UNIT 13  SETS, RELATIONS, FUNCTIONS AND GRAPHS*

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13.1 INTRODUCTION

Mathematics has been defined as the study of sets with structures. Sets provide a language which can be used in the study of Mathematics. This language is sometimes more expressive than the ordinary language. Set language illustrates the use of mathematical symbolism in formulating proofs and illustrating the structures in mathematical logic, probability, Boolean algebra, switching circuits etc.

The concepts of relation and function are fundamental in Mathematics, Sciences and Social Science. In Social Science, the set consists of human beings and the relationships exist between the members of the set of human beings such as father, mother, son, daughter etc. A function essentially shows how one quantity varies with variation of one or more of other quantities. For example, it may show how the area of a rectangle varies with its length and breadth or how the volume of a sphere varies with its radius. The graph of a function gives us a visual picture of the behavior of the function. It can help us in understanding the properties of the function.

13.2 OBJECTIVES

At the end of this unit, you will be able to:

• explain the meaning of basic terms used in set theory;

* A few sections of this unit has been adopted from ES-342, IGNOU, 2000
• apply the concepts of sets for solving daily life problems;
• analyse relations and functions;
• understand the behavior of different functions by drawing their graphs; and
• help students to acquire an in-depth knowledge of sets, relations functions and graphs and its application in day-to-day life.

13.3 SETS

The idea of sets was developed towards the end of the 19th century. George Boole (1815-1864) and George Cantor (1845-1918) were the two mathematicians credited with the development of the idea of sets. Cantor is considered to be the founder of the set theory.

13.3.1 Introduction of Sets

Main Teaching Point

Basic concepts and definitions related to sets: sets and its elements, notations, roster and set builder forms, equal and equivalent sets, finite and infinite sets.

Teaching-Learning Process

Set is a concept to be explained and not to be defined. The word set is synonyms with the words, collection, group, bunch, chain etc. and an element is synonym of the words object, member etc. Hence, a collection of objects in mathematical language is called a set of elements. But, every collection of objects is not a set. Why? It should be well-defined in the sense that we should be able to decisively say whether any particular object belongs to the set or it does not belong to the set.

Explain: A set is a well-defined collection of objects.

Ask: Is, the collection of all tall boys in the class; a set?

The idea of tallness is vague. Given any student of the class, we cannot say whether he is tall or not. Hence, it is not well defined and it is not a set.

Ask: Is ‘the collection of all boys in a class whose height is above 160 cm, a set?

Here, it is well defined that any student whose height is above 160 cm is considered as tall and he is a member of the set. So, in this case, it is a set.

Ask: Is ‘the collection of all odd numbers from 1 to 9, a set?

Here, we can decisively say that 1, 3, 5, 7, 9 are the members of this set. Hence, it is a set.

Explain: Sets are denoted by capital alphabets and all the members are listed and enclosed within parenthesis. The above set can be written as \( A = \{1, 3, 5, 7, 9\} \). The number ‘3 is a member of set A’ is written as \( 3 \in A \). It is also read as ‘3 belongs to the set A’ or 3 is an element of the set A. This form of writing a set when all the elements are listed is called the tabular form or Roster form of expressing a set.
Ask: Consider the set $B = \{a, e, i, o, u\}$. If, $x \in B$, then what can you tell about $x$?

**Explain:** If $x \in A$, then we can say that $x$ is a vowel of English alphabet. Hence, $x$ is a member of set $B$, if $x$ is a vowel of English alphabet, we write, $B = \{x : x$ is a vowel of English alphabet$\}$. This form of writing a set, where some rule is used to define the members of the set, is called the set builder form. Give some exercises to write the sets given in ’set builder form’ into Roster form and vice-versa, so that students become accustomed to using the language of sets.

**Equal and Equivalent Sets:**

**Ask:** Is the team consisting of Anil, Arun and Ravi same as the team consisting of Ravi Anil and Arun? Teams are same; only members are listed in different order.

**Explain:** Any two sets with the same elements are equal, irrespective of the order in which the elements are listed.

\[
\{a, b, c\}, \{a, c, b\}, \{b, c, a\}, \{c, a, b\},
\]

are all equal sets.

**Ask:** Are the sets $\{a, b, c\}$ and $\{p, q, r\}$ equal?

Write answer: No, as they have different elements.

**Ask:** Can we associate the members of the set $A = \{a, b, c\}$ with the members of the set $B = \{p, q, r\}$ such that no elements of $A$ are associated with the same element of $B$?

Write answer: Yes, $a \leftrightarrow p$, $b \leftrightarrow q$, $c \leftrightarrow r$.

**Explain:** This association of elements is called a one-to-one correspondence. Two sets are called equivalent sets, if we can establish a one-to-one correspondence between them. Equivalent sets need not be equal, but equal sets are always equivalent.

**Finite and Infinite Sets, Empty Set:**

**Ask:** How many elements are there in the following sets?

i) Set of natural numbers less than 10;

ii) Set of all natural numbers;

iii) Set of natural numbers between 5 and 6.

Write the answers: i) 9 elements

ii) Countless elements

iii) No elements.

**Explain:** In (i), we can count the number of elements in the set and we call it a finite set. In (ii), we cannot count the number of elements in the set, so it is called an infinite set. In (iii), there is no element in the set. It is called an empty set, null set or void set.

It is denoted by $\{\}$ or alphabet $\emptyset$. (called ‘phi’).

**Methodology:** Discussion method is used alongwith a number of illustrations.
13.3.2 Subsets and Universal Set

Main Teaching Points:

a) Relation between subsets and universal set
b) Power set

Teaching Learning Process:

Let $S$ be the set of all the students of a school.

Let $A$ be the set of all the students in class VI of that school.

Let $B$ be the set of all the students in the Cricket Team of that school.

**Explain:** Set $A$ is such that every element of $A$ is an element of the set $S$. Also, set $B$ is such that every element of $B$ is an element of the set $S$. Sets $A$ and $B$ are called the subsets of the set $S$ and the set $S$ is considered as the Universal set for the sets $A$ and $B$.

Set $A$ is a subset of Set $B$, if every element of set $A$ is in Set $B$. In mathematical notations, it is expressed as:

$A \subseteq B$, if and only if, $a \in A \Rightarrow a \in B$.

**Ask:** Write down all the subsets of $A = \{1, 2, 3\}$.

**Explain:**

Every set is a subset of itself and also empty set is a subset of every set.

So, the set of all the subsets is

$\{ \{ \}, \{ 1 \}, \{ 2 \}, \{ 3 \}, \{ 1, 2 \}, \{ 1, 3 \}, \{ 2, 3 \}, \{ 1, 2, 3 \} \}$

The number of subsets is $8 = 2^3$

**Ask:** Write all the subsets of $B = \{1, 2\}$ and $C \{1\}$.
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**Explain:** The number of subsets of B are \( n = 2^2 \) and the number of subsets of C are \( n = 2^1 \).

Let the students inductively conclude that if a set has \( n \) elements, then the number of subsets is \( 2^n \). The set of all the subsets of a given set is called a Power set. Thus, the set of all subsets of a set of \( n \) elements has \( 2^n \) elements.

**Ask:** Which of the following sets is the largest set?

(i) \( A = \) Set of all the students of the school.
(ii) \( B = \) Set of the students of class IX of the school.
(iii) \( C = \) Set of the students of class X of the school.

**Explain:** \( B \) and \( C \) are subsets of \( A \) and \( A \) is the largest set, which contains all the other sets. This largest set is called the Universal set.

**Explain:** Set \( R \), of real numbers is a Universal set for the sets of rational, integers, irrational and natural numbers.

**Explain:** The Universal set is not unique, whereas the null set is unique.

**Methodology:** Discussion method is mainly used. Inductive method is used to show that the total number of subsets in a set of \( n \) elements is \( 2^n \).

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**Check Your Progress:**

**Notes:**
 a) Write your answers in the space given below each question.
 b) Compare your answer with the one given at the end of the unit.

2) Write the power set of \( \{ a, b, c \} \).

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**13.3.3 Operations on Sets**

**Main Teaching Points**

a) Union and Intersection of Sets
b) \( n (A \cup B) = n (A) + n (B) - n (A \cap B) \)
c) Complement of a Set

**Teaching Learning Process**

In arithmetic we have operations of addition and multiplication. Similarly, in sets we have operations of union and intersection denoted by the symbols ‘\( \cup \)’ and ‘\( \cap \)’ respectively.

Sets are lists of objects and union of sets is like making a single list of all the elements in both the lists.

**Ask:** Team for English debate is the set \( A = \{ Ravi, Anil, Ashok \} \) and the team for Hindi debate is the set \( B = \{ Ashish, Vimal, Anil \} \). Write a set representing the list of students, who are either in English debate or in Hindi debate.

**Answer:** \( C = \{ Ravi, Anil, Ashok, Ashish, Vimal \} \). Note that Anil is common in both the teams, and hence, is listed only once in \( C \).
**Explain:** The name of Anil is written only once. Set C is called the union of sets A and B. We write:

\[ C = A \cup B \]

A \cup B contains all the elements, which are either in A or in B. In Mathematical notation:

\[ x \in A \cup B \iff x \in A \text{ or } x \in B. \]

**Ask:** If \( A = \{1, 2, 3, 4, 5\} \) and \( B = \{3, 4, 5, 6, 7\} \)

Write \( A \cup B = \{1, 2, 3, 4, 5, 6, 7\} \)

\( B \cup A = \{3, 4, 5, 6, 7, 1, 2\} \)

\( A \cup B = B \cup A \) [Since both have the same elements listed in different order.]

Commutative law holds good for union of sets.

We can also use Venn Diagrams to illustrate commutativity of the operation of union. Similarly, using Venn Diagrams and considering different examples, illustrate the following properties of union of sets:

a) **Associative law:** \( A \cup (B \cup C) = (A \cup B) \cup C \),

b) \( A \cup \emptyset = A \)

c) \( A \cup U = U \)

d) \( A \subseteq B \Rightarrow A \cup C \subseteq B \cup C \)

**Ask:** Set \( A = \{\text{Ravi, Anil, Ashok, Ram}\} \) represent the students, who play cricket and the set \( B = \{\text{Ravi, Vimal, Naveen, Ram}\} \) represent the students who play hockey, Write a Set C of students, who play both Cricket and Hockey.

**Answer:** \( C = \{\text{Ravi, Ram}\} \)

**Explain:** Set C is the set of elements, which are common in both the sets A and B. Set C is called the intersection of the Sets A and B, written as:

\[ C = A \cap B \]

A \( \cap \) B contains all the elements, which are in set A and also in set B. In mathematical notation:

\[ x \in A \cap B \iff x \in A \text{ and } x \in B. \]

**Ask:** Write \( A \cap B \) and \( B \cap A \), when

\( A = \{1, 2, 3, 4, 5\} \) and \( B = \{3, 4, 5, 6, 7\} \)

**Answer:** \( A \cap B = \{3, 4, 5\} \) and \( B \cap A = \{3, 4, 5\} \).

\[ \therefore A \cap B \text{ is same as } B \cap A \]

or \( A \cap B = B \cap A \)

Commutative law holds for intersection of sets.

Illustrate the following properties of intersection of sets by considering different examples and using Venn Diagrams:

i) **Associative law:** \( (A \cap B) \cap C = A \cap (B \cap C) \),

ii) \( A \cap \emptyset = \emptyset \)

iii) \( A \cap U = A \)
iv) \( A \subseteq B \iff A \cap C \subseteq B \cap C \)

v) **Distributive Laws:**

\[
A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \\
A \cap (B \cup C) = (A \cap B) \cup (A \cap C)
\]

**Explain:** In number system we have only one distributive law i.e. \( a \times (b + c) = (a \times b) + (a \times c) \), but in sets, we have two distributive laws with the help of Venn Diagrams illustrate that two sets \( A \) and \( B \) are disjoint, if there is no element common between them i.e. when \( A \cap B \) is an empty set.

**Explain:** \( A = \{a, b, c, d, e\} \) and \( B = \{a, e, i, o, u\} \), 

is \( n(A \cup B) = n(A) + n(B) \) ?

**Answer:** \( A \cup B = \{a, b, c, d, e, i, o, u\} \)

\( n(A \cup B) = 8 \), \( n(A) = 5 \) and \( n(B) = 5 \)

\( n(A \cup B) \neq n(A) + n(B) \)

**Explain:** That, in \( n(A) + n(B) \), the common elements \{a, e\} have been counted twice, both in \( A \) and \( B \), whereas in \( n(A \cup B) \), they are counted only once. So, we can conclude that:

\[
n(A \cup B) = n(A) + n(B) - n(A \cap B)
\]

**Explain:** If \( \mathbb{U} \) is the universal set and \( A \) is any subset of \( \mathbb{U} \), then complement of the set \( A \) is the set of elements of \( \mathbb{U} \), which are not in \( A \). Complement of a set \( A \) is denoted by \( A' \) and \( A' = \{x \in \mathbb{U}: x \notin A\} \) or \( x \in A \iff x \notin A' \).

Consider, the whole class as the universal set and let \( A \) be the set of students, who drink tea, then \( A' \) is the set of students in the class who do not drink tea. In this way, the class is divided into two groups \( A \) and \( A' \), which are such that together they constitute the whole class, no student is common in the two groups i.e. they are disjoint and the two groups complement each other.

Mathematically,

\[
(i) \quad (A')' = A \quad (ii) \quad A \cup A' = \mathbb{U} \quad (iii) A \cap A' = \emptyset
\]

**Methodology:** Intuitive logic and discussion is used to get the relation \( n(A \cup B) = n(A) + n(B) - n(A \cap B) \). Use of Venn Diagrams is made to illustrate various properties of union, intersection and complement sets. Teacher should take care to compare the properties of algebra of sets with the properties of algebra of real numbers at every step of his discussion on sets.

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### Check Your Progress:

**Notes:**

a) Write your answer in the space given below each question.

b) Compare your answer with the one given at the end of the Unit.

3. What do the shaded positions in the following diagrams represent?

```
(i) \[ \begin{array}{c}
\mathbb{U} \\
A \\
\end{array} \]

(ii) \[ \begin{array}{c}
A \cap B \\
\mathbb{U} \\
\end{array} \]

(iii) \[ \begin{array}{c}
\mathbb{U} \\
A \cup B \\
\end{array} \]
```
13.3.4 Application of Union and Intersection of Sets

Main Teaching Point

Word problems based on \( n(A \cup B) = n(A) + n(B) - n(A \cap B) \)

Teaching Learning Process

Solving word problems is the best way of understanding the knowledge assimilated. To solve a word problem, students should first read the problem carefully. Analyse the given data and clearly understand what is to be obtained. Encourage students to translate the problem into a mathematical language. Then solve the problem by applying the already learnt formulae and concepts.

Example: 1 In a class of 35 students, 25 play football and 20 play cricket. If each student plays at least one of the two games, how many students play both cricket and football and how many play only cricket?

With the help of students, analyse and translate the problem into a mathematical module.

Let \( A \) be the set of students, who play football, then \( n(A) = 25 \).

Let \( B \) be the set of students who play cricket, then \( n(B) = 20 \).

Since each student plays at least one of the games, \( n(A \cup B) = 35 \).

Now, we are required to find out how many students play both cricket and football i.e. we have to find \( n(A \cap B) \).

Now, we know that
\[
35 = 25 + 20 - n(A \cap B)
\]
\[
\therefore n(A \cap B) = 45 - 35 = 10
\]

The number of students, who play Cricket only
\[ = \text{number of students, who play cricket} - \text{number of students, who play both} \]
\[ = 20 - 10 = 10. \]

This can be better explained with the help of Venn Diagrams.

Example: 2 In a committee, 30 people speak Hindi, 35 speak English and 15 speak both Hindi and English. How many people speak at least one of the two languages?

Analyse: The set \( A \) of people speaking Hindi has 30 elements i.e. \( n(A) = 30 \).

The set \( B \) of people speaking English has 35 elements i.e. \( n(B) = 35 \).

\( A \cap B \) is the set of people, who speak both Hindi and English i.e. \( n(A \cap B) = 15 \).

The set of people, who speak at least one of the two languages is \( A \cup B \), so we have to find \( n(A \cup B) \).

\[
\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)
\]
\[ = 30 + 35 - 15 = 50. \]

Methodology: Heuristic approach is the best to solve the word problems. In solving these problems, Venn Diagrams are also useful.
### 13.3.5 Cartesian Product of Sets

**Main Teaching Point**

a) Cartesian product of sets

b) \( n(A \times B) = n(A) \times n(B) \).

**Teaching Learning Process:**

Let the set \( A = \{a, b, c\} \) be the set of roads a, b and c between city P and city Q and let \( B = \{x, y\} \) be the set of roads between city Q and city R.

Suppose, we take road a to go from P to Q and the road x to go from Q to R, and represent this route from P to R as (a, x).

**Ask:** Write in this way all possible routes that can be taken to go from P to R.

**Answer:** \((a, x), (a, y), (b, x), (b, y), (c, x), (c, y)\)

**Ask:** Similarly write the possible routes for going from R to P.

**Answer:** All possible routes from R to P are \((x, a), (x, b), (x, c), (y, a), (y, b), (y, c)\).

**Ask:** What is the difference between \((a, x)\) and \((x, a)\)?

**Answer:** \((a, x)\) means we first take road a and then road x, whereas \((x, a)\) denotes that we first take road x and then road a.

**Explain:**

1. \((a, x)\) and \((x, a)\) are called ordered pairs
2. \((a, x) \neq (x, a)\)
3. The set \(\{(a, x), (a, y), (b, x), (b, y), (c, x), (c, y)\}\) is the set of all ordered pairs from the set A to the set B. It is called cross-product or Cartesian product of the sets A and B.

\[
\therefore A \times B = \{(x, y) : x \in A, \ y \in B\},
\]

4. \(B \times A = \{(y, x) : x \in A, \ y \in B\},\)
5. Since \((x, y) \neq (y, x)\), note that \(B \times A \neq A \times B\).
6. Note that \(n(A \times B) = n(A) \times n(B) = n(B \times A)\)

**Ask:** What is \(R \times R\) and \(R \times R \times R\), where \(R\) is the set of real numbers?

**Explain:** \(R \times R\) represents the Euclidean plane or XY – plane and \(R \times R \times R\) represents the Euclidean three dimensional space. The word Euclidean indicates that the idea was first conceived of by the Greek Mathematician, Euclid. \(R \times R\) is also written as \(R^2\) and \(R \times R \times R\) is written as \(R^3\).

**Methodology:** Discussion is the best way to deliver the meaning of an ordered pair and defining \(A \times B\).
13.4 Relations

Main Teaching Point

What is a relation?

Teaching Learning Process

In real life, we have seen that two persons may be related to each other and may not be related to each other. When they are related, we have seen different types of relations between them such as ‘is a brother of’, ‘is father of’ ‘is friend of’, ‘is neighbor of and so on. Similarly, given any two sets A and B, some relationship of the elements of A may exist with the elements of B.

Let $A = \{a, b, c, d\}$, $B = \{0, 1, 2, 3\}$

We represent a relationship from A to B by the following arrow diagram:

![Fig. 13.2](image)

In this diagram, arrows are used to represent as to which element of B is related to any element of A.

**Ask:** (i) a is related to which element of B?

(ii) What element of A are related to 2 in B?

**Answer:** (i) a is related to 2 in B.

(ii) a and c in set A are related to 2 in set B. If this relation from set A to the set B is denoted by R, then a is related to 2 can be written as $aR2$ or simply by an ordered pair $(a, 2)$.

**Ask:** Write all the ordered pairs representing the relation of elements of the set A with the elements of the set B.

**Answer:** $(a, 2), (b, 1), (b, 3), (c, 1), (c, 2), (c, 3)$.

**Explain:** Relation $R = \{(x, y) : \ x \in A, \ y \in B\}$ is a relation R from set A to set B. It is the set of ordered pairs $(x, y)$ with $x \in A$ and $y \in B$.

In the above example

$$R = \{(a, 2), (b, 1), (b, 3), (c, 1), (c, 2), (c, 3)\}$$

Note that relation R from A to B is a subset of $A \times B$.

**Ask:** In the above relation, write the set of all the first elements of the ordered pairs in R along with the set of all the second elements of the ordered pairs in R.
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Answer: (i) \( \text{dom } R = P = (a, b, c) \subset A \)

(ii) \( \text{ran } R = Q = (1, 2, 3) \subset B \)

Explain: \( P \) is called the domain of \( R \), denoted by \( \text{dom } R \). It is the subset of the set \( A \) consisting of element of \( A \), which are related to some elements in \( B \). Similarly, \( Q \) is called the range of the relation \( R \) denoted by \( \text{ran } R \). It is the subset of set \( B \) consisting of elements of \( B \), which are related to some elements of \( A \).

Explain: If \( R \) is a relation from \( A \) to \( A \), it is called a relation on \( A \).

Example 3: Let \( A = \{ 2, 3, 4, 5, 6, 7, 8, 9, 10 \} \)

Let \( R \) be a relation on \( A \) defined by \( xRy \). If \( x \) divides \( y \), write \( R \).

Answer: \( R = \{ (2, 2), (2, 4), (2, 6), (2, 8), (2, 10), (3, 3), (3, 6), (3, 9), (4, 4), (4, 8), (5, 5), (5, 10), (6, 6), (7, 7), (8, 8), (9, 9), (10, 10) \} \)

Methodology: Discussion is used to arrive at the mathematical meaning of relation.

13.5 Functions

Main Teaching Point

What is a function?

Teaching Learning Process

A function from set \( A \) to set \( B \) is a relation from the set \( A \) to the set \( B \) such that every element of \( A \) is related to one and only one element of \( B \).

Let \( A = \{ a, b, c \} \) and \( B = \{ 1, 2, 3, 4 \} \)

Ask the students to write down relations from \( A \) to \( B \) and discuss with them whether they are functions or not and why?

For example, consider the relations:

\[
\begin{align*}
\text{f}_1 &= \{ (a, 2), (b, 1), (c, 4), (a, 3) \} \\
\text{f}_2 &= \{ (a, 1), (b, 3), (c, 4) \} \\
\text{f}_3 &= \{ (a, 3), (b, 4) \}
\end{align*}
\]

Discuss: In \( \text{f}_1 \) all the element of \( A \) are related to some elements of \( B \). But \( a \) is related to two different elements 2 and 3 of \( B \). So, it is not a function. But, it is a relation.

Also, explain that if two ordered pairs have the same first entry, then it is not a function.

In \( \text{f}_2 \), every element of \( A \) is related to one and only one element of \( B \). Hence this relation is a function.

In \( \text{f}_3 \), \( c \in A \) and it is not related to any element of \( B \), so it is not a function.

Ask: What is the \( \text{dom } \text{f}_3 \) ?

Answer: \( \text{dom } \text{f}_3 = \{ a, b \} \subset A \)

Explain: Since, \( \text{dom } \text{f}_3 \neq A \), it is a not a function. If \( f \) is a function, then \( \text{dom } f = A \).
So, \( f \) is a function from \( A \) to \( B \), if

(i) \( \text{Dom} \ f = A \) and

(ii) No two ordered pairs have the same first entry.

If \( x \in A \) and \( x \in B \) and \( f : A \rightarrow B \) such that \( f : x \rightarrow y \), then we write it as \( y = f(x) \) and say \( y \) is image of \( x \) or \( f(x) \) is image of \( x \).

If a relation is represented by an arrow diagram, then discuss with the students, whether it is a function or not?

**Explain:** A relation is represented by an arrow diagram. It is not a function, if
- (a) any element of \( A \) is such that no arrow starts from it (fig. (ii) ) or
- (b) any element of \( A \) is such that two or more arrows start from it (fig. (ii)), otherwise it is a function (fig. (iii) ).
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Explain: (i) For any function $f$ from $A$ to $B$, domain of $f$ is $A$ and $B$ is called the co-domain of $f$.

(ii) Range of $f$ is a subset of $B$.

(iii) If the domain and co-domain of a function are the sets (or subsets) of real numbers, the function is called a real function.

Ask: Can you correlate the concept of a function with a daily life situation?

Explain: Explain that the relation between a temperature expressed in Fahrenheit and the temperature expressed in Celsius given by $C = \frac{5}{9} (F - 32)$, is a function. Graph of this function is a straight line. Corresponding to every value of $F$, we can find the value of $C$ and vice-versa. Area of a circle ‘$A$’ is a function of its radius ‘$r$’. This function is given by $A = \pi r^2$. Its graph is a part of a parabola. Similarly, many other real life situations can be explained using the concept of function.

Methodology: Discussion method is used to illustrate the meaning of a function. Mainly, the lecture method is used to define domain, co-domain and range.

Check Your Progress:

Notes: a) Write your answer in the space given below each question.

   b) Compare your answer with the one given at the end of the unit.

4) Which of the following relations is a function from $\{1, 2, 3\}$ to itself:

   i) $\{ (1,2), (2,3), (3,1) \}$

   ii) $\{ (1,1), (1,2), (1,3) \}$

   iii) $\{ (1,1), (2,1), (3,1) \}$

   iv) $\{ (1,1), (2,1), (3,1) \}$

\[ \begin{array}{ccc}
1 & 2 & 3 \\
1 & 2 & 3 \\
\end{array} \]

13.6 GRAPHS

Main Teaching Point

Graphical Representation of Different Functions

Teaching Learning Process:

Rene Descartes, a French Mathematician, first showed the relationship of algebra and geometry by associating number pairs (ordered pairs) with points in plane. The ordered pair associated with a point on the plane is called the Cartesian coordinates of the point after the name of the mathematician.

Students are already familiar with geometrical representation of a linear equation in two variables $x$ and $y$. You can start with recapitulating the idea and explaining that the linear equation in two variables $x$ and $y$ can be written as $y = (linear \ expression \ in \ x)$, and so, it also represents a function.
**Ask:** Draw the graph of \( y = 2x + 1 \).

\[
\begin{array}{c|c|c|c}
 x & 0 & 1 & 2 \\
\hline
 y & 1 & 3 & 5 \\
\end{array}
\]

**Fig. 13.4**

**Explain:** Consider the function \( f : \mathbb{R} \rightarrow \mathbb{R} \),

such that \( f = \{ (x, y) : x \in \mathbb{R}, y \in \mathbb{R}, y = 2x+1 \} \)

Clearly, \( f \) is a real function and the above graph geometrically represents this function. Every ordered pair belonging to \( f \) is represented by a point on this line and ordered pair associated with every point on this line belongs to the given function.

A real function \( f \) such that \( f(x) = ax + b \) is called a linear function because, it represents a straight line, geometrically.

**Ask:** What is the graph of the function \( y = [x] \), where \([x]\) is the greatest integer less than or equal to \( x \).

According to the definition of \([x]\),

If \( 1 \leq x < 2 \), \( y = 1 \), it is a line parallel to x – axis

If \( 2 \leq x < 3 \), \( y = 2 \), it is also a line parallel to x –axis and so on.

**Fig. 13.5**

**Explain:** The value of \( y \) is always an integer and the graph of \( y = [x] \) integer is a line parallel to x –axis.
Therefore, the graph of the function \( y = (x) \) consists of a number of line segments all parallel to x-axis, such that the points \((1, 0), (2, 1), (3, 2)\) which are not included in the graph are represented in the graph by encircling these points.

**Ask:** Draw the graph of \( y = x^2 \), construct the following table:

<table>
<thead>
<tr>
<th>x</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>16</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>

Draw the graph as shown:

![Graph of \( y = x^2 \)](image)

**Explain:** The graph is called a parabola. It is symmetric about y-axis.

**Ask:** The students to draw the graph of \( y = -x^2 \) and other simple quadratic functions.

**Explain:** The function \( f = \{ (x, y) : x \in \mathbb{R}, y \in \mathbb{R}, y=ax^2+bx+c \} \) is called a quadratic function. Its graph is always a parabola with its axis of symmetry as \( x = -\frac{b}{2a} \). The parabola opens upwards if \( a \geq 0 \) and downwards if \( a < 0 \).

**Ask:** Draw the graph of the function.

\[
f = \{ (x, y) : x \in \mathbb{R} - [0], y \in \mathbb{R}, y = \frac{12}{x} \}
\]

To draw the graph of \( y = \frac{12}{x} \), construct the following table:

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>-1</th>
<th>-2</th>
<th>-3</th>
<th>-4</th>
<th>-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>12</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>-12</td>
<td>-6</td>
<td>-4</td>
<td>-3</td>
<td>-2</td>
</tr>
</tbody>
</table>

Plot these points and draw the graph as shown.

\[
y = \frac{12}{x}
\]
Fig. 13.7

**Explain:** The function is not defined at \( x = 0 \).

The graph is called a rectangular hyperbola.

Here \( y \) varies inversely as \( x \) i.e. \( y \) approaches 0 as \( x \) becomes a large number and \( y \) becomes very large as \( x \) approaches 0.

**Methodology:** Skill can be developed only by drill method.

**Check Your Progress:**

**Notes:**

a) Write your answer in the space given below each question.

b) Compare your answer with the one given at the end of the unit.

5) Draw the graphs of the following functions:

(i) \( y = 3 - 2x \)  
(ii) \( y = \frac{x^2}{2} \)  
(iii) \( y = \sqrt{x}, \; x \geq 0 \)

13.7 **LET US SUM UP**

In this unit, we have discussed many types of activities on sets, relations and functions. Many real life problems can be associated with the teaching of sets and functions. Regardless of the ability level of the students, appropriate activities can be found. Teachers should constantly gather material and ideas for teaching of this unit.

The concepts of Cartesian product of sets, relations and functions, their domain and range, and understanding the behavior of functions by graphically
representing them have been studied in this chapter. Remember that students should develop proper understanding of these concepts as they lay a foundation stone for the study of another important branch of Mathematics called ‘calculus’.

### 13.8 UNIT END ACTIVITIES

1) Which of the following collections are not sets and why?
   a) \{x : x^2 - 5x + 6 = 0\}
   b) \{x : x is a good student of the class\}
   c) \{x : x is a capital city of a state in India\}
   d) \{x : x is a fair complexioned student of the class\}.

2) Write the following sets in Roster Form:
   a) \{x : x^2 = 9\}
   b) \{x : x is a letter of the word ‘collection’\}
   c) \{x : x \in \mathbb{N}, x is a prime number less than 20\}
   d) \{x : x \in \mathbb{N}, x is a divisor of 10\}.

3) Which of the following sets are equal?
   a) \{I, G, N, O, U\} and \{N, G, I, O, U\}
   b) \{1, 2, 3, 5\} and \{3, 5, 7, 9\}
   c) \{1, -1\} and \{x, x^2 = 1\}
   d) \{a, b, c, d\} and \{c, b, a, d\}

4) Write true or false:
   a) \{1, 2, 3\} \subseteq \{1, 2, 3\}
   b) Set of integers is a subset of the set of natural numbers.
   c) Empty set is an element of every set.
   d) Empty set is a subset of every set.
   e) \{a, b\} = \{b, a\}
   f) (a, b) = (b, a)
   g) If \(A \subseteq B\) and \(B \subseteq C\), then \(A \subseteq C\).

5) Draw arrow diagrams to illustrate the relation \(R\) from set \(A\) to set \(B\):
   a) \(A = \{1, 2, 3, 4\}, \quad B = \{4, 8, 12, 16, 20\}\) and
      \(R = \{(1, 4), (2, 8), (3, 12), (4, 16)\}\).
   b) \(A = \{1, 2, 3, 4\}, \quad B = \{7, 8, 9\}\) and
      \(R = \{(1, 9), (2, 8), (3, 7), (4, 7)\}\).
   c) \(A = \{1, 2, 3, 4\}, \quad B = \{5, 6, 7\}\) and
      \(R = \{(1, 5), (2, 6), (3, 7)\}\).
   d) \(A = \{1, 2, 3, 4\} \text{ and } B = \{1, 2, 3, 4, 5, 6\}\) and
      \(R = \{(x, y) : x \in A, y \in B \text{ and } y = x + 3\}\).
6) Draw the graphs of the following real functions:
   a) \( f(x) = 3 + 2x \)
   b) \( f(x) = 4 - x^2 \)
   c) \( f(x) = \frac{x}{2} \)
   d) \( f(x) = |x| - x \)

### 13.9 ANSWERS TO CHECK YOUR PROGRESS

1) (i) Not a set
   (ii) A finite set

2) Power set= \{ { }, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \}

3) (i) \( A' \) or \( \sim A \)
   (ii) \( A \cap B \)
   (iii) \( A \cap B' \) i.e. elements, which are A and not in B.

4) (i) and (iii) are functions.

5) (i) \( y = 3 - 2x \)

\[
\begin{array}{c|c|c|c}
\text{x} & 0 & 2 & 4 \\
\text{y} & 3 & -1 & -5 \\
\end{array}
\]

![Graph of y = 3 - 2x]

(ii) \( y = \frac{x^2}{2} \)

\[
\begin{array}{c|c|c|c|c|c|c|c}
\text{x} & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
\text{y} & \frac{9}{2} & 2 & \frac{1}{2} & 0 & \frac{1}{2} & 2 & \frac{9}{2} \\
\end{array}
\]
(iii) \( y = \sqrt{x} \) \( x \geq 0 \)

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>4</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

**13.10 SUGGESTED READINGS**


