UNIT 12 LINEAR EQUATIONS AND INEQUATIONS AND QUADRATIC EQUATIONS*

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12.1 INTRODUCTION

The word equation is within the comprehension of the students. They must be able to differentiate between an expression and an equation. Equations are of different types depending on the number of variables and the degree of variables. Besides, there are many situations which are represented by inequalities. The student is familiar with the solution of linear equations in one variable.

This unit gives various ways to teach linear equations in two variables and quadratic equations in one variable. Linear inequations and their graphical representation are also discussed.

12.2 OBJECTIVES

At the end of this unit, you will be able to:

- explain the distinction between linear equation in one variable and the one in two variables; a system of equations in two variables and quadratic equations;

* A few sections of this unit has been adopted from ES-342, IGNOU, 2000
• show graphically a linear equation in two variables;
• use various methods of solving systems of linear equations in two variables and quadratic equations;
• describe the difference between consistent and inconsistent systems of equations, both graphically and algebraically;
• inculcate problem solving skills, that are:
  i) translate word problems into mathematical models;
  ii) apply mathematical techniques to solve word problems; and
• show graphically the linear inequations; and
• plan and design learning activities that help to teach linear educations, inequations and quadratic equations.

12.3 LINEAR EQUATION IN ONE VARIABLE

Main Teaching Points
a) Recognizing a linear equation in one variable
b) Solution of a linear equation in one variable

Teaching Learning Process
Students are familiar with the terms equation, expression, variable and degree. As a prelude to further study of equations, you may find out whether students can discriminate between an expression and an equation.

You may present them with a number of expressions and equations and ask them to select those which are equations. Also ask them to find out the number of variables in these equations and write the degree of expression in each of the equation.

**Explain:** Equations in which there is only one variable and the degree of the variable is also one, are called equations of degree one in one variable.

They are also called linear equations in one variable. Consider, a linear equation in one variable, say, \( x + 5 = 7 \).

Ask the students whether the given equation is true for \( x = 3 \)?

Students will be able to tell that it is true only for \( x = 2 \).

**Explain:** The value of the variable for which the given equation is true is called the solution of the equation.

Standard form of the linear equation in one variable is \( ax+b=0 \) and its solution is obtained as follows:

\[
ax + b = 0
\]

Adding \(-b\) to both sides, we get

\[
ax + b + (-b) = 0 + (-b)
\]

\[
ax = -b
\]

Multiplying both sides by \( \frac{1}{a} \), we get
Linear Equations, Inequations and Quadratic Equations

\[
\frac{1}{a}(ax) = \frac{1}{a}(-b)
\]

\[
x = -\frac{b}{a}
\]

is the solution of the linear equation \(ax + b = 0\).

**Methodology:** Discussion with various illustrations.

### Check Your Progress

**Notes:**
- a) Write your answer in the space given below each question.
- b) Compare your answer with the one given at the end of the unit.

1) Which of the following are equations?
   
   i) \(2x + 3 = 5\)   
   ii) \(3x - 5\)   
   iii) \(3x \neq 2x - 10\)   
   iv) \(x > 10\)   
   v) \(3x + 2y = 7\)   
   vi) \(2x + 4y < 4\)   
   vii) \(x^2 + 3x - 4 = 0\)   
   viii) \(4x - 3y\)

2) Which of the following are linear equations in one variable. Also write their solutions:
   
   i) \(3x + 2 = 14\)   
   ii) \(2x + 3y = 7\)   
   iii) \(2x - 3 = 7\)   
   iv) \(x^2 + x = 6\)   
   v) \(-2x + 7 = 0\)   
   vi) \(5x - 3 = 3x + 1\)

---

### 12.4 LINEAR EQUATION IN TWO VARIABLES

An equation of degree one involving two variables is discussed in this unit. Such an equation has infinite solutions and its graph is a straight line. The methods of solving system of linear equations in two variables and consistency of equations are the main points which are dealt with here.

**12.4.1 Graph of Linear Equation in two Variables**

**Main Teaching Points**

- Finding various solutions of a linear equation in two variables
- Graph of a linear equation in two variables is a straight line
Teaching Learning Process

Through examples you should bring out inductively that an equation of degree one in two variables has infinite solutions and when plotted on a graph, they lie on a straight line.

After revising the solution of a linear equation in one variable, ask them to find the solutions of a linear equation in two variables. viz. \( x + y = 5 \),

**Explain:** Any set of values of \( x \) and \( y \), which satisfies the given equation, is called the solution of the equation.

Here, \( x=1 \) and \( y=4 \) is a solution of the given equation.

Ask them if they can tell any other solution?

Ask them if they can find a solution of the equation with \( x=10 \)? Let them learn to find the value of \( y \) for a given value of \( x \).

Substituting the value of \( x \) in the given equation,

they get: \( 10 + y = 5 \quad y = -5 \)

\( \therefore \) \( x =10 \) and \( y = -5 \) is a solution of the equation.

**Ask:** How many solutions of the given equation they can find?

**Explain:** Every linear equation in two variables has infinitely many solutions.

Ask them to write down 5 different solutions of the given equation and plot them on a graph.

Say, the 5 solutions of \( x + y = 5 \) are

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Observe that all the points lie on a straight line.

**Explain:** The graph of an equation of degree one in two variables is a straight line. That is the reason it is called a linear equation.

Also, ask them to find any other solution of the equation and plot it on the graph. Note that it also lies on the same line.

Ask them to take any point of the line say, \((5,0)\). Note that \( x = 5 \) and \( y = 0 \) is a solution of the equation.

**Explain:**

i) Every solution of the equation lie on this line and

ii) Every point of this line is a solution of the given equation

Consider, various examples from real life where two variables are linearly related.

**Example 1:** A train is moving with a uniform speed of 60 km/hr. Draw the time distance graph.
Read from the graph the distance travelled in 2.5 hours.

We know that Speed = Distance/Time

Denoting time by x and distance by y,

we get $60 = \frac{y}{x}$  
$60x = y$  
$60x - y = 0$

It is a linear equation in two variables x and y.

Find various solutions of the equation as shown below:

<table>
<thead>
<tr>
<th>Time in Hours (x)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance in km (y)</td>
<td>60</td>
<td>120</td>
<td>180</td>
<td>240</td>
<td>300</td>
</tr>
</tbody>
</table>

Thus, we plot the ordered pairs: (1,60), (2,120), (3,180), (4,240) and (5,300) and join the points marked by a line.

This line represents all the solutions of the given equation.

From the graph, note that when x=2.5, then value of y=150. Indeed you can verify that x=2.5 and y=150 is a solution of the equation $60x - y = 0$. Hence, distance travelled in 2.5 hours is 150 km.

Ask the students to plot the graph of the equation $x = 5$.

As a linear equation in one variable x,

Its graph is a single point, as shown below:
Note: It is a line parallel to Y–axis.

Now, if we consider it as a linear equation in two variables x and y, then it can be expressed as:

\[ x + 0 \cdot y = 5 \]

Which means that the value of \( x \) is 5 for each value of \( y \). So, the various solutions of the equation are as shown below:

<table>
<thead>
<tr>
<th>( x )</th>
<th>5</th>
<th>5</th>
<th>5</th>
<th>5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>–1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Plot these points on a graph and join them. Now you can see through the graph of the equation \( x = 5 \) is a straight line parallel to the y–axis.

Similarly, draw the graphs of \( x = 3 \) and \( x = –2 \).

**Explain:** They are all straight lines parallel to the y-axis.

Note that the equation of y-axis is \( x = 0 \). Similarly let the graphs of \( y = 2 \), \( y = 4 \), \( y = –3 \) etc. be drawn and interpreted by the students.

Let them conclude that graph of \( y=\text{constant} \) is a line parallel to x–axis and the equation of \( x – \text{axis} \) is \( y = 0 \).

**Methodology:** Examples are discovered and the facts are derived inductively from the different examples.
12.4.2 Graph of Linear Equation Involving Absolute Values

Main Teaching Point
To draw the graph of a linear equation involving absolute value.

Teaching Learning Process
The absolute value of $a$ is written as $|a|$ and it is defined as follows:

**Definition:**

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Let us study the graph of a linear equation involving absolute value.

**Example 2:** Draw the graph of $y = 4$.

By definition $|y| = 4 \Rightarrow \begin{cases} y = 4, & \text{when } y \geq 0 \\ y = -4, & \text{when } y < 0 \end{cases}$

The graph of the given equation is, Therefore, the union of graphs of $y = 4$ and $y = -4$ as shown.

![Graph of $y = 4$](image)

**Example 3:** Draw the graph of $|2x - 3| = 4$

By definition $|2x - 3| = \begin{cases} 2x - 3, & \text{when } 2x - 3 \geq 0 \\ -(2x - 3), & \text{when } 2x - 3 < 0 \end{cases}$

$\Rightarrow |2x - 3| = 4 \Rightarrow \begin{cases} 2x - 3 = 4, & \text{when } 2x - 3 \geq 0 \\ -2x + 3 = 4, & \text{when } 2x - 3 < 0 \end{cases}$

or $\begin{cases} x = \frac{7}{2}, & \text{when } x \geq \frac{3}{2} \\ x = -\frac{1}{2}, & \text{when } x < \frac{3}{2} \end{cases}$

$\therefore$ The graph is as shown. It consists of union of graphs of $x = 7/2$ and $x = -1/2$. 
Example 4: Draw the graph of $y = 2|x|$. 

By definition of $|x|$, 

$$y = 2|x|$$ can be written as 

$$y = \begin{cases} 
2x, & \text{when } x \geq 0 \\
-2x, & \text{when } x < 0 
\end{cases}$$

Ask students to point out the difference in the graph of the equation $y = 2x$ and $y = -2x$ when $x \geq 0$.

Which part of the graph of $y = 2x$ can not form a part of the latter graph?

The graph of $y = 2x$, when $x \geq 0$, will be part of the graph of the line $y = 2x$ in the first quadrant.

The graph of $y = -2x$, when $x < 0$, will be part of the graph of the line $y = -2x$ in the second quadrant.

The graph of $y = 2|x|$ is the union of two rays as shown in the figure.

Methodology: Discussion method is used.
Check Your Progress

Notes: a) Write your answer in the space given below each question.
   b) Compare your answer with the one given at the end of the unit.

Draw the graphs of the following equations:

3) \(2x + y = 5\)

4) \(y = |x - 2|\)

12.4.3 System of Linear Equations in Two Variables

Main Teaching Points

a) Finding the solution of a system of linear equations in two variables graphically

b) Conditions for consistency of a system of linear equations in two variables

Teaching Learning Process

Two linear equations in two variables taken together with the help of the connective ‘and’ are called a system of linear equations in two variables. Plot the graph of each of the linear equations on the same XY–plane. The point of intersection of the lines is a solution of both the equations. This common solution is called the solution of the system of linear equations. If the lines coincide, then there are infinitely many common solutions and if they are parallel and do not intersect, then the system has no common solution.

Example 5: Solve the system of linear equations graphically: \(2x – y = 1\) and \(x + 2y = 8\)

![Graph showing the solution of the system of linear equations](image)
Draw the graph of both the equation in the same XOY plane.

Note that the two straight lines intersect at point P(2,3)

So, x = 2 and y = 3 is a solution of each of the two equations.

\[ \therefore x = 2 \text{ and } y = 3 \text{ is the solution of the system of linear equations.} \]

**Explain** A system of linear equations is said to be consistent if they have one and only one solution i.e. They have a unique solution. The graph is a two intersecting lines. Algebraically, the system of equations are in general expressed as

\[
\begin{align*}
    a_1x + b_1y &= c_1 \\
    a_2x + b_2y &= c_2
\end{align*}
\]

In this example, note that

\[
\frac{a_1}{a_2} = \frac{2}{1} \quad \text{and} \quad \frac{b_1}{b_2} = -\frac{1}{2} \quad \text{and so} \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}
\]

\[ \therefore \text{System of equations is consistent having a unique solution if} \ \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \]

**Example 6**: Solve the following system of equations graphically:

\[
\begin{align*}
    2x + y &= 6 \\
    4x + 2y &= 6
\end{align*}
\]

\[
\begin{array}{c|ccc}
    & x & 0 & 1 \\
    & y & 6 & 4 & 2 \\
\hline
    2x + y = 6 & & & \\
    4x + 2y = 6 & & & \\
\end{array}
\]

\[
\begin{array}{c|ccc}
    & x & 0 & 1 & 2 \\
    & y & 3 & 1 & -1 \\
\hline
    2x + y = 6 & & & & \\
    4x + 2y = 6 & & & & \\
\end{array}
\]

![Fig. 12.9](image-url)
Draw the graph of both the equations in the same XY–plane.
Note that the two straight lines are parallel. They do not intersect.
There is no common point and ∴ there is no common solution.
This system of linear equations has no solution.

**Explain:** A system of linear equations is said to be ‘inconsistent’ if it has no solution. The graph is a pair of parallel lines which do not intersect each other.

In this example note that
\[
\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{1}{2} \quad \text{and} \quad \frac{c_1}{c_2} = \frac{6}{6} = 1
\]

∴ \[\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\]

∴ the system of equations’ is inconsistent having no solution if \[\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\]

Observe that according to first equation 2x+y=6, but according to the second equation 2x + y = 3.

∴ If (x, y) satisfies first equation then it will not satisfy the second equation and if (x, y) is such that 2x + y = 3 then 2x + y ≠ 6. The two equations contradict each other.

**Example 7** Solve \[
\begin{align*}
x - 2y & = 1 \\
3x - 6y & = 3
\end{align*}
\]

graphically

\[
x - 2y = 1
\]

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

![Fig. 12.10](image)

The same set of values of x and y satisfy both the equations.
Therefore, the lines are coincident. Each point of this line satisfies both the equations. The system of equations,
Therefore, has infinitely many solutions.

**Explain:** A system of linear equations is said to be consistent, in particular dependent, if they have infinitely many solutions. The graph in this case is coincident lines.

Algebraically note that
\[ \frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-2}{-6} = \frac{1}{3} \quad \text{and} \quad \frac{c_1}{c_2} = \frac{1}{3} \]

\[ \therefore \text{The system of equations is dependent (consistent) having infinitely many solutions, If} \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \]

Note that the second equation can be obtained from the first equation by multiplying it by \(3\cdot \left(\frac{a_2}{a_1}\right)\). Hence, the second equation is not different from the first equation. It actually depends on the first equation.

**Methodology:** Induction method is used. Several examples are discussed and then the results are derived inductively.

**Check Your Progress:**

**Notes:**

a) Write your answer in the space given below each question.

b) Compare your answer with the one given at the end of the unit.

Classify the following system of equations as consistent, dependent or inconsistent:

5) \[2x + 3y = 5 \]
   \[4x + 6y = 10\]

6) \[3x + 4y = 2 \]
   \[6x + 8y = -1\]

7) \[4x - y = 5 \]
   \[2x - y = 3\]

8) Solve \[\begin{cases} x + y = 5 \\ x - y = 3 \end{cases}\] graphically

**12.4.4 Methods of Solving System of Linear Equations**

**Main teaching Point**

Different methods of solving a system of two linear equations in two variables
Teaching Learning Process

1) Method of Elimination

This method consists essentially of eliminating one of the variables from both the equations and thus getting an equation in a single variable which can be solved to get the value of the variable. This value of one variable thus obtained is substituted in any of the two given equations to get the value of the second variable.

Develop with students’ participation by various methods of elimination. They are the following:

i) Method of Comparison

From each of the two equations, find the value of any one variable in terms of the other and equate them.

Example 8  Solve \[
\begin{align*}
3x + 2y &= 8 \\
2x + 3y &= 7
\end{align*}
\]

Finding value of x from each equation, we get

\[3x + 2y = 8 \Rightarrow x = \frac{8 - 2y}{3} \quad \ldots \ldots \ldots \ldots \ldots (1)\]

\[2x + 3y = 7 \Rightarrow x = \frac{7 - 3y}{2} \quad \ldots \ldots \ldots \ldots \ldots (2)\]

Equating (1) and (2), we get

\[\Rightarrow \frac{8 - 2y}{3} = \frac{7 - 3y}{2} \Rightarrow 16 - 4y = 21 - 9y \Rightarrow 5y = 5 \Rightarrow y = 1\]

Putting this value in equation (1), we get \[x = \frac{8 - 2(1)}{3} = 2\]

\[\therefore \text{the solution is } x = 2 \text{ and } y = 1.\]

Ask the students to solve the system of equations by finding the value of y from both the equations and then comparing them and verifying that they get the same result.

ii) Method of Substitution

Find the value of any one variable from and one of the equations in terms of the other and then substitute it in the second equation to eliminate that variable.

Example 9  Solve \[
\begin{align*}
3x + 2y &= 8 \\
2x + 3y &= 7
\end{align*}
\]

\[3x + 2y = 8 \Rightarrow x = \frac{8 - 2y}{3} \quad \ldots \ldots \ldots \ldots \ldots (3)\]

Substituting in the equation (2), we get

\[2\left(\frac{8 - 2y}{3}\right) + 3y = 7 \Rightarrow 2(8 - 2y) + 9y = 21 \]

\[\Rightarrow 16 - 4y + 9y = 21 \Rightarrow 5y = 5 \Rightarrow y = 1\]

Putting this value in (3), we get

\[x = \frac{8 - 2(1)}{3} = 2\]

\[\therefore \text{the solution is } x = 2 \text{ and } y = 1.\]
The equations can also be solved by finding the value of y from (1) and substituting it in (2).

Ask the students to find the value of the variable x from the second equation and substitute it in the first equation and then find the solution.

iii) **Method of Addition or Subtraction**

Make the coefficients of either x or y in the two equations equal by multiplying the equations by suitable constants and then eliminate the variable by addition or subtraction.

**Example 10** Solve

\[
\begin{align*}
3x + 2y &= 8 \quad \text{(1)} \\
2x + 3y &= 7 \quad \text{(2)}
\end{align*}
\]

To make coefficients of x equal in both the equations, multiply equation (1) by 2 and equation (2) by 3 to get:

\[
\begin{align*}
6x + 4y &= 16 \quad \text{(3)} \\
6x + 9y &= 21 \quad \text{(4)}
\end{align*}
\]

Eliminate x by subtracting equation (4) from (3):

\[
-5y = -5
\]

\[
y = 1
\]

Substituting y=1 in equation (1), we get 3x + 2(1) = 8

\[
x = 2.
\]

\[
\therefore \text{ the solution is } x = 2 \text{ and } y = 1.
\]

Ask the students to eliminate y from the two equations by this method and then solve the equations to get the same answer.

2) **The Method of Cross Multiplication**

Let the equations be

\[
\begin{align*}
a_1x + b_1y + c_1 &= 0 \quad \text{(1)} \\
a_2x + b_2y + c_2 &= 0 \quad \text{(2)}
\end{align*}
\]

Multiplying (1) by \(b_2\) and equation (2) by \(b_1\), we get

\[
\begin{align*}
a_1b_2x + b_1b_2y + b_2c_1 &= 0 \quad \text{(3)} \\
a_2b_1x + b_1b_2y + b_1c_2 &= 0 \quad \text{(4)}
\end{align*}
\]

Subtracting equation (4) from equation (3), we get

\[
(a_1b_2 - a_2b_1)x + (b_2c_1 - b_1c_2) = 0
\]

\[
x = \frac{(b_1c_2 - b_2c_1)}{(a_1b_2 - a_2b_1)} \quad \text{(5)}
\]

Similarly, we can find

\[
y = \frac{(c_1a_2 - c_2a_1)}{(a_1b_2 - a_2b_1)} \quad \text{(6)}
\]

From (5) and (6), we get

\[
\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}
\]

Which can be written as below, where denominators are obtained by cross–multiplication as indicated:

\[
\begin{align*}
\frac{x}{(+)(-)} &= \frac{y}{(+)(-)} = \frac{1}{(+)(-)} \quad \text{(a)}
\end{align*}
\]

\[
\begin{align*}
&\quad b_1 \quad c_1 \\
&\quad b_2 \quad c_2
\end{align*}
\]
Explain to the students, the steps given below to use this method.

**Steps**

i) Rewrite the equations in such a way that all the terms are on the left-hand side and zero on the right-hand side. Write the variables $x$ and $y$ in order i.e first $x$, then $y$ and then constants.

Now equations are in the form:

\[
\begin{align*}
& a_1 x + b_1 y + c_1 = 0 \\
& a_2 x + b_2 y + c_2 = 0 \\
\end{align*}
\]

ii) Take the detached coefficients, write them in cyclic order starting with coefficients of $y$ as given below and work the arrows as shown:

\[
\begin{align*}
& b_1 \\
& c_1 \\
& a_1 \\
& b_1 \\
& b_2 \\
& c_2 \\
& a_2 \\
& b_2 \\
\end{align*}
\]

iii) Write the first cross-product below $x$, the second cross-product below $y$ and third cross-product below 1.

iv) Simplify to get the values of $x$ and $y$.

**Example 11** Solve

\[
\begin{align*}
3x + 2y &= 8 \\
2x + 3y &= 7
\end{align*}
\]

by the method of cross-multiplication.

**Step 1** Write equations as

\[
\begin{align*}
& 3x + 2y - 8 = 0 \\
& 2x + 3y - 7 = 0
\end{align*}
\]

**Step 2** Take the detached coefficients and write them cyclic order starting with coefficients of $y$:

\[
\begin{align*}
& 2 \\
& -8 \\
& 3 \\
& -7 \\
& 2 \\
& 3
\end{align*}
\]

**Step 3** Write the cross-products below $x$, $y$ and 1:

\[
\begin{align*}
& x \quad \quad \quad \quad y \quad \quad \quad \quad 1 \\
& \frac{x}{2(-7) - (3)(-8)} = \frac{y}{(-8)(2) - (-7)(3)} = \frac{1}{3(3) - 2(2)}
\end{align*}
\]

\[
\therefore \quad \frac{x}{10} = \frac{y}{5} = \frac{1}{5}
\]

**Step 4** \(\frac{x}{10} = \frac{1}{5} \implies x = 2\) and \(\frac{y}{5} = \frac{1}{5} \implies y = 1\)

\[\therefore \quad x = 2 \text{ and } y = 1 \text{ is the solution.}\]

**Note:** Cross-multiplication method is applicable when the given system is consistent i.e.

\[
\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad \text{or} \quad a_1 b_2 - a_2 b_1 \neq 0
\]

**Methodology:** Discussion method help to involve students in telling different ways of elimination of a variable from two equations.
12.4.5 Solution of Word Problems

Main Teaching Point

Translating a word problem into a mathematical model

Teaching Learning Process

For solving word problems, the main emphasis should be on translating the problem into a mathematical model. Once we obtain two linear equations in two variables, they can be solved by any one of the methods studied earlier.

After reading the problem carefully, determine what are the unknown quantities and denote them by the variables $x$ and $y$. Again read the problem and try to write down relation between the variables and express it as a linear equation in two variables $x$ and $y$.

Example 12 A fraction becomes $\frac{2}{3}$ on subtracting 1 from both the numerator and the denominator. Express it as a mathematical model.

Answer: Ask the students to tell the two unknown quantities in this statement. They can easily conclude that they are numerator and denominator.

Suppose, numerator = $x$ and denominator = $y$.

Then, fraction is $\frac{x}{y}$.

So, according to given statement

$$\frac{x-1}{y-1} = \frac{2}{3} \Rightarrow 3x-3 = 2y-2 \Rightarrow 3x-2y = 1$$

Which is a linear equation in two variables $x$ and $y$.

Example 13 The cost of 3 chairs and 2 tables is Rs 4200. Express it as a linear equation in two variables.

Answer: In this example, we do not know the cost of 1 chair and the cost of 1 table. These are the unknown quantities. Suppose them as $x$ and $y$.

Then according to the given statement, we get

$$3x + 2y = 4200$$

This is a linear equation in two variables $x$ and $y$. Consider a few more such examples of translating a given statement into a mathematical equation.

Now, let us solve a complete problem using this method.

Example 14 The difference between the ages of father and the son is 20 years. After 5 years, father will be twice as old as his son. Find their present ages.

In this problem the unknown quantities are the present ages of the father and the son.

Let the present age of the father be $x$ years and the present age of the son be $y$ years.

Now, it is given that the difference between the ages of father and the son is 20 years.

$\therefore$ The difference between $x$ and $y$ is 20.

$\therefore x - y = 20 \quad -(1)$ (This is the first linear equation in $x$ and $y$)
After 5 years, father will be twice as old as his son.

After 5 years, father will be x+5 years old and the son will be y+5 years old.

\[ x + 5 = 2(y + 5) \Rightarrow x + 5 = 2y + 10 \Rightarrow x - 2y = 5 \quad \text{--(2)} \]

(This is the second linear equation in x and y)

We have obtained two linear equations in two variables x and y i.e.

\[ x - y = 20 \quad \text{--(1)} \]
\[ x - 2y = 5 \quad \text{--(2)} \]

Now, solve equations (1) and (2) by any one of the methods studied earlier to find the values of x and y.

Subtracting (2) from (1), we get y=15

Substituting this value of y in equation (1),

We get \(x - 15 = 20 \Rightarrow x = 20 + 15 = 35.\)

\(\therefore\) age of father = 35 years

And age of son = 15 years.

Similarly, more examples of word problems be solved with the help of the students.

**Methodology:** Heuristic approach is more suitable for solving word problems. However, discussion should be encouraged and the teacher should guide the students towards logical thinking.

**Check Your Progress**

**Notes:**

a) Write your answer in the space given below each equation.

b) Compare your answer with the one given at the end of the unit.

9) A fraction becomes \(\frac{1}{2}\) on adding 1 to both the numerator and the denominator. On adding 2 to the denominator, the fraction becomes \(\frac{1}{3}\). Find the fraction.

10) The difference between the ages of a father and his son is 30 years. Last year (one year ago) the age of the father was four times that of his son. Find their present ages.

11) A boat travels 30 km downstream in one hour and comes back in 1.5 hours. Find the speed of the boat in still water and the speed of the stream.
12.5 INEQUATIONS

It is not always the case that we are faced with problems involving equations. There are also situations when we have to deal with inequations. Consider the problem:

Raju went to a general store to purchase sugar. He found that it was available in 250g and 500g packets. He could not carry more than 2 kg of sugar. How many packets of each type could he buy?

Suppose he buys x packets of 250g and y packets of 500g sugar. Since total weight should not exceed 2000g.

\[-250x + 500y \leq 2000\] or \[x + 2y \leq 8\]

This is a linear inequation in two variables x and y.

In general, a linear inequation in two variables x and y is in one of the following forms:

\[ax + by < c\]
\[ax + by > c\]
\[ax + by \leq c\]
\[ax + by \geq c\]

where a, b, x, y are real numbers and a, b are not simultaneously equal to zero. An ordered pair which satisfies an inequation has infinitely many solutions.

12.5.1 Graphical Representation of Inequation

Main Teaching Point
To represent an inequation graphically

Teaching Learning Process
Represent graphically: \[2x + y \leq 4\], where x, y ∈ R.

Let the students draw the graph of the equation \[2x + y = 4\].

Straight line represent the solution set of the equation \[2x + y = 4\].

Ask: In how many sets, the points in the plane are divided.

Explain: There are three sets of points

i) Set of points which lie on the line.

ii) Set of points which lie on one side of the line, say, Region 1.

iii) Set of points which lie on the other side of the line, say, Region II.

\[
\begin{array}{c|c|c|c}
 x & 0 & 1 & 2 \\
 y & 4 & 2 & 0 \\
\end{array}
\]
Consider, the points (0,5), (2,3) and (3,0) of Region I.

They all satisfy the inequation $2x+y>4$. Consider, the points (0,2), (–1,0) and (1,0) of Region II.

They all satisfy the inequation $2x+y<4$.

Explain that all the points of Region (I) satisfy the inequation $2x+y<4$.

\[ \therefore \text{Region (I) represents the solution set of the inequation } 2x+y>4, \text{ just as the straight line represents the solution set of the equation } 2x+y=4. \text{ Similarly, all the points of Region (II) satisfy the inequation } 2x+y<4. \]

\[ \therefore \text{Region (II) represents the solution set of the inequation } 2x+y<4. \]

Therefore, the straight line together with Region (II) represent $2x+y \leq 4$.

We only shade the required region. To represent the solution set of $2x+y<4$, the line is not included and hence it is drawn dotted and only Region II, will be shaded.
Example 15 Represent graphically: \( |x| \leq 3 \).
First draw the graph of \(|x| = 3\) i.e. \( x = 3 \) or \( x = -3 \).
Note that all the points between the two lines represent \(|x| \leq 3\).
\[ \therefore \] Shaded region together with the two straight lines represent the solution set.
Note that \(|x| \leq 3\) means \( x \leq 3 \) when \( x \geq 0 \) or \( x > -3 \) when \( x < 0 \).

Methodology: Discussion method is suitable as students already know how to draw the graph of an equation.

Check Your Progress
Notes: a) Write your answer in the space given below each question.
      b) Compare your answer with the one given at the end of the unit.

Draw the graphs of the following linear inequations in two variables \( x \) and \( y \):
12) \( |y| \geq 4 \)
13) \( x \geq 2 \)
14) \( y - x \geq 3 \)

12.6 QUADRATIC EQUATIONS IN ONE VARIABLE

An equation of the form \( ax^2 + bx + c = 0 \), where \( a \neq 0 \) is a quadratic equation in one variable \( x \). Students must be able to recognize the quadratic equations form a given set of equations as follows:

(i) \( 3x^2 + 7x + 2 = 0 \)
(ii) \( 2x^2 - 3x = 5 \)
(iii) \( x^2 + 5 = x^3 - 1 \)
(iv) \( 2x^2 + \frac{1}{x^2} = 3 \)
(v) \( x^2 = x + \frac{2}{x} \)
(vi) \( 3x^2 - 5x + 8 = 3x^2 + 2 \)

Only (i) and (ii) are quadratic equations.
You should discuss with the students why others are not quadratic equations.

12.6.1 Solution of a Quadratic Equation

Main Teaching Points

a) Factor method
b) Completing the square method

Teaching Learning Process

a) Factor Method

Express the given quadratic equation in the form \( p(x) = 0 \), where \( p(x) \) is a quadratic polynomial. Factorize \( p(x) \) to get \( p(x) = q_1(x) \cdot q_2(x) \), where \( q_1(x) \) and \( q_2(x) \) are linear polynomials, so that

\[
p(x) = 0 \Rightarrow q_1(x) \cdot q_2(x) = 0 \Rightarrow q_1(x)=0 \text{ or } q_2(x)=0
\]

( using \( ab=0 \Rightarrow a=0 \) or \( b=0 \)).

Solve each of the two linear equations to find the solution set of the quadratic equation.

Example 16 Solve : \( x^2 + 6x + 5 = 0 \)

Factorizing the left hand side, we get

\[(x+1)(x+5)=0\]

\[\Rightarrow x+1=0 \quad \text{or} \quad x+5=0\]

\[\Rightarrow x=-1 \quad \text{or} \quad x=-5.\]

\[\therefore \{ -1, -5 \} \text{ is the solution set of the quadratic equation } x = -1 \text{ and } x = -5 \]

are also called the roots of the equation.

b) Method of Completing the Squares

To solve \( ax^2 + bx + c = 0 \), (\( a \neq 0 \))

Multiplying both sides by \( 4a \), we get

\[4a^2x^2 + 4abx + 4ac = 0\]

\[\therefore 4a^2x^2 + 4abx = -4ac\]

Adding \( b^2 \) to both sides, we get

\[4a^2x^2 + 4abx + b^2 = b^2 - 4ac\]

Note that L.H.S. becomes a perfect square

\[\therefore (2ax+b)^2 = b^2 - 4ac\]

\[\therefore 2ax + b = \pm \sqrt{b^2 - 4ac}\]

\[\therefore 2ax = -b \pm \sqrt{b^2 - 4ac}\]

\[\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
**Explain:** This is called a quadratic formula and it is used to solve a quadratic equation \( p(x) = 0 \), where \( p(x) \) is not easily factorizable. Instead of using this formula, we can use the steps involved in completing the square process.

Moreover, \( b^2 - 4ac \) is called the discriminant. Why?

This is because, it discriminates between different types of roots:

i) If \( b^2 - 4ac > 0 \), the roots are real and unequal.

ii) If \( b^2 - 4ac = 0 \), the roots are real and equal.

iii) If \( b^2 - 4ac < 0 \), the roots are not real.

**Example 17** Solve \( 2x^2 - 3x - 6 = 0 \)

Here \( a = 2, b = -3 \) and \( c = -6 \).

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-6)}}{2(2)}
\]

\[
x = \frac{3 \pm \sqrt{57}}{4}
\]

Or \( 2x^2 - 3x - 6 = 0 \)

Multiplying by \( 4a = 4 \times 2 = 8 \), both sides, we get

\[
16x^2 - 24x - 48 = 0 \quad 16x^2 - 24x = 48
\]

Adding \( b^2 = (-3)^2 = 9 \), both sides, we get

\[
16x^2 - 24x + 9 = 48 + 9
\]

LHS becomes a perfect square.

\[
\therefore (4x - 3)^2 = 57 \quad 4x - 3 = \pm \sqrt{57} \quad 4x = 3 \pm \sqrt{57}
\]

\[
\therefore x = \frac{3 \pm \sqrt{57}}{4}
\]

**Methodology:** Deductive method is used to derive the quadratic formula.

**12.6.2 Relation between Roots and Coefficients**

**Main Teaching Points**

a) Sum of roots = \( \frac{-b}{a} \)

b) Product of roots = \( \frac{c}{a} \)

**Teaching Learning Process**

Ask students to find the roots of the following equations and complete the following table:

<table>
<thead>
<tr>
<th>Equations</th>
<th>Roots</th>
<th>Sum of Roots</th>
<th>a</th>
<th>b</th>
<th>(-b/a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) ( x^2 - 7x + 6 = 0 )</td>
<td>1,6</td>
<td>7</td>
<td>1</td>
<td>-7</td>
<td>7</td>
</tr>
<tr>
<td>ii) ( x^2 - 3x - 4 = 0 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>iii) ( 2x^2 - 5x - 7 = 0 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>iv) ( 3x^2 - 11x + 10 = 0 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>v) ( 2x^2 - 3x - 5 = 0 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
From this table, let them reach the conclusion that Sum of roots $= \frac{-b}{a}$

Again, ask them to complete the following table:

<table>
<thead>
<tr>
<th>Equations</th>
<th>Roots</th>
<th>Sum of Roots</th>
<th>c/a</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) $x^2 - 7x + 6 = 0$</td>
<td>1,6</td>
<td>6</td>
<td>6 1 6</td>
</tr>
<tr>
<td>ii) $x^2 - 3x - 4 = 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>iii) $2x^2 - 5x - 7 = 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>iv) $3x^2 - 11x + 10 = 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>v) $2x^2 - 3x - 5 = 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From this table students will reach the conclusion that

Product of roots $= \frac{c}{a}$

The above results can also be obtained algebraically as follows:

The roots of the quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

ie $x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

and $x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ Sum of Roots $= x_1 + x_2 = \frac{-2b}{2a} = -\frac{b}{a}$

Product of Roots $= x_1x_2$

$= \frac{b}{2a} + \left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{\sqrt{b^2 - 4ac}}{2a}\right) + \frac{b}{2a} - \left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)$

$= \frac{b^2}{4a^2} - \frac{b^2}{4a^2} - \frac{4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$

$\therefore$ If $\alpha$, $\beta$ are the equation $ax^2 + bx + c = 0$, then $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

**Methodology:** Inductive method is used to explain the formulae and then they are derived using a deductive logic.

**Check Your Progress**

Notes: a) Write your answers in the space given below each question.

b) Compare your answers with the one given at the end of the Unit.

15) If $\alpha$ and $\beta$ are the roots of the equation $ax^2 + bx + c = 0$, find the values of the following:

(i) $\frac{1}{\alpha} + \frac{1}{\beta}$

(ii) $\alpha^2 \beta + \alpha \beta^2$
16) For what values of p are the roots of the following equations equal?
   i) $3x^2 - 5x + p = 0$
   ii) $2px^2 - 8x + p = 0$

17) Form a quadratic equation whose roots are:
   i) 2 and 2
   ii) 3 and $-\sqrt{3}$

12.6.3 Equations Reducible to Quadratic Form

Main Teaching Point
How to transform an equation which is not quadratic to a quadratic form in different situations.

Teaching Learning Process
There are equations which are not quadratic but suitable transformation reduces them to a quadratic equation. Such equations are reducible equations.

Example 18 Solve: $2x^4 - 5x^2 + 3 = 0$
This is not a quadratic equation as its degree is 4. Putting $x^2 = z$, we get
$2z^2 - 5z + 3 = 0$
This is a quadratic equation. Now, it can be solved as a normal quadratic equation.
Factorizing LHS we get
$(2z-3)(z-1) = 0$
$2z-3=0$ or $z-1=0$
$z=\frac{3}{2}$ or $z=1$
$x^2=\frac{3}{2}$ or $x^2=1$
$x=\pm \sqrt{\frac{3}{2}}$ or $x=\pm 1$

Example 19 Solve $x + \frac{1}{x} = \frac{13}{6}$, $(x\neq 0)$
This is not a quadratic equation.
Multiplying by \( x \), both sides, we get:

\[
 x^2 + 1 = \frac{13x}{6} \Rightarrow 6x^2 + 6 = 13x \Rightarrow 6x^2 - 13x + 6 = 0
\]

This is a quadratic equation.

Factorizing LHS we get: \((3x - 2)(2x - 3) = 0\)

\[
 3x - 2 = 0 \quad \text{or} \quad 2x - 3 = 0 \quad \Rightarrow x = \frac{2}{3} \quad \text{or} \quad x = \frac{3}{2}
\]

**Example 20** Solve: \( \sqrt{25 - x^2} = x - 1 \)

This is not an quadratic equation.

Squaring both sides, we get

\[
 25 - x^2 = x^2 - 2x + 1 \Rightarrow 2x^2 - 2x - 24 = 0 \Rightarrow x^2 - x - 12 = 0
\]

This is a quadratic equation.

Factorizing LHS we get: \((x - 4)(x + 3) = 0\)

\[
 x - 4 = 0 \quad \text{or} \quad x + 3 = 0 \quad \Rightarrow x = 4 \quad \text{or} \quad x = -3.
\]

**Explain:** A linear equation \( x - 1 = 0 \) has only one solution ie \( x = 1 \)

Now, \( x - 1 = 0 \quad \Rightarrow x = 1 \)

On squaring both sides, we get \( x^2 = 1 \) or \( x^2 - 1 = 0 \).

It is a quadratic equation and it has two roots \( x = \pm 1 \). These are the roots of the quadratic equation but not of the initial linear equation. The root \( x = -1 \) is called an extraneous root. So, whenever we do squaring both sides, we must look out for extraneous roots by verifying the roots from the initial equation.

In this example, initial equation is \( \sqrt{25 - x^2} = x - 1 \)

Check whether \( x = 4 \) is a root of this equation. By substituting \( x = 4 \), we get

\[
 \text{LHS} = \sqrt{25 - 16} = 3, \quad \text{RHS} = 4 - 1 = 3.
\]

\[
 \therefore \text{LHS} = \text{RHS}
\]

\[
 \therefore x = 4 \text{ is a root of this equation.}
\]

Now, verify whether \( x = -3 \) is its root.

\[
 \text{LHS} = \sqrt{25 - 9} = 4, \quad \text{RHS} = (-3) - 1 = -4.
\]

\[
 \therefore \text{LHS} \neq \text{RHS}
\]

\[
 \therefore x = -3 \text{ is not its root.}
\]

It is an extraneous root.

\[
 \therefore x = 4 \text{ is the only root of the given equation.}
\]

**Example 21** Solve \( \sqrt{3x + 10} + \sqrt{6 - x} = 6 \)

This is not a quadratic equation.

Keeping only one root on LHS, we get

\[
 \sqrt{3x + 10} = 6 - \sqrt{6 - x}
\]

Squaring both sides, we get

\[
 3x + 10 = 36 + (6 - x) - 12 \sqrt{6 - x}
\]
Writing root term on LHS, we get

\[12\sqrt{6 - x} = 32 - 4x \text{ or } 3\sqrt{6 - x} = 8 - x\]

Again squaring both sides, we get:

\[9(6 - x) = 64 + x^2 - 16x \text{ or } x^2 - 7x + 10 = 0\]

This is a quadratic equation.

Factorizing LHS, we get:

\[(x - 5)(x - 2) = 0 \Rightarrow x - 5 = 0 \text{ or } x - 2 = 0 \Rightarrow x = 5 \text{ or } x = 2\]

Here, students can verify that both are roots of the given equation.

**Example 22** Solve: \[3\left(x^2 + \frac{1}{x^2}\right) - 16\left(x + \frac{1}{x}\right) + 26 = 0\]

This is not a quadratic equation.

Let \[x + \frac{1}{x} = y\]

On squaring both sides, we get:

\[x^2 + \frac{1}{x^2} + 2 = y^2 \text{ or } x^2 + \frac{1}{x^2} = y^2 - 2\]

Putting these values In the given equation we get:

\[3(y^2 - 2) - 16y + 26 = 0 \text{ or } 3y^2 - 16y + 20 = 0\]

It is a quadratic equation and it can be solved as usual.

Let students solve it and obtain \(x = 1, x = 3\) or \(x = \frac{1}{3}\) as its roots.

**Methodology:** Deductive method is used for teaching the transformation in different situations.

**Check Your Progress**

Notes: a) Write your answer in the space given below each question.

b) Compare your answer with the one given at the end of the Unit.

Solve the following equations:

18) \(\sqrt{x - 2} + \sqrt{x + 1} = 3\)

…………………..

…………………..

…………………..

19) \(\left(x^2 + \frac{1}{x^2}\right) - 4\left(x + \frac{1}{x}\right) + 6 = 0\)

…………………..

…………………..

…………………..
12.6.4 Solving Word Problems

Main Teaching Point
To translate the given word problem into a mathematical model

Teaching Learning Process
There are a few problems which when translated into a mathematical model form a quadratic equation. The main emphasis should be on translation of the language of the problems into an appropriate quadratic equation in one variable.

Example 23
Find two natural numbers whose difference is 3 and sum of their squares is 117.

Let the one natural number be $x$. Since the difference of two numbers is 3. \(\therefore\) The other natural number is $x+3$

Since sum of their squares is 117

\[ \therefore x^2 + (x+3)^2 = 117 \]

On simplification, it leads to the quadratic equation

\[ x^2 + 3x - 54 = 0 \]

It can be solved as usual to get $x = -9$ or $x = 6$, as its roots. Since $x = -9$ is not a natural number

\(\therefore\) the only solution is $x = 6$

\(\therefore\) One number = 6

The other number = 6+3 = 9.

Example 24 The product of two consecutive odd numbers is 15. Find the numbers.

Let the two consecutive odd numbers be $(2x + 1)$ and $(2x + 3)$.

Therefore, $(2x + 1) (2x + 3) = 15$ \(\Rightarrow\) $4x^2 + 8x - 12 = 0$

\[ \therefore x^2 + 2x - 3 = 0 \]

It is a quadratic equation.

Therefore $(x + 3) (x - 1) = 0 \Rightarrow x + 3 = 0$ or $x - 1 = 0$

\[ \therefore x = -3 \text{ or } x = 1 \]

when $x = -3$

The consecutive odd numbers are -5 and -3.

When $x=1$

The consecutive odd numbers are 3 and 5.

Methodology: Heuristic method is used to solve word problems. The teacher should guide the students towards logical thinking.
12.7 LET US SUM UP

In this unit, you have learnt different methods of solving a system of two linear equations in two variables and solving a quadratic equation in one variable. You have also learnt to translate the given word problem into a mathematical model and solving them by using your knowledge of solving equations. The relations between roots of a quadratic equation and its coefficients also helps us in simplifying the solution which is otherwise quite lengthy. A linear inequation in two variables has infinitely many solutions which can be easily represented by a graph.

12.8 UNIT END ACTIVITIES

Discuss with the students the solution of the following types of questions and encourage them to use the knowledge acquired in this unit and apply logical thinking in solving such problems:

1. Solve;
   \[
   \begin{align*}
   ax + by &= a^2 + b^2 \\
   \frac{x}{a} + \frac{y}{b} &= 2
   \end{align*}
   \]

2. Solve:
   \[
   6 \left( y^2 + \frac{1}{y^2} \right) - 25 \left( y - \frac{1}{y} \right) + 12 = 0
   \]

3. Solve:
   \[
   \left( \frac{2x + 1}{x - 1} \right)^4 - 10 \left( \frac{2x + 1}{x - 1} \right)^2 + 9 = 0
   \]

4. Find a quadratic equation whose roots are reciprocals of the roots of the equation \( x^2 - 5x - 14 = 0 \)

5. Find a quadratic equation whose roots are squares of the roots of the equation \( ax^2 + bx + c = 0 \) (\( a \neq 0 \))

6. Find a quadratic equation whose one root is 2 and sum of the roots is -4.

7. Write the nature of the roots of the equation \( 2x^2 - 5x - 1 = 0 \), without actually finding the roots.

8. Find the value of k, so that the equation \( kx^2 + 3kx + 9 = 0 \) has real and equal roots.

9. Draw the graph of \( y \leq |x| \)

12.9 ANSWERS TO CHECK YOUR PROGRESS

1. (i), (v) and (vii) are equations.

2. i) Linear in one variable; \( x = 4 \)
   ii) Linear in two variables.
   iii) Linear in one variable; \( x = 5 \)
   iv) Quadratic in one variable
   v) Linear in one variable; \( x = 7/2 \)
   vi) Linear in one variable; \( x = 2 \)
5. Dependent
6. Inconsistent
7. Consistent
8. From the graph, the solution is $x = 4, y = 1$

9. $\frac{3}{7}$
10. Father’s age = 41 years, Son’s age = 11 years
11. Speed of the boat = 25 km/hour
    Speed of the stream = 5 km/hour
12.
13. 

14. 

15. (i) \(-b/c\)    (ii) \(-bc/a^2\) 

16. (i) \(25/12\)    (ii) \(±2\sqrt{2}\) 

17. (i) \(x^2 - 4x + 4 = 0\) 

(ii) \(x^2 + (-3 + \sqrt{3})x - 3\sqrt{3} = 0\) 

18. \(x=3\) 

19. \(x=1\) 

12.10 SUGGESTED READINGS

