \text{AE} \]
Prove \( \text{AF} \) bisects \( \angle \text{BAD} \).

23. \[ \text{AC} = \text{AD} \]
\[ \text{BC} = \text{BD} \]
\[ \text{AP} = \text{AQ} \]
Prove \( \angle 3 = \angle 4 \).

### 15.14 ANSWERS TO CHECK YOUR PROGRESS

1) None
2) 12
3) 90°
4) i) Infinite
   ii) One line
   iii) Three lines
5) \( \angle 8 = 60^\circ \)
6) \( S_1 \) is not parallelogram to \( S_2 \) because corresponding angles are not equal.
7) i) False
   ii) False
   iii) True
8) The two triangles are similar. Their corresponding angles are equal.
9) The two triangles are similar. Their corresponding angles are equal. The ratio of the third pair of sides is same as the ratio of the first two pairs of sides.

10) \( \angle A = \angle M \quad AB = MN \)
    \( \angle B = \angle N \quad BC = NO \)
    \( \angle C = \angle O \quad CA = OM \)

11) i) For equilateral triangle, the measurement of any one side will be sufficient.
    ii) For Isosceles triangle the measurement of one of the equal side and the included angle.

12) a) Triangles are not congruent because angles are not included between the sides.
    b) \( \Delta PQT \cong \Delta SRT \) — (ASA)
    c) \( \Delta PQR \cong \Delta SUT \) — (AAS)
    d) \( \Delta ABF \cong \Delta CDE \) — (AAS)
    e) \( \Delta BCM \cong \Delta CBN \) — (AAS)
    f) \( \Delta XYL \cong \Delta XZK \) — (AAS)

15.15 SUGGESTED READINGS

UNIT 16  TRIGONOMETRY AND ITS APPLICATION

Structure

16.1  Introduction
16.2  Objectives
16.3  Trigonometric Ratios
   16.3.1  Definitions of Trigonometric Ratios
   16.3.2  Trigonometric Ratios of some Specific Angles
   16.3.3  Trigonometric Ratios of Complementary Angles
16.4  Trigonometric Identities
16.5  Height and Distance
16.6  Let Us Sum Up
16.7  Unit End Activities
16.8  Answers to Check your Progress
16.9  References and Suggested Readings

16.1  INTRODUCTION

Have you ever thought how we can find the height at which the kite is flying or the height of QutabMinar or the distance of a ship from a light house.

In all the situations given above, the distance or height can be found by using some mathematical technique which come under a branch of mathematics called trigonometry.

The word Trigonometry is derived from the Greek words ‘tri’ meaning three, ‘gon’ meaning sides and ‘metron’ meaning measure. These three words together mean triangle measurement. Thus trigonometry is the study of relationships between the sides and angles in a triangle. Trigonometric ratios of angles which always have a unique value for any given angle form the basic tool for the study of these relationships. So in this unit we will study about trigonometric ratio and identities. We will also study about how to apply this knowledge to solve problems on heights and distances. Knowledge of trigonometry is useful in many situations such as navigation of ships or movements of aeroplanes, rockets, astronomical sciences, engineering surveys etc. Thus we will also discuss how trigonometry can be used in our real life situations.
16.2 OBJECTIVES

After going through this unit, you will be able to:-

• make the students understand the importance of Trigonometry;
• demonstrate to the students how the concepts of similarity form the basis of trigonometric ratios;
• develop among students the skill of manipulating trigonometric ratios and appreciate their relationship;
• help the students in using Pythagoras theorem to solve problems involving trigonometric ratios;
• help the students in finding the trigonometric ratios of complementary angles and applying them;
• develop among students the skill of proving trigonometric identities; and
• develop problem solving skills as required to solve problems of height and distance;

16.3 TRIGONOMETRIC RATIOS

16.3.1 Definitions of Trigonometric Ratios

Ask students to consider a right triangle ABC, right angled at B as shown in the figure.

They know that side opposite to the right angle is always called the hypotenuse.

Ask students to consider $\angle A$ or in brief angle A.

It is an acute angle.

Now, for the remaining two sides AB and BC they will see that BC is the side opposite to Angle A, which is perpendicular to point B and AB is the side of the Angle A (Angle A is formed by two sides - AB and the hypotenuse). They can say that side AB as the side adjacent to Angle A.

Students we have already studied the concept of ratio. You can now define certain ratios involving the sides of a right triangle and call them Trigonometric Ratios. These trigonometric ratios express the relationship between the angle and the lengths of its sides. Trigonometric ratios of angle A are defined as:

\[
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AC} \\
\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AB}{AC} \\
\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AB}
\]
In abbreviated form, these ratios are \(\sin A\), \(\cos A\), \(\tan A\). The ratios cosecant \(A\), secant \(A\) and cotangent \(A\) are the reciprocals of ratios \(\sin A\), \(\cos A\) and \(\tan A\) respectively and are written as cosec \(A\), sec \(A\) and cot \(A\).

\[
\text{cosec } A = \frac{1}{\sin A} = \frac{\text{hypotenuse}}{\text{side opposite to angle } A} = \frac{AC}{BC}
\]

\[
\text{sec } A = \frac{1}{\cos A} = \frac{\text{hypotenuse}}{\text{side adjacent to angle } A} = \frac{AC}{AB}
\]

\[
\text{cot } A = \frac{1}{\tan A} = \frac{\text{side adjacent to angle } A}{\text{side opposite to angle } A} = \frac{AB}{BC}
\]

Students can observe that

\[
\tan A = \frac{BC}{AB} = \frac{BC}{AC} \times \frac{AC}{AB} = \frac{\sin A}{\cos A} \text{ and }
\]

\[
\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A}
\]

Now if you define the trigonometric ratios for angle \(C\) in the same right triangle, you will see that \(AC\) remains the hypotenuse as the right angle \(B\) is fixed.

Now in place of angle \(A\), you can take angle \(C\). So the side opposite to angle \(C\) is \(AB\) and side adjacent to angle \(C\) is \(BC\).

Ask your student to write the trigonometric ratios for angle \(C\) yourself.

**Note:-**

1. Note that \(\sin A\) is not the product of \(\sin\) and \(A\), \(\sin\) separated from \(A\) has no meaning. \(\sin\) is always of some angle. The same follows for other Trigonometric ratios also.

2. For the sake of convenience, we may write \(\sin^2 A\), \(\cos^2 A\), etc. in place of \((\sin A)^2\), \((\cos A)^2\) etc. respectively. But we should not write cosec \(A\), which is the reciprocal of \(\sin A\) as cosec \(A = \sin^{-1} A\). We can however write cosec \(A = (\sin A)^{-1}\)

3. Greek letter \(\theta\) (theta) is also sometimes used to denote angle.

4. The trigonometric ratio of an angle is always a real number and so it does not have any unit.

5. The word ‘trigonometric ratios’ is sometimes written briefly as t-ratios.

6. In this unit, you shall always assume that \(0^\circ \leq \theta \leq 90^\circ\).
Ask your students that if, they keep the angle \( A \) same but change the lengths of the sides, will the value of trigonometric ratios of that angle change?

For this, ask them to consider again a right \( \triangle ABC \) right angled at B.

Take a point P on the hypotenuse AC.

Draw PM \( \perp \) AB

Now we want to find whether trigonometric ratios of angle A differ in \( \triangle AMP \) and \( \triangle ABC \).

Student already know that in \( \triangle ABC \), \( \sin A = \frac{BC}{AC} \) \( \quad \) (1)

and they can see that in \( \triangle AMP \), \( \sin A = \frac{PM}{AP} \) \( \quad \) (2)

Now, \( \triangle AMP \sim \triangle ABC \) (AA criterion of similarity).

So, \( \frac{MP}{AP} = \frac{BC}{AC} \) \( \quad \) (3)

So, from (1), (2) and (3) students can conclude that \( \sin A \) in \( \triangle ABC = \sin A \) in \( \triangle AMP \).

In the same way, you can check that if students take any point Q on extended AC and QN \( \perp \) AB extended, the value of \( \sin A \) remains the same in \( \triangle AQN \) also. Similarly, we can show for other trigonometric ratios, so from the above you can conclude that values of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.

When you know any one trigonometric ratio supposes \( \sin A \) in the figure, you also know the lengths of two sides BC and AC.

If your students are able to find the third side they can find the value of all the other trigonometric ratios.

Ask students, do they remember Pythagoras thereom for a right \( \triangle ABC \)?

Student, may reply \( AC^2 = AB^2 + BC^2 \)
From this, they can find side $AB = \sqrt{AC^2 - BC^2}$

So, now, students know the length of all the three sides $AB$, $BC$ and $AC$ and so, they can obtain all other remaining five trigonometric ratios. Also note that as hypotenuse is the longest side in a right triangle, the value of $\sin A$ and $\cos A$ is always less than 1 (or in particular equal to 1). The following examples will make the idea clear.

**Example-1:**
In $\triangle PQR$, right angled at $Q$, if $PQ = 20$ units and $PR= 29$ units, find $\sin P$, $\cos R$, $\cot R$.

**Solution:-**
By Pythagoras theorem students will have

\[
PR^2 = PQ^2 + QR^2
\]
\[
29^2 = 20^2 + QR^2
\]
\[
\text{or } QR^2 = 29^2 - 20^2 = 841 - 400 = 441
\]

So, $QR = 21$ units

Therefore,

\[
\sin P = \frac{QR}{PR} = \frac{21}{29}
\]
\[
\cos R = \frac{QR}{PR} = \frac{21}{29}
\]
\[
\cot R = \frac{PQ}{QR} = \frac{20}{21}
\]

**Example 2:**
Given $\cos A = \frac{3}{5}$, Find other t-ratios of angle $A$.

**Solution:**
Take a right $\triangle ABC$ (Figure)

\[
\cos A = \frac{3}{5} = \frac{AB}{AC}
\]

Note that $\frac{AB}{AC} = \frac{3}{5}$ does not necessarily mean $AB=3$ units and $AC=5$ units. In general, this means $AB = 3k$ units and $AC = 5k$ units, where $K$ is some constant.

By Pythagoras theorem, $AC^2=AB^2+BC^2$

\[
(5k)^2=(3k)^2+BC^2
\]
\[
\text{Or, } BC^2 = (5k)^2 - (3k)^2 = 25k^2 - 9k^2 = 16k^2
\]
\[
\text{So, } BC=4k
\]

Therefore,

\[
\sin A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}
\]
\[
\cosec A = \frac{1}{\sin A} = \frac{5}{4}
\]
Example 3:
In right \( \triangle ABC \), right angled at \( A \), in which \( AB = 5 \) units, \( BC = 13 \) units and \( \angle ABC = \theta \), determine the value of \( \sin^2 \theta - \cos^2 \theta \).

Solution:–

In \( \triangle ABC \),
\[
BC^2 = AB^2 + AC^2
\]
So,
\[
AC^2 = BC^2 - AB^2 = 13^2 - 5^2 = 169 - 25 = 144
\]
So, \( AC = 12 \) units

Therefore, \( \sin \theta = \frac{AC}{BC} = \frac{12}{13} \)
\( \cos \theta = \frac{AB}{BC} = \frac{5}{13} \)

so, \( \sin^2 \theta - \cos^2 \theta = \frac{12^2}{13^2} - \frac{5^2}{13^2} = \frac{144 - 25}{169} = \frac{119}{169} \)

Check Your Progress

Notes:

a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

1) From the figure, write

\[ \sin P = \]
\[ \cos P = \]
\[ \tan R = \]
\[ \csc R = \]
2) From the figure, find \( \tan P - \cot R \)

![Diagram](image1)

3) Given \( \sec \theta = \frac{13}{12} \), find the value of \( \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \)

16.3.2 Trigonometric Ratios of some Specific Angles

In this section we will find the values of Trigonometric Ratios of \( 0^\circ, 30^\circ, 45^\circ, 60^\circ \) and \( 90^\circ \).

Trigonometry ratio of \( 45^\circ \) could be introduced by the following way.

Ask students to consider \( \triangle ABC \), right angled at \( B \), if \( \angle A = 45^\circ \) what can you say about \( \angle C \)?

Students may respond that \( \angle C \) will also be \( 45^\circ \).

The teacher can give measurement of each side to be ‘a’. so

\( AB = BC = a \) (say)

To find the value of all t-ratios of \( A \), you will have to find the third side. Using Pythagoras theorem we have

\[
AC^2 = AB^2 + BC^2
\]

\[
AC^2 = a^2 + a^2 = 2a^2
\]

So,

\[
AC = a\sqrt{2}
\]

Ask students by using the definitions of t-ratios, workout the different trigonometric ratios for angle \( 45^\circ \)

They may bring out that

\[
\sin 45^\circ = \frac{BC}{AC} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}
\]

\[
\cos 45^\circ = \frac{AB}{AC} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}
\]

\[
\tan 45^\circ = \frac{BC}{AB} = \frac{a}{a} = 1
\]
Also, \(\cos 45^\circ = \frac{1}{\sin 45^\circ} = \sqrt{2}\)

\[\sec 45^\circ = \sqrt{2}\]

\[\cot 45^\circ = 1\]

**T-Ratios of 30° and 60°**

You can take an equilateral \(\Delta ABC\)

So, \(\angle A = \angle B = \angle C = 60^\circ\)

Draw, \(AD \perp BC\)

Now \(\Delta ABD \cong \Delta ACD\) (by RHS)

So, \(BD = CD\)

and \(\angle BAD = \angle CAD\)

You know that to find the value of all the t-ratios, you must know all the three sides of the triangle.

In \(\Delta ABD\), let \(AB = 2a\)

As \(BD = CD\) So \(BD = \frac{1}{2}BC = \frac{1}{2} \times 2a = a\)

By Pythagoras theorem,

\[AD^2 = AB^2 - BD^2\]

\[= (2a)^2 - a^2 = 3a^2\]

so, \(AD = a\sqrt{3}\)

Therefore, in \(\Delta ABD\), \(AB = 2a\), \(BD = a\) and \(AD = a\sqrt{3}\)

Now you as well as your students can find all the t-ratios for angle \(60^\circ\) and \(\angle BAD = 30^\circ\)

Ask student to all find the t-ratios yourself.

**T-Ratios of 0° and 90°**

In the figure, in right \(\Delta ABC\),

If \(\angle A\) is made smaller and smaller, the length of the side \(BC\) goes on decreasing and point \(C\) gets closer and closer to point \(B\) and finally when \(\angle A\) becomes very close to 0°, \(BC\) gets very close to 0 and \(AC\) becomes almost the same as \(AB\).
So, \( \sin A = \frac{BC}{AC} \) is very close to 0.

\( \cos A = \frac{AB}{AC} \) is very close to 1.

So, you define

\[
\sin 0^\circ = 0 \\
\cos 0^\circ = 1
\]

Using these, you have

\[
\tan 0^\circ = \frac{\sin 0^\circ}{\cos 0^\circ} = 0 \\
\cot 0^\circ = \frac{1}{\tan 0^\circ} = \infty \text{ which is not defined as division by 0 is not defined.} \\
\sec 0^\circ = \frac{1}{\cos 0^\circ} = 1 \\
\csc 0^\circ = \frac{1}{\sin 0^\circ} = \infty \text{ which again is not defined.}
\]

Similarly, when \( \angle A \) is made larger and larger in \( \triangle ABC \) till it becomes \( 90^\circ \), point A gets closer to point B and side AC almost coincides with side BC.

Thus, you have

\[
\sin 90^\circ = 1 \quad \text{and} \quad \cos 90^\circ = 0
\]

Ask student to find the other Trigonometric ratios of \( 90^\circ \).

**Note:**

It should be noted that there exist values of all the trigonometric ratios for angles other than these angles also. The values can be obtained from trigonometric tables which your student will study in higher classes.

Values of all t-ratios of \( 0^\circ, 30^\circ, 45^\circ, 60^\circ \) and \( 90^\circ \).

<table>
<thead>
<tr>
<th>( \angle A )</th>
<th>( 0^\circ )</th>
<th>( 30^\circ )</th>
<th>( 45^\circ )</th>
<th>( 60^\circ )</th>
<th>( 90^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin A )</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{\sqrt{2}} )</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>1</td>
</tr>
<tr>
<td>( \cos A )</td>
<td>1</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>( \frac{1}{\sqrt{2}} )</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
</tr>
<tr>
<td>( \tan A )</td>
<td>0</td>
<td>( \frac{1}{\sqrt{3}} )</td>
<td>1</td>
<td>( \sqrt{3} )</td>
<td>Not defined</td>
</tr>
</tbody>
</table>
### Table

<table>
<thead>
<tr>
<th></th>
<th>cosec A</th>
<th>sec A</th>
<th>cot A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not defined</td>
<td>$\frac{2}{1}$</td>
<td>$\frac{1}{\sqrt{3}}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{2}{\sqrt{3}}$</td>
<td>$\frac{2}{\sqrt{3}}$</td>
<td>$\frac{1}{\sqrt{3}}$</td>
</tr>
<tr>
<td>Not defined</td>
<td>$\frac{2}{\sqrt{3}}$</td>
<td>$\frac{2}{\sqrt{3}}$</td>
<td>$\frac{1}{\sqrt{3}}$</td>
</tr>
</tbody>
</table>

From the above table, students can observe that as $\angle A$ increases from $0^\circ$ to $90^\circ$, $\sin A$ increases from 0 to 1 and $\cos A$ decreases from 1 to 0 and also that

\[0 \leq \sin A \leq 1\]
\[1 \leq \cos A \leq 0\]

**Example 4:**

Evaluate $\cot^2 45^\circ - \sec^2 60^\circ + \sin^2 60^\circ + \cos^2 90^\circ$

**Solution:** Given expression

\[
= 1^2 - 2^2 + \frac{\sqrt{3}}{2}^2 + 0^2
\]

\[= 1 - 4 + \frac{3}{4}
\]

\[= -\frac{9}{4}
\]

In previous section, you have found all the t-ratios when any two side of the right triangle are given. Now if, your students are given one side of the triangle and one angle of the right triangle, then they can find the other two sides of the triangle. Let us see an example.

**Example 5:**

In $\triangle ABC$, right angled at $B$, if $AB = 5$ cm, $\angle C = 30^\circ$, determine the sides $BC$ and $AC$.

**Solution:**

You are given $AB$ and we want to find $BC$. So we will choose that-ratio, which involves these two sides. Here, it is $\tan C$ (or $\cot C$)

\[\tan 30^\circ = \frac{AB}{BC}\]

Ask student, as they know that $\tan 30^\circ = \frac{1}{\sqrt{3}}$

So, $\frac{1}{\sqrt{3}} = \frac{5}{BC}$

Or, $BC = 5\sqrt{3}$ cm

To determine $AC$, you have
\[
\sin C = \frac{AB}{AC} \\
\sin 30^\circ = \frac{5}{AC} \\
i.e., \quad \frac{1}{2} = \frac{5}{AC}
\]

Or, \( AC = 10 \text{ cm} \)

To determine \( AC \) you could have used \( \cos C \), \( \sec C \) or Pythagoras theorem also.

Now, if your students are given two sides of right triangle they can find its angles also. Let us an example.

**Example 6:-**

In right \( \triangle PQR \), right angled at \( Q \), \( PQ = 3 \text{ cm} \) and \( PR = 6 \text{ cm} \), determine \( \angle P \), \( \angle R \).

**Solution:**

You choose that t-ratio which involves the two given sides.

\[
\sin R = \frac{PQ}{PR} = \frac{3}{6} = \frac{1}{2}
\]

So, \( \angle R = 30^\circ \)

and so, \( \angle P = 60^\circ \)

From the above, you must have noticed that if one of the sides and any other part either an acute angle or any side of a right triangle are given, you can find the remaining sides and angles of the triangle.

**Check Your Progress**

**Notes:**

a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

4. Evaluate \( 4 \cot^2 45^\circ - \sec^2 60^\circ + \sin^2 60^\circ + \cos^2 90^\circ \)

5. In \( \triangle PQR \), right angled at \( Q \),

if \( PQ = 12 \text{ cm}, \quad \angle P = 60^\circ \),

determine \( PR \) and \( QR \).
6. In \( \Delta ABC \) right angled at B, if \( AB = 3 \text{cm} \), \( AC = 2 \sqrt{3} \text{cm} \), find \( \angle A \) and \( \angle C \).

16.3.3 Trigonometric Ratios of Complementary Angles

Ask your student to recall that two angles are complementary angles, if their sum is \( 90^\circ \).

In right \( \Delta ABC \) right angled at B,
\( \angle A + \angle C = 90^\circ \).

For convenience, you can write
\( A + C = 90^\circ \)
i.e. \( C = 90^\circ - A \)

Now,
\[
\sin A = \frac{BC}{AC}, \quad \cos A = \frac{AB}{AC} \\
\sin C = \frac{AB}{AC}, \quad \cos C = \frac{BC}{AC}
\]

You can see that
\[
\sin A = \cos C \quad \text{and} \quad \cos A = \sin C
\]
i.e. \( \sin A = \cos (90^\circ - A) \), \( \cos A = \sin (90^\circ - A) \)

Similarly, you can show other t-ratios.

So, for angles lying between \( 0^\circ \) and \( 90^\circ \), you have
\[
\sin (90^\circ - A) = \cos A \\
\cos (90^\circ - A) = \sin A \\
\tan (90^\circ - A) = \cot A \\
\cos ec (90^\circ - A) = \sec A \\
\sec (90^\circ - A) = \cos ec A \\
\cot (90^\circ - A) = \tan A
\]
so, you have
\[
\sin 30^\circ = \cos 60^\circ \\
\cos 30^\circ = \sin 60^\circ \\
\tan 30^\circ = \cot 60^\circ \\
\cos ec 30^\circ = \sec 60^\circ \\
\sec 30^\circ = \cos ec 60^\circ \\
\cot 30^\circ = \tan 60^\circ
\]
Angle 45° is its own complement. Hence
\[ \sin 45° = \cos 45° \]
\[ \tan 45° = \cot 45° \]
\[ \sec 45° = \cosec 45° \]

**Example 7**

Evaluate: \( \cos 48° - \sin 42° \)

**Solution**

\[
\cos 48° - \sin 42° = \cos 48° - \sin (90° - 48°)
\]
\[
= \cos 48° - \cos 48°
\]
\[
= 0
\]

**Example 8**

If \( \sec 4A = \cosec (A - 20°) \), where 4A is an acute angle, find A.

**Solution**

since, \( \sec 4A = \cosec (90° - 4A) \)

So, \( \cosec (90° - 4A) = \cosec (A - 20°) \)

As 90° - 4A and A - 20°, both are acute angles, you have

\[
90° - 4A = A - 20°
\]

Or

\[
5A = 110°
\]

Or

\[
A = 22°
\]

**Check Your Progress**

**Notes:**

a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

7) Evaluate \( \frac{\sin 18°}{\cos 72°} \)

8) Prove that

\[
\frac{\cos 20°}{\sin 70°} + \frac{\cos \theta}{\sin (90° - \theta)} = 2
\]

9) If \( \sin 3A = \cos (A - 26°) \), where 3A is an acute angle, find the value of A.
16.4 TRIGONOMETRIC IDENTITIES

Can you recall what is an identity?
An equation which is true for all the values of the variables is called an identity.

A trigonometric identity is an equation, which involves Trigonometric ratios and is true for all the values of the angles involved.

Consider Δ ABC right angled at B

\[ AB^2 + BC^2 = AC^2 \]  \(\text{(1)}\)

Dividing each term of (1) by \(AC^2\)

You will have

\[ \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = 1 \]

i.e. \(\left( \frac{AB}{AC} \right)^2 + \left( \frac{BC}{AC} \right)^2 = 1 \)

or \((\cos A)^2 + (\sin A)^2 = 1\)

or \(\cos^2 A + \sin^2 A = 1\)

This is true for all \(A\) such that \(0^\circ \leq A \leq 90^\circ\),

So, this is a Trigonometric Identity.

In (i) above, dividing by \(AB^2\), you will have

\[ \left( \frac{AB}{AB} \right)^2 + \left( \frac{BC}{AB} \right)^2 = \left( \frac{AC}{AB} \right)^2 \]

i.e. \(1 + \tan^2 A = \sec^2 A \) \(\text{(2)}\)

As \(\tan A\) and \(\sec A\) are not defined for \(A = 90^\circ\), so (2) is true for all \(A\) such that \(0^\circ \leq A < 90^\circ\).

Similarly, dividing (i) by \(BC^2\), you can get

\[ \left( \frac{AB}{BC} \right)^2 + \left( \frac{BC}{BC} \right)^2 = \left( \frac{AC}{BC} \right)^2 \]

i.e. \(\cot^2 A + 1 = \cosec^2 A \) \(\text{(3)}\)

(3) is true for all \(A\) such that \(0^\circ < A \leq 90^\circ\).

Thus, you will have

\[ \sin^2 A + \cos^2 A = 1 \]

\[ \sec^2 A = 1 + \tan^2 A \]

\[ \cosec^2 A = 1 + \cot^2 A \]

These three relations are all identities and are called fundamental identities. Each of these identities can be obtained from the other.

If your students know one t-ratio, they can determine other t-ratios using these identities.
Suppose you are given \( \sin A \). From the identity \( \sin^2 A + \cos^2 A = 1 \) you can find \( \cos A \). Also, \( \tan A \) will be obtained from the relations \( \tan A = \frac{\sin A}{\cos A} \).

So, now you know \( \sin A, \cos A \) and \( \tan A \) and the other t-ratio are reciprocals of these.

**Example 9**

Express \( \csc \theta \) in terms of \( \cos \theta \).

**Solution:**

Since, \( \sin^2 \theta + \cos^2 \theta = 1 \)

So, \( \sin^2 \theta = 1 - \cos^2 \theta \)

or, \( \frac{1}{\csc^2 \theta} = 1 - \cos^2 \theta \)

or, \( \csc^2 \theta = \frac{1}{1 - \cos^2 \theta} \)

or, \( \csc \theta = \pm \frac{1}{\sqrt{1 - \cos^2 \theta}} \)

As \( \theta \) is an acute angle, you have

\( \csc \theta = \frac{1}{\sqrt{1 - \cos^2 \theta}} \)

**Example 10**

Prove that \( (\csc \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta} \)

**Solution:**

\[
\text{L.H.S.} = \left( \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2
\]

\[
= \left( \frac{1 - \cos \theta}{\sin \theta} \right)^2 = \frac{(1 - \cos \theta)^2}{\sin^2 \theta}
\]

\[
= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}
\]

\[
= \frac{(1 - \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)} = \frac{1 - \cos \theta}{1 + \cos \theta} = \text{R.H.S.}
\]

**Check Your Progress**

**Notes:**

a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

10) Express \( \sec A \) in terms of \( \sin A \).

………………………………………………………………………

………………………………………………………………………

………………………………………………………………………
11) Simplify \[ \frac{1 + \tan^2 A}{1 + \cot^2 A} \]

………………………………………………………………………
………………………………………………………………………
………………………………………………………………………
………………………………………………………………………
………………………………………………………………………

12) Prove that
\[ \tan^4 A + \tan^2 A = \sec^4 A - \sec^2 A \]
………………………………………………………………………
………………………………………………………………………
………………………………………………………………………
………………………………………………………………………
………………………………………………………………………

16.5 HEIGHT AND DISTANCE

In the earlier section you have studied about trigonometric ratios. In this section, you will study how trigonometry is used in finding the heights and distances of various objects without actually measuring them.

Whenever, an engineer faces problem in determining the width of a river or height of a tower etc., which may not be easily possible to measure with a measuring tape, she/he imagines a big right triangle. Knowing a side and an angle by using any surveying instrument she/he can use the knowledge of trigonometric ratios to calculate the unknown side i.e. width of the river or height of the tower.

Let us consider an observer viewing a certain object

The line drawn from the eye of the observer to the point in the object being viewed is called the line of sight. The angle made by the line of sight with the horizontal is called the angle of elevation or angle of depression depending upon the object viewed is above the horizontal line or below the horizontal line.
If the object being viewed is above the horizontal line, the angle \( \angle BAC \) is the angle of elevation as in fig (i).

If the observer is standing on a balcony and viewed the object C on the road the \( \angle BAC \) is the angle of depression as in the figure (ii). Note here \( \angle CAD \) is not the angle of depression.

Now let us solve problems.

**Example 11**

An Electrician has to repair an electric fault on a pole height 5 m. He needs to reach a point 1.3m below the top of the pole to undertake the repair work what should be the length of the ladder that he should use which, when inclined at an angle of 60\(^{\circ}\) to the horizontal, would enable his to reach the required position? Also, how far from the foot of the pole should he place the foot of the ladder? (take \( \sqrt{3} = 1.73 \))

**Solution:-**

The electrician has to reach to point B on the pole AD

So, \( BD = AD - AB = (5 - 1.3) \text{ m} = 3.7 \text{ m} \)

Here, BC represents the ladder in the right \( \Delta BCD \) and we wants to find its length.

Which t-ratio should be used?

It should be Sin60\(^{\circ}\)

So, \( \frac{BD}{BC} = \sin 60^{\circ} \) or \( \frac{3.7}{BC} = \frac{\sqrt{3}}{2} \)

or, \( BC = \frac{3.7 \times 2}{\sqrt{3}} = 4.28 \text{ m (approx)} \)

i.e. length of the ladder should be 4.28m.

We also want to find how far from the foot of the pole should be the foot of the ladder.

\( \frac{DC}{BC} = \cot 60^{\circ} = \frac{1}{\sqrt{3}} \)

Or, \( \frac{DC}{3.7} = \frac{1}{\sqrt{3}} \)

So, \( DC = \frac{3.7}{\sqrt{3}} = 2.14 \text{ m (approx)} \)

Therefore, she should place the foot of the ladder at a distance of 2.14m from the pole.

**Example 12**

From a point on the ground 40 m away from the foot of a tower, the angle of elevation of the top of the tower is 30\(^{\circ}\). The angle of the elevation to the top of
a water tank on the top of the tower is $45^\circ$. Find the height of the tower and the depth of the tank.

**Solution**

In right $\Delta ACB$, we have

$$\frac{BC}{AC} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

or $$\frac{BC}{40} = \frac{1}{\sqrt{3}}$$

or $$BC = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3} \approx 23.1 \text{m (approx.)}$$

Thus, height of tower is 23.1 m (approx).

To find the depth $BD$ of the water tank you require the length $CD$

In $\Delta ACD$, you have

$$\frac{CD}{AC} = 1 \quad \text{or} \quad \frac{CD}{40} = 1 \quad \text{or} \quad CD = 40 \text{m}$$

Hence, depth of the water tank = $BD \Rightarrow CD - CB \Rightarrow (40 - 23.1) \text{m} \Rightarrow 16.9 \text{m (approx.)}$

**Example 13**

A tree stands vertically on the bank of a river. From a point on the other bank directly opposite the tree, the angle of the elevation of the top of the tree is $60^\circ$.

From a point 20m behind this point on the same bank, the angle of elevation of the top of the tree is $30^\circ$. Find the height of the tree and the width of the river.

**Solution**
Let AB be the height of the tree and let C and D be the two points on the other bank opposite to the tree so that BC measures the width of the river, you want to find AB and BC.

In $\triangle ABC$
\[
\frac{AB}{BC} = \tan 60^\circ
\]
Or $AB = BC \sqrt{3}$  \hspace{1cm} (i)

Now in $\triangle ABD$
\[
\frac{AB}{BD} = \tan 30^\circ = \frac{1}{\sqrt{3}}
\]
i.e. $\frac{AB}{(20 + BC)} = \frac{1}{\sqrt{3}}$

or $AB = \frac{20 + BC}{\sqrt{3}}$  \hspace{1cm} (ii)

From (i) & (ii) you will have
\[
BC \sqrt{3} = \frac{20 + BC}{\sqrt{3}}
\]
or $3BC = 20 + BC$
or, $2BC = 20$
or, $BC = 10$

So, by (i) $AB = 10\sqrt{3} \approx 17.3$ m (approx.)

Hence, height of the tree is 17.3 m approx. and width of the river is 10 m.

**Example 14**

The angels of depression of the top and bottom of an 8m tall building from the top of a multistoried building are $30^\circ$ and $45^\circ$ respectively. Find the height of the multi-storied building and the distance between the two buildings.

**Solution**

In the figure, AB denotes the multi-storied building and CD the 8m tall building.

You want to find out AB and BD.

\[
\angle FAC = \angle ACE \quad \text{and} \quad \angle FAD = \angle ADB
\]

So, $\angle ACE = 30^\circ$ and $\angle ADB = 45^\circ$

Now in right $\triangle ACE$,
\[
\frac{AE}{CE} = \tan 30^\circ = \frac{1}{\sqrt{3}}
\]

So, $CE = AE \sqrt{3}$  \hspace{1cm} (1)
In right \( \triangle ADB \),
\[
\frac{AB}{BD} = \tan 45^\circ = 1
\]
So, \( BD = AB \) \quad \text{----------(2)}
Also \( BD = CE \) \quad \text{----------(3)}
So, by (1), (2) & (3) you have,
\[
CE = AE \sqrt{3} = BD = AB = AE + BE = AE + 8
\]
so, \( AE \sqrt{3} = AE + 8 \)
i.e, \( AE (\sqrt{3} - 1) = 8 \)
so, \( AE = \frac{8}{\sqrt{3} - 1} = \frac{8(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \frac{8(\sqrt{3} + 1)}{3 - 1} = 4(\sqrt{3} + 1) \)
So, height of the multi-storied building
\[
= AB = AE + BE = 4(\sqrt{3} + 1) + 8 = 4(\sqrt{3} + 3) \text{ m}
\]
Distance between two buildings = \( BD = AB = 4(\sqrt{3} + 3) \text{ m} \)

Check Your Progress

Notes: a) Write your answers in the space given below.
    b) Compare your answers with those given at the end of the unit.

13) The length of the shadow of a man is equal to the height of the man. The angle of depression is?

14) From a point 20 m away from the foot of the tower the angle of elevation of the top of the tower is 30°. The height of the tower is:

15) A circus artist is climbing from the ground along a rope stretched from the top of vertical pole and tied at the ground. The height of the pole is 12 m and the angle made by the rope with ground level is 30°. Calculate the distance covered by the artist in climbing to the top of the pole.
16) From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 3m from the banks, find the width of the river.

16.6 LET US SUM UP

In this unit, we have defined six Trigonometric ratios. These ratios are unique for any given angle and form the basic building blocks in the study of trigonometry. T-ratios of some specific angles and the t-ratios of complementary angles are derived. Three fundamental identities using Pythagoras theorem and inter relationship of t-ratios has been used to solve problems on trigonometric identities.

An important use of trigonometry has been illustrated by solving problems on height and distances. You would have realized that for solving height and distance problems, it is very necessary to draw a diagram as per the given information. Thus an elementary treatment of Trigonometry has been provided in this unit since it lays a foundation for further study of subject.

16.7 UNIT END ACTIVITIES

1) In ΔPQR, \( Q = 90° \) and \( \sin R = \frac{3}{5} \), write the value of \( \cos P \).

2) Given that \( 16 \cot A = 12 \), find the value of \( \frac{\sin A + \cos A}{\sin A - \cos A} \).

3) Simplify
\[
\tan^2 60° + 2 \cos^2 45° + 3 (\sec^2 30° + \cos^2 90°)
\]

4) Evaluate
\[
\frac{2 \cos 58°}{\sin 32°} - \sqrt{3} \cdot \frac{\cos 38° \cdot \sec 52°}{\tan 15° \cdot \tan 60° \cdot \tan 75°}
\]

5) If \( \sin 2\theta = \cos (\theta - 36°) \), \( 2\theta \) and \( \theta - 36° \) are acute angles, then find the value of \( \theta \).

6) Prove that:-
\[
\frac{\cos A}{1 - \tan A} + \frac{\cos A}{1 - \cot A} = \cos A, \quad A \neq 45°
\]

7) From a point P on the ground the angle of elevation of the top of a 10m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from the point P. (\( \sqrt{3} = 1.732 \))
8) The angle of elevation of the top of a building from the foot of the tower is $30^\circ$ and the angle of elevation of the top of the tower from the foot of the building is $60^\circ$. If tower is 50m high, find the height of the building.

9) As observed from the top of a 75m high lighthouse from the sea level, the angles of depression of two ships are $30^\circ$ and $45^\circ$. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between two ships.

### 16.8 ANSWERS TO CHECK YOUR PROGRESS

1) $\frac{4}{5}, \frac{3}{5}, \frac{3}{4}, \frac{5}{3}$

2) 0

3) $\frac{119}{169}$

4) $\frac{3}{4}$

5) 24cm, $12\sqrt{3}$cm

6) $30^\circ, 60^\circ$

7) 1

8) 29°

9) $\frac{1}{\sqrt{1-\sin^2 A}}$

10) $\tan^2 A$

11) 45°

12) $20/\sqrt{3}$

13) 24m

14) $3(\sqrt{3}+1)$m

### 16.9 REFERENCES AND SUGGESTED READINGS

UNIT 17  MENSURATION AND COORDINATE GEOMETRY

Structure

17.1  Introduction
17.2  Objectives
17.3  Measurement of Perimeter and Area
   17.3.1  Perimeter of Rectangle, Square and Triangle
   17.3.2  Circumference of the Circle
   17.3.3  Area of Trapezium, Quadrilateral and Polygon
   17.3.4  Surface Area of Cuboid, Cube, Cylinder, Cone and Sphere
17.4  Measurement of Volume
   17.4.1  Volume of Cuboid and Cube
   17.4.2  Volume of Cylinder
   17.4.3  Volume of Cone
17.5  Coordinate Geometry: Basics and Use
17.6  Distance Formula
17.7  Section Formula
17.8  Let Us Sum Up
17.9  Unit End Activities
17.10 Answers to Check Your Progress
17.11 References and Suggested Readings

17.1  INTRODUCTION

Children are familiar with objects like notebook, pencil, lunch box, writing table, bench, desk and so on. When children think of such objects, generally the shape comes to their mind. In addition, the grown ups think about their boundaries, space covered, area, etc. In the case of objects mentioned, each of these objects is in the form of certain geometrical figures i.e. square, rectangle, cylinder, circle, etc. In Mathematics the study of shapes occupies a prominent role as it has relevance in construction of buildings, houses, bridges, play grounds, etc. While constructing a new home, the shape and size matters. The same is experienced when children arrange their bench and desks in the classroom. In such situations, we do take measurements and the plan is executed accordingly. The area concerning measuring various dimensions of geometrical figures is termed as Mensuration. So in first section of this unit, we will discuss concepts of perimeter, area and volume. Then of this unit we will try to explore deductive methods for arriving at formula for perimeter, area and volume for different objects. Further, we will study about basics of co-ordinate geometry and its applications in day-to-day life*

* Few examples and figures of this Unit has been adopted from Mathematics NCERT Textbooks
17.2 OBJECTIVES

After going through this unit, you will be able to:

• help students understand the meaning of area, perimeter and volume;
• use of the methods of measuring area and perimeter;
• determine the volume of various objects;
• appreciate the beauty of doing geometry in algebraic way;
• recall the basics of Cartesian system;
• develop the skill of proving distance formula and section formula;
• apply them in different situations; and
• help students to develop problem solving skills.

17.3 MEASUREMENT OF PERIMETER AND AREA

Meena, a student of eighth class asked her Mathematics teacher, “Madam, each day I am running two rounds in our school playground. But I don’t know how much distance I cover each day? Could you please help me to calculate it?”

The teacher had used this question to initiate chapter on Mensuration in some other classes. To this question, teacher started responding, “Students, we have seen various geometrical shapes such as squares, rectangles, circles, etc.” (Teacher draws various geometrical shapes on black board)

Fig 17.1: Geometrical Shapes

After drawing the figures, she continued asking questions, “Students, how will you list the difference among these figures? Is there something common among the figures? Are they similar?” Few of the students responded but many kept silent. Then teacher continued, “Children, we need to have idea about various geometrical figures and its related dimensions, to compare the figures which would enable one to distinguish.” In the case of geometrical figures, to assess them, we find its dimensions and measurements. Few of such measurements are perimeter, area, surface area and volume. As you know, generally we find two types of figures (shapes) i.e. plane figures and solid figures. The formula for calculating perimeter, area and volume of plane figures and solid figures are different and we would discuss the same in today’s class.

You have seen how the Mathematics teacher has made her first move to introduce the chapter on Mensuration. In Mensuration, the different aspects concerning plane and solid figures are discussed. Being a teacher you may also think of alternative strategies to introduce the topic Mensuration that would create an attention grabbing atmosphere in the classroom. As discussed above, one of the measurements concerning plane figures is the perimeter of the figure. What is perimeter? Perimeter is the distance covered along the boundary forming a closed figure when you go round the figure once. Suppose you start from point A and travel as shown (Figure 17.2) to reach the same point, the distance covered is equal to perimeter of the figure. The
Mensuration and Coordinate Geometry

The perimeter of the plane figure is the distance of the outer boundary of the figure. The knowledge of perimeter is helpful in the following situations:

- Construction of compound wall for houses, educational organizations, etc.
- Partition of cabins and rooms in buildings
- To mark tracks in playgrounds

![Figure 17.2](image) - How will you calculate the perimeter of plane figures? Let us start with a simple figure. In order to calculate the perimeter, you need to have understanding of units and its conversion from one to the other (For example converting cm to m, mm to cm, etc.). In the Figure 17.3, first the distances \( AB, BC, CD, DE, EF \), and \( FA \) are calculated and they are added. The resulting value would be the perimeter of the figure. The calculation is given below:

![Figure 17.3](image) - Perimeter of figure:

\[
\text{Perimeter} = AB + BC + CD + DE + EF + FA = 3\text{cm} + 5\text{cm} + 5\text{cm} + 2\text{cm} + 2\text{cm} + 3\text{cm} = 20\text{cm}
\]

**Check Your Progress**

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

1) Suggest a learning activity to introduce the concept of Mensuration.

2) What is perimeter? How will you calculate the perimeter of plane figures?
17.3.1 Perimeter of Rectangle, Square and Triangle

Consider a rectangle with measurements as shown in the Figure 17.4. In this case the perimeter is found out as follows:

Perimeter of the rectangle = Sum of the lengths of its four sides
\[ = AB + BC + CD + DA \]
\[ = AB + BC + AB + BC \] (Since CD = AB and AD = BC)
\[ = 2 \times AB + 2 \times BC \]
\[ = 2 \times (AB + BC) \]
\[ = 2 \times (7 \text{ cm} + 3 \text{ cm}) \]
\[ = 2 \times (10 \text{ cm}) \]
\[ = 20 \text{ cm} \]

Thus we can say that;
Perimeter of the rectangle = length + breadth + length + breadth

Or Perimeter of a Rectangle = 2 × (Length + Breadth)

Now let us calculate perimeter of few regular closed figures. What is the peculiarity of regular closed figures? Figures that have all sides equal length and all angles of equal measure are known as regular figures. For example, square, equilateral triangle, etc. In the below given box, the perimeter of a square and an equilateral triangle are calculated:

Perimeter of Square = Sum of the lengths of its four sides
\[ = AB + BC + CD + DA \] (Since AB = BC = CD = DA)
\[ = 4 \times AB \]
\[ = 4 \times 4 \text{ cm} \]
\[ = 16 \text{ cm} \]

Therefore we can say that, instead of adding the sides four times, multiply one side by 4, which would give the perimeter of the square. Thus

**Perimeter of Square = 4 × Length of one side**

Perimeter of equilateral triangle = Sum of the lengths of its three sides
\[ = AB + BC + CA \] (Since AB = BC = CA)
\[ = 3 \times AB \]
\[ = 3 \times (3 \text{ cm}) \]
\[ = 9 \text{ cm} \]

Therefore we can say that, instead of adding the sides three times, multiply one side by 3, which would give the perimeter of an equilateral triangle. Thus

**Perimeter of Equilateral Triangle = 3 × Length of one side**
In the above section we have derived the formula for finding perimeter of a regular figure. Now the question is what would be the perimeter of a regular closed figure having five sides? For example, a pentagon. In the case of pentagon or figures having equal sides (regular polygons), we may deduce that, the perimeter is:

**Perimeter of a regular polygon= Number of sides × Length of one side.**

Check Your Progress
Note:  a) Write your answers in the space given below.
     b) Compare your answers with those given at the end of the Unit.

3) Find the perimeter of the following figure:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>9 cm</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>9 cm</td>
<td>C</td>
<td></td>
</tr>
</tbody>
</table>

17.3.2 Circumference of the Circle

So far we discussed perimeter of few shapes such as rectangle, square, equilateral triangle etc. Now let us investigate the question put forwarded by Meena. She wants to know the distance covered as she takes two rounds in the playground (The playground is circular in shape). The distance around a circular region is called **circumference of the circle**. So if we want to calculate the distance covered by Meena, we need to calculate the circumference of the circular ground and then multiply by two. How circumference of circle is calculated? We may use a string for the same. Stretch the string from point A and move along till it reaches the same point (Fig 17.7).

Then measure the length of the string using a scale (cm or m scale may be used) which would give the circumference. Such a measurement is practically impossible, if the circle is big and hence we have to arrive at a formula which can be used to find the circumference of the circle.

In order to arrive at the formula for the circumference of the circle, let us examine the following table:

<table>
<thead>
<tr>
<th>Circle</th>
<th>Radius in cm</th>
<th>Diameter in cm</th>
<th>Circumference in cm</th>
<th>Ratio of Circumference to Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.5</td>
<td>7</td>
<td>22</td>
<td>22/7=3.14</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>14</td>
<td>44</td>
<td>44/14=3.14</td>
</tr>
<tr>
<td>3</td>
<td>10.5</td>
<td>21</td>
<td>66</td>
<td>66/21=3.14</td>
</tr>
<tr>
<td>4</td>
<td>21</td>
<td>42</td>
<td>132</td>
<td>132/42=3.14</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>10</td>
<td>32</td>
<td>32/10=3.2</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>30</td>
<td>94</td>
<td>94/30=3.13</td>
</tr>
</tbody>
</table>

(Source: 7th Class Mathematics Text Book, NCERT)
Here the circumference is calculated using string method. From the table, it is found that, the circumference is approximate three times diameter and the ratio of circumference to diameter is constant. This constant is denoted by the term \( \pi \) (pi) and its approximate value is \( 22/7 = 3.14 \). Thus we can deduce that;

\[
\frac{C}{d} = \pi \quad \text{(Where 'C' is the circumference of the circle and 'd' is the diameter)}
\]

Therefore, \( C = \pi d = \pi 2r \) (Since \( d = 2r \) i.e. diameter of the circle is twice the radius)

Or \( C = 2\pi r \)

Thus circumference of the circle is given by the formula, \( C = 2\pi r \)

Now let us discuss a practical application of measuring perimeter. Consider the following example:

**Example 1:** A farmer wishes to fence his farmland. The length of the sides of the land is 82m, 222m, 104m and 282 m. If it costs ₹ 50 per metre to fence, what will be total cost required to fence the farmland?

![Fig. 17.8](image)

**Solution**

In this case, firstly, we have to find the perimeter of the farmland and then it has to be multiplied with the money required for one metre. The calculation goes like this:

Perimeter of the farmland = 82m + 222m + 104m + 282 m = 690m

Cost required = ₹50 × 690 = ₹34500

**17.3.3 Area of Trapezium, Quadrilateral and Polygon**

You as a teacher have the knowledge that, children studied few facts related to the concept area in their earlier classes and they have understood area as the region occupied by the closed figure or amount of surface enclosed by a closed figure. How will you start the topic measurement of area? A simple method would be, you may distribute different leaves to students and ask them to find the area. Or else even you can ask children to measure area of their notebook/textbooks. A more interesting strategy would be, split children into different groups, provide them objects of different shapes and direct them to find the area of each these objects. At this point, you may help your students to recall the following points;

- Area is the amount of surface enclosed by a closed figure.
• To calculate area using squared paper/graph paper, following conventions are used:
  a) Ignore portions of the area that are less than half a square.
  b) If more than half a square is in a region, count it as one square.
  c) If exactly half the square is counted, take its area as ½ sq. units.
• Area of rectangle=length × breadth
• Area of square=side × side
• Area of parallelogram=base × height
• Area of triangle=1/2(area of the parallelogram generated form it)
  =1/2 × base × height
• All the congruent triangles are equal in area but the triangles equal in area need not be congruent.
• Area of circle=$\pi r^2$ (where r is the radius of the circle)

Let us recall, the following facts (Teacher shows the following chart in the classroom)

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Shape</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Rectangle</td>
<td>axb</td>
</tr>
<tr>
<td>b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>Square</td>
<td>axa</td>
</tr>
<tr>
<td>a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>Triangle</td>
<td>½ bxh</td>
</tr>
<tr>
<td>b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>Parallelogram</td>
<td>bxh</td>
</tr>
<tr>
<td>b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>r</td>
<td>Circle</td>
<td>$\pi r^2$</td>
</tr>
</tbody>
</table>

In the coming sections we will explore formula to find area of few other shapes. Let us do it one by one and later look at its practical applications.
Area of Trapezium

Consider a trapezium as shown in the Figure 17.9. To find the area of the trapezium, draw EC ∥ AB. Then we get a rectangle ABCE and triangle ECD right angled at C.

Therefore, Area of trapezium ABDE = Area of triangle + Area of rectangle

Area of trapezium = Area of Δ ECD + Area of ABCE

\[ \text{Area of trapezium} = \frac{1}{2} (h \times c) + \frac{1}{2} (h \times a) \]

\[ = h \left( \frac{c+2a}{2} \right) = h \left( \frac{c+a+a}{2} \right) \]

\[ = \frac{h(b+a)}{2} \]

i.e. Area of Trapezium =
\[ \text{height} \times \frac{\text{(sum of parallel sides)}}{2} \]

Or Area of trapezium =
\[ \text{perpendicular distance between the parallel sides} \times \frac{\text{(sum of parallel sides)}}{2} \]

So to find the area of a trapezium we need to know the length of the parallel sides and the perpendicular distance between these two parallel sides. Half the product of the sum of the lengths of parallel sides and the perpendicular distance between them gives the area of trapezium.

Let us see an example to make idea clear about it.

Example 2. Find the area of the following figure ABCD.

Solution:

Area of trapezium =
\[ \text{height} \times \frac{\text{(sum of parallel sides)}}{2} \]

\[ = 3 \times \frac{(8+6)}{2} = 21 \text{ cm}^2 \]

Area of General Quadrilateral

Consider a quadrilateral ABCD (children should be convinced that parallelogram is also a quadrilateral). To find the area, draw any diagonal to get two triangles as shown in the Figure 17.10. This process is known as ‘triangulation’. At this stage children should made to recall the process of triangulation, which they have come across in their previous classes.
Thus area of quadrilateral $ABCD = \text{Area of } \triangle ABC + \text{Area of } \triangle ADC$
\[
= \left( \frac{1}{2} AC \times h_1 \right) + \left( \frac{1}{2} AC \times h_2 \right) \\
= \frac{1}{2} AC \times (h_1 + h_2) \\
= \frac{1}{2} d(h_1 + h_2)
\]
Where $d =$ Length of the diagonal $AC$

**Area of Special Quadrilateral (Rhombus)**

Consider a rhombus $ABCD$ as shown in the Figure 17.11. To calculate the area of rhombus, draw two diagonals resulting in $\triangle ABC$ and $\triangle ACD$. The two diagonals are perpendicular bisectors.

Then, area of rhombus $= \text{Area of } \triangle ADC + \text{Area of } \triangle ABC$
\[
= \frac{1}{2} (AC \times OD) + \frac{1}{2} (AC \times OB) \\
= \frac{1}{2} AC \times (OD+OB) \\
= \frac{1}{2} AC \times BD \\
= \frac{1}{2} d_1 \times d_2
\]

Area of Rhombus$= \frac{1}{2} (d_1 \times d_2)$
where $d_1$ and $d_2$ are length of diagonals

**Area of a Polygon**

Apart form the general shapes, children come across various shapes that have multiple sides and we call them polygon. In order to calculate area of any polygon, we divide them into various triangles and quadrilateral and area is separately calculated for each of the triangles and quadrilateral. Then after, these areas are added to get the final area. For example, consider a polygon as shown in the Figure 17.12 Here the polygon $ABCDE$ is divided into three triangles namely $\triangle ABC$, $\triangle ACD$ and $\triangle AED$. Thus the area of the polygon $ABCDE$ is given by

Area of polygon $ABCDE = \text{Area of } \triangle ABC + \text{Area of } \triangle ACD + \text{Area of } \triangle AED$

We have explored formula for the finding area of various geometrical figures. Let see few practical applications.

**Example 3.** Find the area of the quadrilateral $ABCD$.

**Solution:**

Area of quadrilateral $= \frac{1}{2} d (h_1 \times h_2)$

Here $d = 6 \text{ cm}$, $h_1 = 3\text{ cm}$, $h_2 = 2.5 \text{ cm}$
Area of quadrilateral $ABCD = \frac{1}{2} \times (3 + 2.5) = 16.5 \text{ cm}^2$

**Example 4:** Find the area of the given rhombus.

**Solution:**

Area of rhombus $= \frac{1}{2} (d_1 \times d_2)$

Here $d_1 = 8 \text{ cm}$, $d_2 = 7 \text{ cm}$

$\therefore$ Area of rhombus $= \frac{1}{2} (8 \times 7) = 28 \text{ cm}^2$

**Example 5:** Find the area of the given polygon,

**Solution:**

Area of polygon $PQRST = \text{Area of } \triangle PXQ + \text{Area of Trapezium } QRYX + \text{Area of } \triangle RYS + \text{Area of } \triangle PST$

Area of $\triangle PXQ = \frac{1}{2} \times PX \times QX = \frac{1}{2} \times 2.5 \times 2.5 = 3.125 \text{ cm}^2$

Area of Trapezium $QRYX = \frac{XY}{2} \times (QX + RY) = \frac{3}{2} \times (2.5 + 3) = 8.25 \text{ cm}^2$.

Area of $\triangle RYS = \frac{1}{2} \times YS \times RY = \frac{1}{2} \times 2.5 \times 3 = 3.75 \text{ cm}^2$

Area of $\triangle PST = \frac{1}{2} \times PS \times ZT = \frac{1}{2} \times 8 \times 2.5 = 10 \text{ cm}^2$

$\therefore$ Area of Polygon $PQRST = 3.125 \text{ cm}^2 + 8.25 \text{ cm}^2 + 3.75 \text{ cm}^2 + 10 \text{ cm}^2 = 25.125 \text{ cm}^2$

**Example 6:** The area of a rhombus is $360 \text{ cm}^2$. If the length of one of the diagonals is $26 \text{ cm}$, what will be the length of the second diagonal?

**Solution:**

We know, area of rhombus is given by the formula $\frac{1}{2} (d_1 \times d_2)$

Here area $= 360 \text{ cm}^2$, $d_1 = 26 \text{ cm}$ and $d_2 = x$ (to be found out).

Substituting values, we get,$\quad 360 = \frac{1}{2} (26 \times x)$
\[
360 = \frac{26x}{2} \\
\therefore x = \frac{360 \times 2}{26} = 27.69 \text{ cm} \approx 28 \text{ cm}
\]

Length of second diagonal = 28 cm

So far we have discussed the formula to find the area of quadrilateral and polygons and mostly these are plane figures. Remember that children must have seen match boxes, text books, lunch boxes and many such objects. Are they plane figures? Of course not! Such objects are solid objects and we classify them into solid figures which are three dimensional in nature. Unlike area for plane figures, we use the term surface area for solid figures. Now let us establish the formula used for finding surface area of solid shapes such as cuboids, cubes and cylinders.

**Check Your Progress**

Note:  
(a) Write your answers in the space given below.  
(b) Compare your answers with those given at the end of the Unit.

4) ‘To calculate the area of any polygon, one should have mastery in calculation of area of triangles’. Prove the statement with an example.

\..............................
\..............................
\..............................
\..............................
\..............................
\..............................

**17.3.4 Surface Area of Cuboid, Cube, Cylinder, Cone and Sphere**

**Surface Area of Cuboid**

A cuboidal box (Fig 17.13a), when opened will look similar to the figure (Fig 17.13b) shown below. In Fig 17.3 b, we can see that, the outer surface of cuboidal box is made out of six rectangles (Rectangle I to VI) and there are three pairs of identical faces. The rectangular regions are called as the faces of the cuboid. In order to find the surface area of cuboidal box, find the area of each of the rectangles from I to VI and add them.

(Source: Mathematical Textbook for class VIII, pp 180, NCERT)
Surface area of the cuboid = Area of I + Area of II + Area of III + Area of IV + Area of V + Area of VI
= h × l + b × l + h × h + b × h + h × b
= 2(h × l + b × h + b × l)
= 2(lb + bh + hl)

**Surface Area of Cuboid = 2(lb + bh + hl)**;

Where l = length, b = breadth and h = height

**Lateral Surface Area of Cuboid**

In the case of cuboids, the area corresponding to four faces (Area I to IV in Figure 17.13b) is referred to as lateral surface area. Lateral surface area of a cuboid of length ‘l’, breadth ‘b’ and height ‘h’ is given by the formula;

\[ \text{Lateral Surface Area} = 2lh + 2bh \]

**Surface Area of Cube**

As we have done in the case of cuboids, children must be asked to open up a cube and the resulting figure would be one shown in Figure 17.14b.

![Fig. 17.14a](image)

![Fig. 17.14b](image)

What will be the surface area of cube then? Students must be provided learning experiences to recognize that; cube is a form of cuboid with all six faces having same length. Thus the surface area (or we call total surface area) of cube is given by the formula \( 2(l^2 + 2l^2 + 2l^2) = 2(3l^2) \)

i.e. **Surface area of Cube = 6l^2**

Let us solve some examples.

**Example 7:** Calculate the surface area of a matchbox having height 5cm, length 2cm and width 3.5cm.

**Solution:** The match box is in cuboidal shape.

Area of match box = 2(lb + bh + hl)

Here, h = 5 cm, l = 2 cm, b = 3.5 cm

Surface Area = 2(2 × 3.5 + 3.5 × 5 + 5 × 2)

= 2(7 + 17.5 + 10) = 69 cm^2

**Example 8:** What is the surface area of the cube measuring each side 6cm?

**Solution:**

Surface area of cube = 6l^2

Here l = 6 cm

Surface Area = 6 × (6)^2 = 216 cm^2
Surface Area of a Cylinder

Students have seen objects like tins, lunch boxes, oil bottles, water bottles, water pipes, etc. Such objects may be used as teaching aids to transact the concept of surface area of cylinder. You may ask them to draw the picture of water pipe and compare with that given below in Figure 17.15a. Then after, you may explain that, the same picture can be drawn as shown in Figure 17.15b. Teacher continues that a cylinder has two circular faces (whose radius is ‘r’) and a rectangular area (whose height is ‘h’). Thus the total surface area of the cylinder (or we call total surface area) will be the area of two circular faces plus area of rectangle.

\[ \text{Surface Area of Cylinder} = \text{Area of Circle 1} + \text{Area of Rectangle} + \text{Area of Circle 2} \]

\[ = \pi r^2 + 2\pi rh + \pi r^2 \]

\[ = 2\pi r^2 + 2\pi rh \]

\[ = 2\pi r(r+h) \]

\[ \text{Surface Area of Cylinder} = 2\pi r(r+h); \text{ Where } r= \text{radius of cylinder, } h=\text{height of the cylinder} \]

(Note: Similar to cuboid, cylinders do have lateral surface area. Lateral surface area is also known as a curved surface area. Lateral (curved) surface area of cylinder is the surface area of the curved part of the cylinder and is given by the formula \(2\pi rh\))

Example 9: Rajeev brought a cylindrical pipe having radius 2cm and total surface area 2640cm². Calculate the height of the cylinder pipe.

Solution:

Radius of the cylindrical pipe \(r = 2\text{cm}\)

Total surface area of the cylindrical pipe \(= 2640 \text{ cm}^2\)

Height of the cylindrical pipe \(= h \) (to be found out)

Total surface area of cylinder \(= 2\pi (r + h)\)

\[ \therefore 2640 = 2 \times \frac{22}{7} \times 2(2 + h) \]

\[ 2640 = \frac{88}{7}(2 + h) \]
\[2 + h = \frac{2640 \times 7}{88}\]
\[2 + h = 210\]
\[h = 208 \text{ cm}\]

Or height if cylindrical pipe = 208 cm = 2.08 m

**Surface Area of Right Circular Cone**

Let us move on our discussion to finding area of right circular cone. Generally we find right circular and non right circular cones. You may start by showing the following figures. The Figure 17.16 a is right circular cone; where ‘A’ is vertex, ‘AB’ the height, ‘BC’ radius and ‘AC’ is the slant height of the cone. The height, radius and slant height are usually denoted by ‘h’, ‘r’ and ‘l’ respectively and ‘B’ is the centre of circular base of the cone. Figure 17.16b is not a right circular cone because the base is not circular. After brainstorming about preliminary knowledge of circular cones, you may help children to arrive at formula for finding surface area of cone.

![Fig. 17.16(a)](image1)
![Fig. 17.16(b)](image2)

(Source: Mathematics Textbook for class IX, pp 218, NCERT)

To find the surface area of cone we may redraw a cone as shown in Figure 17.17(a). If we touch points A and B, it forms the circular base of the cone. As shown in Figure 17.17(b), the curved surface of the cone is made of tiny triangles and hence the curved surface area will be the sum of areas of each triangle.

Thus, surface area of right circular cone = Surface area of curved surface + Surface area of circular region

\[
\frac{1}{2} b_1 l + \frac{1}{2} b_2 l + \frac{1}{2} b_3 l + \ldots + \pi r^2
\]

\[
= \frac{1}{2} l (b_1 + b_2 + b_3 + \ldots) + \pi r^2
\]

\[
= \frac{1}{2} l \times 2\pi r + \pi r^2
\]

\[
= \pi rl + \pi r^2
\]

\[
= \pi (l + r) \quad \text{(Since } l = \sqrt{r^2 + h^2} \text{, By applying Pythagoras Theorem in Fig 17.17c)}
\]

i.e Total Surface Area of Cone = \(\pi r (l + r)\)
Example 10: The diameter of the base and height of a cone are 24 cm and 16 cm respectively. Calculate the following (Take value of $\pi = 3.14$)

a) Curved surface area of cone.

b) Surface area of the cone.

Solution:

Curved surface area of cone $= \pi rl$

Surface area of cone $= \pi rl + \pi r^2$

We have diameter, $d = 24$ cm so radius $r = 12$ cm

Also $l^2 = h^2 + r^2$

So $l = \sqrt{h^2 + r^2}$

$l = \sqrt{16^2 + 12^2} = 20$ cm

So curved surface area $= \pi rl = 3.14 \times 12 \times 20 = 753.6$ cm$^2$

Surface Area $= \pi rl + \pi r^2 = 753.6 + 3.14(12)^2 = 1205.76$ cm$^2$

Surface Area of Sphere

Students of secondary classes have come across various spherical shapes in their life. Being a Mathematics teacher, you may elicit few of such shapes before you arrive at the formula for measuring its surface area. Also, you may ask your students to define sphere, which would help them in deducing its surface area. How will you define sphere? A sphere is a three dimensional figure (solid figure), which is made up of all points in the space, which lie at a constant distance called the radius, from a fixed point called the centre of the sphere.

How can we teach children the formula to find surface area of sphere? The following activity will help you. Ask them to take a long string and rubber ball. Then measure the diameter of the ball from which you may calculate the radius. After that, fix a nail and start winding the string over the ball as shown in Fig. 17.18a. Then mark the start and end point of the string. Now draw four circles with radius equal to the radius of the ball. Unwind the string and fill each circle with the string that have been used (as given in Fig. 17.18b). Through this activity, children will realise that, the string that completely covered the surface of the ball is completely used to fill four circles. Thus we can deduce that,

The surface area of sphere $= 4$ times the area of a circle of radius $r = 4 \times (\pi r^2)$

Or Surface Area of Sphere $= 4\pi r^2$

(Source: Mathematics Textbook for class IX, pp 223, NCERT)
Note that, the formula for finding surface area of sphere, may be used to final formula for surface area of hemisphere. You may do it as an exercise. Hemisphere is half portion of the sphere and is obtained if we cut sphere along its centre. Hemisphere consists of a flat and curved surface. Thus the total surface area of hemisphere is obtained by adding surface area of curved surface and area of circular region.

\[ \text{Surface area of hemisphere} = 2\pi r^2 + \pi r^2 = 3\pi r^2 \]

**Example 11:** Find the surface area of the sphere in the following cases:

a) Sphere with radius 12 cm  
b) Sphere with diameter 49 cm

**Solution:** We know that the Surface Area of Sphere = \(4\pi r^2\)

Case (a) \(r=12\) cm  
Surface Area of Sphere = \(4\pi r^2\)
\[
= 4\times3.14\times(12)^2 = 1810.28 \text{ cm}^2
\]

Case (b) diameter=49 cm so radius=diameter/2=49/2=24.5 cm  
Surface Area of Sphere = \(4\pi r^2\)
\[
= 4\times3.14\times(24.5)^2 = 7546 \text{ cm}^2
\]

**Check Your Progress**

Note:  
a) Write your answers in the space given below.  
b) Compare your answers with those given at the end of the Unit.

5) How will you help your students to realise surface area of cylinder is \(2\pi(r+h)\)? Where \(r\) = radius of cylinder, \(h\) = height of the cylinder. Suggest one learning activity.
6) If the radius of a hemisphere is 8 cm, find its surface area.

17.4 MEASUREMENT OF VOLUME

As students reach secondary stage, they hear about volume and later distinguish among volume and capacity. You can think of interesting situations and examples to introduce the concept volume. Here is one. Sangeeta, a secondary Mathematics teacher brought two containers, almost similar in size but really not. Then she raised a question to her students, if I keep the containers in an Almirah, which one would consume more space? It was so simple to answer. Children said the big container. Then Sangita asked, why do think that big container would consume more space. This time few students struggled, but in between Stephen said, “If the container is big in size and it would require more space”. After discussions, teacher concluded saying ‘space occupied by a three dimensional object is called volume’. We will explore the formula for finding volume of various solid figures in today’s class.

You may start with simple solid shapes as shown below. As students start finding volume, the point that, ‘cubic units’ are used to find the volume of objects unlike ‘square units’ for area. To find the volume of the given figure, it is divided into different cubit units measuring 1 cm each side. Thus the volume of the given figure 17.20 is 8 cubic units. Also students should be made aware of the point that, the commonly used cubical units are:

- 1 cubic cm = 1 cm × 1 cm × 1 cm = 1 cm³
- 1 cubic mm = 1 mm × 1 mm × 1 mm = 1 mm³
- 1 cubic m = 1 m × 1 m × 1 m = 1 m³

A word of caution: Take a glass container and keep it in shelf. In such case, the space occupied by the container is called volume. We know that, the interior of the glass bottle can be filled with any fluids and imagine presently it is filled with water. Then the volume of the substance that can fill the interior is called the capacity of the container. In short, the volume of an object is the measure of the space it occupies, and the capacity of an object is the volume of substance its interior can accommodate. As a teacher,
you need to provide learning activities to distinguish the concept of volume and capacity.

17.4.1 Volume of Cuboid and Cube

Show a cuboid; constructed out of 36 cubes (of equal size) to your students and ask them to find the volume. Probably they may not be finding difficulty to arrive at the answer 36 cubic units. After that, you may pose the question, “Can you deduce a formula (which connect length, breadth and height of the cuboid) that will give you result 36”? You may show the Figure 17.21b as a clue. This may be practised as a group exercise. At the end of discussions, students would come up with the formula i.e. \( l \times b \times h = 36 \). As concluding remark, teacher can say, “Yes, you are right, the formula you have found is correct. Thus we can say that, the formula for finding the volume of a cuboid is given by the formula \( l \times b \times h \). Since \( l \times b \) gives area of base, we can also say that, volume of cuboid is area of base x height.”

**Volume of Cuboid**

\[ Volume \text{ of Cuboid} = l \times b \times h = \text{Area of Base} \times \text{Height} \]

![Fig. 17.21a](image.png) ![Fig. 17.21b](image.png)

(Source: NCERT, Mathematics Textbook, class IX, pp 188)

Students know that cube is a form of cuboid with all the faces having same length (\( l \)). Thus the volume of cube is deduced form the formula for volume of cuboid by substituting ‘\( l \)’. i.e. Volume of cuboid = \( l \times b \times h = l \times l \times l = l^3 \) (Since \( l = b = h = l \)). Therefore, volume of cube is;

**Volume of Cube**

\[ Volume \text{ of Cube} = l^3; \text{ Where } l = \text{ Length of the side of cube} \]

17.4.2 Volume of Cylinder

Vinod, a Mathematics teacher was teaching the formula required to find the volume of cylinder. Suddenly, one of his students asked, “Sir, can we use the formula of cuboid to find the volume of cylinder”? Yes of course! Vinod replied. Then Vinod asked the student to explain the procedure of arriving at the formula for volume of cylinder. The explanation of the student goes like this;

“Sir, similar to cuboid, cylinders do have a base and particular height. I will draw it on the board. For cuboid we use the following formula;

i.e. Volume of Cuboid= \( l \times b \times h = \text{Area of Base} \times \text{Height} \)

In the case of cylinder, we can write volume= Area of Base \times Height

\[ = \pi r^2 \times h = \pi r^2 h (\text{ Since Area of base of cylinder} = \pi r^2) \]
After finding at the formula, Vinod appreciated his student. Then after, he concluded the class by restating the formula. Thus the volume of cylinder is given by the formula;

\[
\text{Volume of cylinder}= \pi r^2 h; \text{ Where } r=\text{radius of cylinder, } h=\text{height of cylinder}
\]

Let see an example to make idea clear about it.

**Example 12:** If the lateral surface area of cylindrical vessel is 34 cm\(^2\) and its height is 3 cm. Then find the following:

a) Radius of the cylindrical vessel

b) Volume of the cylindrical vessel

**Solution :**

a) We have lateral surface area of cylindrical vessel = 34 cm\(^2\)

Height of cylindrical vessel=3 cm

We know that the lateral surface area of cylinder = \(2\pi rh\)

\[
34=2\times3.14\times r \times 3
\]

\[
r = \frac{34}{2 	imes 3.14 \times 3}
\]

\[
r = \frac{34}{18.84} = 1.80 \text{ cm}
\]

b) Volume of the cylindrical vessel = \(\pi r^2 h\); Where \(r=\text{radius of cylinder, } h=\text{height of cylinder}\)

\[
= 3.14 \times (1.8)^2 \times 3 = 34.00 \text{ cm}^3
\]

**17.4.3 Volume of Cone**

Now let us discuss about the volume of cone. You may think of interesting activities to achieve the same. One of such activity for cone is discussed below:
Ask children to bring a hollow cone and a cylinder having same base area. Then fill the cone with water and empty it an to the cylinder. They may observe that it fills up only a part of cylinder. Tell them to repeat the same exercise. Now they may observe that the cylinder is not full. Tell them to repeat the same exercise. Now they can find that cylinder is full with water. Thus we come to the conclusion that, three times the volume of cone is equal to the volume of the cylinder. Therefore volume of cone is three times the volume of cylinder, i.e.

\[
\text{Volume of Cone} = \frac{1}{3} \pi r^2 h; \quad \text{Where } r=\text{base radius, } h=\text{height of the cone}
\]

Now you can give few practical applications to your students.

---

**Check Your Progress**

Note:  
- a) Write your answers in the space given below.  
- b) Compare your answers with those given at the end of the Unit.

7) What are the probable mistakes children commit as they find the volume of cone?

……………………………………………………………………………
……………………………………………………………………………
……………………………………………………………………………
……………………………………………………………………………
……………………………………………………………………………

8) A cuboidal water tank is 5 m long, 3 m wide and 4 m deep. Find its volume.

……………………………………………………………………………
……………………………………………………………………………
……………………………………………………………………………
……………………………………………………………………………
……………………………………………………………………………

9) Find the volume of circular cone of radius 4 cm and height 8 cm.

……………………………………………………………………………
……………………………………………………………………………
……………………………………………………………………………
……………………………………………………………………………
……………………………………………………………………………

---

**17.5 COORDINATE GEOMETRY: BASICS AND USE**

Rene Descartes (1596-1650), the great French Mathematician of the seventeenth century described the position of a point in a plane. This has given rise to a very important branch of Mathematics known as Coordinate Geometry. In honour of Descartes, the system used for describing the position of a point in a plane is also known as the Cartesian system.
Coordinate Geometry has been developed as an algebraic tool for studying Geometry of figures. It helps us to study by using Algebra. Coordinate Geometry is widely applied in various fields such as Physics, Navigation, Seismology, art etc.

In Algebra, you were introduced to the coordinate system, plotting ordered pairs, graphing lines and solving equations. These tools are used in geometry as well. Algebra and Geometry are used hand-in-hand to solve many real world problems. Points, lines, line segments and angles as well as other geometric shapes can be graphed on the coordinate plane. Let us recall some basics.

In the Cartesian or coordinate plane, the two perpendicular lines are the coordinate axes – x axis and y-axis. The point of intersection of these axes is the origins and these axes divide the plane in four quadrants. Each point in the Cartesian system has an x-coordinate or abscissa representing its horizontal distance and a y-coordinate or ordinate representing its vertical positions. These are typically written as the ordered pairs (x, y) and are the coordinates of that point. The coordinates of a point on the x-axis are in the form (x, 0) and of the point on the y-axis in the form (0, y) and the coordinates of origin are (0, 0). The coordinates of a point are of the form (+, +) in the first quadrant, (–, +) in the second quadrant, (–, –) in the third quadrant and (+, –) in the fourth quadrant where + denotes a positive real number and – denotes a negative real number.

### Check Your Progress

**Note:**
Choose the correct answer and compare your answer with those given at the end of the Unit.

10. P is a point on x-axis at a distance of 3 units from y-axis to its left. The coordinates of P are:
   (a) (3, 0)   (b) (0, 3)   (c) (–3, 0)   (d) (0, –3)

11. The distance of point P (3, –2) from y-axis is:
   (a) 3 units   (b) 2 units   (c) –2 units   (d) $\sqrt{13}$ units

12. If the coordinates of two points are P(–2, 3) and Q (–3, 5), then (abscissa of P) – (abscissa of Q) is
   (a) –5   (b) 1   (c) –1   (d) –2

### 17.6 DISTANCE FORMULA

Discuss the following situation with the students, if Neha’s house is situated at point A and her school is situated at Point B as in the figure. Her house is located at A(2, 1) and the school is located at B(6, 4) then how far is her school from her house if 1 unit = 1 km.

![Fig. 17.24](image-url)
To give answer to this question, let us first consider two points lying on the x-axis say P(2, 0) and Q(6, 0). Then the distance between P and Q is the length of line segment PQ.

So, \(PQ = OQ - OP = 6 - 2 = 4\) units

Similarly, R is (0, 1) and S (0, 4) lying on y-axis then the distance between them is

\(RS = OS - OR = 4 - 1 = 3\) units.

Thus, we see that if two points lie on the x-axis or the y-axis we can easily find the distance between them.

Now, what if two points do not lie on the coordinate axes. How can we find the distance between them?

Take coordinate axes and mark two such points \(P(x_1, y_1)\) and \(Q(x_2, y_2)\).

Draw PA and QB perpendicular to x-axis and PR \(\perp QB\) as in the figure. We want to find the distance between P and Q i.e. length of the line segment PQ. In \(\triangle PQR\), right angled at R, we can apply Pythagoras theorem to find PQ.

For that we must have PR and QR.

Now A and B are points on x-axis. So,

\[OA = x_1, \quad OB = x_2\]

and so, \(AB = OB - OA = x_2 - x_1\)

\[PR = AB = x_2 - x_1\]

Also, \(QR = QB - RB = QB - AP = y_2 - y_1\)

Using, Pythagoras theorem, we have

\[PQ^2 = PR^2 + QR^2\]

\[= (x_2 - x_1)^2 + (y_2 - y_1)^2\]

So, \(PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\) .... (1)

Note that here we have taken only the positive square root. It is so because PQ is the length of line segment and so will always be non-negative. Thus, we have - :

Distance between P and Q = \(PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\)
This result is known as **Distance Formula**.

**Remarks:**

1) We can also write PQ as \( \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \).

2) In particular, the distance between the origin and \( P(x, y) = OP = \sqrt{x^2 + y^2} \).

3) The points P and Q may lie in any quadrant, the distance PQ is given by the same formula as given in (1) above.

Now, we come back to the situation which we considered in the beginning of this section. For distance of Neha’s school from her house.

\[
AB = \sqrt{(6-2)^2 + (4-1)^2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5 \text{ units.}
\]

As 1 unit = 1 km., the distance of her school from her house is 5 km.

**Example 13**

Find the distance between A \((1, -2)\) and B\((3, -5)\).

**Solution:**

\[
AB = \sqrt{(3-1)^2 + (-5-(-2))^2} = \sqrt{2^2 + (-5+2)^2} = \sqrt{2^2 + (-3)^2} = \sqrt{4 + 9} = \sqrt{13}
\]

**Example 14**

Raghav’s grandmother, mother and himself visited a temple. The priest asked them to sit in a line. They occupied the seats at \( A(-3, 2), B(1, -2)\) and \( C(9, -10)\). Do you think they are seated in a line? Justify.

**Solution:**

Three points will be collinear or lie in a line of sum of lengths of the two line segments is equal to the third.

Here,

\[
AB = \sqrt{(1+3)^2 + (-2-2)^2} = \sqrt{4^2 + (-4)^2} = 4\sqrt{2}
\]
\[
BC = \sqrt{(9-1)^2 + (-10+2)^2} = \sqrt{8^2 + (-8)^2} = 8\sqrt{2}
\]
\[
AC = \sqrt{(9+3)^2 + (-10-2)^2} = \sqrt{12^2 + (-12)^2} = 12\sqrt{2}
\]

As \( AB + BC = 4\sqrt{2} + 8\sqrt{2} = 12\sqrt{2} = AC \), the points A, B, C are collinear i.e. grandmother, mother and himself are seated in a line.

**Example 15**

Find a point on the x-axis which is equidistant from the points \((5, 4)\) and \((-2, 3)\).

**Solution:**

Let \(P(x, y)\) be the required point. Since it is on the x-axis, its ordinate is zero.

So, coordinates of P are \((x, 0)\)

Let A and B denote the points \((5, 4)\) and \((-2, 3)\) respectively.

We are given that \(AP = BP\)

So, \(AP^2 = BP^2\)
i.e., \((x - 5)^2 + (0 - 4)^2 = (x + 2)^2 + (0 - 3)^2\)

or, \(x^2 - 10x + 25 + 16 = x^2 + 4x + 4 + 9\)

or, \(-14x = -28\) or, \(x = 2\)

Thus, the required point is \((2, 0)\).

**Example 16**

What type of quadrilateral do the points \(A(2, -2), B(7, 3), C(11, -1)\) and \(D(6, -6)\) taken in that order form?

**Solution:**

\[AB = \sqrt{(7 - 2)^2 + (3 + 2)^2} = \sqrt{5^2 + 5^2} = 5\sqrt{2}\]

\[BC = \sqrt{(11 - 7)^2 + (-1 - 3)^2} = \sqrt{4^2 + (-4)^2} = 4\sqrt{2}\]

\[CD = \sqrt{(6 - 11)^2 + (-6 + 1)^2} = \sqrt{(-5)^2 + (-5)^2} = 5\sqrt{2}\]

\[AD = \sqrt{(6 - 2)^2 + (-6 + 2)^2} = \sqrt{4^2 + (-4)^2} = 4\sqrt{2}\]

As, \(AB = CD\) and \(BC = AD\) i.e. opposite sides are equal, so \(ABCD\) is a parallelogram.

Diagonal \(AC = \sqrt{(11 - 2)^2 + (-1 + 2)^2} = \sqrt{9^2 + 1^2} = \sqrt{81 + 1} = \sqrt{82}\)

Diagonal \(BD = \sqrt{(6 - 7)^2 + (-6 - 3)^2} = \sqrt{(-1)^2 + (-9)^2} = \sqrt{1 + 81} = \sqrt{82}\)

As diagonals \(AC = BD\), therefore, parallelogram \(ABCD\) is a rectangle.

**Check Your Progress**

Note:  
 a) Write your answers in the space given below  
 b) Compare your answers with those given at the end of the Unit

13. Find distance between the points:

   (i) \(A(1, -3), B(4, 1)\)

   ........................................................................................................

   ........................................................................................................

   (ii) \(A(5, -8), B(-7, -3)\)

   ........................................................................................................

   ........................................................................................................

14. Determine whether the points \((8, 4), (5, 7)\) and \((-1, 1)\) are the vertices of a right triangle.

   ........................................................................................................

   ........................................................................................................

   ........................................................................................................

15. Check whether the points \((1, 2), (5, 3)\) and \((18, 6)\) are collinear.

   ........................................................................................................

   ........................................................................................................

   ........................................................................................................
17.7 SECTION FORMULA

Suppose two friends Aditya and Guninder are seated at A and B (Figure). Their third friend Akhtar wants to sit in between Aditya and Guninder at P in such a way that the distance of Akhtar from Guninder is three times his distance Aditya.

![Figure 17.27](image1)

Points A, B and P lie on the same line so we can say that P divides AB in the ratio 1 : 3. We know the coordinates of A and B and we want to know the coordinates of P where their third friend should sit.

For that, let us consider the following:

Consider two points A\((x_1, y_1)\) and B\((x_2, y_2)\) and assume that P\((x, y)\) divides AB internally in the ratio \(m_1 : m_2\) i.e.

\[
\frac{PA}{PB} = \frac{m_1}{m_2}
\]

By internal division we mean that the point of division P lies on the line AB between A and B. If point P lies on the line AB outside of the line segment AB and \(\frac{PA}{PB} = \frac{m_1}{m_2}\) we say that P divides externally the line segment joining the points A and B.

In this section, we will study the section formula for internal division only.

Draw AR, PS, BT perpendicular to x-axis and AQ, PC parallel to x-axis.

![Figure 17.28](image2)

Then, \(\triangle PAQ \sim \triangle BPC\) (AA similarity criterion)

So,

\[
\frac{PA}{BP} = \frac{AQ}{PC} = \frac{PQ}{BC} \quad \ldots \quad (1)
\]
Here,

\[ \begin{align*}
AQ &= RS = OS - OR = x - x_1 \\
PC &= ST = OT - OS = x_2 - x \\
PQ &= PS - QS = PS - AR = y - y_1 \\
BC &= BT - CT = BT - PS = y_2 - y
\end{align*} \]

Also, \( \frac{PA}{PB} = \frac{m_1}{m_2} \)

Putting these values in (1) we have

\[ \begin{align*}
m_1 &= \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}
\end{align*} \]

Solving for \( x \) we have

\[ \frac{x - x_1}{x_2 - x} = \frac{m_1}{m_2} \]

or, \( m_1x - m_2x_1 = m_2x_2 - m_1x \)

or, \( (m_2 + m_1)x = m_1x_2 + m_2x_1 \)

\[ x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \]

Similarly, solving for \( y \) we have

\[ \begin{align*}
y - y_1 &= \frac{m_1}{m_2} \\
y_2 - y &= \frac{m_2}{m_2}
\end{align*} \]

\[ X = y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \]

So, \( P(x, y) \) is \( \left( \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right) \)

So, the coordinates of the point \( P(x, y) \) which divides the line segment joining the points \( A(x_1, y_1) \) and \( B(x_2, y_2) \) internally in the ratio \( m_1 : m_2 \) are:

\[ \left( \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right) \]

This is known as **Section Formula**.

As a special case, if \( P \) is the midpoint of \( AB \) then the ratio is 1 : 1. So, coordinates of the midpoint \( P \) will be

\[ \left( \frac{1 \times x_1 + 1 \times x_2}{1 + 1}, \frac{1 \times y_1 + 1 \times y_2}{1 + 1} \right) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

**Example 17**

Find the coordinates of the point which divides the join of \( (7, -4) \) and \( (1, 5) \) in the ratio 1: 2 internally.

**Solution:**

Let \( P(x, y) \) be the required point.

Using Section formula, we have:
\[
x = \frac{1 \times 1 + 2 \times 7}{1 + 2} = \frac{15}{3} = 5
\]
\[
y = \frac{1 \times 5 + 2(-4)}{1 + 2} = \frac{5 - 8}{3} = \frac{-3}{3} = -1
\]

So, (5, -1) is the required point.

**Example 18**

Find the ratio in which the line segment joining the points A(−3, 10) and B(6, −8) as divided by (−1, 6).

**Solution:**

Let (−1, 6) divide AB in the ratio \(m_1 : m_2\) internally using section formula, we have

\[
(-1, 6) = \left(\frac{6m_1 - 3m_2}{m_1 + m_2}, \frac{-8m_1 + 10m_2}{m_1 + m_2}\right)
\]

or, \(\frac{6m_1 - 3m_2}{m_1 + m_2} = -1,\)

\(\frac{-8m_1 + 10m_2}{m_1 + m_2} = 6\)

or, \(6m_1 - 3m_2 = -m_1 - m_2,\)

\(-8m_1 + 10m_2 = 6m_1 + 6m_2\)

or, \(7m_1 = 2m_2\)

\(-14m_1 = -4m_2\)

or, \(\frac{m_1}{m_2} = \frac{2}{7}\)

or, \(\frac{m_1}{m_2} = \frac{4}{14} = \frac{2}{7}\)

Thus, the ratio is 2 : 7.

Alternatively

The ratio \(m_1 : m_2\) can also be written as

\(\frac{m_1}{m_2} : 1\) or \(k : 1\). Thus, we have

\[
(-1, 6) = \left(\frac{6k - 3}{k + 1}, \frac{-8k + 10}{k + 1}\right)
\]

So, \(\frac{6k - 3}{k + 1} = -1\)

or, \(6k - 3 = -k - 1\)

or, \(7k = 2\)

or, \(k = \frac{2}{7}\)

Thus, the ratio is \(\frac{2}{7} : 1\) or \(2 : 7\).

We could have checked for coordinate also. We see that it is convenient to find the ratio by assuming the ratio to be \(k : 1\).
Example 19
Find the coordinates of point A where AB is the diameter of a circle whose centre is (2, –3) and B is (1, 4).

Solution:
Let A be (x, y). AB is the diameter of the circle and centre is the midpoint of the diameter.
Thus, the midpoint of the diameter AB is:
\((-1, 6) = \left( \frac{x + 1}{2}, \frac{y + 4}{2} \right) = (2, -3)\)

So, \(\frac{x + 1}{2} = 2, \quad \frac{y + 4}{2} = -3\)

Or, \(x = 4 - 1 = 3, \quad y = -6 - 4 = -10\)

So, coordinates of A are (3, –10).

Thus, from the above examples we observe that in section formula of any three out of the following four are given we can find the fourth.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

16. Find the coordinates of the point which divides the line segment joining the points (–7, 4) and (–6, –5) internally in the ratio 7 :2.

17. Coordinates of A and B are (1, 2) and (2, 3). Find the coordinates of R so that \(\frac{AR}{RB} = \frac{4}{3}\).

18. Find the coordinates of the mid-point of the line segment joining the points (22, –20) and (0, 16).
19. Find the ratio in which the line segment joining the points (6, 4) and (1, –7) is divided internally by the x-axis.

..........................................................................................................................................................................
..........................................................................................................................................................................
..........................................................................................................................................................................
..........................................................................................................................................................................

17.8 LET US SUM UP

In this unit, we discussed briefly how the need for Mensuration arose as civilisation progressed. We discussed the concept of perimeter and area of plane figures and shown usefulness of deductive approach in arriving at areas of different plane figures. We also derived formulas for measuring volume of cuboid, cube, cylinder, sphere and cone.

In the other section of this unit, we introduced how Algebra and Geometry go hand-in-hand in Coordinate Geometry. We recalled some basics of Coordinate Geometry studied in Algebra. We derived distance formula for obtaining distance between two points and in particular, obtained the formula for distance of a point from origin. We also derived formula known as section formula and as its special case obtained the coordinates of the mid-point of the line segment joining the two given points.

17.9 UNIT END ACTIVITIES

1) Find the length of the longest road that can be placed in a room 12 m long, 9 m broad, 8 m high.

2) A wire when bent in the form of an equilateral triangle encloses an area of 121 cm$^2$. If the same wire is bent into the form of a circle then find the diameter of the circle.

3) Find the surface area and volume of a sphere whose diameter is 21 m. Find the diameter of a sphere whose surface area is 5544 cm$^2$.

4) The diameter of a roller 120 m long is 84 m. If it takes 500 complete revolutions to level a ground. Find the cost of levelling the ground at the rate of 30 paise per square meter.

5) The fourth vertex D of a parallelogram ABCD whose three vertices are A(–2, 3), B(6, 7) and C(8, 3) is
   (a) (0, 1) (b) (0, –1) (c) (–1, 0) (d) (1, 0)

6) The houses of Khyati and Shahnaz are situated at the points with coordinates (7, 3) and (4, –3) respectively. They study in the same school which is situated at a point (2, 2). Both start for the school at the same time in the morning and reach the school at the same time. Who walks fast?

7) State whether the following statement is true or false. Points A (3, 1), B(12, –2) and C (0, 2) cannot be the vertices of a triangle.

8) The points A($x_1$, $y_1$), B($x_2$, $y_2$) and C($x_3$, $y_3$) are the vertices of $\triangle ABC$. AD is the median and P is a point on AD such that $AP : PD = 2 : 1$. Find the coordinates of P.
9) If C is a point lying on the line segment AB joining A(1, 1) and B(2, –3) such that 3AC = BC. Find the coordinates of C.

10) Find the coordinates of the points of trisection of the line segment joining the points A(2,–2) and B (–7, 4).

### 17.10 ANSWERS TO CHECK YOUR PROGRESS

1) Bring card boards of different sizes and ask children to find the bigger one among them.

2) Do it yourself (Hint: Section 17.3)

3) Perimeter of a Rectangle = 2× (Length+Breadth) =2(5+5)=20cm

4) Draw a polygon having any number of sides. Then calculate the area by dividing into different rectangles (Refer section 17.3)

5) Refer the sub section describing “Surface area of cylinder” in section 17.3

6) 603.72 cm²

7) List the mistakes from your own experiences.

8) 60 m³

9) Volume of Cone=\(\frac{1}{3}\pi r^2 h\); Where r= base radius, h=height of the cone

\[=\left(\frac{3.14\times4^2\times8}{3}\right) =133.97 \text{ cm}^3\]

10) (c)

11) (a)

12) (b)

13) (i) 5 (ii) 13

14) Yes

15) No

16) \(\left(\frac{-56}{9},-3\right)\)

17) \(\left(\frac{11}{7},\frac{18}{7}\right)\)

18) (11, –2)

19) 4 : 7

### 17.11 REFERENCES AND SUGGESTED READINGS

- http://epathshala.nic.in/e-pathshala-4/flipbook/