UNIT 11 POLYNOMIALS : BASIC CONCEPTS AND FACTORING*

Structure

11.1 Introduction
11.2 Objectives
11.3 Basic Concepts of Polynomials
11.4 Polynomials: Concepts and Definitions
   114.1 Polynomials in One Variable
   114.2 Polynomials in Two Variables
11.5 Operations on Polynomials
   115.1 Value of Polynomials and Zeroes of a Polynomial
   115.2 Addition of Polynomials
   115.3 Subtraction of Polynomials
   115.4 Multiplication of Polynomials
   115.5 Division of Polynomials
11.6 Factorization of Polynomials
   116.1 Basic Concepts
   116.2 Factoring a Quadratic Polynomial
   116.3 Method of Splitting the Middle Term
   116.4 Method of Splitting the Middle Term, (contd.)
11.7 Remainder Theorem
   117.1 Use of Remainder Theorem in Factorization
11.8 Let Us Sum Up
11.9 Unit End Activities
11.10 Points for Discussion
11.11 Answers to Check Your Progress
11.12 Suggested Readings

11.1 INTRODUCTION

This unit assumes that the students have already understood the basic ideas regarding dividend, divisor, quotient and remainder in arithmetic in earlier units. Also, they are familiar with the concept of numbers. They also know factorization of numbers into prime factors in arithmetic. However, a brief recapitulation by the teacher can help them to revise their basic concepts before you start actual teaching of this unit. The concepts to be learnt in this unit will be effectively learnt if they are related to the previous knowledge of the students. The concepts and techniques learnt in this unit would form the foundation for learning algebra later.

11.2 OBJECTIVES

At the end of this unit, you will be able to:

- clarify basic concepts of variables constant, algebraic expression and their numerical values, and zeroes of a polynomial;

* A few sections of this unit has been adopted from ES-342, IGNOU, 2000
• explain the meanings of degree of a polynomial, monomials and binomials, coefficients and constant terms;
• develop the skill of performing fundamental operations on polynomials;
• clarify the meaning and purpose of factorization of polynomials in algebra;
• develop skill to factorize quadratic polynomials by using basic algebraic identities;
• find ways of helping children to perform fundamental operations on polynomials;
• clarify the meaning of the statement of the remainder theorem;
• enable to compute remainder factorize quadratic and to polynomials x-a;
• enable to use remainder theorem to factorize polynomials; and
• use constructivist approach for teaching the concept and use of remainder theorem.

11.3 BASIC CONCEPTS OF POLYNOMIALS

Main Teaching Points
a) Meaning of a variable, terms, and expression
b) Value of an expression for given values of the variables

Teaching Learning Process
Teaching of algebra should begin with the relationship between arithmetic and algebra.

Variables
In algebra, we use alphabets to represent value, size and quantity in different situations. For example, the fact that the perimeter of a rectangle can be obtained by adding the length of its four sides, can be written algebraically as follows:

\[ P = 2a + 2b \quad \text{or} \quad P = 2(a + b) \]

Where \( P \) represents the perimeter of the rectangle and 'a' and 'b' represent its length and breadth respectively. Here, a and b will take different values for different rectangles. Similarly, value of P will be determined by the values of a and b.

In algebra, the letters like P, a and b, which are used to represent different elements and which take different values in different situations are referred to as unknowns or variables. Numbers and variables together act as alphabets of the language of algebra.

Terms
A combination of variables and numbers joined by operations of multiplication and division is known as a ‘term’. For examples:

\[ 3xy, -2x^2yz, \frac{7x}{y}, \frac{1}{2z}, 5x^3 \] are all terms. Terms are like words in English language.
Algebraic Expression

A combination of terms connected by operations of addition or subtraction is called an algebraic expression. For example:

\[ 2x + y, \ x^2 - 3xy, \ 3x^2 + 2x + 4, \ \frac{x}{y} + 4xy - 7 \]

are all algebraic expressions connected by multiplication and division are also algebraic expressions. Several algebraic expressions connected by multiplication and division are also algebraic expressions.

For example, \( (x^2 + 5)(x + 2), \frac{2x + 3}{1 - y}, \ (x + 1) - (x + 2)(x + 3) \) are also algebraic expressions.

Number of Variables in an Algebraic Expression.

The expressions

\[ 2x^2 - 3x + 1, \ \frac{3x + 1}{2x - 3}, \ (2x + 5)^2 \]

are algebraic expressions in one variable \( x \). Also,

\[ 3y + y^2 \]

is an algebraic expression in one variable \( y \). \( 2x + y, \ x^2 - xy, \ \frac{x}{y} + 4xy - 7 \)

are algebraic expressions in two variables \( x \) and \( y \).

Similarly, \( 2x + y + 3z - 1 \) is an algebraic expression in 3 variables \( x, y \) and \( z \).

Value of an Algebraic Expression

An algebraic expression becomes a number when all its variables, such as \( x, y, z \), are assigned definite numerical values and indicated operations are performed with those numerical values. Such a number is called the value of the algebraic expression.

For example, consider the algebraic expression \( 2x + y \). When \( x = 2 \) and \( y = 3 \), the value of the expression is \( 2(2) + 3 = 7 \). Thus, 7 is the value of the expression.

Consider another algebraic expression \( 3x^2 - 2xy + y^2 \). It’s value, when \( x = -1 \) and \( y = 2 \) is computed as follows:

\[ 3x^2 - 2xy + y^2 = 3(-1)^2 - 2(-1)(2) + (2)^2 = 3 + 4 + 4 = 11. \]

\[ \therefore 11 \] is the value of the expression when \( x = -1 \) and \( y = 2 \).

The value of the expression will change if we change the values of \( x \) and \( y \).

Methodology: Discussion method to be used.

11.4 POLYNOMIALS : CONCEPTS AND DEFINITIONS

Main Teaching Points

Definition of

a) Polynomial

b) Monomial, Bionomial, and Trinomial
Consider several algebraic expressions. Write them such that in all the terms, variable is neither in the denominator nor under the root sign. Then ask the students to identify the algebraic expression in which powers of the variables are only positive integral numbers.

Explain:

An algebraic expression in which the variables in the terms are raised to positive integral powers only is called a polynomial.

Example 1: Consider the Following algebraic expressions:

(i) \(3x^2\)  
(ii) \(5x - 1\)  
(iii) \(x^3 - 2x + 3\)  
(iv) \(3x^2 - 2xy + 4y^2\)  
(v) \(x + \frac{1}{x}\) or \(x + x^{-1}\)  
(vi) \(x^3 - 3\sqrt{x} + 2\) or \(x^3 - 3x^{1/2} + 2\)  
(vii) \(\frac{3x + y}{x - y}\) or \((3x + y)(x - y)^{-1}\)  
(viii) \(\sqrt{x + y} = (x + y)^{1/2}\)

In algebraic expression (i) - (iv), the powers of the variables are 1, 2 or 3. These algebraic expressions are polynomials.

In expression (v), the power of variable is –1

In expression (vi), the power of variable is \(\frac{1}{2}\)

In expression (vii) and (viii) also, the power of variable is not a positive integer.

Therefore, the algebraic expressions (v), (vi), (vii) and (viii) are not polynomials.

Polynomials consisting of a single term are called monomials. For examples, \(2x\), \(3x^2\), \(-7y\), 5 are all monomials.

Polynomials having two terms are called Binomials. The expressions \(3x + 2y\), \(5x - 3\), \(7 - 2y\), and \(x^2 + 3xyz\) are all binomials.

Similarly, a trinomial is a polynomial having three terms. For examples, \(3x^2 + 2x + 1\), \(x^3 - xy + y^3\), \(2x^2 + 3x + 2y\), \(x + y + 2\) are all trinomials.

Methodology: Inductive method using different examples is used.

### 11.4.1 Polynomials in one Variable

**Main Teaching Points:**

a) Degree of the polynomial

b) Standard form of a polynomial

c) Constant polynomial

d) Linear, Quadratic and Cubic polynomials.

**Teaching Learning Process**

The polynomial involving one variable is called a polynomial in one variable. Such polynomials \(x\), \(y\), \(s\) and \(t\) respectively may be written as:
p(x) = 3x^2 – 2x + 5
p(y) = 3y + 2
p(s) = s^2 – 2
p(t) = t^3 – 3t^2 + 2t + 4

Ask: What is the highest value of the exponent of the variable in each of the above polynomials. In the first polynomial there are three terms. The exponent of x is highest in the term 3x^2. It is 2. In p(y), the highest exponent of the variable y is 1.

Explain:
The highest exponent of the variable in a polynomial is called the degree of the polynomial. The degree of p(x) is 2, the degree of p(y) is 1, the degree of p(s) is 2 and the degree of p(t) is 3. The term which has the highest exponent of the variable is written first and then the other terms are written in the decreasing order of the exponent of the variable.

Explain:
The standard form of a polynomial of degree n in one variable x is:

\[ p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + + + + + + + + a_{n-1}x + a_n. \]

\( a_0, a_1, a_2, \ldots, a_n \) are constants and n is a positive integer.

Linear Polynomial  A polynomial of degree 1 is called a linear polynomial. For example, 3x + 2, 2y – 1, t + 3 are all linear polynomials.

Quadratic Polynomial A polynomial of degree 2 is called a quadratic polynomial. For example, \( x^2 – 2x + 3, 3y^2 – 2y + 1, s^2 + 3s + 5 \) are quadratic polynomials in variables x, y and s respectively.

Cubic Polynomials A polynomial of degree 3 is called a cubic polynomial. For example, \( x^3 – 2x^2 + 3x + 2, x^3 – 1, 4y^3, s^3 + s^2 + s \) are all cubic polynomials in variables x, y and s respectively.

Constant polynomials A polynomial consisting of only constant term is known as a constant polynomial. For example, p(x) = 3 is a constant polynomial. It is a polynomial of degree 0 as 3 can also be written as 3x^0.

Methodology: After giving various examples, inductive reasoning is used to generalize the concepts.

11.4.2 Polynomials in Two Variables

Main Teaching Points
a) Degree of a term in case of more than one variable
b) Linear and quadratic polynomials in the variables

Teaching Learning Process
Degree of a term with more than one variable is the sum of the exponents of all the variables x and y. In the term 2xy^2z^2 is 1 + 2 + 2 = 5, because the exponents of the variables x, y and z are 1, 2 and 2 respectively.
Discuss the degree of terms by considering different examples of terms in several variables with positive integral exponents.

Explain: A linear polynomial in two variables \(x\) and \(y\) is a polynomial in which the maximum degree of any of its term is one. Consider various examples of polynomials in two variables \(x\) and \(y\) such that -

\[
\begin{align*}
a) & \quad x^2 + y^2 \\
b) & \quad x + 3y + 2 \\
c) & \quad 2xy - 5 \\
d) & \quad 3x + 5y \\
e) & \quad 3x + 2xy^2 + y^3 \\
\end{align*}
\]

In polynomial (a), degree of each of its term is 2 so, it is not a linear polynomial whereas in polynomial (b) degree of the terms are 1,1 and 0 respectively. Since the highest degree of any of its term is 1, it is linear polynomial. Discuss that (c) and (e) are not linear polynomials but (d) is a linear polynomial. Similarly, explain that if in a polynomial, the highest degree of any of its term is two then it is a quadratic polynomial. Standard form of a linear polynomial in two variable is \(p(x, y) = ax + by + c\).

The quadratic polynomial in two variables is

\[
p(x, y) = ax^2 + bxy + cy^2 + dx + ey + f
\]

where \(a, b, c, d, e\) and \(f\) are constants.

**Methodology:** After giving various examples, inductive reasoning is used to generalize the concepts.

While arithmetic is mainly concerned with the techniques of performing the four fundamental operations with numbers, algebra is concerned with general principles and properties of these operations. In other words, algebra generalizes arithmetical concepts.

### 11.5 OPERATIONS ON POLYNOMIALS

Before teaching the operations on polynomials, the teacher should clarify to his/her students that there is a lot of similarity between basic operations on integers and on polynomials. Like numbers, polynomials can also be added, subtracted, multiplied or divided. As in integers, the result of adding subtracting or multiplying polynomials is also a polynomial. But, when we divide one polynomial by another polynomial, we get a quotient and a remainder just as in the case of division in integers.

#### 11.5.1 Value of a Polynomial and Zeroes of a Polynomial

**Main Teaching Points**

- a) Value of a polynomial for given values of the variables
- b) Meaning of zero of a polynomial

**Teaching Learning Process**

The value of a polynomial for given values of the variables is obtained by substituting these values of the variables in the polynomial and simplifying the numerical result.
Ask: How do you find the value of a polynomial for some assigned value of the variable?

Explain: Illustrate it by considering several examples as follows:

Let \( p(x) = 3x^3 - 5x^2 - 4x + 4 \) be a polynomial in \( x \). Find the value of \( p(x) \) for \( x = 0 \)

The value of \( p(x) \) for \( x = 0 \) is obtained by substituting \( x = 0 \) in the given polynomial and it is denoted by \( p(0) \).

\[
p(0) = 3(0)^3 - 5(0)^2 - 4(0) + 4 = 0 + 0 = 4
\]

Thus, the value of the given polynomial for \( x = 0 \) is 4.

Ask: Can you obtain the value of the polynomial for \( x = 1 \)?

Let them do and obtain the result.

\[
p(1) = 3(1)^3 - 5(1)^2 - 4(1) + 4 = 3 - 5 - 4 + 4 = -2
\]

Similarly, ask them to find out \( p(2) \).

\[
p(2) = 3(2)^3 - 5(2)^2 - 4(2) + 4 = 24 - 20 - 8 + 4 = 0
\]

Explain: The value of polynomial changes as the value assigned to \( x \) changes. Also, explain that the value of the polynomial for which \( p(x) \) becomes 0 is called the zero of the polynomial.

In the above example \( x = 2 \) is a zero of the given polynomial \( p(x) = 3x^3 - 5x^2 - 4x + 4 \) because for \( x = 2 \), the value of \( p(x) \) is zero.

There are as many zeroes of a polynomial as the degree of the polynomial. However, these may be real numbers or complex numbers. Let the students verify themselves that for the above polynomial other two zeroes are \( x = -1 \) and \( x = \frac{-2}{3} \). The teacher should carefully select a few polynomials with one or more integral zeroes. Students should be asked to find the integral zeroes for the polynomials given to them.

Methodology: Through illustrations students should be taught to find the value of a polynomial and then inductively define zeroes of a polynomial.

11.5.2 Addition of Polynomials

Main Teaching Point

a) Similar terms or like terms
b) How to add two polynomials

Teaching Learning Process

Ask the students to add 5 pens and 3 pens.

5 pens + 3 pens = 8 pens.

Now, ask them to add 5 pens and 3 shirts. It can only be written as 5 pens + 3 shirts. In the first case, two things that we want to add are alike. So, we were able to add them. But in the second instance pens and shirts are different things and so we could not add them up. Similarly, in polynomials ask them to add \( 2x^2 \) and \( 3x^2 \) and let them tell that it is \( 5x^2 \). Again, ask them to add \( 2x^2 \) and \( 3x^4 \). Let them write it as \( 2x^2 + 3x^4 \).
Explain: Terms having the same variable part are called similar terms or like terms. We can add similar terms to get one single term. But when we have to add two terms which are not similar as \(2x^2\) and \(3x^4\), we can only express it as \(2x^2 + 3x^4\) (it is referred to as ‘indicated sum’)

Provide the students with several terms, and ask to add them up.

Illustration: Given: \(3x^2\), \(5x^3\), \(2x^3\), \(7x\), \(8x^2\), \(-2x\)

Identify similar terms and add them up.

\(3x^2\) and \(8x^2\) are similar, so, \(3x^2 + 8x^2 = 11x^2\)

\(5x^3\) and \(2x^3\) are similar, so, \(5x^3 + 2x^3 = 7x^3\)

\(7x\) and \(-2x\) are similar, so, \(7x + (-2x) = 5x\)

Explain: in the case of polynomials, similar terms in the polynomials are added and other terms are shown as indicated addition.

Example 2: Add the polynomials \(p(x) = x^3 + 2x^2 + 3\) and \(q(x) = 3x^2 + 5x + 2\)

In both the polynomials identity the similar terms and add them up and write the other terms as indicated addition.

In these two polynomials

\(2x^2\) and \(3x^2\) are similar and therefore \(2x^2 + 3x^2 = 5x^2\)

\(3\) and \(2\) are similar terms and \(3 + 2 = 5\)

So, \(p(x) + q(x) = (x^3 + 2x^2 + 3) + (3x^2 + 5x + 2)\)

\[= x^3 + (2x^2 + 3x^2) + 5x + (3 + 2)\]

\[= x^3 + 5x^2 + 5x + 5\]

Example 3: Add \(x^2 + 3x - 5\) and \(2x^2 - 2x + 3\)

\[(x^2 + 3x - 5) + (2x^2 - 2x + 3)\]

\[= (x^2 + 2x^2) + [(3x) - (2x)] + [(-5) + (3)]\]

\[= 3x^2 + x - 2\]

This method of addition is described as row method of addition. However, polynomials can also be added by writing one polynomial under the other polynomial such that similar terms are written one below the other, as in the method of number addition.

Explain: Write the first polynomial, then write the second polynomial such that similar terms are written in the same column, then add the two polynomials. Example 2 can be done as follows also:

\[
\begin{align*}
x^3 + 2x^2 + 3 \\
+ 3x^2 + 5x + 2 \\
\hline
x^3 + 5x^2 + 5x + 5
\end{align*}
\]

Example 3 can be done as follows:

\[
\begin{align*}
x^2 + 3x - 5 \\
2x^2 - 2x + 3 \\
\hline
3x^2 + x - 2
\end{align*}
\]

The teacher should give a few more similar examples to clarify the process of addition of polynomials to the students. In the end, a few problems may be
given as exercises. Students should be asked to practice both column method of addition and row method of addition.

**Methodology:** Lecture-cum-discussion method is used to illustrate that only similar terms can be added.

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### Check Your Progress

**Notes:**

a) Write your answer in the space given below each question.

b) Compare your answer with the one given at the end of the unit.

Add the following polynomials by row method:

1) \( p(x) = 2x^2 + 3x + 5 \) \( q(x) = 3x^2 - 4x - 7 \)

2) \( p(x) = x^3 - 2x - 3 \) \( q(x) = x^2 - 3x + 1 \)

Add the following polynomials by column method:

3) \( p(t) = 2t^2 + t - 1 \) \( q(t) = 3t - 5 - 3t^2 \) \( r(t) = 1 - 3t - 3t^2 \)

4) \( p(k) = k^4 + k^2 + 2 \) \( q(k) = k^2 + 2k + 1 \) \( r(k) = k^3 - 3k + 3 \)

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### 11.5.3 Subtraction of Polynomials

**Main Teaching Point**

To subtract one polynomial from another.

**Teacher Learning Process**

**Explain:** The subtraction of polynomials involves the same principles and process as the subtraction of integers. Remind them that \( 7 - (-3) = 7 + 3 = 10 \) and \( 7 - (3) = 4 \).

Just as in the case of addition of polynomials, we subtract only like terms. For example, to subtract \(-3x\) from \(7x\). We write as \(7x - (-3x) = 7x + 3x = 10x\) and to subtract \(3x\) from \(7x\) we write as \(7x - (3x) = 7x - 3x = 4x\). Note the sign of the term to be subtracted by row method or by column method. In both the ways, the sign of every term of the polynomial to be subtracted is changed.

**Example 4:** Subtract \(x - y\) from \(2x + 3y\)

**Row method:**

\[
\begin{align*}
(2x + 3y) - (x - y) &= 2x + 3y - x + y \\
&= [2x + (-x)] + [3y + y] \\
&= x + 4y
\end{align*}
\]

(sign of both the terms in polynomial to be subtracted is changed and then added as usual)
**Column Method:**

\[
\begin{array}{c}
2x + 3y \\
-(x - y)
\end{array}
\]

Polynomials are written one below the other

\[
\begin{array}{c}
2x + 3y \\
-x + y
\end{array}
\]

Sign of each term of the polynomial to be subtract is changed

\[
\begin{array}{c}
x + 4y
\end{array}
\]

The answer is the \( x + 4y \).

**Example 5:** Subtract \( 2x^2 + 2y^2 - 6 \) from \( 3x^2 - 7y^2 + 9 \)

**Row Method**

\[
(3x^2 - 7y^2 + 9) - (2x^2 + 2y^2 - 6)
\]

\[
= 3x^2 - 7y^2 + 9 - 2x^2 - 2y^2 + 6
\]

\[
= (3x^2 - 2x^2) + (-7y^2 - 2y^2) + (9 + 6)
\]

**Column Method**

\[
\begin{array}{c}
3x^2 - 7y^2 + 9 \\
-(2x^2 + 2y^2 - 6)
\end{array}
\]

(Changing the signs of the terms of polynomial to be subtracted.)

\[
\begin{align*}
&3x^2 - 7y^2 + 9 \\
&-2x^2 - 2y^2 + 6 \\
&x^2 - 9y^2 + 15
\end{align*}
\]

The answer is \( x^2 - 9y^2 + 15 \)

Let \( p(x) = 3x^2 - 4x + 7 \)

\( q(x) = 2x^3 + x^2 - 5x - 3 \)

and \( r(x) = 3x^2 - 2x + 1 \)

Ask the students to compute

\[
p(x) - q(x), q(x) - p(x), [p(x) - q(x)] - r(x) \quad \text{and} \quad p(x) - [q(x) - r(x)]
\]

**Then ask:** Is \( p(x) - q(x) = q(x) - p(x) \)?

Let them tell that the two answers are different.

**Explain:** Subtraction is not commutative.

**Ask:** Is \( [p(x) - q(x)] - r(x) = p(x) - [q(x) - r(x)] \)?

**Explain:** Subtraction is not associative. Point out that whereas addition of polynomials is both commutative and associative. The subtraction is neither associate nor commutative.

**Methodology:** Lecture-cum-discussion method is used. Sufficient practice should be provided so that the students develop the skill.
Check Your Progress
Notes: a) Write your answer in the space given below each question.
   b) Compare your answer with the one given at the end of the unit.
5) Perform the subtractions given below:
   i. \(3x - y - (2x - y)\)
   ii. \(a^2 - 3b + c^2 - (a^2 + 5b + 3c^2)\)

6) Subtract by row method:
   \((-6x - 8y) - (-3x + 8y)\)

7. Subtract \(x^2 - 7y^2 + 1\) from \(2x^2 + 3y^2 - 3\) by row method.

8. What should be subtracted from \(x^4 - 1\) to get \(3x^2 - 2x + 1 + x^4\)?

9. What should be added to \(x^3 + 3x^2 + 1\) to get \(x^5 - 2x^2 - 3x\)?

11.5.4 Multiplication of Polynomials
Main Teaching Points
a) To multiply a monomial by a monomial
b) To multiply a polynomial by a monomial
c) To multiply a polynomial by a polynomial

Teaching Learning Process
Three fundamental properties of operations are most commonly used in the process of multiplication of polynomials.

Commutative law which tells us \(xy = yx\)

Associative law Which expresses the fact that \((xy)z = x(yz)\) and

Distributive law Which states that \(x(y + z) = xy + xz\). When a monomial is multiplied by another monomial, laws of exponents are also useful.

In particular, \(x^m \cdot x^n = x^{m+n}\)

Explain: To multiply a monomial by another monomial, we arrange the numbers together and the variables together using commutative and associative properties. Then using law of exponents, find the product.
Example 6: Multiply $3x^3y$ by $5xy^3z$

Solution

$$(3x^3y)(5xy^3z)$$

$$(3\cdot 5)(x^3 \cdot x)(y \cdot y^3) \cdot z$$

$= 15x^4y^4z$$  
(using commutative and associative properties)

Example 7: Multiply $-2ay^2z$ by $4xyz^2$

Solution

$$(-2ay^2z)(4xyz^2)$$

$$= (-2)(4)(a)(x)(y^2)(y)(z)(z^2)$$

$$= -8axy^3z^3$$

**Explain:** To multiply a polynomial by a monomial, we multiply each term of the polynomial by the monomial just as in distribution law.

Example 8: Multiply $2x^2 - 3x - 5$ by $4x$

Solution

$$4x(2x^2 - 3x - 5)$$

$$= (4x)(2x^2) - (4x)(3x) - (4x)(5)$$

$$= (4 \cdot 2)(x \cdot x^2) - (4 \cdot 3)(x \cdot x) - (4 \cdot 5)x$$

$$= 8x^3 - 12x^2 - 20x$$

In actual practice, we straight away write the last step doing it orally without showing the intermediary steps.

For example: $-2x (5x^2 - 3xy + 1) = -10x^3 + 6x^2y - 2x$

**Explain:** To multiply a polynomial by another polynomial, we multiply each term of the second polynomial by each term of the first polynomial just as in distributive law.

Example 9: Multiply $2x + 3y$ by $2x^3 - xy + 2y^2$

Solution

$$(2x + 3y)(2x^3 - xy + 2y^2)$$

$$= 2x(2x^3 - xy + 2y^2) + 3y(2x^3 - xy + 2y^2)$$

$$= (4x^4 - 2x^2y + 4xy^2) + (6x^3y - 3xy^2 + 6y^3)$$

$$= 4x^4 - 2x^2y + 4xy^2 + 6x^3y - 3xy^2 + 6y^3$$

$$= 4x^4 - 2x^2y + xy^2 + 6x^3y + 6y^3$$

Example 10: Multiply $x^2 - 3x - 2$ by $3x^2 + x - 1$

Solution

$$(x^2 - 3x - 2)(3x^2 + x - 1)$$

$$= x^2(3x^2 + x - 1) - 3x(3x^2 + x - 1) - 2(3x^2 + x - 1)$$

$$= 3x^4 + x^3 - x^2 - 9x^3 - 3x^2 - 6x^2 - 2x + 2$$

$$= 3x^4 + (x^3 - 9x^3) + (-x^2 - 3x^2 - 6x^2) + (3x - 2x) + 2$$

$$= 3x^4 - 8x^3 - 10x^2 + x + 2$$

**Methodology:** The method of multiplication is illustrated through various examples and sufficient practice should be provided to develop the skill.
Check Your Progress
Notes: a) Write your answers in the space given below each question.
   b) Compare your answer with the one given at the end of the Unit

10) Find the following products:
   \((2x + 3) \cdot (x + 4)\)

11) \((x^3 - x^2 - 1) (2x^2 + x + 3)\)

12) \((x + y) (x^2 - xy + 2y^2)\)

13. \((x - y) (x + y) (x^2 + y^2)\)

11.5.5 Division of Polynomials

Main Teaching Points
   a) To divide a monomial by a monomial
   b) To divide a polynomial by a polynomial
   c) Dividend = Divisor \times Quotient + Remainder

Teaching Learning Process

Here we will be restricting ourselves to division of polynomials in one variable only. The process of division in the case of polynomials is the same as in arithmetic. The similarity between the two processes should be made clear to the pupils by giving suitable examples.

In case of numbers, we can only divide a bigger number by a smaller number. Similarly in case of polynomials a higher degree polynomial can be divided by a polynomial of lower degree.

Example 11: Divide \(12x^3\) by \(5x\)

Now, \(12x^3 \div 5x = \frac{12x^3}{5x} = \frac{12 \cdot x \cdot x}{5 \cdot x} = \frac{12}{5} x^2\)

or we can use the law of exponents : \(\frac{x^m}{x^n} = x^{m-n}\). Note that in integers, when a smaller integer is divided by a larger integer, the answer is not an integer.

For example, \(3 \div 7 = \frac{3}{7}\), it is not an integer. Similarly, if we divide a monomial of smaller degree by a monomial of higher degree, then the answer is not a polynomial clearly, \(3x \div x^3 = \frac{3x}{x^3} = \frac{3}{x^2}\); it is not a polynomial.
The process of dividing a polynomial by another polynomial is quite similar to long division for integers. Here, we write the dividend and the divisor both in decreasing powers of the variables and then divide as illustrated in the following example:

**Example 12:** Divide \(3x^2 - 9x + 6\) by \(3x\)

Step 1: both the polynomials should be written in decreasing powers of \(x\). The polynomials are already in this form

\[
\begin{array}{c}
\text{3x}^2 - 9x + 6 \\
\hline
3x \quad | \\
\end{array}
\]

- \[
\begin{array}{c}
3x^2 \\
- \\
\hline
-9x + 6 \\
\hline
-9x \\
+ \\
\hline
6 \\
\end{array}
\]

Step 2: Divide the first term of the dividend by \(3x^2 ÷ 3x = \frac{3x^2}{3x} = x\) write it as the first term of the quotient.

Step 3: Multiply the divisor by the quotient of step 2 and write it below the dividend and subtract to get \(-9x + 6\).

Step 4: Divide \(-9x\) by \(3x\) the result in \(-3\), write it as the second term of the quotient.

Step 5: Multiply the divisor by the quotient of step 4 and write it below the divided and subtract to get 6.

Step 6: Since the degree of 6 is smaller than the degree of \(3x\), the process stops. Thus, when \(3x^2 - 9x + 6\) is divided by \(3x\), the quotient is \(x - 3\) and the remainder is 6.

Ask the students to verify that \(3x^2 - 9x + 6 = 3x \times (x - 3) + 6\)

(\text{Dividend} = \text{quotient} \times \text{divisor} + \text{remainder})

**Example 13:** Divide \(2x^3 - 3x^2 + 5x - 1\) by \(x - 2\)

Step 1: Write both the polynomials is standard form.

Step 2: Divide first term of dividend by the first term of divisor \(\frac{2x^3}{x} = 2x^2\) write it as first term of the quotient.

\[
\begin{array}{c}
2x^2 + x + 7 \\
\hline
(x-2) \\
\end{array}
\]

\[
\begin{array}{c}
2x^3 - 3x^2 + 5x - 1 \\
- (2x^3 - 4x^2) \\
\hline
x^2 + 5x - 1 \\
- (x^2 - 2x) \\
\hline
7x - 14 \\
- (7x - 14) \\
\hline
13 \\
\end{array}
\]
Step 3: Multiply the divisor by the quotient in step 2, write it below the dividend and subtract to get $x^2 + 5x - 1$

Step 4: Divide the first term of this polynomial by the first term of the divisor.

$$x^2 \div x = \frac{x^2}{x} = +x$$

(with proper sign) write it as the second term (with proper sign) of the quotient.

Step 5: Multiply the divisor by the quotient in step 4, write it below the new dividend and subtract to get $7x - 1$.

Step 6: Divide the first term of this new dividend by the first term of the divisor.

$$7x \div x = \frac{7x}{x} = +7$$

write it as the third term (with a proper sign) of the quotient.

Step 7: Multiply the divisor by the quotient in step 6 writes it below the new dividend and subtract to get 13.

Step 8: Since the degree of 13 is less than the degree of the divisor, the process stops.

When $2x^3 - 3x^2 + 5x - 1$ is divided by $x - 2$, the quotient is $2x^2 + x + 7$ and the remainder is 13.

Ask the students to verify that

Dividend = Quotient x Divisor + Remainder

or $2x^3 - 3x^2 + 5x - 1 = (x-2) (2x^2 + x + 7) + 13$

Check Your Progress

Notes: a) Write your answer in the space given below.

b) Compare your answer with the one given at the end of the unit.

Divide and write the quotient and remainder in each of the following:

14) $(6m^2 - 17m - 3) \div (m - 3)$

15) $(8y^2 - 6y - 5) \div (2y + 1)$

16) $(27x^3 - 1) \div (3x - 1)$
11.6 FACTORIZATION OF POLYNOMIALS

In the preceding section you were introduced to the methods of multiplying polynomials to obtain polynomials of higher degree. Now, we shall consider the reverse procedure, i.e., techniques of writing a polynomial as a product of two or more polynomials of lower degree.

11.6.1 Basic Concepts

Main Teaching Points
a) What is a factor?
b) Meaning of factorization of polynomials

Teaching Learning Process

Students have already learnt in arithmetic, the concept of factors and prime factorization of numbers. In algebra, the concept of factor is the same, given a polynomial, we express it as a product of immediate polynomials and the process is called the factorization of the polynomial.

For example, we know that
1) \((2x) \cdot (3x^2) = 6x^3\)
2) \((2xy) \cdot (x + y) = 2x^2 y + 2xy^2\)
3) \((x^2+x) \cdot (2x + 1) = 2x^3 + 3x^2 + x\)

In examples 1, \(6x^3\) is the product of \(2x\) and \(3x^2\). \(2x\) and \(3x^2\) are called the factors of \(6x^3\).

So, in these examples, we get—

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Polynomial</th>
<th>Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(6x^3)</td>
<td>(2x) and (3x^2)</td>
</tr>
<tr>
<td>2.</td>
<td>(2x^2 y + 2xy^2)</td>
<td>(2xy) and (x + y)</td>
</tr>
<tr>
<td>3.</td>
<td>(2x^3 + 3x^2 + x)</td>
<td>(x^2 + x) and (2x + 1)</td>
</tr>
</tbody>
</table>

Explain: If a polynomial can be expressed as a product of two or more polynomials of lower degree, then these polynomials of lower degree are called its factors.

Ask: In the above examples, which of the factors can be further factorized?

\(2x = 2 \cdot x\)  
\(3x^2 = 3 \cdot x \cdot x\)  

Similarly,
\(2xy = 2 \cdot x \cdot y\)  
\(x^2 + x = x(x + 1)\)

\(x + y\) and \(2x + 1\) cannot be factorized further.

Therefore,
\(6x^3 = 2 \cdot 3 \cdot x \cdot x \cdot x\) (it has 5 factors)
\(2x^2 y + 2xy^2 = 2 \cdot x \cdot y \cdot (x + y)\) (it has 4 factors)
\(2x^3 + 3x^2 + x = x(x + 1) \cdot (2x + 1)\) (it has 3 factors)
Polynomials: Basic Concepts and Factoring

**Explain:** These factors are such that they cannot be factorized further.

A polynomial which can not be factorized is called **irreducible**.

Expressing a polynomial as a product of irreducible factors is similar to prime factorization of numbers.

**Methodology:** Discussion with inductive reasoning is used to clarify that concept.

### 11.6.2 Factoring a Quadratic Polynomial

**Main Teaching Points**

To factorize using algebraic identities

**Teaching learning Process**

Students have already learnt in lower classes

That \((x + a)^2 = x^2 + 2ax + a^2\) ............................................(i)

\((x – a)^2 = x^2 – 2ax + a^2\) .............................................(ii)

\((x + a) . (x – a) = x^2 – a^2\) ...................................................(iii)

A quadratic polynomial is of the form \(p(x) = ax^2 + bx + c\), where \(a\), \(b\) and \(c\) are real numbers, \(a \neq 0\). The students may be led to discover that the right hand side expressions given in the above equations are quadratic polynomials and the left hand side expressions are their factors, since \((x + a)^2 = (x + a)(x + a)\) and \((x – a)^2 = (x – a)(x – a)\).

Note that the first term and the last term of the quadratic polynomials in the above equations are perfect squares. So, whenever a given quadratic polynomial in its standard form is such that the first and the last term are perfect squares, then it might be possible to express it in any one of the above forms. The following examples may be given to clarify the method.

**Example 14:** Factorize \(x^2 + 10x + 25\)

The first and the last terms are \(x^2\) and 25. They are perfect squares. The quadratic polynomial may be written as-

\[x^2 + 10x + 25 = (x)^2 + 2(x) (5) + (5)^2\]

\[= (x + 5)^2 \] ..................................................... (using identity(i))

**Example 15:** Factorize \(9x^2 + 12x + 4\)

The first and the last terms are \(9x^2\) and 4. Clearly they are perfect squares. The quadratic polynomial may be written as:

\[9x^2 + 12x + 4 = (3x)^2 + 2(3x) (2) + (2)^2\]

\[= (3x+2)^2 \] ..................................................... (using identity (i))

**Example 16:** Factorize \(25x^2 – 30x + 9\)

The first and the last terms are \(25x^2\) and 9, which are perfect squares. The quadratic polynomial may be written as

\[25x^2 – 30x + 9 = (5x)^2 – 2(5x) (3) + (3)^2\]

\[= (5x-3)^2 \] ..................................................... (using identity (ii))
Example 17: Factorize $9x^2 - 16$

The first and the last terms are $9x^2$ and $16$ which are perfect squares. The quadratic polynomial may be written as:

$$9x^2 - 16 = (3x)^2 - (2)^2$$

$$= (3x + 4)(3x - 4) \quad \text{(using identity (iii))}$$

After giving a few more similar examples, the teacher may give some exercises for practice.

Methodology: Method is illustrated using various examples and sufficient drill should be provided to develop the skill.

11.6.3 Method of Splitting the Middle Term

Main Teaching Point

To factorize a quadratic polynomial of the type $x^2 + (p + q) x + pq$ by splitting the middle term.

Teaching Learning Process

A polynomial of the form $x^2 + (p + q) x + pq$ can be factorized into linear factors by splitting the middle term as follow:

$$x^2 + (p + q) x + pq = x^2 + px + qx + pq$$

$$= x(x + p) + q(x + p)$$

$$= (x + p)(x + q)$$

Explain: If a given polynomial of the form $x^2 + bx + c$ is such that $b = p + q$ and $c = pq$ for some numbers $p$ and $q$, then the given polynomial can be factorized by splitting the middle term as shown above. The numbers $p$ and $q$ are determined by hit and trial. The method should be illustrated through examples as follows:

Example 18: Factorize $x^2 + 7x + 12$ Note that $p + q = 7$ and $pq = 12$ Ask the students to find two numbers $p$ and $q$ such that $p + q = 7$ and $pq = 12$ by hit and trial.

The pupils should be made to think as follows: Find two numbers whose product is 12 or break it up as product of two numbers. As $12 = 1 \times 12 \text{ or } 2 \times 6 \text{ or } 3 \times 4$

Out of these, 3 and 4, are such that $3 + 4 = 7$

$\therefore$ $p$ and $q$ are 3 and 4.

Now, we can factorize the polynomial.

As follow: $x^2 + 7x + 12 = x^2 + (4 + 3) x + (4.3)$

$$= x^2 + 4x + 3x + 4.3$$

$$= x(x + 4) + 3(x + 4)$$

$$= (x + 4)(x + 3)$$

Which are the required factors?

Example 19: Factorize $x^2 + x - 6$.

Here, $p + q = 1$ and $pq = -6$. 
Now, break up –6 into product of the two numbers.

\[-6 = 1 \times (-6) \text{ or } (-1) \times 6 \text{ or } 2 \times (-3) \text{ or } (-2) \times 3.\]

Out of these –2 and 3 are such that \((-2) + 3 = 1\).

\[\therefore \text{ p and q are } -2 \text{ and } 3.\]

\[\therefore \text{ } x^2 + x - 6 = x^2 + [(-2) + 3)] x + (-2) (3)
\]
\[= x^2 - 2x + 3x + (-2) (3)
\]
\[= x(x-2) + 3(x-2)
\]
\[= (x-2)(x+3)
\]

Which are the required factors.

**Example 20:** Factorize \(x^2 - 7x + 10\)

Here, \(p + q = -7\) and \(pq = +10\).

Now, break up 10 into product of the two numbers.

10 = 1 × 10 or \((-1) \times (-10)\) or 2 × 5 or \((-2) \times (-5)\).

Out of these –2 and –5 are such that \((-2) + (-5) = -7\).

\[\therefore \text{ p and q are } -2 \text{ and } -5.\]

\[\therefore \text{ } x^2 - 7x + 10 = x^2 + [(-2) + (-5)]x + (-2) (-5)
\]
\[= x^2 - 2x + (-5)x +(-2) (-5)
\]
\[= x(x-2) + (-5)(x-2)
\]
\[= (x-2)(x-5)
\]

Which are the required factors.

**Methodology:** The method is illustrated using several examples and then practice should be provided to develop the skill.

11.6.4 **Method of Splitting the Middle Term (Contd.)**

**Main Teaching Point**

To factorize a quadratic polynomial of the type \(ax^2 + bx + c\) by splitting the middle term.

**Teaching Learning Process.**

When a quadratic polynomial is of the form \(ax^2 + bx + c\) such as \(2x^2 + 9x + 4\) or \(3x^2 - 14x + 8\), the factors are of the form \(px + q\) and \(rx + s\). Multiplying \(px+q\) and \(rx + s\) we get \((px + q)(rx + s) = prx^2 + (ps + qr)x + qs\) comparing it with \(ax^2 + bx + c\), we get \(a = pr\), \(b = ps + qr\) and \(c = qs\).

It is also seen that \((ps)(qr) = (pr)(qs) = ac\).

\[\therefore \text{ Coefficient of } x = b\]

\[= \text{sum of two numbers ps and qr whose product is ac}\]

So, the method of factoring \(ax^2 + bx + c\) is similar to the one for the polynomial \(x^2 + bx + c\) except for now we have to find two numbers whose sum is \(b\) and their product is \(ac\).

Consider the following examples:
**Example 21.** Factorize $2x^2 + 9x + 4$

Here, we have to find two numbers $p$ and $q$ such that $p + q = 9$ and $pq = 2 \times 4 = 8$

Now, break up 8 into product of two numbers $8 = 1 \times 8$ or $(-1) \times (-8)$ or $2 \times 4$ or $(-2) (-4)$

Out of these pairs 1 and 8 are such that $1 + 8 = 9$

$\therefore$ $p$ and $q$ are 1 and 8.

$\therefore 2x^2 + 9x + 4 = 2x^2 + (1 + 8)x + 4$

$= 2x^2 + x + 8x + 4$

$= x(2x + 1) + 4 (2x + 1)$

$= (2x + 1) (x + 4)$

Which are the required factors.

**Example 22:** Factorize $3x^2 - 14x + 8$

Here, we have to find two numbers $p$ and $q$ such that $p+q = (-14)$ and $pq = 3 \times 8 = 24$

Now, break up 24 into product of two numbers $24 = 1 \times 24$ or $2 \times 12$ or $3 \times 8$ or $4 \times 6$

Or $(-1) \times (-24)$ or $(-2) \times (-12)$ or $(-3) \times (-8)$ or $(-4) \times (-6)$

Out of these pairs, –2 and –12 are such that $(-2) + (-12) = -14$

$\therefore$ $p$ and $q$ are $-2$ and $-12$.

$\therefore 3x^2 - 14x + 8 = 3x^2 + (-2-12)x + 8$

$= 3x^2 - 2x - 12x + 8$

$= x(3x - 2) - 4 (3x - 2)$

$= (3x - 2) (x - 4)$

Which are the required factors.

**Example 23:** Factorize $6x^2 - 11x - 2$

Here, we have to find two numbers $p$ and $q$ such that $p + q = (-11)$ and $pq = 6 \times (-2) = -12$

Now, break up $-12$ into product of two numbers $-12 = 1 \times (-12)$ or $(-1) \times 12$ or $3 \times (-4)$ or $(-3) \times 4$

Or $2 \times (-6)$ or $(2) \times 6$

Out of these pairs 1 and $-12$ are such that $1 + (-12) = -11$

$\therefore$ $p$ and $q$ are $1$ and $-12$.

$\therefore 6x^2 - 11x - 2 = 6x^2 + (1 - 12)x - 2$

$= 6x^2 + x - 12x - 2$

$= x(6x + 1) - 2 (6x+1)$

$= (6x + 1) (x - 2)$

Which are the required factors.

**Methodology:** The method is illustrated by various examples and sufficient practice is required to develop the skill.
11.7 REMAINDER THEOREM

Main Teaching Points
a) Statement of remainder theorem
b) Use of remainder theorem to find remainder

Teaching Learning Process
You have already taught your students the process of dividing a polynomial by another polynomial. If a polynomial $p(x)$ is divided by $q(x)$ to give quotient $s(x)$ and remainder $r(x)$, then verify that:

$$p(x) = q(x) \cdot s(x) + r(x)$$

Also, remind them that degree of remainder $r(x)$ is smaller than the degree of the divisor. Now, as a special case, $p(x)$ divided by $x - a$ to give quotient $q(x)$ and remainder $r(x)$, then-

$$p(x) = (x - a) q(x) + r(x)$$

and degree of $r(x)$ is smaller than the degree of the divisor $x - a$, which is 1.

$\therefore$ degree of $r(x)$ should be zero as $r(x)$ is a constant polynomial ($r$), independent of $x$.

$\therefore$ we get $p(x) = (x - a) q(x) + r$
**Ask:** Find the value of the polynomial
\[ p(x) \text{ at } x = a \]
\[ p(a) = (a – a) q(a) + r = r \]

This leads to an important theorem, known as remainder theorem which states as

"If \( p(x) \) is a polynomial and it is divided by \( (x-a) \), where \( a \) is a real number, the remainder will be \( p(a) \)"

Illustrate through examples, how remainder theorem is useful in finding the remainder without actual division.

**Example 24:** Find the remainder when \( 3x^2 + 2x + 1 \) is divided by \( x – 4 \).

Here, \( p(x) = 3x^2 + 2x + 1 \),
Comparing \( x – 4 \) with \( x – a \), we get \( a = 4 \).
According to remainder theorem,

\[
\text{Remainder, } r = p(4) = 3(4)^2 + 2(4) + 1
\]
\[ = 48 + 8 + 1 = 57 \]

**Example 25:** Find the remainder when \( x^3 – 2x^2 + x – 5 \) is divided by \( x + 2 \).

Here, \( p(x) = x^3 – 2x^2 + x – 5 \),
Comparing \( x + 2 \) with \( x – a \), we get \( a = -2 \).
According to remainder theorem,

\[
\therefore \text{Remainder} = p(-2) = (-2)^3 - 2(-2)^2 + (-2) - 5
\]
\[ = -8 - 8 - 2 - 5
\]
\[ = -23 \]

**Methodology:** Deductive method is used to explain the meaning of the statement of the remainder theorem. Than various illustrations are given to find the remainder.

### 11.7.1 Use of Remainder Theorem in Factorization

**Teaching Points**

a) \( x – a \) is a factor of \( p(x) \) if \( p(a) = 0 \)
b) Factorizing a cubic polynomial using the factor theorem

**Main Teaching Process**

**Ask:** When \( p(x) \) is divided by \( x – a \), the remainder is \( p(a) \). If \( p(a) \) turns out to be zero, what do you conclude?

**Explain:** If the remainder is zero, then \( p(x) = (x – a) \). \( q(x) \) and therefore \( x – a \) is a factor of \( p(x) \).

\[
\therefore \text{If } p(a) = 0, \text{ for some number } a, \text{ than } x \text{-a is a factor of } p(x). \text{ This statement is called the factor theorem.}
\]

**Example 26:** Find if \( x – 3 \) is a factor of \( x^3 – 3x^2 + 4x – 10 \).

Here, \( p(x) = x^3 – 3x^2 + 4x – 10 \)

\[
p(3) = (3)^3 - 3(3)^2 + 4(3) - 10
\]
\[ = 27 - 27 + 12 - 10
\]
\[ = 2 \neq 0 \]

\( x – 3 \) is not a factor of \( p(x) \)
Example 27: Find the factor of $x^3 - x^2 + x - 1$.

Here, $p(x) = x^3 - x^2 + x - 1$

$p(1) = 1^3 - 1^2 + 1 - 1$
\[= 1 - 1 + 1 - 1 = 0\]

Now, divide $p(x)$ by $x - 1$ to get the other factor

\[
\begin{array}{c|cc}
& x^2 + 1 \\
\hline
x - 1 & x^3 - x^2 + x - 1 \\
\hline
& x^3 - x^2 \\
& - + \\
& x - 1 \\
& x - 1 \\
& - + \\
& 0
\end{array}
\]

\[\therefore \ x^3 - x^2 + x - 1 = (x - 1) (x^2 + 1)\]

since, $(x^2 + 1)$ is irreducible

\[\therefore \] These are the required factors.

Example 28: Find the factors of $x^3 + 3x^2 - 4x - 12$

Here, $p(x) = x^3 + 3x^2 - 4x - 12$

$p(1) = 1 + 3 - 4 - 12 = -12 \neq 0$

\[\therefore \] (x - 1) is not a factor of $p(x)$

$p(-1) = (-1)^3 + 3(-1)^2 - 4(-1) - 12$
\[= -1 + 3 + 4 - 12 = 7 - 13 = -6 \neq 0\]

\[\therefore \] (x + 1) is not a factor of $p(x)$

$p(2) = (2)^3 + 3(2)^2 - 4(2) - 12$
\[= 8 + 12 - 8 - 12 = 0\]

\[\therefore \] (x-2) is a factor of $p(x)$

Now, divide $p(x)$ by $(x - 2)$:

\[
\begin{array}{c|cc}
& x^2 + 5x + 6 \\
\hline
x - 2 & x^3 + 3x^2 - 4x - 12 \\
\hline
& x^3 - 2x^2 \\
& - + \\
& 5x^2 - 4x - 12 \\
& 5x^2 - 10x \\
& - + \\
& 6x - 12 \\
& 6x - 12 \\
& - + \\
& 0
\end{array}
\]
Example 29: For what value of k is \( x^3 - kx^2 + 4x - 12 \) divisible by \( x - 3 \).

Here, \( p(x) = x^3 - kx^2 + 4x - 12 \)

Comparing \( x - 3 \) with \( x - a \), we get \( a = 3 \)

\[
\begin{align*}
\text{Remainder} & = p(3) = (3)^3 - k(3^2) + 4(3) - 12 \\
& = 27 - 9k + 12 - 12 \\
& = 27 - 9k
\end{align*}
\]

For divisibility, \( p(3) = 0 \Rightarrow 27 - 9k = 0 \Rightarrow k = 3 \)

**Methodology:** Inductive logic is used to illustrate that \( x - a \) is a factor of \( p(x) \) if \( p(a) = 0 \)

---

**Check Your Progress**

**Notes:**
- a) Write your answer in the space given below each question.
- b) Compare your answer with the one given at the end of the Unit.

21) Find whether \( p(x) \) is divisible by \( q(x) \) for \( p(x) = x^3 - 2x^2 + x + 3 \) and \( q(x) = x - 4 \).

22) For what value of \( k \), \( q(x) \) will be a factor of \( p(x) \) in the following:

\( p(x) = x^3 + 2x^2 + kx - 2 \) and \( q(x) = x + 2 \)

---

**11.8 LET US SUM UP**

In this unit you have learnt some strategies and approaches to clarify certain important concepts such as constants, variables, algebraic expression, value of an algebraic expression and polynomials. You have also learnt how to teach effectively, the concepts of basic operations involving polynomials and impart
skills to perform these operations. You have also studied the techniques of factorizing quadratic polynomials. The use of remainder theorem is algebra to find the remainder without performing the division was also presented. It was also shown that the remainder theorem may be used to factorize polynomials of higher degree. In short, this unit provides you a foundation for further development in teaching algebra.

11.9 UNIT END ACTIVITIES

At the end of this unit you may:
1) Give your students some more exercises on basic operations with polynomials and factorization;
2) Evaluate the learning outcomes of your students after this unit has been taught;
3) Prepare lesson plans or develop teaching aids for teaching factorization and multiplication of polynomials; and
4) Visit your nearby schools and observe the teaching of polynomials to see whether the ideas and techniques learnt in this unit find a place in classroom teaching.

11.10 POINTS FOR DISCUSSION

1) Can we correlate the ideas involved in this unit with corresponding concepts in arithmetic?
2) Can we correlate the concepts of products and factors involved in this unit to the concepts of area and volume in mensuration and develop some novel teaching aids.
3) Can remainder theorem be used to develop a general technique of factorization?

11.11 ANSWER TO CHECK YOUR PROGRESS

1) $5x^2 - x - 2$
2) $x^3 + x^2 + x - 2$
3) $-4t^2 + t - 5$
4) $k^4 + k^3 + 2k^2 - k + 6$
5) (i) $x$ (ii) $-8b - 2c^2$
6) $-3x - 16y$
7) $x^2 + 10y^2 - 4$
8) $-3x^2 + 2x - 2$
9) $x^3 - x^2 - 5x^2 - 3x - 1$
10) $2x^2 + 11x + 12$
11) $2x^5 - x^4 + 2x^3 - 5x^2 - x - 3$
12) $x^3 + xy^2 + 2y^3$
13) $x^4 - y^4$
14) Quotient $= -6m + 1$, Remainder $= 0$
15) Quotient $= 4y - 5$, Remainder $= 0$
16) Quotient $= 9x^2 + 3x + 1$, Remainder $= 0$
17) $(x + 3) (x + 5)$
18) $(x - 3) (x + 2)$
19) $5(x + 2) (x + 6)$
20) $(9x - 4) (6x + 1)$
21) Not divisible
22) $k = -1$
11.12 SUGGESTED READINGS


