UNIT 5  APPROACHES AND STRATEGIES FOR LEARNING MATHEMATICS

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5.1 INTRODUCTION

Mathematics is one of the few subjects that have the practical, cultural and
disciplinary value. Mathematics has the potential to range across all the three
values, but due to inappropriate teaching-learning process, its potential is not
being utilized to its optimum level. Unfortunately, the current focus of
Mathematics education is mostly on memorizing formulae to solve specific
problems and applying them to examination questions. To make Mathematics
an instrument of all the three values, it must be taught in an engaging,
interesting and interacting manner. According to National Curriculum
Framework-2005, the main goal of Mathematics education in school is the
mathematisation of the child’s thought process. Mathematics relies on logic,
reasoning, problem solving, creativity and mathematical way of thinking.
These skills can be useful in many other subjects.

In this unit, we will discuss various approaches, strategies and techniques of
teaching-learning of Mathematics. We will also discuss the shift with paradigm
of learning Mathematics from traditional behaviorist approach of rote learning
and drill to constructive approach, where learner constructs his/her own
knowledge.
5.2 OBJECTIVES

After going through the Unit, you will be able to:

- analyze the pedagogical shift in teaching Mathematics from behaviorist to constructivist;
- illustrate the constructivist approach for teaching-learning Mathematics;
- explain about various strategies for teaching-learning Mathematics;
- differentiate among various strategies for teaching-learning Mathematics;
- identify and apply different strategies of teaching;
- help learners to apply problem-solving skills in solving mathematical problems; and
- apply various techniques for transacting Mathematics.

5.3 PEDAGOGICAL SHIFT: FROM BEHAVIORIST TO CONSTRUCTIVIST

Mathematics has always been praised for its usefulness and significance in life. It plays a key role in deciding how individuals deal with various problems of life. But, at the same time, learners find it hard to understand how functions, equations, or geometric shapes, which can help them in everyday life. They consider it as a difficult subject due to its nature of being abstract which may be one of the reasons why Mathematics is not popular amongst many learners. Even today, as in the past, many learners still struggle with Mathematics. In recent years, there has been a debate over the different approaches by which Mathematics is being taught in the schools. In the past, Mathematics was taught by the traditional methods with direct instruction and rote-memorization of facts and procedures. The recent Mathematics initiatives shift, teaching and learning away from a traditional on learning rules for manipulating symbols to active engagement of learners in learning Mathematics. In other words, teaching-learning Mathematics has started shifting from behaviorist approach to constructive approach. Constructivism's success may be due, in part to the frustrations that educators experienced with behaviorist educational practices. Beginning in the 1960s, behaviorism swept from the arena of psychology into education with an air of authority that was startling. Schooling became structured around the premise that if teachers provided the correct stimuli, then learners would not only learn, but also their learning could be measured through observations of learner behaviors. (Jones & Brader-Araje, 2002).

Skinner and Watson were the two major proponents of behaviorism. According to behaviorists, all behavior is the result of an individual’s responses to external stimuli i.e. the external environment contributes to learning. It emphasizes on the effects of external conditions such as rewards and punishments in determining the learning of learners. It focuses mainly on objectively observable behaviors and consequently, does not count mental activities. In contrast to the beliefs of behaviorists, the constructivists viewed learning as a search for meaning.

Piaget and Vygotsky were strong proponents of constructivism who opined that knowledge is constructed by the learner. The Skinner, a behaviorist, constructed a teaching machine in 1958. Skinner’s teaching machine was a rote-and-drill machine where a chunk of information was presented before the
individual in the form of programmed instruction. Constructivist model believes that learning occurs as an internal cognitive activity wherein learners construct knowledge (models) from their classroom experience. Constructivists believe that children develop their knowledge through active participation in their learning. However, Behaviorists believed that meaning exists in the world separate from personal experience. In behaviorist model all instructional goals are framed in specific, behavioral, and observable terms. Like the traditional approach, the instructor is in the centre of the presentation and interaction and the role of the learner is to absorb the presentation and material whereas constructivist suggests that learning activities must have the characteristics of active engagement, inquiry, problem solving, and collaboration with others’ real life. They consider that the teacher is a guide and facilitator, who encourages learners to take part in discussions and formulate their own ideas, opinions, and reach the conclusions.

Behaviorist supports deductive approach and Constructivist supports inductive approach of teaching. Therefore, Constructivists focus on a different aspect of education than Behaviorists. Constructivist sees how learners learn on their own when learners are presented with stimuli and Behaviorists focus more on how learners respond to positive and negative reinforcement provided by teacher.

Check Your Progress

Note: a) Space is given below to write your answer.
   b) Compare your answer with the one given at the end of this Unit.

1. Who were the major proponents of behaviorism?
   …………………………………………………………………………………
   …………………………………………………………………………………
   …………………………………………………………………………………
   …………………………………………………………………………………
   …………………………………………………………………………………

2. What is the overall philosophy of constructivism?
   …………………………………………………………………………………
   …………………………………………………………………………………
   …………………………………………………………………………………
   …………………………………………………………………………………
   …………………………………………………………………………………

3. Write down the any three differences between constructivist and behaviorist approaches.
   …………………………………………………………………………………
   …………………………………………………………………………………
   …………………………………………………………………………………
4. Fill in the blanks.

I. For ………………………… learners are not blank-slates.

II. ……………………………………supports inductive approach and ……………….supports deductive approach of teaching.

5.4 CONSTRUCTIVIST APPROACH FOR TEACHING LEARNING MATHEMATICS

As we discussed that constructivism assumes that knowledge cannot be transmitted to learner but is constructed by him/her. It is constructed by the learner on the basis of experiences. In the process of knowledge generation new experiences, talking to others and reflective thinking could be helpful. Many times learning includes change in existing conception. There are three ways for meaningful learning to take place which are as follows:

1) Addition to the existing knowledge,
2) Small modification to the existing knowledge,
3) Major changes in the existing knowledge.

Conceptual change does not take place easily. For conceptual change three conditions are necessary, which are as follow:

1) Learner must encounter a situation which he/she is not able to understand using existing knowledge, thereby producing dissatisfaction in the learner.
2) Learner must come across some knowledge, which is intelligible to him/her and seems plausible.
3) The new knowledge help learner to understand some new situations which were beyond his/her reach earlier.

There are many shades of constructivism. It is not unique monolith philosophy. As we discussed, Piaget the first constructivist laid emphasis on action by the learner on the object which results in accommodation and assimilation. Vygotsky another important constructivist is proponent of socio-cultural perspective. For him zone of proximal development, scaffolding and peer learning are three important considerations. Zone of Proximal Development (ZPD) is that stage of development of a child where he cannot solve a problem of his/her own but a sight hint and help by teacher or some other able person, is sufficient to enable him to solve the problem. Scaffolding is the support provided by an expert to a novice in initial stage of learning. Slowly as the progress is made by the learner, support is withdrawn gradually peer learning could take place in three ways peer tutoring, cooperative learning and peer collaboration.

As constructive teacher, you should have a clear idea about learners’ previous knowledge. You can use appropriate strategies to assess the previous knowledge since this will be very important for designing suitable activities for working in their ZPD.

Whenever learners find difficulties within the zone, it is the duty of the teacher to provide assistance or support in the process. This can either be done by the teacher or with the help of a more competent peer. This process of
assisting is technically known as scaffolding. Thus a constructivist teacher is required to create opportunities for peer scaffolding and teacher-directed scaffolding in order to stimulate knowledge construction.

The important task of a teacher is to design appropriate activities so that the learners can work on it and construct expected knowledge with confidence and a feeling of success. This of course needs ingenuity and creativity.

Creation of a learning environment which is stimulating, interactive, and enlightening for the learners is the most challenging task of a constructivist teacher. Is it possible to create a stimulating environment? Of course you can, then how? Consider the following example where a teacher wants to help the learners to comprehend the identity \((a+b)^2 = a^2+2ab+b^2\) by using constructivist approach.

As an introductory activity teacher can give the following task with the aim to assess and strengthen the prerequisites needed for the proposed learning.

Task: The learners are asked to draw squares of different sizes on given graph paper individually.

- Subsequently the teacher initiates the whole class discussion by providing points for discussion:
  - “Count the number of smaller squares inside the bigger square you constructed”
  - “Can you establish any relationship between the number so obtained and the size of the side?”
  - “Can you relate this with any other concept you learnt earlier?”
Learners may recognize that the area of the whole square is the sum of the areas of all the four parts. They may find that areas of the square parts are $3^2$ and $4^2$ respectively, and those of the rectangles are $3\times4$ and $4\times3$ respectively.

Then learner may conclude that $(3+4)^2 = 3^2 + 4^2 + 2\times3\times4$ = smaller square + bigger square + one rectangle + other rectangle

So, $(a+b)^2 = a^2 + 2ab + b^2$

In this way teacher can stimulate the learners to participate actively in the teaching learning process. The feelings of success as well as enjoyment will naturally motivate the learners.

The task should provide insight to the learners to reach the relationship between the side and area of a square.

Similarly by providing ample opportunities for learners to engage in dialogue, both with the teacher and with one another; teacher can create an interactive classroom. The experience of success during different stages will automatically enlighten the learners both scholastically and co scholastically.

While organising different activities give opportunities to the learners to initiate discussions, to ask questions, to work independently etc.

From the conceptual meaning of constructivism as learning theory you could observe that knowledge construction is not the product of successive pouring of information through teacher talk, but a natural consequence of personal experience, inquiry, reflection and insight. Therefore, thoughtful and open-ended questions revealing learners’ prior knowledge and experiences are asked by the teachers in constructivist classrooms.

### Activity For Practice:

1. You studied the concept of constructivism and its importance in the classroom teaching learning process. Make a list of principles to be considered by you while structuring classroom learning activities.
2. Select any topic from your choice from class IX Mathematics textbook. And design activities to transact that topic through constructivist approach.

3. You are teaching the concept of formation of simple linear equation. Design one interesting example which connects to life situation of the learner to teach formation of simple linear equation.

Check Your Progress
Note: a) Space is given below to write your answer.

b) Compare your answer with the one given at the end of this Unit.

5. Who is the originator of Constructivist philosophy?

…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
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6. What do you understand by constructivist pedagogy? Explain its benefits in teaching learning in Mathematics?

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…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………
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5.5 STRATEGIES FOR TEACHING-LEARNING MATHEMATICS

We have discussed constructivist approach of teaching learning of Mathematics. There are some strategies of teaching Mathematics like inductive – deductive, analysis-synthesis, problem solving, discovery, activity etc. which help the learner in constructing their knowledge. The purpose of these strategies is to make teaching-learning more interactive as well as effective. You can select a particular strategy based on the needs of learners as well as its relevance to the content. Some strategies of teaching-learning Mathematics are as follows:
5.5.1 Inductive – Deductive

It is a combination of inductive and deductive approach. Let us first discuss the Inductive approach.

a) Inductive Approach

Inductive approach is based on the process of induction i.e. reasoning from specific facts to general principles. Therefore, it proceeds from particular to general, from concrete to abstract. It is a method of constructing a formula with the help of a sufficient number of concrete and specific examples. Learners arrive at the formula or general rule through the examples of particular cases. It is based on actual observation and experiments. Inductive approach is a much more learner-centered approach. The learners are encouraged to devise the formula on their own. This approach is psychological in nature. It develops scientific attitude, comprehension ability and logical thinking among learners. The teacher’s role is only to facilitate the use of appropriate questions. Inductive approach is suitable in the following situation:

- Introduction of new topic
- Formulation of rules
- Derivation of formulas
- Generalization

Mathematically speaking, inductive reasoning might take this form:

Step1 - Shows that something is true for specific items. (Particular concept).
Step2 - Shows that if it is true for one and more, then it must be true for the rest. (General concept).

Let us discuss some examples to make clear idea about it.

Example 1: Sum of two odd numbers is even.

Solution:
Teacher: Draw the following table in your notebook. Write two odd numbers and then sum it. Now observe the table and try to find out mathematical relationship between the numbers.

<table>
<thead>
<tr>
<th>First odd number</th>
<th>Second odd number</th>
<th>Sum of these numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
<td>..</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
<td>..</td>
</tr>
</tbody>
</table>
General concept: Learners may conclude that the sum of two odd numbers is even.

Example 2: Sum of the angles of the triangles is equal to 180°.

Solution:
Teacher: Draw a few triangles.
Measure and sum up the angles in triangle A.
Measure and sum up the angles in triangle B.
Measure and sum up the angles in triangle C.
and so on
Draw the following table in your notebook and fill up the values of various angles of triangles.
Now observe the table and try to find out relationship between angles of triangle.

**Particular concept:**

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Measure of angle 1</th>
<th>Measure of angle 2</th>
<th>Measure of angle 3</th>
<th>Sum of angles of a triangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50</td>
<td>50</td>
<td>80</td>
<td>180</td>
</tr>
<tr>
<td>B</td>
<td>45</td>
<td>50</td>
<td>85</td>
<td>180</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>60</td>
<td>90</td>
<td>180</td>
</tr>
<tr>
<td>D</td>
<td>..</td>
<td>..</td>
<td>..</td>
<td>..</td>
</tr>
<tr>
<td>E</td>
<td>..</td>
<td>..</td>
<td>..</td>
<td>..</td>
</tr>
<tr>
<td>F</td>
<td>..</td>
<td>..</td>
<td>..</td>
<td>..</td>
</tr>
</tbody>
</table>

General concept: Learners conclude that the sum of the angles of the triangles is equal to 180° or two right angles.

b) Deductive Approach

Deductive approach is based on deduction. It is just the opposite of inductive approach. It proceeds from abstract to concrete, from general rule to particular or specific instances, and from formula to examples, from unknown to known. This approach is relatively more teacher-centered. In this approach, rules are initially given by teachers and then learners are asked to apply these rules to solve more problems of similar nature. This approach is mainly used in Algebra, Geometry and Trigonometry because different relations, laws and formulae are used in these sub branches of Mathematics. It is more useful for teaching Mathematics in higher classes. This method is useful for revision and drill work. It enhances speed and efficiency.

Example 1: Learners are told that ‘the sum of angles in a triangle is 180°’.

Let ABC be a triangle.

Construct a line DE
Parallel to BC through
Here \( \angle ABC = \angle BAD \)
\( \angle ACB = \angle CAE \)
But $\angle DAB + \angle BAC + \angle CAE = 180^\circ$

So $\angle ABC + \angle BAC + \angle ACB = 180^\circ$

**Example 2**: Find $a^5 \times a^8 = ?$

**Solution**:

General concept: First teacher told the formula that $“a^m \times a^n = a^{m+n}”$

Then learners solve the problem.

Particular concept: $a^5 \times a^8 = a^{5+8} = a^{13}$

### Differences between Inductive and deductive approaches

<table>
<thead>
<tr>
<th>SN.</th>
<th>Inductive Approach</th>
<th>Deductive Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Process of learning from specific facts to general principles.</td>
<td>Process of learning and reasoning from general principles to specific facts.</td>
</tr>
<tr>
<td>2</td>
<td>Certain complex and complicated formulae cannot be generated so this method is limited in range and is not suitable for all topics.</td>
<td>It is suitable for all topics.</td>
</tr>
<tr>
<td>3</td>
<td>It is time consuming and laborious method</td>
<td>It is a short and time saving method.</td>
</tr>
<tr>
<td>4</td>
<td>It is a scientific method.</td>
<td>It does not impart any training in scientific method.</td>
</tr>
<tr>
<td>5</td>
<td>It does not burden the mind. Formula becomes easy to remember.</td>
<td>It puts more emphasis on memory.</td>
</tr>
<tr>
<td>6</td>
<td>It is a learner centric approach.</td>
<td>Learners are only passive listeners. It is more teacher-centric.</td>
</tr>
<tr>
<td>7</td>
<td>It gives new knowledge.</td>
<td>It does not give any new knowledge.</td>
</tr>
<tr>
<td>8</td>
<td>It is a method of discovery.</td>
<td>It is a method of verification.</td>
</tr>
<tr>
<td>9</td>
<td>It is an upward process of thought and leads to principles.</td>
<td>It is a downward process of thought and leads to useful results.</td>
</tr>
</tbody>
</table>

Inductive – Deductive approach is a combination of both Inductive and Deductive approaches. This method can be used in totality for realizing the desired goals of mathematical learning. Through, Inductive approach, rules and generalization are established and formulae are derived, while Deductive approach is helpful in applying the deduced results. It also helps in improving skills and efficiency in problem solving. No induction is possible without deduction and no deduction results without induction.

### 5.5.2 Analytic - Synthetic

We have seen that in its early stages, most mathematics originates in ideas and concepts in logico-deductive form. The ability to understand and workout a rigorous deductive structure using logic or reasoning is of great importance.
Analyses and synthesis are approaches which use reasoning and arguments to discover relationships. Let us discuss both the approaches separately.

a) Analytic Approach

Analysis is the process of breaking a complex topic or substance into smaller parts in order to gain a better understanding of it. In this method, a problem is analyzed into smaller/simpler problems. All the related facts are analyzed to seek help in proceeding to the known conclusion. The purpose of breaking it into smaller parts is to figure out the hidden aspects of the problem. So, basically this approach moves from unknown to known. This method helps learners in discovering the things himself. It is a psychological method based on the principle of interest, which inculcates the spirit of inquiry and investigation in the learners (Katozai, 2002 as quoted by Asif, Khan and Zaman (2010). It facilitates comprehension and strengthens the urge to discover new facts. It also provides opportunities to learners to tackle the problem confidently and intelligently. But it is not applicable equally well for all topics.

Example 1: If $a^2 + b^2 = 14ab$ prove that $2\log (a + b) = 2 \log 4 + \log a + \log b$

Proof:

To prove this using analytic method, begin with the unknown.

The unknown is $2 \log (a+b) = 2 \log 4 + \log a + \log b$

Now, $2\log (a + b) = 2 \log 4 + \log a + \log b$ is true

If $\log (a + b)^2 = \log 4^2 + \log a + \log b$ is true

If $\log (a + b)^2 = \log 16 + \log ab$ is true

If $\log (a + b)^2 = \log 16ab$ is true

If $(a + b)^2 = 16ab$ is true

if $a^2 + b^2 = 14ab$ which is known and true

Thus, if $a^2 + b^2 = 14ab$, it is proved that $2\log (a + b) = 2 \log 4 + \log a + \log b$

Example 2: If $\frac{a}{b} = \frac{c}{d}$, prove that $\frac{(ac - 4b^2)}{b} = \frac{(c^2 - 4bd)}{d}$

Proof:

The unknown part is $\frac{(ac - 4b^2)}{b} = \frac{(c^2 - 4bd)}{d}$ is true,

If $ac - 4b^2d = bc^2 - 4b^2d$ is true,

If $ac = bc^2$ is true,

If $ad = bc$ is true

That is, if $\frac{a}{b} = \frac{c}{d}$ is true,

Which is known and true.

Thus if $\frac{a}{b} = \frac{c}{d}$, prove that $\frac{(ac - 4b^2)}{b} = \frac{(c^2 - 4bd)}{d}$
Synthetic Approach

Synthesis refers to a combination of two or more entities that together form something new. In this method we move from known to unknown and from hypothesis to conclusion. It is just the opposite of analytic method. It is an approach in which we collect and combine various facts to find out the unknown result. It presents the facts in a systematic way and can be applied to majority of topics in teaching of Mathematics. According to (Katozai, 2002 as quoted by Asif, Khan and Zaman: 2010) it is the process of putting together known bits of information to reach the point where unknown formation becomes obvious and true.

Example 1:

If \(a^2 + b^2 = 14ab\) prove that \(2\log (a + b) = 2\log 4 + \log a + \log b\)

Proof:

To prove this using synthetic approach, begin from the known.

The known is \(a^2 + b^2 = 14ab\)

Now, \(a^2 + b^2 + 2ab = 14ab + 2ab\)

So, it becomes \((a + b)^2 = 16ab\)

Taking log on both side,

\[
\log (a + b)^2 = \log 16ab
\]

\[
2\log (a + b) = \log 16 + \log ab
\]

\[
2\log (a + b) = \log 4^2 + \log ab
\]

\[
2\log (a + b) = 2 \log 4 + \log a + \log b
\]

So if \(a^2 + b^2 = 14ab\), \(2\log (a + b) = 2 \log 4 + \log a + \log b\)

Example 2:

If \(\frac{a}{b} = \frac{c}{d}\), prove that \(\frac{(ac - 4b^2)}{b} = \frac{(c^2 - 4bd)}{d}\)

Proof:

The known part is \(\frac{a}{b} = \frac{c}{d}\)

Subtract \(4b/c\) from both sides

\[
\frac{a}{b} - \frac{4b}{c} = \frac{c}{d} - \frac{4b}{c}
\]

\[
\frac{(ac - 4b^2)}{bc} = \frac{(c^2 - 4bd)}{cd}
\]

i.e \(\frac{(ac - 4b^2)}{b} = \frac{(c^2 - 4bd)}{d}\) which is unknown..
## Differences between Analytic and Synthetic Methods

<table>
<thead>
<tr>
<th>S N</th>
<th>Analytic Method</th>
<th>Synthetic Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>This process refers to breaking down a bigger problem into smaller components.</td>
<td>This process combines many small known components to derive something new.</td>
</tr>
<tr>
<td>2</td>
<td>It leads from unknown to known. In other words it leads from conclusions to hypothesis.</td>
<td>It leads from known to unknown. In other words it leads from hypothesis to conclusion.</td>
</tr>
<tr>
<td>3</td>
<td>This approach is lengthy and time consuming.</td>
<td>It is short and time saving method.</td>
</tr>
<tr>
<td>4</td>
<td>It is known as psychological method.</td>
<td>It is known as logical method.</td>
</tr>
<tr>
<td>5</td>
<td>This method encourages thinking and reasoning. This approach promotes meaningful learning.</td>
<td>It puts more emphasis on rote learning. It promotes memory work.</td>
</tr>
<tr>
<td>6</td>
<td>This approach is informal in nature and it is disorganized.</td>
<td>This approach is very formal and systematic.</td>
</tr>
<tr>
<td>7</td>
<td>In this approach learners can recall and reconstruct any step easily, if forgotten.</td>
<td>It is very difficult for the learners to reconstruct the steps, if forgotten.</td>
</tr>
<tr>
<td>8</td>
<td>It is based on heuristic lines.</td>
<td>This approach doesn’t cater to heuristic approach.</td>
</tr>
</tbody>
</table>

Both these methods look like opposite to each other but they go together. They support and complement each other. It can be said that analysis lead to synthesis and synthesis makes analysis complete as well as clear. That’s why it is desirable that teacher should use analytic method while teaching Mathematics and motivate the learners to use synthetic approach for presentation.

### Activity for Practice:

4. Write a plan to conduct a topic through analysis – synthesis method in your class. Try it out in your class. Reflect on the process and write the merits and demerits of the process.

### 5.5.3 Problem Solving Approach

Learners learn mathematical thinking most effectively through applying concepts and skills in interesting and realistic contexts which are personally meaningful to them. Thus, Mathematics is best taught by helping learners to solve problems drawn from their own experience. Real-life problems are not always closed, nor do they necessarily have only one solution. The solutions to problems which are worth solving seldom involve only one item of mathematical understanding or only one skill. Rather than learners remembering the single correct method, problem-solving requires them to search the information for clues and to make connections to the various pieces of Mathematics and other knowledge, experiences and skills that they have
already learned. Such problems encourage thinking rather than mere recall. What is the most common thing that learners do when they encounter a Mathematical problem which they don’t know how to solve? Some give up very quickly while some of them ask other learners or their teachers for help. This doesn’t help the learner in the long run as each and every individual some time or other will encounter problem, which they have not been solved earlier and their problem solving ability will play an important role in these scenarios.

According to NCTM(2000) “Problem solving means engaging in a task for which the solution is not known in advance.” Any mathematical situation can be a problem for a learner if learner has not previously learned about how to solve that. Once the learner learns how to solve a problem, it becomes an exercise. Teaching through problem solving and teaching problem solving are two different approaches. Teaching problem solving usually works on guess and check, working backward etc. methods.

In teaching through problem solving, teacher will setup the context and explain the problem. Now, learners work on the problem and the teacher monitors their progress. After stipulated time each learner of the class shares his/her ideas with the whole class and then they compare as to which idea is best for solving that particular problem. In this way learner learns many new mathematical ideas and procedures.
Let us see an illustration where a teacher used problem solving strategy.

**Illustration**

**Teacher** : Look at the following series and tell me the next number: 2, 4, 8, 16, 32, …

**Mani** : 34

**Teacher** : How did you find?

**Mani** : Next number is multiple of 2 because all numbers in the series are multiples of 2.

**Teacher** : According to your guess the series is 2, 4, 8, 16, 32, 34…… Then, 34 is correct answer. Whether it is correct? Let us check. Tell me all the multiples of 2.

**Mani** : 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, etc.

**Teacher** : Tell me, whether all these numbers are coming in the series.

**Mani** : No madam

**Teacher** : So, your guess is not correct and hence 34 is not the right answer. Any other guess (put the question to whole class)?

**Kanjan** : 64

**Teacher** : Very good, how did you find?

**Kanjan** : When observed the series, I found that every number is the double of previous number. So, next number may be double of pervious number.

**Teacher** : Let us check your answer

\[
\begin{align*}
2 \times 2 &= 4 \\
4 \times 2 &= 8 \\
8 \times 2 &= 16 \\
16 \times 2 &= 32 \\
32 \times 2 &= 64
\end{align*}
\]

**Teacher** : Your answer is correct. Do you agree?

**Learners** : Yes, madam (All learners with loud voice).

From above illustration, we have seen that teacher has given a problem to learners then learners have analyzed the problem, searched the expected mathematical relation, found the solution. At that time the teacher monitored their progress. After stipulated time each learner of the class has shared his/her ideas and find the best solution of the problem.

**Activity for Practice:**

5) How would transact the topic ‘linear equations’ through problem solving strategy.
Check Your Progress

Note:  
a) Space is given below to write your answer.
   
b) Compare your answer with the one given at the end of this Unit.

7. Fill in the blanks.
   
i.  Inductive approach proceeds from………..to………….. and 
    ………to………….. .

   ii. Synthesis refers to a …………………….of two or more entities.

   iii. …………………. approach moves from unknown to known.

8. Write any three differences between inductive and deductive approach of teaching learning of Mathematics.

   ........................................................................................................................
   ........................................................................................................................
   ........................................................................................................................
   ........................................................................................................................

9. In what ways analysis and synthesis approaches are looks like opposite to each other.

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   ........................................................................................................................
   ........................................................................................................................
   ........................................................................................................................

5.6 TECHNIQUES FOR TRANSACTING MATHEMATICS CURRICULUM

There are various techniques of transacting Mathematics Curriculum in an effective manner. Some of the techniques are as follows-

5.6.1 Drill and Practice

Drill is one of the most essential ways (or methods) of learning in mathematics. The controlling purpose of all teaching activity is to reduce necessary learning to habit. Gaining mastery requires acquisition of habits, hence drill/practice plays an important role in acquiring mastery. By and large, practice lessons are of three types. The first category of lessons for mastery is of basic subject matter, e.g., multiplication tables, addition combinations, fractional equivalents of decimals and percentages, factorization, construction in geometry, etc. These include subject matter which must be thoroughly mastered so that speed and accuracy is ensured on which future learning can be based.

The second category includes lessons for the mastery of procedures. In Mathematics one has to adhere to a systematic arrangement of steps, follow correct algorithms to scrutinize and check the correctness of each step, label
approach parts in a diagram, sort out data, translate problems into symbolic form, practice short cuts, etc.

The third category consists of lessons which strive to develop the power of thinking and reasoning, and increase the concentration and interest of the learner. Such lessons include quizzes, puzzles and historical material which does not form part of a regular lesson.

Although, a certain amount of formal drill is inevitable, preference should be given to functional or meaningful drill. Meaningful drill implies prior understanding of content and its appropriate application. This drill is purposeful and is determined by need as well as by use. An effective drill lesson should be organized keeping in view the following considerations:

1. Drill should follow learning and understanding of basics. It should not encourage rote memorization without understanding the subject matter.
2. Drill should be varied. Some routine procedures make learning monotonous and uninteresting.
3. Drill should be individualized and rewarding to each learner. Each learner should see its purpose and utility.
4. Drill periods should be short and the learner’s achievement should be frequently tested.
5. Drill should not be planned merely to keep learners “busy”. It should be based upon thought-provoking situations to avoid the repetition of any process mechanically.
6. Drill may also provide diagnostic information about learners.

5.6.2 Play Way

Play way technique is a child-centered informal method of teaching which suits the interest of the child and improves its academic proficiency effortlessly. This method helps to develop interest in Mathematics, motivates learners to learn more, and reduces the abstract nature of the subject to some extent. (Patel, nd). Play way can be an effective way of teaching Mathematics to learner. Although only some concepts can be taught through games, the most important benefit of games like mathematical quiz, puzzle, tricks, riddles, guessing game, etc. is the oral practice of various mathematical concepts. Let us see an example of guessing game where learners need to decode the structure like the one given below.

**Game: Guess the number**

Think of a number. Multiply it by 5 and add 10. Tell me the number, I will tell you your number.

Unknowingly in the process learners will discover the structure of the problem in terms of operations of multiplication and addition and reversal of these processes in terms of subtraction and then division.

For example, in this problem if the number given is 90, then original number can be calculated by subtracting 10 and dividing by 5. The number which one has thought will be 18.
Some of the examples of mathematical puzzles, tricks, riddles and guessing games has been given in the unit 6 of this block. Refer and use it in day-to-day teaching.

**Activity for Practice:**

6. Design a game for introducing the concept of ‘ratio’ for class VII learners. Reflect on whether the task is a game or not.

### 5.6.3 Home Work

Homework refers to tasks assigned to learners by their teachers to be completed outside the class. The purpose of home work is to encourage learners to review, apply, practice and integrate what he/she has learnt in the classroom. Siddhu (2006) quotes that *school time is insufficient to exhaust everything provided in the curriculum of Mathematics. Homework has to be given regularly to provide for application and practice and to supplement classroom teaching.* It provides an opportunity for learners to make, discover, and correct mistakes so that they can learn from them. It is intended to engage learners in exploration of concept beyond the class time. It helps learners in long lasting learning and preparing for the next class. The fundamental purpose of homework to learners is the same as schooling in general.

Homework may consist of open-ended questions or closed-ended questions. Open-ended questions are developed to prompt learners to apply concepts, solve problems, and make mathematical connections. No routine or prescribed methods are given for responding to open questions. Therefore, it makes teachers able to observe the strategies, skills, logic, concepts, and connections used by the learners to solve the questions. Open questions may be classified as open-development, open-process, and open-ended questions.

Let us see some examples of various type of questions.

<table>
<thead>
<tr>
<th>Type of question</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open-ended development question</td>
<td>Write a challenging story problem for the equation n +3 = 7.</td>
</tr>
<tr>
<td>Open-ended process question</td>
<td>The length of a field is 3 times to its width. Its perimeter is 4800m. What are the dimensions of the field? Show your work.</td>
</tr>
<tr>
<td>Open-ended question</td>
<td>The average mass of 15 fishes caught in a pond is 2.5 kg. The mode is 3 kg. What are the possible masses of the 15 fishes? Explain your thinking.</td>
</tr>
</tbody>
</table>

Home work has some drawback also. It can destroy curiosity and love of learning in the learner. Studies have shown that it takes time away from independent study and extracurricular, family, and social activities important to childhood development. So it is very necessary for the teacher to carefully monitor the amount of home work so that learner can get enough time for social activities. It is also necessary to make sure that homework has appropriate level of difficulty so that learner can complete the homework independently. A rule of thumb for homework might be that "all daily
homework assignments combined should take about as long to complete as 10 minutes multiplied by the learners' grade level" and "when required reading is included as a type of homework, the 10-minute rule might be increased to 15 minutes" (Cooper, 2007, cited in Marzano & Pickering, 2007, p. 77).

Homework review in class is an important part of Mathematics teaching-learning. It should be well graded. It should be assessed as a part of the overall assessment of a learner. Home work should be duly checked and corrected. If it is not checked, the learners may fall into the bad habit of evading it or copying it.

5.6.4 Assignments

An assignment is a task or work allotment. In this technique, the learners are provided with the responsibility for his/her own learning. The teacher acts as an advisor and guide in case of any difficulty encountered. The method has several advantages. It encourages initiative and independence, and provides learners with the maximum amount of individual practice. Teacher should keep in mind that assignment should always be a task which is within the capability of the learner and has some interest for him. It is the major important part of every learner life. The learners are encouraged to keep their completed assignments for future references.

Characteristics of good assignment

- Assigned task must be clearly defined.
- Assigned work must have correlation with previous knowledge and experiences.
- Assigned work must be stimulating and directing the learning experiences and activities.
- It should be precise as well as have sufficient information to enable the learners to complete the task.
- Newer topics for the assignments must be proposed with the earlier learning experiences.
- Teacher must know what they want from the learners to gain from the experience.
- Assigned task must be interesting to be completed within the stipulated time by the learners.
- Library facilities and other reference resources are mandatory for completion of assignments.

Check Your Progress

**Note:** a) Space is given below to write your answer.
   b) Compare your answer with the one given at the end of this Unit.

10) Fill in the blanks.

   i. ............... may also provide diagnostic information about learner.

   ii. School time is not sufficient therefore...............is required.

   iii. ...............technique is child centered.
11) Discuss the importance of homework in Mathematics.

......................................................................................................................
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12) Write any four characteristics of good assignments.

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......................................................................................................................

5.7 LET US SUM UP

The following concepts/issues are dealt in this unit:

✓ Teaching and learning Mathematics has started shifting from behaviorist approach to constructive approach. Constructivism states that individual construct their own knowledge and understanding of the world through experiencing and reflection.

✓ Constructivist approach in Mathematics believes that learner can construct knowledge by active participation rather than acquiring knowledge by observing lectures delivered by teachers.

✓ Inductive approach is based on the process of induction i.e. reasoning from specific facts to general principles. Therefore, it proceeds from particular to general, and from concrete to abstract.

✓ Deductive method is based on deduction. It proceeds from abstract to concrete, from general rule to particular or specific instances, and from formula to examples, and from unknown to known.

✓ Analysis is the process of breaking a complex topic or substance into smaller parts in order to gain a better understanding of it.

✓ Synthesis refers to a combination of two or more entities that together form something new. In this method we move from known to unknown and from hypothesis to conclusion.

✓ According to Problem solving approach, learners learn mathematical thinking most effectively through applying concepts and skills in interesting and realistic contexts which are personally meaningful to them.

✓ Drill method gives learners an opportunity to learn certain concepts quickly and effectively.

✓ Play way method can be used for teaching Mathematics by activities such as mathematical games, checkers, magic squares, puzzles and building blocks.

✓ The purpose of home work is to encourage learners to review, apply, practice and integrate what he/she has learnt in the classroom.

✓ An assignment is a task or work allotment. In this technique, the learners are provided with the responsibility for his/her own learning. The teacher acts as an advisor and guide in case of any difficulty encountered.
5.8 UNIT END EXERCISES

1) Analyze the behaviorist approach of learning. How does it differ from constructivist approach? Discuss the significance of constructivist approach.

2) Which is the best method of teaching Mathematics according to your view? Justify your preference with suitable examples and arguments.

3) Illustrate and discuss the inductive–deductive method of teaching Mathematics.

4) Describe the use of analytic and synthetic methods in the teaching of Mathematics.

5) What do you understand by the problem solving method? How will you employ it in teaching Mathematics?

6) What is place and value of drill and practices in the teaching of Mathematics?

7) Illustrate the constructivist approach for teaching learning Mathematics.

8) How will you employ the techniques of assignment? How is it different from homework.

5.9 ANSWERS TO CHECK YOUR PROGRESS

1) Skinner and Watson were the major proponents of behaviorism.

2) The overall philosophy of constructivists holds that learners construct their own understanding based on their unique experiences and furnish meaning on the world.

3) The difference between behaviorists and constructivists are as follows:
   - Behaviorism supports traditional teacher-centered approach whereas for constructivists, learner is at the center of teaching learning process.
   - Behaviorists focus more on how learners respond to positive and negative reinforcement provided by the teacher, whereas Constructivists see how learners learn on their own when learners are presented with stimuli.
   - Behaviorist supports deductive approach and Constructivist supports inductive approach of teaching.

4) (i) Constructivists (ii) Constructivist, Behaviorist

5) Jean Piaget

6) Refer section 5.4

7) (i) from particular to general and from concrete to abstract (ii) combination (iii) Analytic

8) Refer section 5.5.1

9) Refer section 5.5.2

10) (i) Drill (ii) home work (iii) Play way

11) Refer section 5.6.3
5.10 REFERENCES AND SUGGESTED READINGS


12) Refer section 5.6.4